DESCHANEL'S NATURAL PHILOSOPHY.

BY PROFESSOR J. D. EVERETT, D.C.L., F.R.S

LITERARY OPINIONS.

"We have no work in our own scientific literature to be compared with it, and we are glad that the translation has fallen into such good hands as those of Professor Everett. . . The book is a valuable contribution to our scientific literature; it will form an admirable text-book for special science classes in schools."—*Quarterly Journal of Science*.

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ELEMENTARY TREATISE

ON

NATURAL PHILOSOPHY.

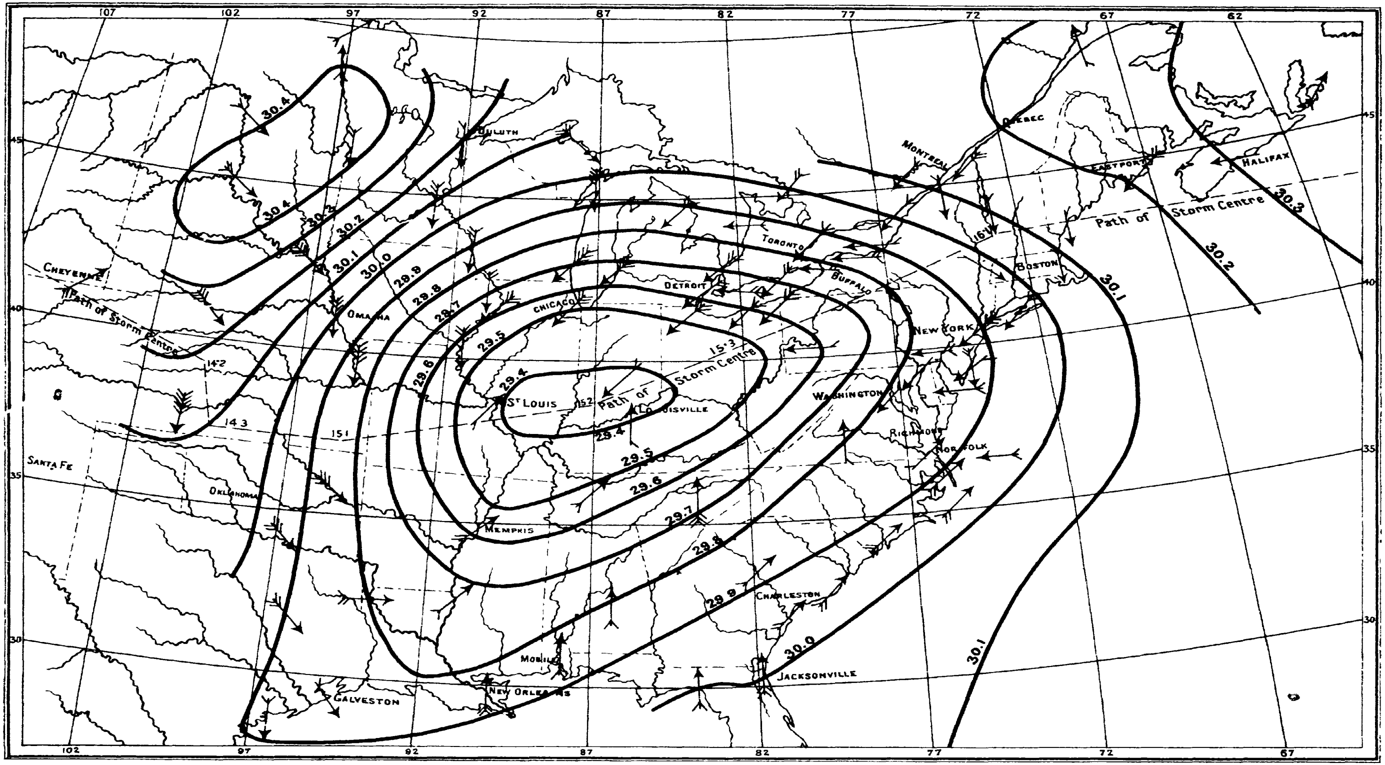

CHART OF PRESSURE AND WIND FOR THE STORM OF JAN. 15, 1877.

DRAWN BY PROFESSOR LOOMIS.

For explanation see page 165.

ELEMENTARY TREATISE

ON

NATURAL PHILOSOPHY

BASED ON THE TRAITÉ DE PHYSIQUE OF

A. PRIVAT DESCHANEL

FORMERLY PROFESSOR OF PHYSICS IN THE LYCÉE LOUIS-LE GRAND,
INSPECTOR OF THE ACADEMY OF PARIS

BY

J. D. EVERETT, M.A., D.C.L., F.R.S.

PROFESSOR OF NATURAL PHILOSOPHY IN THE
QUEEN'S COLLEGE, BELFAST

THIRTEENTH EDITION.

LONDON
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GLASGOW, EDINBURGH, AND DUBLIN
1894

PREFACE

TO THE THIRTEENTH EDITION.

In this thirteenth edition the work has been entirely recast with the exception of the portions relating to the comparatively stationary subjects of Mechanics, Hydrostatics, and Acoustics. Much of the old matter has been rearranged for greater lucidity, new matter has been largely introduced into old chapters, and four entirely new chapters on special modern subjects have been added—one in Part II., one in Part IV., and two in Part III.

Previous editions are now superseded, as the following account of the changes will show.

PART I. contains the latest information respecting weather prediction and weather charts.

PART II. contains a new chapter on Thermodynamics, in which free use is made of the methods of the Differential Calculus. Entropy is explained, and several examples are given of the deduction of physical relations by changing the order of differentiation.

Among the additions in other parts of the book are:—

Bunsen's calorimeter figured and described,

Dewar's experiments on liquid oxygen;

Rowland's determination of the mechanical equivalent of heat, and of the specific heat of water at various temperatures, the minimum specific heat being attained at about 30° C.;

Thomson and Joule's experiments on forcing gases through a cotton wool plug, to determine the difference between the cooling effect of expansion and the work done in the expansion;

Van der Waals' theory with respect to the departure of gases from Boyle's law.

To prevent the book from becoming too large, the account of Melloni's experiments is curtailed; and a number of details respecting steam-engines are omitted.

The pages and sections of Part II. are now numbered from 1 onwards instead of making the numbers consecutive to those in Part I

The rapid progress of electrical science in recent years has rendered sweeping changes necessary in PART III., which has accordingly been thoroughly recast. Two completely new chapters have been added, and large additions have been made in other places; while room has been gained by striking out matter which was of merely antiquarian interest.

The most important additions relate to electromagnetic induction, which, from the prominent part that it plays in the theory of the modern dynamo, has received much attention in recent years. In conformity with the plan of this treatise, which aims at teaching science from a liberal rather than a technical stand-point, we have endeavoured to lay the foundations of this subject clearly, and to insert every step of the somewhat elaborate reasoning which leads to the rules required for practice.

As a necessary preliminary, the relations between intensity of magnetization and density of free magnetism are first stated. The distinction between the two forces H and B is then very carefully explained; and it is shown, as a consequence of first principles, that the normal component of B does not change in passing from iron to air—in other words, that the flux out of the first medium is equal to the flux into the second. In a subsequent chapter the subject is resumed, and the analogy between flux of magnetic induction and flux of heat is not merely stated but proved. This leads to the analogy with Ohm's law, and the modern theory of the "magnetic circuit". By limiting our discussion of these points to isotropic material, we have been able to treat them in a manner intelligible to students who are not masters of analysis.

Approaching the subject from another side, the theory of magnetic shells and their relation to currents is expounded, leading to the theory of line-integrals of magnetic force round closed paths, the tendencies of circuits to motion in magnetic fields, the e.m.f. produced by such motion, and coefficients of induction between circuits.

Some important differences have been pointed out between a field produced by steel magnets and a field produced by currents, as regards the effect of introducing soft iron.

The influence of self-induction upon the amplitudes and phases of alternating currents, is worked out by a brief method suggested by § 46 of Lord Rayleigh's treatise on Sound.

Among other items of new matter may be mentioned:—

The effect of the electrostatic coefficient K upon attractions and repulsions, and its analogy to thermal conductivity;
a fuller account of Wimshurst's machine;
a fuller statement and illustration of Ohm's law;
Kirchhoff's laws;
Foster's method of comparing resistances;

measurement of resistance of electrolytes;
Tesla's experiments on rapidly alternating discharges;
the production of a rotating field by combining two alternating currents;
series and shunt dynamos;
series and shunt motors;
accumulators and transformers;
electric welding.

A brief account has been given of modern views on the seat of the energy of a field, and also of the modern method of harmonizing the two sets of electric dimensions.

In PART IV. very extensive changes have been made in the optical portion.

They comprise a recasting of the chapters on concave and convex mirrors and lenses, and considerable changes in the chapter on refraction. There are additions relating to the curvature of the image of a slit formed by a prism, to the action of lenses and mirrors on converging rays (leading to a practical method of measuring the focal length of a concave lens or a convex mirror), and to direct vision spectroscopes. A new focometer is described and figured.

The chapter on the wave-theory has been greatly enlarged, chiefly by additions relating to interference, including the colours of thin films, and the fringes produced by Fresnel's mirrors or biprism, or by Young's two holes. Instructions are given for the practical measurement of wave-lengths by means of a plane reflection grating mounted on a spectrometer; and Rowland's arrangement for measuring them by means of his concave gratings are fully discussed. A simple investigation is given of the effect of the relative width of bars and spaces upon brightness, and the chapter concludes with a discussion (borrowed from Mascart) of the effect of diffraction upon the image of a star in a telescope.

In the final chapter (on polarization and double refraction) "plane of best reflection" is suggested as an alternative name for "plane of polarization"; Huygens' construction is discussed in its application to internal reflection, and to both internal and external conical refraction, as well as to the specially simple cases of refraction into uniaxal crystals; and a description is given of the biquartz analyser.

Besides these improvements, a new chapter has been introduced dealing with the modern subject of systems of coaxal lenses. Our aim here has been to establish in the simplest manner the existence of certain properties, a knowledge of which is essential for appreciating and specifying the resultant effect of an optical instrument. such as the human eye, or a microscope or telescope. One of our examples (the first of section 209)

explains the well-known but little-understood fact that a telescope makes objects look nearer and smaller than they really are.

The collection of unworked examples in optics has been greatly enlarged, and Part IV. may now claim to be a suitable text-book for classes of mathematical physics.

We have endeavoured to exclude all superfluous matter, but some increase of size has been unavoidable.

BELFAST, *February, 1894.*

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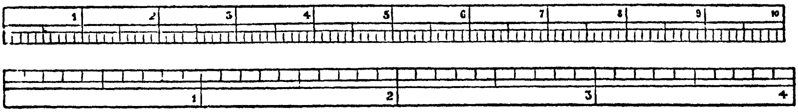
FRENCH AND ENGLISH MEASURES.

A DECIMETRE DIVIDED INTO CENTIMETRES AND MILLIMETRES.



INCHES AND TENTHS.

REDUCTION OF FRENCH TO ENGLISH MEASURES.

LENGTH.

1 millimetre = ·03937 inch, or about $\frac{1}{25}$ inch.
1 centimetre = ·3937 inch.
1 decimetre = 3·937 inch.
1 metre = 39·37 inch = 3 281 ft. = 1·0936 yd.
1 kilometre = 1093 6 yds , or about $\frac{5}{8}$ mile.

More accurately, 1 metre = 39·370432 in. = 3·2808693 ft. = 1·09362311 yd.

AREA.

1 sq. millim. = ·00155 sq. in.
1 sq. centim. = ·155 sq. in.
1 sq. decim. = 15·5 sq in.
1 sq. metre = 1550 sq. in. = 10·764 sq. ft. = 1·196 sq. yd.

VOLUME.

1 cub. millim. = ·000061 cub. in.
1 cub. centim. = ·061025 cub. in.
1 cub. decim. = 61·0254 cub in.
1 cub. metre = 61025 cub. in. = 35·3156 cub. ft. = 1·308 cub. yd.

The Litre (used for liquids) is the same as the cubic decimetre, and is equal to 1·7617 pint, or ·22021 gallon.

MASS AND WEIGHT.

1 milligramme = ·01543 grain.
1 gramme = 15·432 grain.
1 kilogramme = 15432 grains = 2·205 lbs. avoir.
More accurately, the kilogramme is 2 20462125 lbs.

MISCELLANEOUS.

1 gramme per sq. centim. = 2 0481 lbs. per sq. ft.
1 kilogramme per sq. centim. = 14·223 lbs. per sq. in.
1 kilogrammetre = 7·2331 foot-pounds.

1 force de cheval = 75 kilogrammetres per second, or 542½ foot-pounds per second nearly, whereas 1 horse-power (English) = 550 foot-pounds per second.

REDUCTION TO C.G.S. MEASURES. (See page 48.)

[*cm.* denotes centimetre(s); *gm.* denotes gramme(s).]

LENGTH.

1 inch = 2 54 centimetres, nearly.
1 foot = 30·48 centimetres, nearly.
1 yard = 91·44 centimetres, nearly.
1 statute mile = 160933 centimetres, nearly.

More accurately, 1 inch = 2·5399772 centimetres.

AREA.

1 sq. inch = 6·45 sq. cm., nearly.
1 sq. foot = 929 sq. cm., nearly.
1 sq. yard = 8361 sq. cm., nearly.
1 sq. mile = $2{\cdot}59 \times 10^{10}$ sq. cm., nearly.

VOLUME.

1 cub. inch = 16·39 cub. cm., nearly.
1 cub. foot = 28316 cub. cm., nearly.
1 cub. yard = 764535 cub. cm , nearly.
1 gallon = 4541 cub. cm., nearly.

MASS.

1 grain = ·0648 gramme, nearly.
1 oz. avoir. = 28·35 gramme, nearly.
1 lb. avoir. = 453 6 gramme, nearly.
1 ton = $1{\cdot}016 \times 10^6$ gramme, nearly

More accurately, 1 lb. avoir. = 453·59265 gm.

VELOCITY.

1 mile per hour = 44·704 cm. per sec.
1 kilometre per hour = 27·7 cm. per sec.

DENSITY.

1 lb. per cub. foot = ·016019 gm. per cub cm.
62·4 lbs. per cub. ft. = 1 gm. per cub. cm.

FORCE (assuming $g=981$). (See p. 48.)

Weight of 1 grain $=63{\cdot}57$ dynes, nearly.
,, 1 oz. avoir. $=2{\cdot}78\times10^4$ dynes, nearly.
,, 1 lb. avoir. $=4{\cdot}45\times10^5$ dynes, nearly.
,, 1 ton $=9{\cdot}97\times10^8$ dynes, nearly.
,, 1 gramme $=981$ dynes, nearly.
,, 1 kilogramme $=9{\cdot}81\times10^5$ dynes, nearly.

WORK (assuming $g=981$). (See p. 48.)

1 foot-pound $=1{\cdot}356\times10^7$ ergs, nearly.
1 kilogrammetre $=9{\cdot}81\times10^7$ ergs, nearly.
Work in a second by one theoretical "horse." $=7{\cdot}46\times10^9$ ergs, nearly.

STRESS (assuming $g=981$).

1 lb. per sq. ft. $=479$ dynes per sq. cm., nearly.
1 lb. per sq. inch $=6{\cdot}9\times10^4$ dynes per sq. cm., nearly.
1 kilog. per sq. cm. $=9{\cdot}81\times10^5$ dynes per sq. cm., nearly.
760 mm. of mercury at 0°C. $=1{\cdot}014\times10^6$ dynes per sq. cm., nearly.
30 inches of mercury at 0° C. $=1{\cdot}0163\times10^6$ dynes per sq. cm., nearly
1 inch of mercury at 0° C. $=3{\cdot}388\times10^4$ dynes per sq. cm., nearly.

TABLE OF DENSITIES, IN GRAMMES PER CUBIC CENTIMETRE.

LIQUIDS.	
Pure water at 4° C.,	1·000
Sea water, ordinary,	1·026
Alcohol, pure,	·791
,, proof spirit,	·916
Ether,	·716
Mercury at 0° C.,	13·596
Naphtha,	·848
SOLIDS.	
Brass, cast,	7·8 to 8·4
,, wire,	8 54
Bronze,	8·4
Copper, cast,	8 6
sheet,	8·8
hammered,	8 9
Gold,	19 to 19·6
Iron, cast,	6·95 to 7 3
,, wrought,	7·6 to 7·8
Lead,	11·4
Platinum,	21 to 22
Silver,	10·5
Steel,	7·8 to 7 9
Tin,	7·3 to 7·5
Zinc,	6·8 to 7·2
Ice,	·92
Basalt,	3 00
Brick,	2 to 2·17
Brickwork,	1·8
Chalk,	1·8 to 2·8
Clay,	1 92
Glass, crown,	2·5
,, flint,	3·0
Quartz (rock-crystal),	2 65
Sand,	1·42
Fir, spruce,	·48 to ·7
Oak, European,	·69 to ·99
Lignum-vitæ,	·65 to 1·33
Sulphur, octahedral,	2 05
,, prismatic,	1 98
GASES, at 0° C. and a pressure of a million dynes per sq. cm (See p. 142.)	
Air, dry,	·0012756
Oxygen,	·0014107
Nitrogen,	·0012393
Hydrogen,	·00008837
Carbonic acid,	·0019509

TABLE OF CONSTANTS.

The velocity acquired in falling for one second in vacuo, in any part of Great Britain, is about 32 2 feet per second, or 9 81 metres per second

The pressure of one atmosphere, or 760 millimetres (29·922 inches) of mercury, is 1 033 kilogramme per sq centimetre, or 14·73 lbs. per square inch.

The weight of a litre of dry air, at this pressure (at Paris) and 0° C., is 1 293 gramme.

The weight of a cubic centimetre of water is about 1 gramme

The weight of a cubic foot of water is about 62·4 lbs.

The equivalent of a unit of heat, in gravitation units of energy, is—

772 for the foot and Fahrenheit degree.
1390 for the foot and Centigrade degree.
424 for the metre and Centigrade degree.
42400 for the centimetre and Centigrade degree

In absolute units of energy, the equivalent is –

41·6 millions for the centimetre and Centigrade degree;
or 1 gramme-degree is equivalent to 41·6 million ergs.

UNITS EMPLOYED BY PRACTICAL ELECTRICIANS.

The *ohm* is (or was intended to be) 10^9 C. G. S. electro-magnetic units of resistance.

The *volt* is 10^8 C. G. S. electro-magnetic units of electro-motive force.

The *weber* is $\frac{1}{10}$ of the C. G. S. electro-magnetic unit of quantity; and a current of 1 weber per second is produced by an electro-motive force of 1 volt in a circuit whose resistance is 1 ohm.

The *farad* is 10^{-9} of the C. G. S. electro-magnetic unit of capacity, and the *microfarad* is the millionth part of the farad. A charge of 1 weber given to a condenser of capacity 1 farad would raise its potential by 1 volt.

PART I.

MECHANICS, HYDROSTATICS, AND PNEUMATICS.

ELEMENTARY TREATISE

ON

NATURAL PHILOSOPHY.

CHAPTER I.

INTRODUCTORY.

1. Natural Science, in the widest sense of the term, comprises all the phenomena of the material world. In so far as it merely describes and classifies these phenomena, it may be called Natural History; in so far as it furnishes accurate quantitative knowledge of the relations between causes and effects it is called Natural Philosophy. Many subjects of study pass through the natural history stage before they attain the natural philosophy stage; the phenomena being observed and compared for many years before the quantitative laws which govern them are disclosed.

2. There are two extensive groups of phenomena which are conventionally excluded from the domain of Natural Philosophy, and regarded as constituting separate branches of science in themselves; namely:—

First. Those phenomena which depend on vital forces; such phenomena, for example, as the growth of animals and plants. These constitute the domain of Biology.

Secondly. Those which depend on elective attractions between the atoms of particular substances, attractions which are known by the name of chemical affinities. These phenomena are relegated to the special science of Chemistry.

Again, Astronomy, which treats of the nature and movements of the heavenly bodies, is, like Chemistry, so vast a subject, that it forms a special science of itself; though certain general laws, which its phenomena exemplify, are still included in the study of Natural Philosophy.

3. Those phenomena which specially belong to the domain of Natural Philosophy are called *physical;* and Natural Philosophy itself is called *Physics.* It may be divided into the following branches.

I. DYNAMICS, or the general laws of force and of the relations which exist between force, mass, and velocity. These laws may be applied to solids, liquids, or gases. Thus we have the three divisions, *Mechanics, Hydrostatics,* and *Pneumatics.*

II. THERMICS; the science of Heat.

III. The science of ELECTRICITY, with the closely related subject of MAGNETISM.

IV. ACOUSTICS; the science of Sound.

V. OPTICS; the science of Light.

The branches here numbered I. II. III. are treated in Parts I. II. III. respectively, of the present Work. The two branches numbered IV. V. are treated in Part IV.

CHAPTER II.

FIRST PRINCIPLES OF DYNAMICS. STATICS.

4. **Force.**—Force may be defined as that which tends to produce motion in a body at rest, or to produce change of motion in a body which is moving. A particle is said to have uniform or unchanged motion when it moves in a straight line with constant velocity; and every deviation of material particles from uniform motion is due to forces acting upon them.

5. **Translation and Rotation.**—When a body moves so that all lines in it remain constantly parallel to their original positions (or, to use the ordinary technical phrase, *move parallel to themselves*), its movement is called a *pure translation.* Since the lines joining the extremities of equal and parallel straight lines are themselves equal and parallel, it can easily be shown that, in such motion, all points of the body have equal and parallel velocities, so that the movement of the whole body is completely represented by the movement of any one of its points.

On the other hand, if one point of a rigid body be fixed, the only movement possible for the body is *pure rotation,* the axis of the rotation at any moment being some straight line passing through this point.

Every movement of a rigid body can be specified by specifying the movement of one of its points (any point will do) together with the rotation of the body about this point.

6. Force which acts uniformly on all the particles of a body, as gravity does sensibly in the case of bodies of moderate size on the earth's surface (equal particles being urged with equal forces and in parallel directions), tends to give the body a movement of pure translation.

In elementary statements of the laws of force, it is necessary, for

the sake of simplicity, to confine attention to forces tending to produce pure translation.

7. Instruments for Measuring Force.—We obtain the idea of force through our own conscious exercise of muscular force, and we can approximately estimate the amount of a force (if not too great or too small) by the effort which we have to make to resist it; as when we try the weight of a body by lifting it.

Dynamometers are instruments in which force is measured by means of its effect in bending or otherwise distorting elastic springs, and the spring-balance is a dynamometer applied to the measurement of weights, the spring in this case being either a flat spiral (like the mainspring of a watch), or a helix (resembling a corkscrew).

A force may also be measured by causing it to act vertically downwards upon one of the scale-pans of a balance and counterpoising it by weights in the other pan.

8. Gravitation Units of Force.—In whatever way the measurement of a force is effected, the result, that is, the magnitude of the force, is usually stated in terms of weight; for example, in pounds or in kilogrammes. Such units of force (called gravitation units) are to a certain extent indefinite, inasmuch as gravity is not exactly the same over the whole surface of the earth; but they are sufficiently definite for ordinary commercial purposes.

9. Equilibrium, Statics, Kinetics.—When a body free to move is acted on by forces which do not move it, these forces are said to be *in equilibrium*, or to *equilibrate* each other. They may equally well be described as *balancing* each other. Dynamics is usually divided into two branches. The first branch, called *Statics*, treats of the conditions of equilibrium. The second branch, called *Kinetics*, treats of the movements produced by forces not in equilibrium.

10. Action and Reaction.—Experiment shows that force is always a mutual action between two portions of matter. When a body is urged by a force, this force is exerted by some other body, which is itself urged in the opposite direction with an equal force. When I press the table downwards with my hand, the table presses my hand upwards; when a weight hangs by a cord attached to a beam, the cord serves to transmit force between the beam and the weight, so that, by the instrumentality of the cord, the beam pulls the weight upwards and the weight pulls the beam downwards. Electricity

and magnetism furnish no exception to this universal law. When a magnet attracts a piece of iron, the piece of iron attracts the magnet with a precisely equal force.

11. Specification of a Force acting at a Point.—Force may be applied over a finite area, as when I press the table with my hand; or may be applied through the whole substance of a body, as in the case of gravity; but it is usual to begin by discussing the action of forces applied to a *single particle*, in which case each force is supposed to act along a mathematical straight line, and the particle or material point to which it is applied is called its *point* of application. A force is completely specified when its *magnitude*, its *point of application*, and its *line of action* are all given.

12. Rigid Body. Fundamental Problem of Statics.—A force of finite magnitude applied to a mathematical point of any actual solid body would inevitably fracture the body. To avoid this complication and other complications which would arise from the bending and yielding of bodies under the action of forces, the fiction of a perfectly rigid body is introduced, a body which cannot bend or break under the action of any force however intense, but always retains its size and shape unchanged.

The fundamental problem of Statics consists in determining the conditions which forces must fulfil in order that they may be in equilibrium when applied to a rigid body.

13. Conditions of Equilibrium for Two Forces.—In order that two forces applied to a rigid body should be in equilibrium, it is necessary and sufficient that they fulfil the following conditions:—

1st. Their lines of action must be one and the same.

2nd. The forces must act in opposite directions along this common line.

3rd. They must be equal in magnitude.

It will be observed that nothing is said here about the points of application of the forces. They may in fact be anywhere upon the common line of action. *The effect of a force upon a rigid body is not altered by changing its point of application to any other point in its line of action.* This is called the principle of the *transmissibility of force.*

It follows from this principle that the condition of equilibrium for any number of forces with the same line of action is simply that the sum of those which act in one direction shall be equal to the sum of those which act in the opposite direction.

14. Three Forces Meeting in a Point. Triangle of Forces.—If three forces, not having one and the same line of action, are in equilibrium, their lines of action must lie in one plane, and must either meet in a point or be parallel. We shall first discuss the case in which they meet in a point.

From any point A (Fig. 1) draw a line AB parallel to one of the two given forces, and so that in travelling from A to B we should be travelling in the same direction in which the force acts (not in the opposite direction). Also let it be understood that the length of AB represents the magnitude of the force.

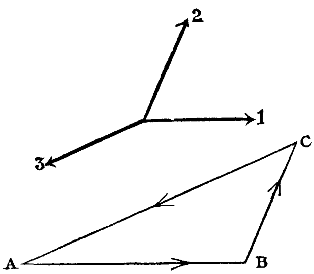


Fig. 1.—Triangle of Forces.

From the point B draw a line BC representing the second force in direction, and on the same scale of magnitude on which AB represents the first.

Then the line CA will represent both in direction and magnitude the third force which would equilibrate the first two.

The principle embodied in this construction is called the *triangle of forces*. It may be briefly stated as follows:—*The condition of equilibrium for three forces acting at a point is, that they be represented in magnitude and direction by the three sides of a triangle, taken one way round.* The meaning of the words "taken one way round" will be understood from an inspection of the arrows with which the sides of the triangle in Fig. 1 are marked. If the directions of all three arrows are reversed the forces represented will still be in equilibrium. The arrows must be so directed that it would be possible to travel completely round the triangle by moving along the sides in the directions indicated.

When a line is used to represent a force, it is always necessary to employ an arrow or some other mark of direction, in order to avoid ambiguity between the direction intended and its opposite. In naming such a line by means of two letters, one at each end of it, the order of the letters should indicate the direction intended. The direction of AB is from A to B; the direction of BA is from B to A.

15. Resultant and Components.—Since two forces acting at a point can be balanced by a single force, it is obvious that they are equivalent to a single force, namely, to a force equal and opposite to that which would balance them. This force to which they are equivalent

is called their *resultant.* Whenever one force acting on a rigid body is equivalent to two or more forces, it is called their resultant, and they are called its *components.* When any number of forces are in equilibrium, a force equal and opposite to any one of them is the resultant of all the rest.

The "triangle of forces" gives us the resultant of any two forces acting at a point. For example, in Fig. 1, AC (with the arrow in the figure reversed) represents the resultant of the forces represented by AB and BC.

16. Parallelogram of Forces.—The proposition called the "parallelogram of forces" is not essentially distinct from the "triangle of forces," but merely expresses the same fact from a slightly different point of view. It is as follows:—*If two forces acting upon the same rigid body in lines which meet in a point, be represented by two lines drawn from the point, and a parallelogram be constructed on these lines, the diagonal drawn from this point to the opposite corner of the parallelogram represents the resultant.*

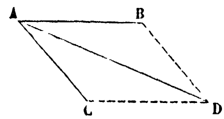


Fig. 2.—Parallelogram of Forces

For example, if AB, AC, Fig. 2, represent the two forces, AD will represent their resultant.

To show the identity of this proposition with the triangle of forces, we have only to substitute BD for AC (which is equal and parallel to it). We have then two forces represented by AB, BD (two sides of a triangle) giving as their resultant a force represented by the third side AD. We might equally well have employed the triangle ACD, by substituting CD for AB.

17. Gravesande's Apparatus.—An apparatus for verifying the parallelogram of forces is represented in Fig. 3. ACDB is a light frame in the form of a parallelogram. A weight P″ can be hung at A, and weights P, P′ can be attached, by means of cords passing over pulleys, to the points B, C. When the weights P, P′, P″ are proportional to AB, AC and AD respectively, the strings attached at B and C will be observed to form prolongations of the sides, and the diagonal AD will be vertical.

18. Resultant of any Number of Forces at a Point.—To find the resultant of any number of forces whose lines of action meet in a point, it is only necessary to draw a crooked line composed of straight lines which represent the several forces. The resultant will be represented by a straight line drawn from the beginning to the

end of this crooked line. For by what precedes, if ABCDE be a crooked line such that the straight lines AB, BC, CD, DE represent four forces acting at a point, we may substitute for AB and BC

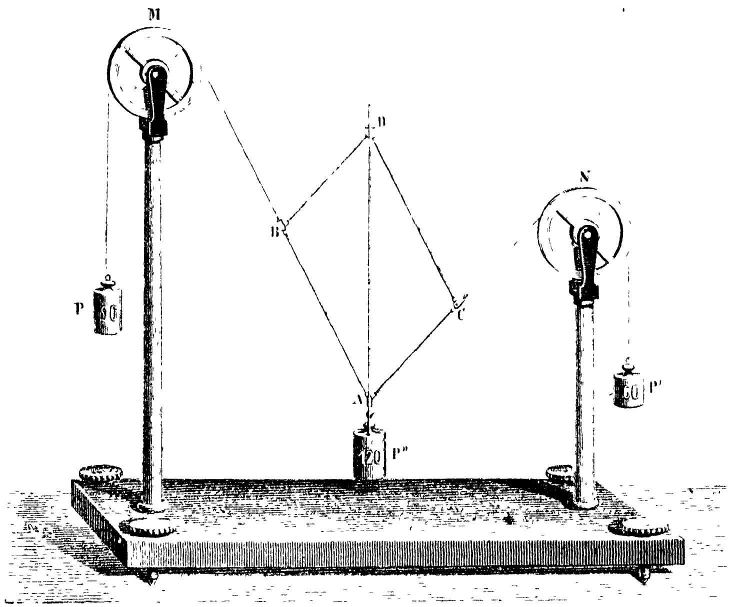


Fig 3.—Gravesande's Apparatus

the straight line AC, since this represents their resultant. We may then substitute AD for AC and CD, and finally AE for AD and DE.

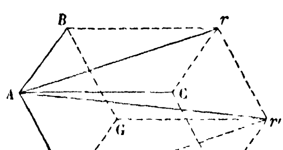


Fig. 4.—Parallelepiped of Forces.

One of the most important applications of this construction is to three forces not lying in one plane. In this case the crooked line will consist of three edges of a parallelepiped, and the line which represents the resultant will be the diagonal. This is evident from Fig. 4, in which AB, AC, AD represent three forces acting at A. The resultant of AB and AC is A*r*, and the resultant of A*r* and AD is A*r'*. The crooked line whose parts represent the forces, may be either AB*rr'*, or ABG*r'*, or ADG*r'*, &c., the total number of alternatives being six, since three things can be taken in six different orders. We have here an excellent illustration of the fact that the same final resultant is obtained in whatever order the forces are combined.

o **19. Equilibrium of Three Parallel Forces.**—If three parallel forces, P, Q, R, applied to a rigid body, balance each other, the following conditions must be fulfilled:—

1. The three lines of action AP, BQ, CR, Fig. 5, must be in one plane.

2. The two outside forces P, R, must act in the opposite direction to the middle force Q, and their sum must be equal to Q.

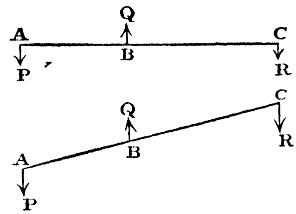


Fig. 5.

3. Each force must be proportional to the distance between the lines of action of the other two; that is, we must have

$$\frac{P}{BC} = \frac{Q}{AC} = \frac{R}{AB}. \tag{1}$$

The figure shows that AC is the sum of AB and BC; hence it follows from these equations, that Q is equal to the sum of P and R, as above stated.

c **20. Resultant of Two Parallel Forces.**—Any two parallel forces being given, a third parallel force which will balance them can be found from the above equations; and a force equal and opposite to this will be their resultant. We may distinguish two cases.

1. Let the two given forces be in the same direction. Then their resultant is equal to their sum, and acts in the same direction, along a line which cuts the line joining their points of application into two parts which are inversely as the forces.

2. Let the two given forces be in opposite directions. Then their resultant will be equal to their difference, and will act in the direction of the greater of the two forces, along a line which cuts the production of the line joining their points of application on the side of the greater force; and the distances of this point of section from the two given points of application are inversely as the forces.

21. Centre of Two Parallel Forces.—In both cases, if the points of application are not given, but only the magnitudes of the forces and their lines of action, the magnitude and line of action of the resultant are still completely determined; for all straight lines which are drawn across three parallel straight lines are cut by them in the same ratio; and we shall obtain the same result whatever points of application we assume.

If the points of application are given, the resultant cuts the line

joining them, or this line produced, in a definite point, whose position depends only on the magnitudes of the given forces, and not at all on the angle which their direction makes with the joining line. This result is important in connection with centres of gravity. The point so determined is called the centre of the two parallel forces. If these two forces are the weights of two particles, the "centre" thus found is their centre of gravity, and the resultant force is the same as if the two particles were collected at this point.

22. Moments of Resultant and of Components Equal.—The following proposition is often useful. Let any straight line be drawn across the lines of action of two parallel forces P_1, P_2 (Fig. 6). Let O be any point on this line, and x_1, x_2 the distances measured from O to the points of section, distances measured in opposite directions being distinguished by opposite signs, and forces in opposite directions being also distinguished by opposite signs. Also let R denote the resultant of P_1 and P_2, and $\bar{x}$ the distance from O to its intersection with the line; then we shall have

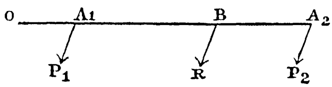


Fig. 6.

$$P_1 x_1 + P_2 x_2 = R \bar{x}.$$

For, taking the standard case, as represented in Fig. 6, in which all the quantities are positive, we have $OA_1 = x_1$, $OA_2 = x_2$, $OB = \bar{x}$, and by § 19 or § 20 we have

$$P_1 . A_1B = P_2 . BA_2,$$

that is,

$$P_1 (\bar{x} - x_1) = P_2 (x_2 - \bar{x}),$$

whence

$$(P_1 + P_2) \bar{x} = P_1 x_1 + P_2 x_2,$$

that is,

$$R \bar{x} = P_1 x_1 + P_2 x_2. \qquad (2)$$

23. Any Number of Parallel Forces in One Plane.—Equation (2) affords the readiest means of determining the line of action of the resultant of several parallel forces lying in one plane. For let P_1, P_2, P_3, &c., be the forces, R_1 the resultant of the first two forces P_1, P_2, and R_2 the resultant of the first three forces P_1, P_2, P_3. Let a line be drawn across the lines of action, and let the distances of the points of section from an arbitrary point O on this line be expressed according to the following scheme:—

Force	P_1	P_2	P_3	R_1	R_2
Distance	x_1	x_2	x_3	$\bar{x}_1$	$\bar{x}_2$

Then, by equation (2) we have

$$R_1 \bar{x}_1 = P_1 x_1 + P_2 x_2.$$

Also since R_2 is the resultant of R_1 and P_3, we have

$$R_2 \bar{x}_2 = R_1 \bar{x}_1 + P_3 x_3,$$

and substituting for the term $R_1 \bar{x}_1$, we have

$$R_2 \bar{x}_2 = P_1 x_1 + P_2 x_2 + P_3 x_3.$$

This reasoning can evidently be extended to any number of forces, so that we shall have finally

$$R\bar{x} = \text{sum of such terms as } Px,$$

where R denotes the resultant of all the forces, and is equal to their algebraic sum; while $\bar{x}$ denotes the value of x for the point where the line of action of R cuts the fixed line. It is usual to employ the Greek letter Σ to denote "the sum of such terms as." We may therefore write

$$R = \Sigma (P)$$

$$R\bar{x} = \Sigma (Px)$$

whence

$$\bar{x} = \frac{\Sigma (Px)}{\Sigma (P)} \qquad (3)$$

24. Moment of a Force about a Point.—When the fixed line is at right angles to the parallel forces, the product Px is called the moment of the force P about the point O. More generally, the *moment of a force about a point* is the *product of the force by the length of the perpendicular dropped upon it from the point.* The above equations show that for parallel forces in one plane, the *moment of the resultant* about any point in the plane *is the sum of the moments of the forces* about the same point.

If the resultant passes through O, the distance $\bar{x}$ is zero; whence it follows from the equations that the algebraical sum of the moments vanishes.

The moment of a force about a point measures the tendency of the force to produce rotation about the point. If one point of a body be fixed, the body will turn in one direction or the other according as the resultant passes on one side or the other of this point (the direction of the resultant being supposed given). If the resultant passes through the fixed point, the body will be in equilibrium.

The moment Px of any force about a point, changes sign with P and also with x; thereby expressing (what is obvious in itself) that

the direction in which the force tends to turn the body about the point will be reversed if the direction of P is reversed while its line of action remains unchanged, and will also be reversed if the line of action be shifted to the other side of the point while the direction of the force remains unchanged.

25. Experimental Illustration.—Fig. 7 represents a simple apparatus (called the *arithmetical lever*) for illustrating the laws of par-

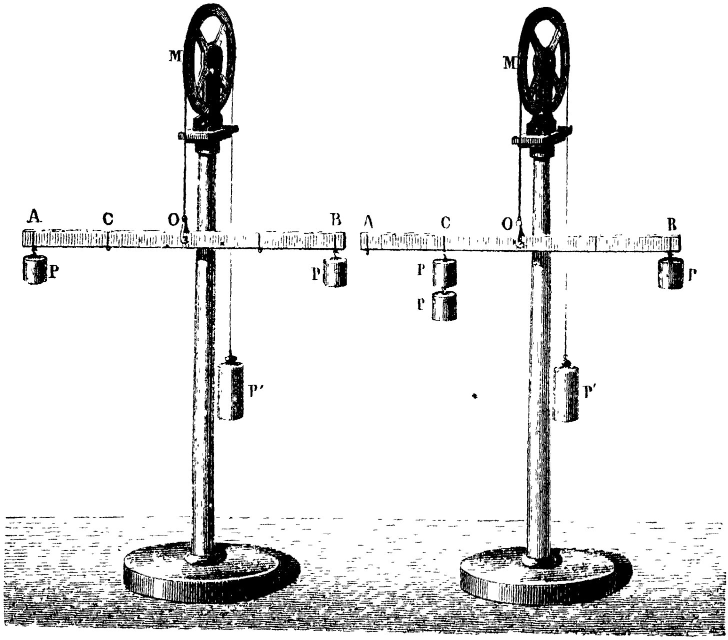


Fig. 7.—Composition of Parallel Forces.

allel forces. The lever AB is suspended at its middle point by a cord, so that when no weights are attached it is horizontal. Equal weights P, P are hung at points A and B equidistant from the centre, and the suspending cord after being passed over a very freely moving pulley M, has a weight P′ hung at its other end sufficient to produce equilibrium. It will be found that P′ is equal to the sum of the two weights P together with the weight required to counterpoise the lever itself.

In the second figure, the two weights hung from the lever are not equal, but one of them is double of the other, P being hung at B, and 2 P at C; and it is necessary for equilibrium that the distance OB be double of the distance OC. The weight P′ required

to balance the system will now be 3 P together with the weight of the lever.

26. Couple.—There is one case of two parallel forces in opposite directions which requires special attention; that in which the two forces are equal.

To obtain some idea of the effect of two such forces, let us first suppose them not exactly equal, but let their difference be very small compared with either of the forces. In this case, the resultant will be equal to this small difference, and its line of action will be at a great distance from those of the given forces. For in § 19 if Q is very little greater than P, so that Q-P, or R is only a small fraction of P, the equation $\frac{P}{BC}=\frac{R}{AB}$ shows that AB is only a small fraction of BC, or in other words that BC is very large compared with AB.

If Q gradually diminishes until it becomes equal to P, R will gradually diminish to zero; but while it diminishes, the product R . BC will remain constant, being always equal to P . AB.

A very small force R at a very great distance would have sensibly the same moment round all points between A and B or anywhere in their neighbourhood, and the moment of R is always equal to the algebraic sum of the moments of P and Q.

When Q is equal to P, they compose what is called *a couple*, and the algebraic sum of their moments about any point in their plane is constant, being always equal to P . AB, which is therefore called the moment of the couple.

A couple consists of two equal and parallel forces in opposite directions applied to the same body. The distance between their lines of action is called the arm of the couple, and the product of one of the two equal forces by this arm is called the moment of the couple.

27. Composition of Couples. Axis of Couple.—A couple cannot be balanced by a single force; but it can be balanced by any couple of equal moment, opposite in sign, if the plane of the second couple be either the same as that of the first or parallel to it.

Any number of couples in the same or parallel planes are equivalent to a single couple whose moment is the algebraic sum of theirs.

The laws of the composition of couples (like those of forces) can be illustrated by geometry.

Let a couple be represented by a line perpendicular to its plane, marked with an arrow according to the convention that if an

ordinary screw were made to turn in the direction in which the couple tends to turn, it would advance in the direction in which the arrow points. Also let the length of the line represent the moment of the couple. Then the same laws of composition and resolution which hold for forces acting at a point will also hold for couples. A line thus drawn to represent a couple is called the *axis* of the couple.

Just as any number of forces acting at a point are either in equilibrium or equivalent to a single force, so any number of couples applied to the same rigid body (no matter to what parts of it) are either in equilibrium or equivalent to a single couple.

28. Resultant of Force and Couple in Same Plane.—The resultant of a force and a couple in the same plane is a single force. For the couple may be replaced by another of equal moment having its equal forces P, Q, each equal to the given force F, and the latter couple may then be turned about in its own plane and carried into such a position that one of its two forces destroys the force F, as represented in Fig. 8. There will then only remain the force P, which is equal and parallel to F.

Fig. 8.

By reversing this procedure, we can show that a force P which does not pass through a given point A is equivalent to an equal and parallel force F which does pass through it, together with a couple; the moment of the couple being the same as the moment of the force P about A.

29. General Resultant of any Number of Forces applied to a Rigid Body.—Forces applied to a rigid body in lines which do not meet in one point are not in general equivalent to a single force. By the process indicated in the concluding sentence of the preceding section, we can replace the forces by forces equal and parallel to them, acting at any assumed point, together with a number of couples. These couples can then be reduced (by the principles of § 27) to a single couple, and the forces at the point can be replaced by a single force; so that we shall obtain, as the complete resultant, a single force applied at any point we choose to select, and a couple.

We can in general make the couple smaller by resolving it into two components whose planes are respectively perpendicular and parallel to the force, and then compounding one of these components (the latter) with the force as explained in § 28, thus moving the

force parallel to itself without altering its magnitude. This is the greatest simplification that is possible. The result is that we have a single force and a couple whose plane is perpendicular to the force. Any combination of forces that can be applied to a rigid body is reducible to a force acting along one definite line and a couple in a plane perpendicular to this line. Such a combination of a force and couple is called a *wrench*, and the "one definite line" is called the *axis* of the wrench. The point of application of the force is not definite, but is any point of the axis.

30. Application to Action and Reaction.—Every action of force that one body can exert upon another is reducible to a wrench, and the law of reaction is that the second body will, in every case, exert upon the first an equal and opposite wrench. The two wrenches will have the same axis, equal and opposite forces along this axis, and equal and opposite couples in planes perpendicular to it.

31. Resolution the Inverse of Composition.—The process of finding the resultant of two or more forces is called *composition.* The inverse process of finding two or more forces which shall together be equivalent to a given force, is called *resolution,* and the two or more forces thus found are called *components.*

The problem to resolve a force into two components along two given lines which meet it in one point and are in the same plane with it, has a definite solution, which is obtained by drawing a triangle whose sides are parallel respectively to the given force and the required components. The given force and the required components will be proportional to the sides of this triangle, each being represented by the side parallel to it.

The problem to resolve a force into three components along three given lines which meet it in one point and are not in one plane, also admits of a definite solution.

32. Rectangular Resolution.—In the majority of cases which occur in practice the required components are at right angles to each other, and the resolution is then said to be rectangular. When "the component of a force along a given line" is mentioned, without anything in the context to indicate the direction of the other component or components, it is always to be understood that the resolution is rectangular. The process of finding the required component in this case is illustrated by Fig. 9. Let AB represent the given force F, and let AC be the line along which the component of F is required. It is only necessary to drop from B a

perpendicular BC on this line; AC will represent the required component. CB represents the other component, which, along with AC, is equivalent to the given force. If the total number of rectangular components, of which AC represents one, is to be three, then the other two will lie in a plane perpendicular to AC, and they can be found by again resolving CB. The magnitude of AC will be the same whether the number of components be two or three, and the component along AC will be F $\frac{AC}{AB}$, or in trigonometrical language,

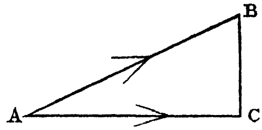


Fig. 9.—Component along a given Line.

$$F \cos . BAC.$$

We have thus the following rule:—*The component of a given force along a given line is found by multiplying the force by the cosine of the angle between its own direction and that of the required component.*

CHAPTER III.

CENTRE OF GRAVITY.

33. Gravity is the force to which we owe the names "up" and down." The direction in which gravity acts at any place is called the downward direction, and a line drawn accurately in this direction is called *vertical;* it is the direction assumed by a plumb-line. A plane perpendicular to this direction is called *horizontal,* and is parallel to the surface of a liquid at rest. The verticals at different places are not parallel, but are inclined at an angle which is approximately proportional to the distance between the places. It amounts to 180° when the places are antipodal, and to about 1′ when their distance is one geographical mile, or to about 1″ for every hundred feet. Hence, when we are dealing with the action of gravity on a body a few feet or a few hundred feet in length, we may practically regard the action as consisting of parallel forces.

34. Centre of Gravity.—Let A and B be any two particles of a rigid body, let w_1 be the weight of the particle A, and w_2 the weight of B. These weights are parallel forces, and their resultant divides the line AB in the inverse ratio of the forces. As the body is turned about into different positions, the forces w_1 and w_2 remain unchanged in magnitude, and hence the resultant cuts AB always in the same point. This point is called the centre of the parallel forces w_1 and w_2, or the centre of gravity of the two particles A and B. The magnitude of the resultant will be w_1+w_2, and we may substitute it for the two forces themselves; in other words, we may suppose the two particles A and B to be collected at their centre of gravity. We can now combine this resultant with the weight of a third particle of the body, and shall thus obtain a resultant $w_1+w_2+w_3$, passing through a definite point in the line which joins

the third particle to the centre of gravity of the first two. The first three particles may now be supposed to be collected at this point, and the same reasoning may be extended until all the particles have been collected at one point. This point will be the *centre of gravity* of the whole body. From the manner in which it has been obtained, it possesses the property that *the resultant of all the forces of gravity on the body passes through it, in every position in which the body can be placed.* The resultant force of gravity upon a rigid body is therefore a single force passing through its centre of gravity.

35. Centres of Gravity of Volumes, Areas, and Lines.—If the body is homogeneous (that is composed of uniform substance throughout), the position of the centre of gravity depends only on the figure, and in this sense it is usual to speak of the centre of gravity of a figure. In like manner it is customary to speak of the centres of gravity of areas and lines, an area being identified in thought with a thin uniform plate, and a line with a thin uniform wire.

It is not necessary that a body should be rigid in order that it may have a centre of gravity. We may speak of the centre of gravity of a mass of fluid, or of the centre of gravity of a system of bodies not connected in any way. The same point which would be the centre of gravity if all the parts were rigidly connected, is still called by this name whether they are connected or not.

36. Methods of Finding Centres of Gravity.—Whenever a homogeneous body contains a point which bisects all lines in the body that can be drawn through it, this point must be the centre of gravity. The centres of a sphere, a circle, a cube, a square, an ellipse, an ellipsoid, a parallelogram, and a parallelepiped, are examples.

Again, when a body consists of a finite number of parts whose weights and centres of gravity are known, we may regard each part as collected at its own centre of gravity.

When the parts are at all numerous, the final result will most readily be obtained by the use of the formula

$$\bar{x} = \frac{\Sigma (Px)}{\Sigma (P)}, \qquad (3)$$

where P denotes the weight of any part, x the distance of its centre of gravity from any plane, and $\bar{x}$ the distance of the centre of gravity of the whole from that plane. We have already in § 23

proved this formula for the case in which the centres of gravity lie in one straight line and x denotes distance from a point in this line; and it is not difficult, by the help of the properties of similar triangles, to make the proof general.

37. Centre of Gravity of a Triangle.—To find the centre of gravity of a triangle ABC (Fig. 10), we may begin by supposing it divided into narrow strips by lines (such as bc) parallel to BC. It can be shown, by similar triangles, that each of these strips is bisected by the line AD drawn from A to D the middle point of BC. But each strip may be collected at its own centre of gravity, that is at its own middle point; hence the whole triangle may be collected on the line AD; its centre of gravity must therefore be situated upon this line. Similar reasoning shows that it must lie upon the line BE drawn from B to the middle point of AC. It is therefore the intersection of these two lines. If we join DE we can show that the triangles AGB, DGE, are similar, and that

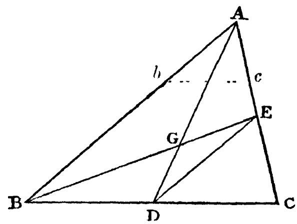


Fig 10

$$\frac{AG}{GD} = \frac{AB}{DE} = 2.$$

DG is therefore one third of DA. The centre of gravity of a triangle therefore lies upon the line joining any corner to the middle point of the opposite side, and is at one-third of the length of this line from the end where it meets that side.

It is worthy of remark that if three equal particles are placed at the corners of any triangle, they have the same centre of gravity as the triangle. For the two particles at B and C may be collected at the middle point D, and this double particle at D, together with the single particle at A, will have their centre of gravity at G, since G divides DA in the ratio of 1 to 2.

38. Centre of Gravity of a Pyramid.—If a pyramid or a cone be divided into thin slices by planes parallel to its base, and a straight line be drawn from the vertex to the centre of gravity of the base, this line will pass through the centres of gravity of all the slices, since all the slices are similar to the base, and are similarly cut by this line.

In a tetrahedron or triangular pyramid, if D (Fig. 11) be the centre of gravity of one face, and A be the corner opposite to this

face, the centre of gravity of the pyramid must lie upon the line AD. In like manner, if E be the centre of gravity of one face, the centre of gravity of the pyramid must lie upon the line joining E with the opposite corner B. It must therefore be the intersection G of these two lines. That they do intersect is otherwise obvious, for the lines AE, BD meet in C, the middle point of one edge of the pyramid, E being found by taking CE one third of CA, and D by taking CD one third of CB.

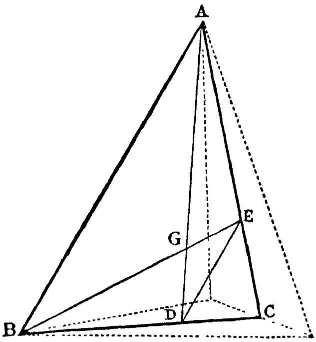


Fig. 11.—Centre of Gravity of Tetrahedron

If D, E be joined, we can show that the joining line is parallel to BA, and that the triangles AGB, DGE are similar. Hence

$$\frac{AG}{GD} = \frac{AB}{DE} = \frac{BC}{DC} = 3.$$

That is, the line AD joining any corner to the centre of gravity of the opposite face, is cut in the ratio of 3 to 1 by the centre of gravity G of the triangle. DG is therefore one-fourth of DA, and the distance of the centre of gravity from any face is one-fourth of the distance of the opposite corner.

A pyramid standing on a polygonal base can be cut up into triangular pyramids standing on the triangular bases into which the polygon can be divided, and having the same vertex as the whole pyramid. The centres of gravity of these triangular pyramids are all at the same perpendicular distance from the base, namely at one-fourth of the distance of the vertex, which is therefore the distance of the centre of gravity of the whole from the base. The centre of gravity of any pyramid is therefore found by joining the vertex to the centre of gravity of the base, and cutting off one-fourth of the joining line from the end where it meets the base. The same rule applies to a cone, since a cone may be regarded as a polygonal pyramid with a very large number of sides.

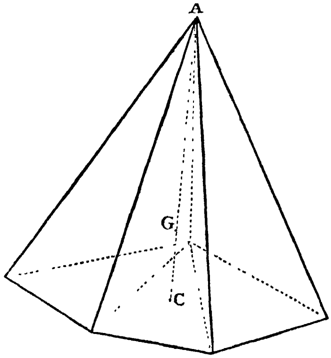


Fig. 12.—Centre of Gravity of Pyramid.

39. If four equal particles are placed at the corners of a triangular pyramid, they will have the same centre of gravity as the pyramid. For three of them may, as we have seen (§ 37) be collected at the centre of gravity of one face; and the centre of gravity of the four particles will divide the line which joins this point to the fourth, in the ratio of 1 to 3.

40. Condition of Standing or Falling.—When a heavy body stands on a base of finite area, and remains in equilibrium under the action of its own weight and the reaction of this base, the vertical through its centre of gravity must fall within the base. If the body is supported on three or more points, as in Fig. 13, we are to understand by the base the convex[1] polygon whose corners are the points of support; for if a body so supported turns over, it must turn about the line joining two of these points.

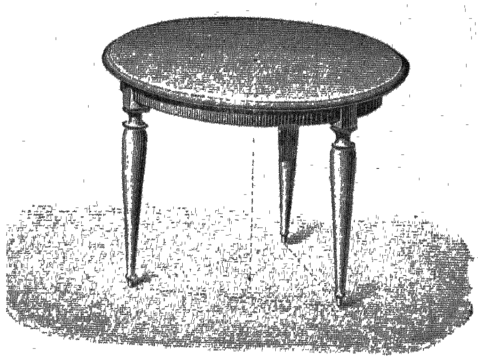


Fig. 13.—Equilibrium of a Body supported on a Horizontal Plane at three or more Points.

41. Body supported at one Point.—When a heavy body supported at one point remains at rest, the reaction of the point of support equilibrates the force of gravity. But two forces cannot be in equilibrium unless they have the same line of action; hence the vertical through the centre of gravity of the body must pass through the point of support. If instead of being supported at a point, the heavy body is supported by an axis about which it is free to turn, the vertical through the centre of gravity must pass through this axis.

42. Stability and Instability.—When the point of support, or axis of support, is vertically *below* the centre of gravity, it is easily seen that, if the body were displaced a little to either side, the forces acting upon it would turn it still further away from the position of equilibrium. On the other hand, when the point or axis of support is vertically *above* the centre of gravity, the forces which would

[1] The word *convex* is inserted to indicate that there must be no re-entrant angles. Any points of support which lie within the polygon formed by joining the rest, must be left out of account.

act upon it if it were slightly displaced would tend to restore it. In the latter case the equilibrium is said to be *stable*, in the former *unstable*.

When the centre of gravity coincides with the point of support, or lies upon the axis of support, the body will still be in equilibrium when turned about this point or axis into any other position. In this case the equilibrium is neither stable nor unstable but is called *neutral*.

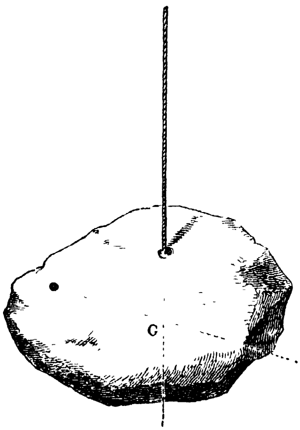


Fig. 14.—Experimental Determination of Centre of Gravity.

43. Experimental determination of Centre of Gravity.—In general, if we suspend a body by any point, in such a manner that it is free to turn about this point, it will come to rest in a position of stable equilibrium. The centre of gravity will then be vertically beneath the point of support. If we now suspend the body from another point, the centre of gravity will come vertically beneath this. The intersection of these two verticals will therefore be the centre of gravity (Fig. 14).

44. To find the centre of gravity of a flat plate or board (Fig. 15), we may suspend it from a point near its circumference, in such a manner that it sets itself in a vertical plane. Let a plumb-line be at the same time suspended from the same point, and made to leave its trace upon the board by chalking and "snapping" it. Let the board now be suspended from another point, and the operation be repeated. The two chalk lines will intersect each other at that point of the face which is opposite to the centre of gravity; the centre of gravity itself being of course in the substance of the board.

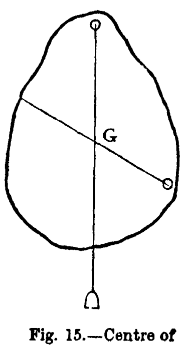


Fig. 15.—Centre of Gravity of Board.

45. Work done against Gravity.—When a heavy body is raised, work is said to be done against gravity, and the amount of this work is reckoned by multiplying together the weight of the body and the height through which it is raised. Horizontal movement does not count, and when a body is raised obliquely, only the vertical component of the motion is to be reckoned.

Suppose, now, that we have a number of particles whose weights

are w_1, w_2, w_3 &c., and let their heights above a given horizontal plane be respectively h_1, h_2, h_3 &c. We know by equation (3), § 23, that if $\bar{h}$ denote the height of their centre of gravity we have

$$(w_1 + w_2 + \&c.)\,\bar{h} = w_1\,h_1 + w_2\,h_2 + \&c. \qquad (4)$$

Let the particles now be raised into new positions in which their heights above the same plane of reference are respectively H_1, H_2, H_3 &c. The height $\bar{H}$ of their centre of gravity will now be such that

$$(w_1 + w_2 + \&c.)\,\bar{H} = w_1\,H_1 + w_2\,H_2 + \&c. \qquad (5)$$

From these two equations, we find, by subtraction

$$(w_1 + w_2 + \&c.)\,(\bar{H} - \bar{h}) = w_1\,(H_1 - h_1) + w_2\,(H_2 - h_2) + \&c. \qquad (6)$$

Now $H_1 - h_1$ is the height through which the particle of weight w_1 has been raised; hence the work done against gravity in raising it is $w_1\,(H_1 - h_1)$ and the second member of equation (6) therefore expresses the whole amount of work done against gravity. But the first member expresses the work which would be done in raising all the particles through a uniform height $\bar{H} - \bar{h}$, which is the height of the new position of the centre of gravity above the old. The work done against gravity in raising any system of bodies will therefore be correctly computed by supposing all the system to be collected at its centre of gravity. For example, the work done in raising bricks and mortar from the ground to build a chimney, is equal to the total weight of the chimney multiplied by the height of its centre of gravity above the ground.

46. The Centre of Gravity tends to Descend.—When the forces which tend to move a system are simply the weights of its parts, we can determine whether it is in equilibrium by observing the path in which its centre of gravity would travel if movement took place. If we suppose this path to represent a hard frictionless surface, and the centre of gravity to represent a heavy particle placed upon it, the conditions of equilibrium will be the same as in the actual case. The centre of gravity tends to run down hill, just as a heavy particle does. There will be stable equilibrium if the centre of gravity is at the bottom of a valley in its path, and unstable equilibrium if it is at the top of a hill. When a rigid body turns about a horizontal axis, the path of its centre of gravity is a circle in a vertical plane. The highest and lowest points of this circle are the positions of the centre of gravity in unstable and stable equilibrium respectively:

except when the axis traverses the centre of gravity itself, in which case the centre of gravity can neither rise nor fall, and the equilibrium is neutral.

A uniform sphere or cylinder lying on a horizontal plane is in neutral equilibrium, because its centre of gravity will neither be raised nor lowered by rolling. An egg balanced on its end as in Fig. 16, is in unstable equilibrium, because its centre of gravity is at the top of a hill which it will descend when the egg rolls to one side. The position of equilibrium shown in Fig. 17 is stable as regards rolling to left or right, because the path of its centre of gravity in

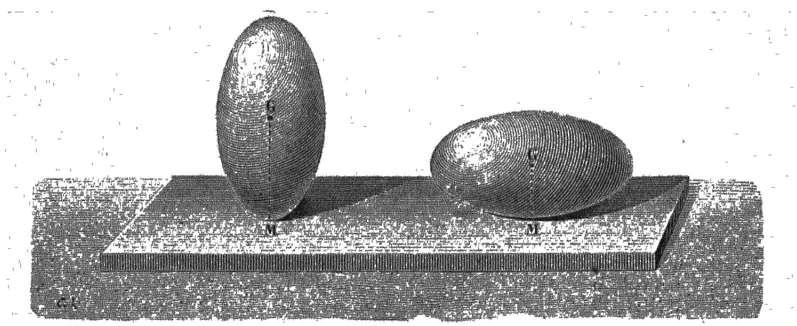


Fig. 16.—Unstable Equilibrium. Fig. 17.—Stable Equilibrium.

such rolling would be a curve whose lowest point is that now occupied by the centre of gravity. As regards rolling in the direction at right angles to this, if the egg is a true solid of revolution, the equilibrium is neutral.

47. **Work done by Gravity.**—When a heavy body is lifted, the lifting force does work against gravity. When it descends gravity does work upon it; and if it descends to the same position from which it was lifted, the work done by gravity in the descent is equal to the work done against gravity in the lifting; each being equal to the weight of the body multiplied by the vertical displacement of its centre of gravity. The tendency of the centre of gravity to descend is a manifestation of the tendency of gravity to do work; and this tendency is not peculiar to gravity.

48. **Work done by any Force.**—A force is said to do work when its point of application moves in the direction of the force, or in any direction making an acute angle with this, so as to give a component displacement in the direction of the force; and the amount of work done is the product of the force by this component. If F denote

the force, a the displacement, and θ the angle between the two, the work done by F is

$$F\,a\cos\theta,$$

which is what we obtain either by the above rule or by multiplying the whole displacement by the effective component of F, that is the component of F in the direction of the displacement. If the angle θ is obtuse, $\cos\theta$ is negative and the force F does negative work. If θ is a right angle F does no work. In this case F neither assists nor resists the displacement. When θ is acute, F assists the displacement, and would produce it if the body were constrained by guides which left it free to take this displacement and the directly opposite one, while preventing all others.

If θ is obtuse, F resists the displacement, and would produce the opposite displacement if the body were constrained in the manner just supposed.

49. **Principle of Work.**—If any number of forces act upon a body which is only free to move in a particular direction and its opposite, we can tell in which of these two directions it will move by calculating the work which each force would do. Each force would do positive work when the displacement is in one direction, and negative work when it is in the opposite direction, the absolute amounts of work being the same in both cases if the displacements are equal. The body will upon the whole be urged in that direction which gives an excess of positive work over negative. If no such excess exists, but the amounts of positive and negative work are exactly equal, the body is in equilibrium. This principle (which has been called the principle of *virtual velocities*, but is better called the *principle of work*) is often of great use in enabling us to calculate the ratio which two forces applied in given ways to the same body must have in order to equilibrate each other. It applies not only to the "mechanical powers" and all combinations of solid machinery, but also to hydrostatic arrangements; for example to the hydraulic press. The condition of equilibrium between two forces applied to any frictionless machine and tending to drive it opposite ways, is that in a small movement of the machine they would do equal and opposite amounts of work. Thus in the screw-press (Fig. 30) the force applied to one of the handles, multiplied by the distance through which this handle moves, will be equal to the pressure which this force produces at the foot of the screw, multiplied by the distance that the screw travels.

This is on the supposition of no friction. A frictionless machine gives out the same amount of work which is spent in driving it. The effect of friction is to make the work given out less than the work put in. Much fruitless ingenuity has been expended upon contrivances for circumventing this law of nature and producing a machine which shall give out more work than is put into it. Such contrivances are called "perpetual motions."

50. General Criterion of Stability.—If the forces which act upon a body and produce equilibrium remain unchanged in magnitude and direction when the body moves away from its position, and if the velocities of their points of application also remain unchanged in direction and in their ratio to each other, it is obvious that the equality of positive and negative work which subsists at the beginning of the motion will continue to subsist throughout the entire motion. The body will therefore remain in equilibrium when displaced. Its equilibrium is in this case said to be neutral.

If the forces which are in equilibrium in a given position of the body, gradually change in direction or magnitude as the body moves away from this position, the equality of positive and negative work will not in general continue to subsist, and the inequality will increase with the displacement. If the body be displaced with a constant velocity and in a uniform manner, the rate of doing work, which is zero at first, will not continue to be zero, but will have a value, whether positive or negative, increasing in simple proportion to the displacement. Hence it can be shown that the whole work done is in a small movement proportional to the square of the displacement, for when we double the displacement we, at the same time, double the mean working force.

If this work is positive, the forces assist the displacement and tend to increase it; the equilibrium must therefore have been unstable.

On the other hand, if the work is negative in all possible displacements from the position of equilibrium, the forces oppose the displacements and the equilibrium is stable.

51. Illustration of Stability.—A good example of stable equilibrium of this kind is furnished by Gravesande's apparatus (Fig. 3) simplified by removing the parallelogram and employing a string to support the three weights, one of them P″ being fastened to it at a point A near its middle, and the others P, P′ to its ends. The point A will take the same position as in the figure, and will return to it again when displaced. If we take hold of the point A and

move it in any direction whether in the plane of the string or out of it, we feel that at first there is hardly any resistance and the smallest force we can apply produces a sensible disturbance; but that as the displacement increases the resistance becomes greater. If we release the point A when displaced, it will execute oscillations, which will become gradually smaller, owing to friction, and it will finally come to rest in its original position of equilibrium.

The centre of gravity of the three weights is in its lowest position when the system is in equilibrium, and when a small displacement is produced the centre of gravity rises by an amount proportional to its square, so that a double displacement produces a quadruple rise of the centre of gravity.

In this illustration the three forces remain unchanged, and the directions of two of them change gradually as the point A is moved. Whenever the circumstances of stable equilibrium are such that the forces make no abrupt changes either in direction or magnitude for small displacements, the resistance will, as in this case, be proportional to the displacement (when small), and the work to the square of the displacement, and the system will oscillate if displaced and then left to itself.

52. **Stability where Forces vary abruptly with Position.**—There are other cases of stable equilibrium which may be illustrated by the example of a book lying on a table. If we displace it by lifting one edge, the force which we must exert does not increase with the displacement, but is sensibly constant when the displacement is small, and as a consequence the work will be simply proportional to the displacement. The reason is, that one of the forces concerned in producing equilibrium, namely, the upward pressure of the table, changes *per saltum* at the moment when the displacement begins. In applying the principle of work to such a case as this, we must employ, instead of the actual work done by the force which changes abruptly, the work which it would do if its magnitude and direction remained unchanged, or only changed gradually.

53. **Illustrations from Toys.**—The stability of the "balancer" (Fig. 18) depends on the fact that, owing to the weight of the two leaden balls, which are rigidly attached to the figure by stiff wires, the centre of gravity of the whole is below the point of support. If the figure be disturbed it oscillates, and finally comes to rest in a position in which the centre of gravity is vertically under the toe on which the figure stands.

The "tumbler" (Fig. 19) consists of a light figure attached to a hemisphere of lead, the centre of gravity of the whole being between the centre of gravity of the hemisphere and the centre of the sphere to which it belongs. When placed upon a level table, the lowest position of the centre of gravity is that in which the figure is upright, and it accordingly returns to this position when displaced.

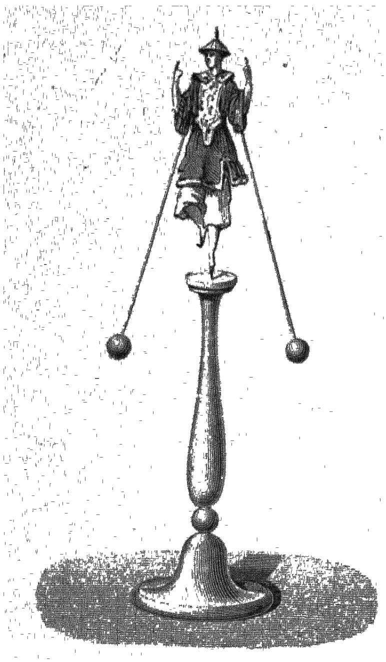


Fig 18 —Balancer.

54. Limits of Stability.—In the foregoing discussion we have employed the term "stability" in its strict mathematical sense. But there are cases in [illegible] though small displace[illegible]ely produce [illegible]ver displ[illegible]he body, [illegible] fall entirely [illegible] given position [illegible]is may be express[illegible]hat the equilibrium is stable for displacements lying [illegible]ain limits, but unstable for displacements beyond these lim[illegible] the equilibrium

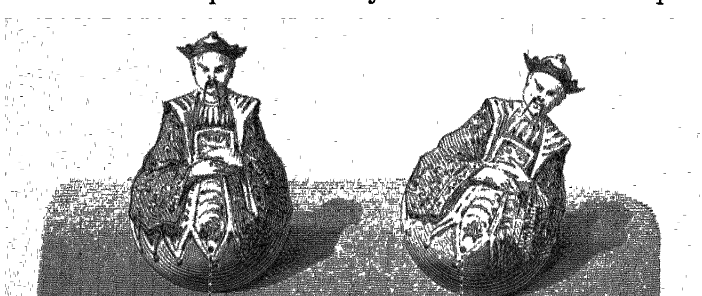


Fig 19 —Tumblers

of a system is *practically* unstable when the displacements which it is likely to receive from accidental disturbances lie beyond its limits of stability.

CHAPTER IV.

THE MECHANICAL POWERS.

55. We now proceed to a few practical applications of the foregoing principles; and we shall begin with the so-called "mechanical powers," namely, the *lever*, the *wheel and axle*, the *pulley*, the *inclined plane*, the *wedge*, and the *screw*.

56. Lever.—Problems relating to the lever are usually most conveniently solved by taking moments round the fulcrum. The general condition of equilibrium is, that the moments of the power and the weight about the fulcrum must be in opposite directions, and must be equal. When the power and weight act in parallel directions, the conditions of equilibrium are precisely those of three parallel forces (§ 19), the third force being the reaction of the fulcrum.

It is usual to distinguish three "orders" of lever. In levers of the first order (Fig. 20) the fulcrum is between the power and the

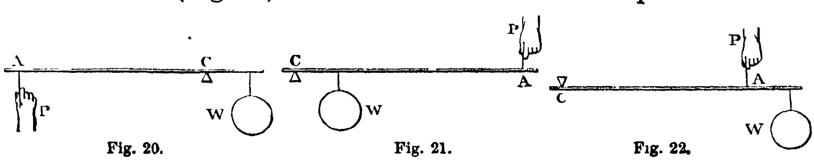


Fig. 20. Fig. 21. Fig. 22.

Three Orders of Lever.

weight. In those of the second order (Fig. 21) the weight is between the power and the fulcrum. In those of the third order (Fig. 22) the power is between the weight and the fulcrum.

In levers of the second order (supposing the forces parallel), the weight is equal to the sum of the power and the pressure on the fulcrum; and in levers of the third order, the power is equal to the sum of the weight and the pressure on the fulcrum; since the middle one of three parallel forces in equilibrium must always be equal to the sum of the other two.

57. Arms.—The *arms of a lever* are the two portions of it intermediate, respectively, between the fulcrum and the power, and between the fulcrum and the weight. If the lever is bent, or if, though straight, it is not at right angles to the lines of action of the power and weight, it is necessary to distinguish between the arms of the lever as above defined (which are parts of the lever), and the *arms of the power and weight* regarded as forces which have moments round the fulcrum. In this latter sense (which is always to be understood unless the contrary is evidently intended), the arms are the perpendiculars dropped from the fulcrum upon the lines of action of the power and weight.

58. Weight of Lever.—In the above statements of the conditions of equilibrium, we have neglected the weight of the lever itself. To take this into account, we have only to suppose the whole weight of the lever collected at its centre of gravity, and then take its moment round the fulcrum. We shall thus have three moments to take account of, and the sum of the two that tend to turn the lever one way, must be equal to the one that tends to turn it the opposite way.

59. Mechanical Advantage.—Every machine when in action serves to transmit *work* without altering its amount; but the *force* which the machine gives out (equal and opposite to what is commonly called the *weight*) may be much greater or much less than that by which it is driven (commonly called the *power*). When it is greater, the machine is said to confer *mechanical advantage*, and the mechanical advantage is measured by the ratio of the weight to the power for equilibrium. In the lever, when the power has a longer arm than the weight, the mechanical advantage is equal to the quotient of the longer arm by the shorter.

60. Wheel and Axle.—The wheel and axle (Fig. 23) may be regarded as an endless lever. The condition of equilibrium is at once given by taking moments round the common axis of the wheel and axle (§ 24). If we neglect the thickness of the ropes, the condition is that the power multiplied by the radius of the wheel must equal the weight multiplied by the radius of the axle; but it is more exact to regard the lines of action of the two forces as coinciding with the axes of the two ropes, so that each of the two radii should be increased by half the thickness of its own rope. If we neglect the thickness of the ropes, the

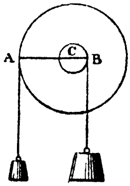


Fig. 23.

mechanical advantage is the quotient of the radius of the wheel by the radius of the axle.

61. **Pulley.**—A pulley, when fixed in such a way that it can only turn about a fixed axis (Fig. 24), confers no mechanical advantage. It may be regarded as an endless lever of the first order with its two arms equal.

The arrangement represented in Fig. 25 gives a mechanical advantage of 2; for the lower or movable pulley may be regarded as an endless lever of the second order, in which the arm of the power is the diameter of the pulley, and the arm of the weight is

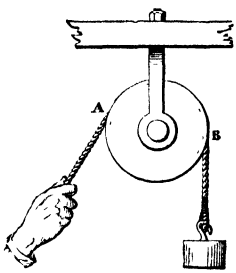


Fig 24.

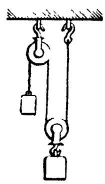


Fig 25

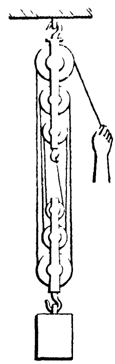


Fig 26.

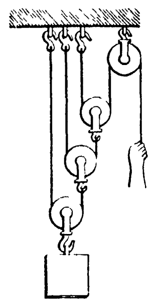


Fig 27.

half the diameter. The same result is obtained by employing the principle of work; for if the weight rises 1 inch, 2 inches of slack are given over, and therefore the power descends 2 inches.

62. In Fig. 26 there are six pulleys, three at the upper and three at the lower block, and one cord passes round them all. All portions of this cord (neglecting friction) are stretched with the same force, which is equal to the power; and six of these portions, parallel to one another, support the weight. The power is therefore one-sixth of the weight, or the mechanical advantage is 6.

63. In the arrangement represented in Fig. 27, there are three movable pulleys, each hanging by a separate cord. The cord which supports the lowest pulley is stretched with a force equal to half the weight, since its two parallel portions jointly support the weight. The cord which supports the next pulley is stretched with a force half of this, or a quarter of the weight; and the next cord with a force half of this, or an eighth of the weight; but this cord is directly attached to the power. Thus the power is an eighth of the

weight, or the mechanical advantage is 8. If the weight and the block[1] to which it is attached rise 1 inch, the next block rises 2 inches, the next 4, and the power moves through 8 inches. Thus, the work done by the power is equal to the work done upon the weight.

In all this reasoning we neglect the weights of the blocks themselves; but it is not difficult to take them into account when necessary.

64. Inclined Plane.—We now come to the inclined plane. Let AB (Fig. 28) be any portion of such a plane, and let AC and BC be drawn vertically and horizontally. Then AB is called the *length*, AC the *height*, and CB the *base* of the inclined plane. The force of gravity upon a heavy body M resting on the plane, may be represented by a vertical line MP, and may be resolved by the parallelogram of forces (§ 16) into two components, MT, MN, the former parallel and the latter perpendicular to the plane. A force equal and opposite to the component MT will suffice to prevent the body from slipping down the plane. Hence, if the power act parallel to the plane, and the weight be that of a heavy body resting on the plane, the power is to the weight as MT to MP; but the two triangles MTP and ACB are similar, since the angles at M and A are equal, and the angles at T and C are right angles; hence MT is to MP as AC to AB, that is, as the height to the length of the plane.

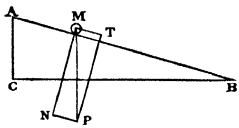


Fig. 28.

65. The investigation is rather easier by the principle of work (§ 49). The work done by the power in drawing the heavy body up the plane, is equal to the power multiplied by the length of the plane. But the work done upon the weight is equal to the weight multiplied by the height through which it is raised, that is, by the height of the plane. Hence we have

$$\text{Power} \times \text{length of plane} = \text{weight} \times \text{height of plane; or}$$
$$\text{power} : \text{weight} :: \text{height of plane} : \text{length of plane.}$$

66. If, instead of acting parallel to the plane, the power acted parallel to the base, the work done by the power would be the product of the power by the base; and this must be equal to the product of the weight by the height; so that in this case the condition of equilibrium would be—

[1] The "pulley" is the revolving wheel. The pulley, together with the frame in which it is inclosed, constitute the "block."

Power : weight : : height of plane : base of plane.

67. **Wedge.**—In these investigations we have neglected friction. The wedge may be regarded as a case of the inclined plane; but its practical action depends to such a large extent upon friction and impact[1] that we cannot profitably discuss it here.

68. **Screw.**—The screw (Fig. 29) is also a case of the inclined plane. The length of one convolution of the thread is the length of the corresponding inclined plane, the step of the screw, or distance between two successive convolutions (measured parallel to the axis of the screw), is the height of the plane, and the circumference of

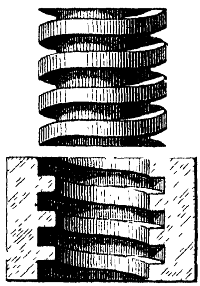


Fig. 29.

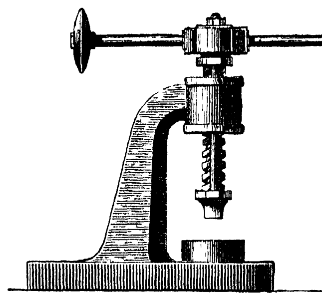


Fig. 30.

the screw is the base of the plane. This is easily shown by cutting out a right-angled triangle in paper, and bending it in cylindrical fashion so that its base forms a circle.

69. **Screw Press.**—In the screw press (Fig. 30) the screw is turned by means of a lever, which gives a great increase of mechanical advantage. In one complete revolution, the pressures applied to the two handles of the lever to turn it, do work equal to their sum multiplied by the circumference of the circle described (approximately) by either handle (we suppose the two handles to be equidistant from the axis of revolution); and the work given out by the machine, supposing the resistance at its lower end to be constant, is equal to this resistance multiplied by the distance between the threads. These two products must be equal, friction being neglected.

[1] An *impact* (for example a blow of a hammer) may be regarded as a very great (and variable) force acting for a very short time. The magnitude of an impact is measured by the momentum which it generates in the body struck.

CHAPTER V.

THE BALANCE.

70. General Description of the Balance.—In the common *balance* (Fig. 31) there is a stiff piece of metal, A B, called the *beam*, which turns about the sharp edge O of a steel wedge forming part of the beam and resting upon two hard and smooth supports. There are two other steel wedges at A and B, with their edges upwards, and upon these edges rest the hooks for supporting the scale pans. The three edges (called knife-edges) are parallel to one another and perpendicular to the length of the beam, and are very nearly in one plane.

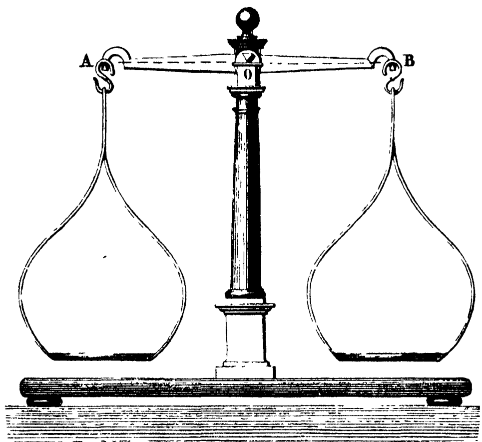


Fig 31.—Balance.

71. Qualities Requisite.—The qualities requisite in a balance are:

1. That it be consistent with itself; that is, that it shall give the same result in successive weighings of the same body. This depends chiefly on the trueness of the knife-edges.

2. That it be just. This requires that the distances A O, O B, be equal, and also that the beam remain horizontal when the pans are empty. Any inequality in the distances A O, O B, can be detected by putting equal (and tolerably heavy) weights into the two pans. This adds equal moments if the distances are equal, but unequal

moments if they are unequal, and the greater moment will preponderate.

3. Delicacy or sensibility (that is, the power of indicating inequality between two weights even when their difference is very small).

This requires a minimum of friction, and a very near approach to neutral equilibrium (§ 40). In absolutely neutral equilibrium, the smallest conceivable force is sufficient to produce a displacement to the full limit of neutrality; and in barely stable equilibrium a small force produces a large displacement. The condition of stability is that if the weights supported at A and B be supposed collected at these edges, the centre of gravity of the system composed of the beam and these two weights shall be below the middle edge O. The equilibrium would be neutral if this centre of gravity exactly coincided with O, and it is necessary as a condition of delicacy that its distance below O be very small.

4. Facility for weighing quickly is desirable, but must sometimes be sacrificed when extreme accuracy is required.

The delicate balances used in chemical analysis are provided with a long pointer attached to the beam. The end of this pointer moves along a graduated arc as the beam vibrates; and if the weights in the two pans are equal, the excursions of the pointer on opposite sides of the zero point of this arc will also be equal. Much time is consumed in watching these vibrations, as they are very slow; and the more nearly the equilibrium approaches to neutrality, the slower they are. Hence quick weighing and exact weighing are to a certain extent incompatible.

72. Double Weighing.—Even if a balance be not just, yet if it be consistent with itself, a correct weighing can be made with it in the following manner:—Put the body to be weighed in one pan, and counterbalance it with sand or other suitable material in the other. Then remove the body and put in its place such weights as are just sufficient to counterpoise the sand. These weights are evidently equal to the weight of the body. This process is called *double weighing,* and is often employed (even with the best balances) when the greatest possible accuracy is desired.

73. Investigation of Sensibility.—Let A and B (Fig. 32) be the points from which the scale-pans are suspended, O the axis about which the beam turns, and G the centre of gravity of the beam. If when the scale-pans are loaded with equal weights, we put into one

of them an excess of weight p, the beam will become inclined, and will take a position such as A′B′, turning through an angle which we will call a, and which is easily calculated.

In fact let the two forces P and P + p act at A′ and B′ respectively, where P denotes the less of the two weights, including the weight of the pan. Then the two forces P destroy each other in consequence of the resistance of the axis O; there is left only the force p applied at B′, and the weight π of the beam applied at G′, the new position of the centre of gravity. These two forces are parallel, and are in equilibrium about the axis O, that is, their resultant passes through the point O. The distances of the points of application of the forces from a vertical through O are therefore inversely proportional to the forces themselves, which gives the relation

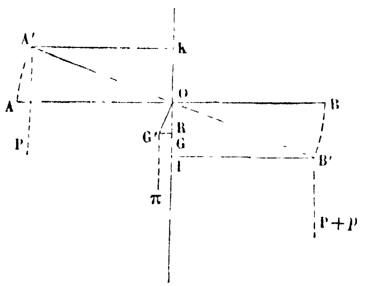


Fig. 32.

$$\pi.\ G'R = p.\ B'L.$$

But if we call half the length of the beam l, and the distance OG r we have

$$G'R = r \sin a, \quad B'L = l \cos a.$$

whence $\pi r \sin a = pl \cos a$, and consequently

$$\tan a = \frac{pl}{\pi r}. \tag{a}$$

This formula shows the conditions of sensibility when the points A, O, B are in one straight line. We see, in the first place, that tan a increases with the excess of weight p, which was evident beforehand. We see also that the sensibility increases as l increases and as π diminishes, or, in other words, as the beam becomes longer and lighter. At the same time it is obviously desirable that, under the action of the weights employed, the beam should be stiff enough to undergo no sensible change of shape. The problem of the balance then consists in constructing a beam of the greatest possible length and lightness, which shall be capable of supporting the action of given forces without bending.

Fortin, whose balances are justly esteemed, employed for his beams bars of steel placed edgewise; he thus obtained great rigidity, but

certainly not all the lightness possible. At present the makers of balances employ in preference beams of copper or steel made in the form of a frame, as shown in Fig 33. They generally give them the shape of a very elongated lozenge, the sides of which are connected by bars variously arranged. The determination of the best shape is, in fact, a special problem, and is an application on a small scale of that principle of applied mechanics which teaches us that hollow pieces have greater resisting power in proportion to their weight than solid pieces, and consequently, for equal resisting power, the former are lighter than the latter. Aluminium, which with a rigidity nearly equal to that of copper, has less than one-fourth of its density, seems naturally marked out as adapted to the construction of beams. It has as yet, however, been little used.

The formula (a) shows us, in the second place, that the sensibility increases as r diminishes; that is, as the centre of gravity approaches the centre of suspension. These two points, however, must not coincide, for in that case for any excess of weight, however small, the beam would deviate from the horizontal as far as the mechanism would permit, and would afford no indication of approach to equality in the weights. With equal weights it would remain in equilibrium in any position. In virtue of possessing this last property, such a balance is called *indifferent*. Practically the distance between the centre of gravity and the point of suspension must not be less than a certain amount depending on the use for which the balance is designed. The proper distance is determined by observing what difference of weights corresponds to a division of the graduated arc along which the needle moves. If, for example, there are 20 divisions on each side of zero, and if 2 milligrammes are necessary for the total displacement of the needle, each division will correspond to an excess of weight of $\frac{2}{20}$ or $\frac{1}{10}$ of a milligramme. That this may be the case we must evidently have a suitable value of r, and the maker is enabled to regulate this value with precision by means of the screw which is shown in the figure above the beam, and which enables him slightly to vary the position of the centre of gravity.

74. Weighing with Constant Load.—In the above analysis we have supposed that the three points of suspension of the beam and of the two scale-pans are in one straight line; in which case the value of tan α does not include P, that is, the sensibility is independent of the weight in the pans. This follows from the fact that the resultant of the two forces P passes through O, and is thus destroyed, because

the axis is fixed. This would not be the case if, for example, the points of suspension of the pans were above that of the beam; in this case the point of application of the common load is above the point O, and, when the beam is inclined, acts in the same direction as the excess of weight; whence the sensibility increases with the load up to a certain limit, beyond which the equilibrium becomes unstable.[1] On the other hand, when the points of suspension of the pans are below that of the beam, the sensibility increases as the load diminishes, and, as the centre of gravity of the beam may in this case be above the axis, equilibrium may become unstable when the load is less than a certain amount. This variation of the sensibility with the load is a serious disadvantage; for, as we have just shown, the displacement of the needle is used as the means of estimating weights, and for this purpose we must have the same displacement corresponding to the same excess of weight. If we wish to employ

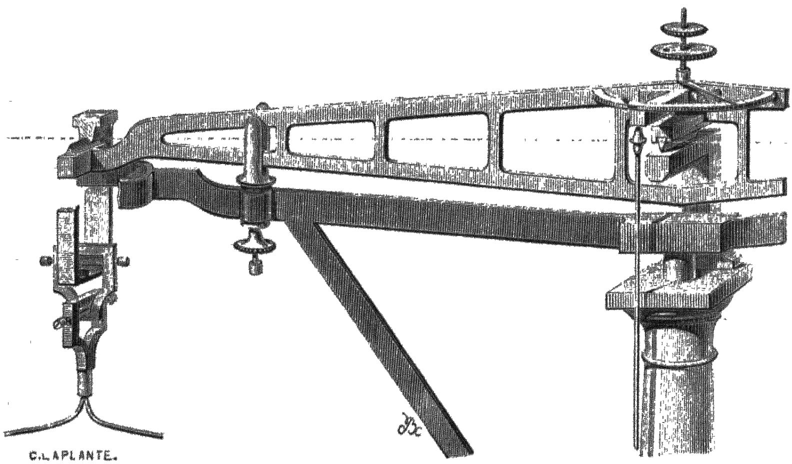


Fig. 33.—Beam of Balance.

either of the two above arrangements, we should weigh with a constant load. The method of doing so, which constitutes a kind of double weighing, consists in retaining in one of the pans a weight equal to this constant load. In the other pan is placed the same load subdivided into a number of marked weights. When the body

[1] This is an illustration of the general principle, applicable to a great variety of philosophical apparatus, that a maximum of sensibility involves a minimum of stability; that is, a very near approach to instability. This near approach is usually indicated by excessive slowness in the oscillations which take place about the position of equilibrium.

to be weighed is placed in this latter pan, we must, in order to maintain equilibrium, remove a certain number of weights, which evidently represent the weight of the body.

We may also remark that, strictly speaking, the sensibility always depends upon the load, which necessarily produces a variation in the friction of the axis of suspension. Besides, it follows from the nature

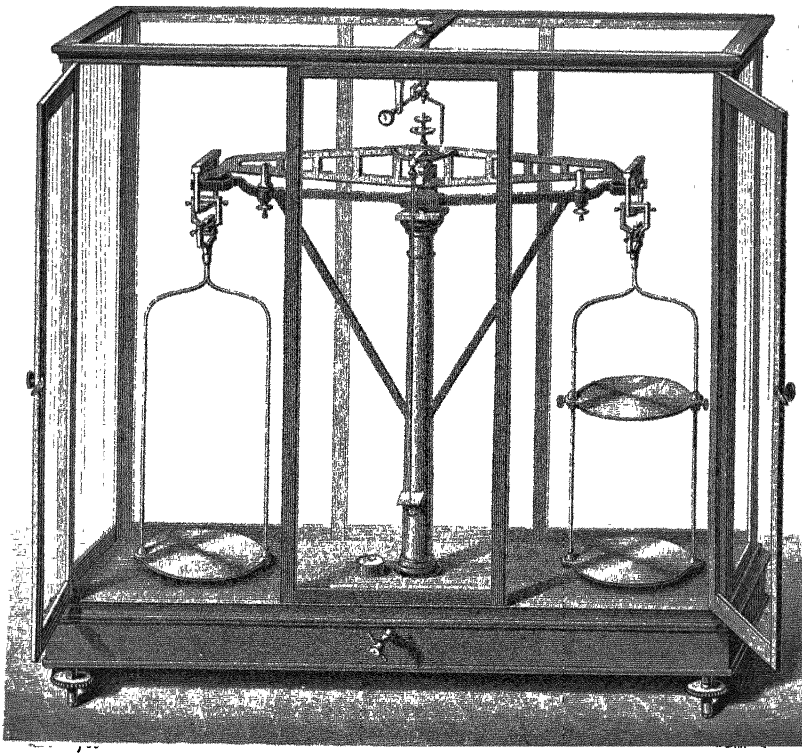


Fig. [illegible].—Balance for Purposes of Accuracy.

of bodies that there is no system that does not yield somewhat even to the most feeble action. For these reasons, there is a decided advantage in operating with constant load.

75. Details of Construction.—A fundamental condition of the correctness of the balance is, that the weight of each pan and of the load which it contains should always act exactly at the same point, and therefore at the same distance from the axis of suspension. This important result is attained by different methods. The arrangement represented in Fig. 33 is one of the most effectual. At the

extremities of the beam are two knife-edges, parallel to the axis of rotation, and facing upwards. On these knife-edges rests, by a hard plane surface of agate or steel, a stirrup, the front of which has been taken away in the figure. On the lower part of the stirrup rests another knife-edge, at right angles to the former, the two being together equivalent to a universal joint supporting the scale-pan and its contents. By this arrangement, whatever may be the position of the weights, their action is always reduced to a vertical force acting on the upper knife-edge.

Fig. 34 represents a balance of great delicacy, with the glass case that contains it. At the bottom is seen the extremity of a lever, which enables us to raise the beam, and thus avoid wearing the knife-edge when not in use. At the top may be remarked an arrangement employed by some makers, consisting of a horizontal graduated circle, on which a small metallic index can be made to travel; its different displacements, whose value can be determined once for all, are used for the final adjustment to produce exact equilibrium.

76. Steelyard.—The steelyard (Fig. 35) is an instrument for weighing bodies by means of a single weight, P, which can be hung at any point of a graduated arm O B. As P is moved further from the fulcrum O, its moment round O increases, and therefore the weight which must be hung from the fixed point A to counterbalance it increases. Moreover, equal movements of P along the arm produce equal additions to its moment, and equal additions to the weight at A produce equal additions to the opposing moment. Hence the divisions on the arm (which indicate the weight in the pan at A) must be equidistant.

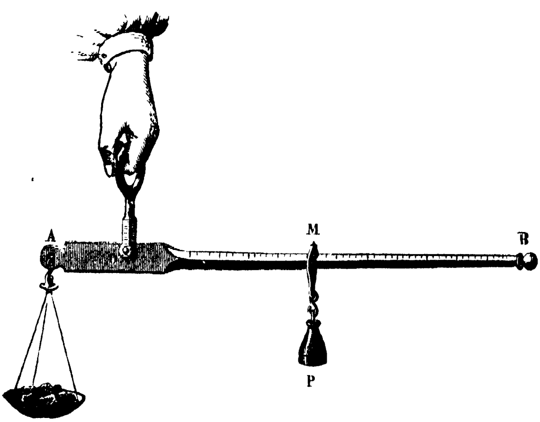


Fig. 35

CHAPTER VI.

FIRST PRINCIPLES OF KINETICS.

77. Principle of Inertia.—A body not acted on by any forces, or only acted on by forces which are in equilibrium, will not commence to move; and if it be already in motion with a movement of pure translation, it will continue its velocity of translation unchanged, so that each of its points will move in a straight line with uniform velocity. This is Newton's first law of motion, and is stated by him in the following terms:—

"Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it is compelled by impressed forces to change that state."

The tendency to continue in a state of rest is manifest to the most superficial observation. The tendency to continue in a state of uniform motion can be clearly understood from an attentive study of facts. If, for example, we make a pendulum oscillate, the amplitude of the oscillations slowly decreases and at last vanishes altogether. This is because the pendulum experiences resistance from the air which it continually displaces; and because the axis of suspension rubs on its supports. These two circumstances combine to produce a diminution in the velocity of the apparatus until it is completely annihilated. If the friction at the point of suspension is diminished by suitable means, and the apparatus is made to oscillate *in vacuo*, the duration of the motion will be immensely increased.

Analogy evidently indicates that if it were possible to suppress entirely these two causes of the destruction of the pendulum's velocity, its motion would continue for an indefinite time unchanged.

This tendency to continue in motion is the cause of the effects which are produced when a carriage or railway train is suddenly stopped. The passengers are thrown in the direction of the motion,

in virtue of the velocity which they possessed at the moment when the stoppage occurred. If it were possible to find a brake sufficiently powerful to stop a train suddenly at full speed, the effects of such a stoppage would be similar to the effects of a collision.

Inertia is also the cause of the severe falls which are often received in alighting incautiously from a carriage in motion; all the particles of the body have a forward motion, and the feet alone being reduced to rest, the upper portion of the body continues to move, and is thus thrown forward.

When we fix the head of a hammer on the handle by striking the end of the handle on the ground, we utilize the inertia of matter. The handle is suddenly stopped by the collision, and the head continues to move for a short distance in spite of the powerful resistances which oppose it.

78. Second Law of Motion.—Newton's second law of motion is that "Change of motion is proportional to the impressed force and is in the direction of that force."

Change of motion is here spoken of as a quantity, and as a directed quantity. In order to understand how to estimate change of motion, we must in the first place understand how to compound motions.

When a boat is sailing on a river, the motion of the boat relative to the shore is compounded of its motion relative to the water and the motion of the water relative to the shore. If a person is walking along the deck of the boat in any direction, his motion relative to the shore is compounded of three motions, namely the two above mentioned and his motion relative to the boat.

Let X, Y and Z be any three bodies or systems. The motion of X relative to Y, compounded with the motion of Y relative to Z, is the motion of X relative to Z. This is to be taken as the definition of what is meant by compounding two motions; and it leads very directly to the result that

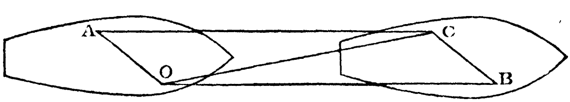


Fig. 36.—Composition of Motions

two rectilinear motions are compounded by the parallelogram law. For if a body moves along the deck of a ship from O to A (Fig. 36), and the ship in the meantime advances through the distance OB, it is obvious that, if we complete the parallelogram OBCA, the point A of the ship will be brought to C, and the movement of the body in space will be from O to C. If the motion along OA is uniform

and the motion of the ship is also uniform, the motion of the body through space will be a uniform motion along the diagonal OC. Hence, *if two component velocities be represented by two lines drawn from a point, and a parallelogram be constructed on these lines, its diagonal will represent the resultant velocity.*

It is obvious that if OA in the figure represented the velocity of the ship and OB the velocity of the body relative to the ship, we should obtain the same resultant velocity OC. This is a general law; the interchanging of velocities which are to be compounded does not affect their resultant.

Now suppose the velocity OB to be changed into the velocity OC, what are we to regard as the change of velocity? The change of velocity is that velocity which compounded with OB would give OC It is therefore OA. The same force which, in a given time, acting always parallel to itself, changes the velocity of a body from OB to OC, would give the body the velocity OA if applied to it for the same time commencing from rest. Change of motion, estimated in this way, depends only on the acting force and the body acted on by the force; it is entirely independent of any previous motion which the body may possess. No experiments on forces and motions inside a carriage or steamboat which is travelling with perfect smoothness in a straight course, will enable us to detect that it is travelling at all. We cannot even assert that there is any such thing as absolute rest, or that there is any difference between absolute rest and uniform straight movement of translation.

As change of motion is independent of the initial condition of rest or motion, so also is the change of motion produced by one force acting on a body independent of the change produced by any other force acting on the body, provided that each force remains constant in magnitude and direction. The actual motion will be that which is compounded of the initial motion and the motions due to the two forces considered separately. If AB represent one of these motions, BC another, and CD the third, the actual or resultant motion will be AD.

The change produced in the motion of a body by two forces acting jointly can therefore be found by compounding the changes which would be produced by each force separately. This leads at once to the "parallelogram of forces," since the changes of motion produced in one and the same body are proportional to the forces which produce them, and are in the directions of these forces.

In case any student should be troubled by doubt as to whether the "changes of motion" which are proportional to the forces, are to be understood as distances, or as velocities, we may remark that the law is equally true for both, and its truth for one implies its truth for the other, as will appear hereafter from comparing the formula for the distance $s = \frac{1}{2}ft^2$, with the formula for the velocity $v = ft$, since both of these expressions are proportional to f.

79. Explanation of Second Law continued.—It is convenient to distinguish between the *intensity* of a force and the *magnitude* or *amount* of a force. The intensity of a force is measured by the change of velocity which the force produces during the unit of time; and can be computed from knowing the motion of the body acted on, without knowing anything as to its mass. Two bodies are said to be of equal *mass* when the same change of motion (whether as regards velocity or distance) which is produced by applying a given force to one of them for a given time, would also be produced by applying this force to the other for an equal time. If we join two such bodies, we obtain a body of double the mass of either; and if we apply the same force as before for the same time to this double mass, we shall obtain only half the change of velocity or distance that we obtained before. Masses can therefore be compared by taking the inverse ratio of the changes produced in their velocities by equal forces.

The velocity of a body multiplied by its mass is called the *momentum* of the body, and is to be regarded as a directed magnitude having the same direction as the velocity. The change of velocity, when multiplied by the mass of the body, gives the change of momentum; and the second law of motion may be thus stated:—

The change of momentum produced in a given time is proportional to the force which produces it, and is in the direction of this force. It is independent of the mass; the change of velocity in a given time being inversely as the mass.

80. Proper Selection of Unit of Force.—If we make a proper selection of units, the change of momentum produced *in unit time* will be not only proportional but numerically *equal* to the force which produces it; and the change of momentum produced in any time will be the product of the force by the time. Suppose any units of length, time, and mass respectively to have been selected. Then the unit velocity will naturally be defined as the velocity with which unit length would be passed over in unit time; the unit momentum will be the momentum of the unit mass moving with this velocity;

and the unit force will be that force which produces this momentum in unit time. We define the unit force, then, as *that force which acting for unit time upon unit mass produces unit velocity.*

81. Relation between Mass and Weight.—The *weight* of a body, strictly speaking, is the force with which the body tends towards the earth. This force depends partly on the body and partly on the earth. It is not exactly the same for one and the same body at all parts of the earth's surface, but is decidedly greater in the polar than in the equatorial regions. Bodies which, when weighed in a balance *in vacuo,* counterbalance each other, or counterbalance one and the same third body, have equal *weights* at that place, and will also be found to have equal weights at any other place. Experiments which we shall hereafter describe (§ 89) show that such bodies have equal masses; and this fact having been established, the most convenient mode of comparing masses is by weighing them. A pound of iron has the same mass as a pound of brass or of any other substance. A pound of any kind of matter tends to the earth with different forces at different places. The weight of a pound of matter is therefore not a definite standard of force. But the pound of matter itself is a perfectly definite standard of mass. If we weigh one and the same portion of matter in different states; for instance water in the states of ice, snow, liquid water, or steam; or compare the weight of a chemical compound with the weights of its components; we find an exact equality; hence it has been stated that the mass of a body is a measure of the quantity of matter which it contains; but though this statement expresses a simple fact when applied to the comparison of different quantities of one and the same substance, it expresses no known fact of nature when applied to the comparison of different substances. A pound of iron and a pound of lead tend to the earth with equal forces; and if equal forces are applied to them both their velocities are equally affected. We may if we please agree to measure "quantity of matter" by these tests; but we must beware of assuming that two things which are essentially different in kind can be equal in themselves.

82. Third Law of Motion. Action and Reaction.—Forces always occur in pairs, every exertion of force being a mutual action between two bodies. Whenever a body is acted on by a force, the body from which this force proceeds is acted on by an equal and opposite force. The earth attracts the moon, and the moon attracts the earth. A magnet attracts iron and is attracted by iron. When two

boats are floating freely, a rope attached to one and hauled in by a person in the other, makes each boat move towards the other. Every exertion of force generates equal and opposite momenta in the two bodies affected by it, since these two bodies are acted on by equal forces for equal times.

If the forces exerted by one body upon the other are equivalent to a single force, the forces of reaction will also be equivalent to a single force, and these two equal and opposite resultants will have the same line of action. We have seen in § 29 that the general resultant of any set of forces applied to a body is a *wrench;* that is to say it consists of a force with a definite line of action (called the *axis*), accompanied by a couple in a perpendicular plane. The reaction upon the body which exerts these forces will always be an equal and opposite wrench; the two wrenches having the same axis, equal and opposite forces along this axis, and equal and opposite couples in the perpendicular plane.

83. **Motion of Centre of Gravity Unaffected.**—A consequence of the equality of the mutual forces between two bodies is, that these forces produce no movement of the common centre of gravity of the two bodies. For if A be the centre of gravity of a mass m_1, and B the centre of gravity of a mass m_2, their common centre of gravity C will divide AB inversely as the masses. Let the masses be originally at rest, and let them be acted on only by their mutual attraction or replusion. The distances through which they are moved by these equal forces will be inversely as the masses, that is, will be directly as AC and BC; hence if A′ B′ are their new positions after any time, we have

$$\frac{AC}{BC} = \frac{AA'}{BB'} = \frac{AC \pm AA'}{BC \pm BB'} = \frac{A'C}{B'C}.$$

The line A′B′ is therefore divided at C in the same ratio in which the line AB was divided; hence C is still the centre of gravity.

84. **Velocity of Centre of Gravity.**—If any number of masses are moving with any velocities, and in any directions, but so that each of them moves uniformly in a straight line, their common centre of gravity will move uniformly in a straight line.

To prove this, we shall consider their component velocities in any one direction,

let these component velocities be	u_1	u_2	u_3	&c.,
the masses being	m_1	m_2	m_3	&c.,

and the distances of the bodies (strictly speaking the distances of

their respective centres of gravity) from a fixed plane to which the given direction is normal, be x_1 x_2 x_3 &c.

The formula for the distance of their common centre of gravity from this plane is

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + \&c.}{m_1 + m_2 + \&c.} \qquad (1)$$

In the time t, x_1 is increased by the amount $u_1 t$, x_2 by $u_2 t$, and so on; hence the numerator of the above expression is increased by

$$m_1 u_1 t + m_2 u_2 t + \&c.,$$

and the value of $\bar{x}$ is increased in each unit of time by

$$\frac{m_1 u_1 + m_2 u_2 + \&c.}{m_1 + m_2 + \&c.}, \qquad (2)$$

which is therefore the component velocity of the centre of gravity in the given direction. As this expression contains only given constant quantities, its value is constant; and as this reasoning applies to all directions, the velocity of the centre of gravity must itself be constant both in magnitude and direction.

We may remark that the above formula (2) correctly expresses the component velocity of the centre of gravity at the instant considered, even when u_1, u_2, &c., are not constant.

85. Centre of Mass.—The point which we have thus far been speaking of under the name of "centre of gravity," is more appropriately called the "centre of mass," a name which is at once suggested by formula (1) § 84. When gravity acts in parallel lines upon all the particles of a body, the resultant force of gravity upon the body is a single force passing through this point; but this is no longer the case when the forces of gravity upon the different parts of the body (or system of bodies) are not parallel.

86. Units of Measurement.—It is a matter of importance, in scientific calculations, to express the various magnitudes with which we have to deal in terms of units which have a simple relation to each other. The British weights and measures are completely at fault in this respect, for the following reasons:—

1. They are not a decimal system; and the reduction of a measurement (say) from inches and decimals of an inch to feet and decimals of a foot, cannot be effected by inspection.

2. It is still more troublesome to reduce gallons to cubic feet or inches.

3. The weight (properly the mass) of a cubic foot of a substance in lbs., cannot be written down by inspection, when the specific gravity of the substance (as compared with water) is given.

87. The C.G.S. System.—A committee of the British Association, specially appointed to recommend a system of units for general adoption in scientific calculation, have recommended that the *centimetre* be adopted as the unit of length, the *gramme* as the unit of mass, and the *second* as the unit of time. We shall first give the rough and afterwards the more exact definitions of these quantities.

The centimetre is approximately $\frac{1}{10^9}$ of the distance of either pole of the earth from the equator; that is to say 1 followed by 9 zeros expresses this distance in centimetres.

The gramme is approximately the mass of a cubic centimetre of cold water. Hence the same number which expresses the specific gravity of a substance referred to water, expresses also the mass of a cubic centimetre of the substance, in grammes.

The second is $\frac{1}{24 \times 60 \times 60}$ of a mean solar day.

More accurately, the centimetre is defined as one hundredth part of the length, at the temperature 0° Centigrade, of a certain standard bar, preserved in Paris, carefully executed copies of which are preserved in several other places; and the gramme is defined as one thousandth part of the mass of a certain standard which is preserved at Paris, and of which also there are numerous copies preserved elsewhere.

For brevity of reference, the committee have recommended that the system of units based on the Centimetre, Gramme, and Second, be called the C.G.S. system.

The unit of area in this system is the square centimetre.

The unit of volume is the cubic centimetre.

The unit of velocity is a velocity of a centimetre per second.

The unit of momentum is the momentum of a gramme moving with a velocity of a centimetre per second.

The unit force is that force which generates this momentum in one second. It is therefore that force which, acting on a gramme for one second, generates a velocity of a centimetre per second. This force is called the *dyne*, an abbreviated derivative from the Greek δύναμις (force).

The unit of work is the work done by a force of a dyne working through a distance of a centimetre. It might be called the dyne-centimetre, but a shorter name has been provided and it is called the *erg*, from the Greek ἔργον (work).

CHAPTER VII.

LAWS OF FALLING BODIES.

88. Effect of the Resistance of the Air.—In air, bodies fall with unequal velocities; a sovereign or a ball of lead falls rapidly, a piece of down or thin paper slowly. It was formerly thought that this difference was inherent in the nature of the materials; but it is easy to show that this is not the case, for if we compress a mass of down or a piece of paper by rolling it into a ball, and compare it with a piece of gold-leaf, we shall find that the latter body falls more slowly than the former. The inequality of the velocities which we observe is due to the resistance of the air, which increases with the extent of surface exposed by the body.

It was Galileo who first discovered the cause of the unequal rapidity of fall of different bodies. To put the matter to the test, he prepared small balls of different substances, and let them fall at the same time from the top of the tower of Pisa; they struck the ground almost at the same instant. On changing their forms, so as to give them very different extents of surface, he observed that they fell with very unequal velocities. He was thus led to the conclusion that gravity acts on all substances with the same intensity, and that in a vacuum all bodies would fall with the same velocity.

This last proposition could not be put to the test of experiment in the time of Galileo, the air-pump not having yet been invented. The experiment was performed by Newton, and is now well known as the "guinea and feather" experiment. For this purpose a tube from a yard and a half to two yards long is used, which can be exhausted of air, and which contains bodies of various densities, such as a coin, pieces of paper, and feathers. When the tube is full of air and is inverted, these different bodies are seen to fall with very unequal velocities; but if the experiment is repeated after the tube

has been exhausted of air, no difference can be perceived between the times of their descent.

89. Mass and Gravitation Proportional.—This experiment proves that bodies which have equal weights are equal in mass. For equal masses are defined to be those which, when acted on by equal forces, receive equal accelerations; and the forces, in this experiment, are the weights of the falling bodies.

Newton tested this point still more severely by experiments with pendulums (*Principia*, book III. prop. vi.). He procured two round wooden boxes of the same size and weight, and suspended them by threads eleven feet long. One of them he filled with wood, and he placed very accurately in the centre of oscillation of the other the same weight of gold. The boxes hung side by side, and, when set swinging in equal oscillations, went and returned together for a very long time. Here the forces concerned in producing and checking the motion, namely, the force of gravity and the resistance of the air, were the same for the two pendulums, and as the movements produced were the same, it follows that the masses were equal. Newton remarks that a difference of mass amounting to a thousandth part of the whole could not have escaped detection. He experimented in the same way with silver, lead, glass, sand, salt, water, and wheat, and with the same result. He therefore infers that universally bodies of equal mass gravitate equally towards the earth at the same place. He further extends the same law to gravitation generally, and establishes the conclusion that the mutual gravitating force between any two bodies depends only on their masses and distances, and is independent of their materials.

The time of revolution of the moon round the earth, considered in conjunction with her distance from the earth, shows that the relation between mass and gravitation is the same for the material of which the moon is composed as for terrestrial matter; and the same conclusion is proved for the planets by the relation which exists between their distances from the sun and their times of revolution in their orbits.

90. Uniform Acceleration.—The fall of a heavy body furnishes an illustration of the second law of motion, which asserts that the change of momentum in a body in a given time is a measure of the force which acts on the body. It follows from this law that if the same force continues to act upon a body the changes of momentum in successive equal intervals of time will be equal. When a heavy

body originally at rest is allowed to fall, it is acted on during the time of its descent by its own weight and by no other force, if we neglect the resistance of the air. As its own weight is a constant force, the body receives equal changes of momentum, and therefore of velocity, in equal intervals of time. Let g denote its velocity in centimetres per second, at the end of the first second. Then at the end of the next second its velocity will be $g+g$, that is $2g$; at the end of the next it will be $2g+g$, that is $3g$, and so on, the gain of velocity in each second being equal to the velocity generated in the first second. At the end of t seconds the velocity will therefore be tg. Such motion as this is said to be *uniformly accelerated,* and the constant quantity g is the measure of the acceleration. Acceleration is defined as the gain of velocity per unit of time.

91. Weight of a Gramme in Dynes. Value of g.—Let m denote the mass of the falling body in grammes. Then the change of momentum in each second is mg, which is therefore the measure of the force acting on the body. The weight of a body of m grammes is therefore mg dynes, and the weight of 1 gramme is g dynes. The value of g varies from 978·1 at the equator to 983·1 at the poles; and 981 may be adopted as its average value in temperate latitudes. Its value at any part of the earth's surface is approximately given by the formula

$$g = 980\ 6056 - 2\ 5028 \cos 2\lambda - \cdot 000{,}003h,$$

in which λ denotes the latitude, and h the height (in centimetres) above sea-level.[1]

In § 79 we distinguished between the intensity and the amount of a force. The amount of the force of gravity upon a mass of m grammes is mg dynes. The intensity of this force is g dynes per gramme. The intensity of a force, in dynes per gramme of the body acted on, is always equal to the change of velocity which the force produces per second, this change being expressed in centimetres per second. In other words the intensity of a force is equal to the acceleration which it produces. The best designation for g is *the intensity of gravity.*

92. Distance fallen in a Given Time.—The distance described in a given time by a body moving with uniform velocity is calculated by multiplying the velocity by the time; just as the area of a rectangle is calculated by multiplying its length by its breadth. Hence if we draw a line such that its ordinates AA′, BB′, &c., represent the

[1] For the method of determination see § 120.

velocities with which a body is moving at the times represented by OA, OB (time being reckoned from the beginning of the motion), it can be shown that the whole distance described is represented by the area OB′B bounded by the curve, the last ordinate, and the base line. In fact this area can be divided into narrow strips (one of which is shown at AA′, Fig. 37) each of which may practically be regarded as a rectangle, whose height represents the velocity with which the body is moving during the very small interval of time represented by its base, and whose area therefore represents the distance described in this time.

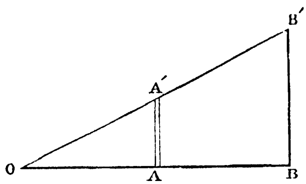


Fig. 37

This would be true for the distance described by a body moving from rest with any law of velocity. In the case of falling bodies the law is that the velocity is simply proportional to the time; hence the ordinates AA′, BB′, &c., must be directly as the abscissæ OA, OB; this proves that the line OA′ B′ must be straight; and the figure OB′ B is therefore a triangle. Its area will be half the product of OB and BB′. But OB represents the time t occupied by the motion, and BB′ the velocity gt at the end of this time. The area of the triangle therefore represents half the product of t and gt, that is, represents $\frac{1}{2}gt^2$, which is accordingly the distance described in the time t. Denoting this distance by s, and the velocity at the end of time t by v, we have thus the two formulæ

$$v = gt, \qquad (1)$$
$$s = \tfrac{1}{2}gt^2, \qquad (2)$$

from which we easily deduce

$$gs = \tfrac{1}{2}v^2. \qquad (3)$$

93. Work spent in Producing Motion.—We may remark, in passing, that the third of these formulæ enables us to calculate the work required to produce a given motion in a given mass. When a body whose mass is 1 gramme falls through a distance s, the force which acts upon it is its own weight, which is g dynes, and the work done upon it is gs ergs. Formula (3) shows that this is the same as $\frac{1}{2}v^2$ ergs. For a mass of m grammes falling through a distance s, the work is $\frac{1}{2}mv^2$ ergs. *The work required to produce a velocity* v (centimetres per second) *in a body of mass* m (grammes) originally at rest *is* $\frac{1}{2}mv^2$ (ergs).

94. Body thrown Upwards.—When a heavy body is projected ver-

tically upwards, the formulæ (1) (2) (3) of § 92 will still apply to its motion, with the following interpretations:—

v denotes the velocity of projection.

t denotes the whole time occupied in the ascent.

s denotes the height to which the body will ascend.

When the body has reached the highest point, it will fall back, and its velocity at any point through which it passes twice will be the same in going up as in coming down.

95. Resistance of the Air.—The foregoing results are rigorously applicable to motion in vacuo, and are sensibly correct for motion in air as long as the resistance of the air is insignificant in comparison with the force of gravity. The force of gravity upon a body is the same at all velocities; but the resistance of the air increases with the velocity, and increases more and more rapidly as the velocity becomes greater; so that while at very slow velocities an increase of 1 per cent. in velocity would give an increase of 1 per cent. in the resistance, at a higher velocity it would give an increase of 2 per cent., and at the velocity of a cannon-ball an increase of 3 per cent.[1] The formulæ are therefore sensibly in error for high velocities. They are also in error for bodies which, like feathers or gold-leaf, have a large surface in proportion to their weight.

96. Projectiles.—If, instead of being simply let fall, a body is projected in any direction, its motion will be compounded of the motion of a falling body and a uniform motion in the direction of projection. Thus if OP (Fig. 38) is the direction of projection, and OQ the vertical through the point of projection, the body would move along OP keeping its original velocity unchanged, if it were not disturbed by gravity. To find where the body will be at any time t, we must lay off a length OP equal to Vt, V denoting the velocity of projection, and must then draw from P the vertical line PR downwards equal to $\frac{1}{2}gt^2$, which is the distance that the body would have fallen in the time if simply dropped. The point R thus determined, will be the actual position of the body. The velocity of the body at any time will in like manner be found by compounding the initial

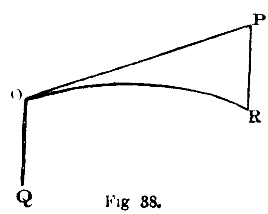


Fig 38.

[1] This is only another way of saying that the resistance varies approximately as the velocity when very small, and approximately as the cube of the velocity for velocities like that of a cannon-ball.

velocity with the velocity which a falling body would have acquired in the time.

The path of the body will be a curve, as represented in the figure, OP being a tangent to it at O, and its concavity being downwards. The equations above given, namely

$$OP = Vt, \; PR = \tfrac{1}{2}gt^2,$$

show that PR varies as the square of OP, and hence that the path (or *trajectory* as it is technically called) is a parabola, whose axis is vertical.

97. Time of Flight, and Range.—If the body is projected from a point at the surface of the ground (supposed level) we can calculate the time of flight and the range in the following way.

Let α be the angle which the direction of projection makes with the horizontal. Then the velocity of projection can be resolved into two components, $V \cos \alpha$ and $V \sin \alpha$, the former being horizontal, and the latter vertically upward. The horizontal component of the velocity of the body is unaffected by gravity and remains constant. The vertical velocity after time t will be compounded of $V \sin \alpha$ upwards and gt downwards. It will therefore be an upward velocity $V \sin \alpha - gt$, or a downward velocity $gt - V \sin \alpha$. At the highest point of its path, the body will be moving horizontally and the vertical component of its velocity will be zero; that is, we shall have

$$V \sin \alpha - gt = 0; \text{ whence } t = \frac{V \sin \alpha}{g}.$$

This is the time of attaining the highest point; and the time of flight will be double of this, that is, will be $\frac{2V \sin \alpha}{g}$.

As the horizontal component of the velocity has the constant value $V \cos \alpha$, the horizontal displacement in any time t is $V \cos \alpha$ multiplied by t. The range is therefore

$$\frac{2V^2 \sin \alpha \cos \alpha}{g} \text{ or } \frac{V^2 \sin 2\alpha}{g}.$$

The range (for a given velocity of projection) will therefore be greatest when $\sin 2\alpha$ is greatest, that is when $2\alpha = 90°$ and $\alpha = 45°$.

We shall now describe two forms of apparatus for illustrating the laws of falling bodies.

98. Morin's Apparatus.—Morin's apparatus consists of a wooden cylinder covered with paper, which can be set in uniform rotation about its axis by the fall of a heavy weight. The cord which sup-

ports the weight is wound upon a drum, furnished with a toothed wheel which works on one side with an endless screw on the axis of the cylinder, and on the other drives an axis carrying fans which serve to regulate the motion.

In front of the turning cylinder is a cylindro-conical weight of cast-iron carrying a pencil whose point presses against the paper, and having ears which slide on vertical threads, serving to guide it in its fall. By pressing a lever, the weight can be made to fall at a chosen moment. The proper time for this is when the motion of the cylinder has become sensibly uniform. It follows from this arrangement that during its vertical motion the pencil will meet in succession the different generating lines[1] of the revolving cylinder, and will consequently describe on its surface a certain curve, from the study of which we shall be able to gather the law of the fall of the body which has traced it. With this view, we describe (by turning the cylinder while the pencil is stationary) a circle passing through the commencement of the curve, and also draw a vertical line through this point. We cut the paper along this latter line and develop it (that is, flatten

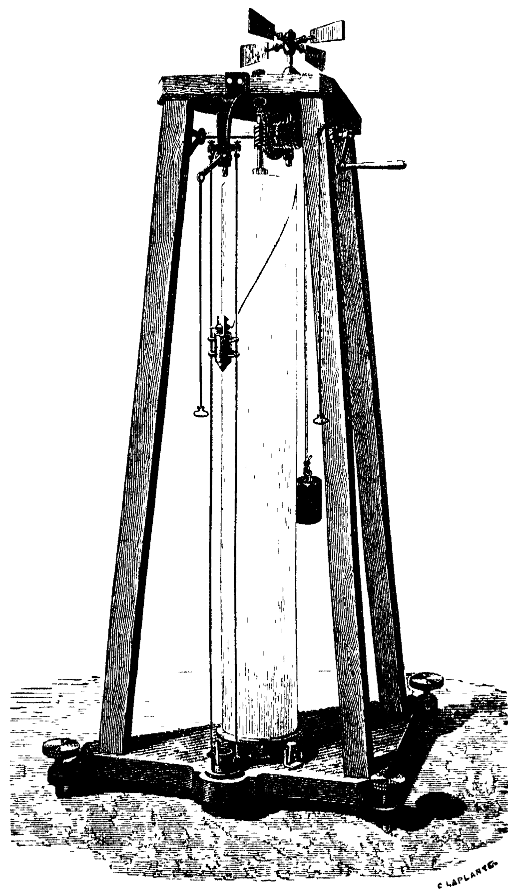


Fig 39 —Morin's Apparatus

[1] A cylindric surface could be swept out or "generated" by a straight line moving round the axis and remaining always parallel to it. The successive positions of this generating line are called the "generating lines of the cylinder."

it out into a plane). It then presents the appearance shown in Fig. 40.

If we take on the horizontal line equal distances at 1, 2, 3, 4, 5 . . . , and draw perpendiculars at their extremities to meet the curve, it is evident that the points thus found are those which were traced by the pencil when the cylinder had turned through the distances 1, 2, 3, 4, 5. . . . The corresponding verticals represent the spaces traversed in the times 1, 2, 3, 4, 5. . . . Now we find, as the figure shows, that these spaces are represented by the numbers 1, 4, 9, 16, 25 . . . , thus verifying the principle that the spaces described are proportional to the squares of the times employed in their description.

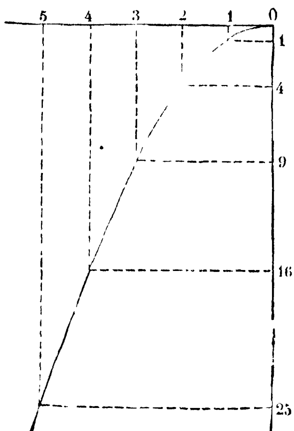


Fig. 40.—Parabolic Trace.

We may remark that the proportionality of the vertical lines to the squares of the horizontal lines shows that the curve is a parabola. The parabolic trace is thus the consequence of the law of fall, and from the fact of the trace being parabolic we can infer the proportionality of the spaces to the squares of the times.

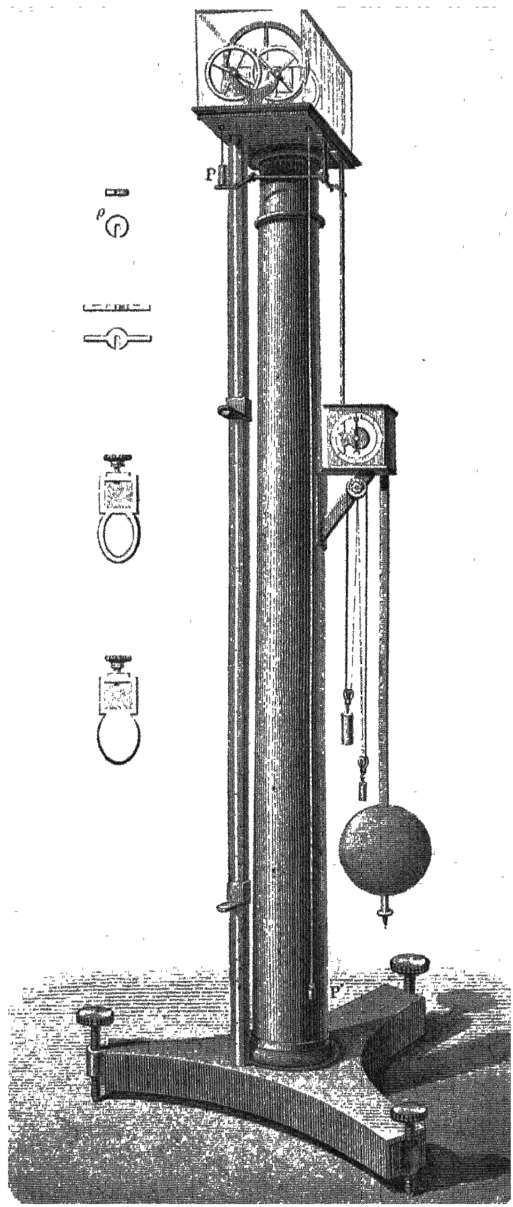
The law of velocities might also be verified separately by Morin's apparatus; we shall not describe the method which it would be necessary to employ, but shall content ourselves with remarking that the law of velocities is a logical consequence of the law of spaces.[1]

99. Atwood's Machine.—Atwood's machine, which affords great facilities for illustrating the effects of force in producing motion, consists essentially of a very freely moving pulley over which a fine cord passes, from the ends of which two equal weights can be suspended. A small additional weight of flat and elongated form is laid upon one of them, which is thus caused to descend with uniform *acceleration*, and means are provided for suddenly removing

[1] Consider, in fact, the space traversed in any time t; this space is given by the formula $s = Kt^2$; during the time $t+\theta$ the space traversed will be $K(t+\theta)^2 = Kt^2 + 2Kt\theta + K\theta^2$, whence it follows that the space traversed during the time θ after the time t is $2Kt\,\theta + K\theta^2$. The average velocity during this time θ is obtained by dividing the space by θ, and is $2Kt + K\theta$, which, by making θ very small, can be made to agree as accurately as we please with the value $2Kt$. This limiting value $2Kt$ must therefore be the velocity at the end of time t.—*D.*

this additional weight at any point of the descent, so as to allow the motion to continue from this point onward with uniform *velocity*.

The machine is represented in Fig. 41. The pulley over which the string passes is the largest of the wheels shown at the top of the apparatus. In order to give it greater freedom of movement, the ends of its axis are made to rest, not on fixed supports, but on the circumferences of four wheels (two at each end of the axis) called friction-wheels, because their office is to diminish friction. Two small equal weights are shown, suspended from this pulley by a string passing over it. One of them P′ is represented as near the bottom of the supporting pillar, and the other P as near the top. The latter is resting upon a small platform, which can be suddenly dropped when it is desired that the motion shall commence. A little lower down and vertically beneath the platform, is seen a ring,



large enough to let the weight pass through it without danger of

contact. This ring can be shifted up or down, and clamped at any height by a screw; it is represented on a larger scale in the margin. At a considerable distance beneath the ring, is seen the stop, which is also represented in the margin, and can like the ring be clamped at any height. The office of the ring is to intercept the additional weight, and the office of the stop is to arrest the descent. The upright to which they are both clamped is marked with a scale of equal parts, to show the distances moved over. A clock with a pendulum beating seconds, is provided for measuring the time; and there is an arrangement by which the movable platform can be dropped by the action of the clock precisely at one of the ticks. To measure the distance fallen in one or more seconds, the ring is removed, and the stop is placed by trial at such heights that the descending weight strikes it precisely at another tick. To measure the velocity acquired in one or more seconds, the ring must be fixed at such a height as to intercept the additional weight at one of the ticks, and the stop must be placed so as to be struck by the descending weight at another tick.

100. Theory of Atwood's Machine.—If M denote each of the two equal masses, in grammes, and m the additional mass, the whole moving mass (neglecting the mass of the pulley and string) is $2M+m$, but the moving force is only the weight of m. The acceleration produced, instead of being g, is accordingly only $\frac{m}{2M+m}\,g$. In order to allow for the inertia of the pulley and string, a constant quantity must be added to the denominator in the above formula, and the value of this constant can be determined by observing the movements obtained with different values of M and m. Denoting it by C, we have

$$\frac{m}{m+2M+C}\,g \qquad \text{(A)}$$

as the expression for the acceleration. As m is usually small in comparison with M, the acceleration is very small in comparison with that of a freely falling body, and is brought within the limits of convenient observation. Denoting the acceleration by a, and using v and s, as in § 92, to denote the velocity acquired and space described in time t, we shall have

$$v=at, \qquad (1)$$
$$s=\tfrac{1}{2}at^2, \qquad (2)$$
$$as=\tfrac{1}{2}v^2, \qquad (3)$$

and each of these formulæ can be directly verified by experiments with the machine.

101. Uniform Motion in a Circle.—A body cannot move in a curved path unless there be a force urging it towards the concave side of the curve.

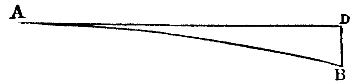


Fig. 42

We shall proceed to investigate the intensity of this force when the path is circular and the velocity uniform. We shall denote the velocity by v, the radius of the circle by r, and the intensity of the force by f. Let AB (Figs. 42, 43) be a small portion of the path, and BD a perpendicular upon AD the tangent at A. Then, since the arc AB is small in comparison with the whole circumference, it is sensibly equal to AD, and the body would have been found at D instead of at B if no force had acted upon it since leaving A. DB is accordingly the distance due to the force; and if t denote the time from A to B, we have

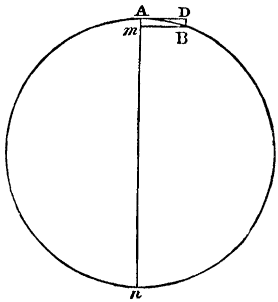


Fig 43

$$\mathrm{AD} = vt \tag{1}$$

$$\mathrm{DB} = \tfrac{1}{2}ft^2. \tag{2}$$

The second of these equations gives

$$f = \frac{2\mathrm{DB}}{t^2}$$

and substituting for t from the first equation, this becomes

$$f = \frac{2\mathrm{DB}}{\mathrm{AD}^2}v^2, \tag{3}$$

But if An (Fig. 43) be the diameter at A, and Bm the perpendicular upon it from B, we have, by Euclid, $\mathrm{AD}^2 = m\mathrm{B}^2 = \mathrm{A}m\,.\,mn = 2r\,.\,\mathrm{A}m$ sensibly, $= 2r\,.\,\mathrm{DB}$.

Therefore $\frac{2\mathrm{DB}}{\mathrm{AD}^2} = \frac{1}{r}$, and hence by (3)

$$f = \frac{v^2}{r}. \tag{4}$$

Hence the force necessary for keeping a body in a circular path without change of velocity, is a force of intensity $\frac{v^2}{r}$ directed towards the centre of the circle. If m denote the mass of the body, the amount of the force will be $\frac{mv^2}{r}$. This will be in dynes, if m be in grammes, r in centimetres, and v in centimetres per second.

If the time of revolution be denoted by T, and π as usual denote the ratio of circumference to diameter, the distance moved in time

T is $2\pi r$; hence $v = \frac{2\pi r}{T}$, and another expression for the intensity of the force will be

$$f = \left(\frac{2\pi}{T}\right)^2 r. \qquad (5)$$

102. Deflecting Force in General.—In general, when a body is moving in any path, and with velocity either constant or varying, the force acting upon it at any instant can be resolved into two components, one along the tangent and the other along the normal. The intensity of the tangential component is measured by the rate at which the velocity increases or diminishes, and the intensity of the normal component is given by formula (4) of last article, if we make r denote the radius of curvature.

103. Illustrations of Deflecting Force.—When a stone is swung round by a string in a vertical circle, the tension of the string in the lowest position consists of two parts:—

(1) The weight of the stone, which is mg if m be the mass of the stone.

(2) The force $m\frac{v^2}{r}$ which is necessary for deflecting the stone from a horizontal tangent into its actual path in the neighbourhood of the lowest point.

When the stone is at the highest point of its path, the tension of the string is the difference of these two forces, that is to say it is

$$m\left(\frac{v^2}{r} - g\right),$$

and the motion is not possible unless the velocity at the highest point is sufficient to make $\frac{v^2}{r}$ greater than g.

The tendency of the stone to persevere in rectilinear motion and to resist deflection into a curve, causes it to exert a force upon the string, of amount $m\frac{v^2}{r}$, and this is called *centrifugal force*. It is not a force acting upon the stone, but a force exerted by the stone upon the string. Its direction is *from* the centre of curvature, whereas the deflecting force which acts upon the stone is *towards* the centre of curvature.

104. Centrifugal Force at the Equator.—Bodies on the earth's surface are carried round in circles by the diurnal rotation of the earth upon its axis. The velocity of this motion at the equator is about 46,500 centimetres per second, and the earth's equatorial radius is about $6{\cdot}38 \times 10^8$ centimetres. Hence the value of $\frac{v^2}{r}$ is found to be about 3·39. The case is analogous to that of the stone

at the highest point of its path in the preceding article, if instead of a string which can only exert a pull we suppose a stiff rod which can exert a push upon the stone. The rod will be called upon to exert a pull or a push at the highest point according as $\frac{v^2}{r}$ is greater or less than g. The force of the push in the latter case will be

$$m\left(g - \frac{v^2}{r}\right),$$

and this is accordingly the force with which the surface of the earth at the equator pushes a body lying upon it. The push, of course, is mutual, and this formula therefore gives the apparent weight or apparent gravitating force of a body at the equator, mg denoting its true gravitating force (due to attraction alone). A body falling in vacuo at the equator has an acceleration 978·10 relative to the surface of the earth in its neighbourhood; but this portion of the surface has itself an acceleration of 3·39, directed towards the earth's centre, and therefore in the same direction as the acceleration of the body. The absolute acceleration of the body is therefore the sum of these two, that is 981·49, which is accordingly the intensity of true gravity at the equator.

The apparent weight of bodies at the equator would be *nil* if $\frac{v^2}{r}$ were equal to g. Dividing 3·39 into 981·49, the quotient is approximately 289, which is $(17)^2$. Hence this state of things would exist if the velocity of rotation were about 17 times as fast as at present.

Since the movements and forces which we actually observe depend upon *relative* acceleration, it is usual to understand, by the value of g or the intensity of gravity at a place, the *apparent* values, unless the contrary be expressed. Thus the value of g at the equator is usually stated to be 978·10.

105. Direction of Apparent Gravity.—The total amount of centrifugal force at different places on the earth's surface, varies directly as their distance from the earth's axis; for this is the value of r in the formula (5) of § 101, and the value of T in that formula is the same for the whole earth. The direction of this force, being perpendicular to the earth's axis, is not vertical except at the equator; and hence, when we compound it with the force of true gravity, we obtain a resultant force of apparent gravity differing in direction as well as in magnitude from true gravity. What is always understood by a *vertical*, is the direction of *apparent* gravity; and a plane perpendicular to it is what is meant by a horizontal plane.

CHAPTER VIII.

THE PENDULUM.

106. The Pendulum.—When a body is suspended so that it can turn about a horizontal axis which does not pass through its centre of gravity, its only position of stable equilibrium is that in which its centre of gravity is in the same vertical plane with the axis and below it (§ 42). If the body be turned into any other position, and left to itself, it will oscillate from one side to the other of the position of equilibrium, until the resistance of the air and the friction of the axis gradually bring it to rest. A body thus suspended, whatever be its form, is called a pendulum. It frequently consists of a rod which can turn about an axis O (Fig. 44) at its upper end, and which carries at its lower end a heavy lens-shaped piece of metal M called the bob; this latter can be raised or lowered by means of the screw V. The applications of the pendulum are very important: it regulates our clocks, and it has enabled us to measure the intensity of gravity in different parts of the world; it is important then to know at least the fundamental points in its theory. For explaining these, we shall begin with the consideration of an ideal body called the *simple pendulum.*

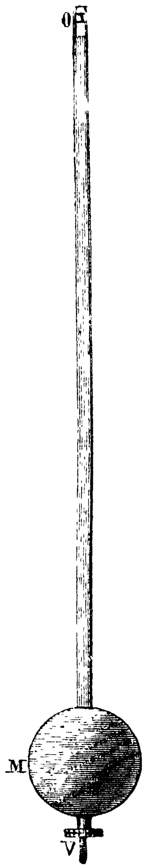


Fig. 44.—Pendulum.

107. Simple Pendulum.—This is the name given to a pendulum consisting of a heavy particle M (Fig. 45) attached to one end of an inextensible thread without weight, the other end of the thread being fixed at A. When the thread is vertical, the weight of the particle acts in the direction of its length, and there is equilib-

rium. But suppose it is drawn aside into another position, as AM. In this case, the weight MG of the particle can be resolved into two forces MC and MH. The former, acting along the prolongation of the thread, is destroyed by the resistance of the thread; the other, acting along the tangent MH, produces the motion of the particle. This effective component is evidently so much the greater as the angle of displacement from the vertical position is greater. The particle will therefore move along an arc of a circle described from A as centre, and the force which urges it forward will continually diminish till it arrives at the lowest point M′. At M′ this force is zero, but, in virtue of the velocity acquired, the particle will ascend on the opposite side, the effective component of gravity being now opposed to the direction of its motion; and, inasmuch as the magnitude of this component goes through the same series of values in this part of the motion as in the former part, but in reversed order, the velocity will, in like manner, retrace its former values, and will become zero when the particle has risen to a point M″ at the same height as M. It then descends again and performs an oscillation from M″ to M precisely similar to the first, but in the reverse direction. It will thus continue to vibrate between the two points M, M″ (friction being supposed excluded), for an indefinite number of times, all the vibrations being of equal extent and performed in equal periods.

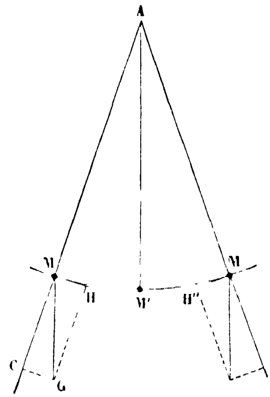


Fig 45 —Motion of Simple Pendulum.

The distance through which a simple pendulum travels in moving from its lowest position to its furthest position on either side, is called its *amplitude*. It is evidently equal to half the complete arc of vibration, and is commonly expressed, not in linear measure, but in degrees of arc. Its numerical value is of course equal to that of the angle MAM′, which it subtends at the centre of the circle.

The *complete period* of the pendulum's motion is the time which it occupies in moving from M to M″ and back to M, or more generally, is the time from its passing through any given position to its next passing through the same position *in the same direction*.

What is commonly called the time of vibration, or the time of a single vibration, is the half of a complete period, being the time of

passing from one of the two extreme positions to the other. Hence what we have above defined as a complete period is often called a double vibration.

When the amplitude changes, the time of vibration changes also, being greater as the amplitude is greater; but the connection between the two elements is very far from being one of simple proportion. The change of time (as measured by a ratio) is much less than the change of amplitude, especially when the amplitude is small; and when the amplitude is less than about 5°, any further diminution of it has little or no sensible effect in diminishing the time. *For small vibrations*, then, *the time of vibration is independent of the amplitude*. This is called the law of *isochronism*.

108. Law of Acceleration for Small Vibrations.—Denoting the length of a simple pendulum by l, and its inclination to the vertical at any moment by θ, we see from Fig. 45 that the ratio of the effective component of gravity to the whole force of gravity is $\frac{\text{MH}}{\text{MG}}$, that is $\sin\theta$; and when θ is small this is sensibly equal to θ itself as measured by $\frac{\text{arc}}{\text{radius}}$. Let s denote the length of the arc MM′ intervening between the lower end of the pendulum and the lowest point of its swing, at any time; then θ is equal to $\frac{s}{l}$, and the intensity of the effective force of gravity when θ is small is sensibly equal to $g\theta$, that is to $\frac{gs}{l}$. Since g and l are the same in all positions of the pendulum, this effective force varies as s. Its direction is always *towards* the position of equilibrium, so that it accelerates the motion during the approach to this position, and retards it during the recess; the acceleration or retardation being always in direct proportion to the distance from the position of equilibrium. This species of motion is of extremely common occurrence. It is illustrated by the vibration of either prong of a tuning-fork, and in general by the motion of any body vibrating in one plane in such a manner as to yield a simple musical tone.

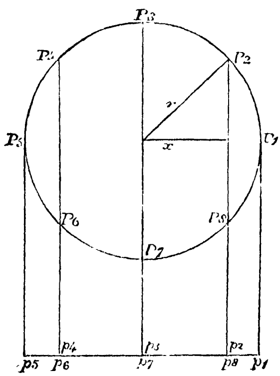


Fig. 46.—Projection of Circular Motion.

109. General Law for Period.—Suppose a point P to travel with uniform velocity round a circle (Fig. 46), and from its successive

positions P_1, P_2, &c., let perpendiculars P_1p_1, P_2p_2, &c., be drawn to a fixed straight line in the plane of the circle. Then while P travels once round the circle, its projection p executes a complete vibration.

The acceleration of P is always directed towards the centre of the circle, and is equal to $\left(\frac{2\pi}{T}\right)^2 r$ (§ 101). The component of this acceleration parallel to the line of motion of p, is the fraction $\frac{x}{r}$ of the whole acceleration (x denoting the distance of p from the middle point of its path), and is therefore $\left(\frac{2\pi}{T}\right)^2 x$. This is accordingly the acceleration of p, and as it is simply proportional to x we shall denote it for brevity by μx. To compute the periodic time T of a complete vibration, we have the equation $\mu = \left(\frac{2\pi}{T}\right)^2$, which gives

$$T = \frac{2\pi}{\sqrt{\mu}}. \tag{1}$$

110. Application to the Pendulum.—For the motion of a pendulum in a small arc, we have

$$\text{acceleration} = \frac{g}{l} s,$$

where s denotes the displacement in linear measure. We must therefore put $\mu = \frac{g}{l}$, and we then have

$$T = 2\pi \sqrt{\frac{l}{g}}, \tag{2}$$

which is the expression for the time of a complete (or double) vibration. It is more usual to understand by the "time of vibration" of a pendulum the half of this, that is the time from one extreme position to the other, and to denote this time by T. In this sense we have

$$T = \pi \sqrt{\frac{l}{g}}. \tag{3}$$

To find the length of the seconds' pendulum we must put $T = 1$. This gives

$$\pi^2 . \frac{l}{g} = 1, \quad l = \frac{g}{\pi^2} = \frac{g}{9{\cdot}87} \text{ nearly.}$$

If g were 987 we should have $l = 100$ centimetres or 1 metre. The actual value of g is everywhere a little less than this. The length of the seconds' pendulum is therefore everywhere rather less than a metre.

111. Simple Harmonic Motion.—Rectilinear motion consisting of vibration about a point with acceleration μx, where x denotes

distance from this point, is called *Simple Harmonic Motion*, or Simple Harmonic Vibration. The above investigation shows that such vibration is isochronous, its period being $\frac{2\pi}{\sqrt{\mu}}$ whatever the amplitude may be.

To understand the reason of this isochronism we have only to remark that, if the amplitude be changed, the velocity at corresponding points (that is, points whose distances from the middle point are the same fractions of the amplitudes) will be changed in the same ratio. For example, compare two simple vibrations in which the values of μ are the same, but let the amplitude of one be double that of the other. Then if we divide the paths of both into the same number of small equal parts, these parts will be twice as great for the one as for the other; but if we suppose the two points to start simultaneously from their extreme positions, the one will constantly be moving twice as fast as the other. The number of parts described in any given time will therefore be the same for both.

In the case of vibrations which are not simple, it is easy to see (from comparison with simple vibration) that if the acceleration increases in a greater ratio than the distance from the mean position, the period of vibration will be shortened by increasing the amplitude; but if the acceleration increases in a less ratio than the distance, as in the case of the common pendulum vibrating in an arc of moderate extent, the period is increased by increasing the amplitude.

112. Experimental Investigation of the Motion of Pendulums.—The preceding investigation applies to the simple pendulum; that is to say to a purely imaginary existence; but it can be theoretically demonstrated that every rigid body vibrating about a horizontal axis under the action of gravity (friction and the resistance of the air being neglected), moves in the same manner as a simple pendulum of determinate length called the *equivalent simple pendulum.* Hence the above results can be verified by experiments on actual pendulums.

The discovery of the experimental laws of the motion of pendulums was in fact long anterior to the theoretical investigation. It was the earliest and one of the most important discoveries of Galileo, and dates from the year 1582, when he was about twenty years of age. It is related that on one occasion, when in the cathedral of Pisa, he was struck with the regularity of the oscillations of a lamp suspended from the roof, and it appeared to him

that these oscillations, though diminishing in extent, preserved the same duration. He tested the fact by repeated trials, which confirmed him in the belief of its perfect exactness. This law of isochronism can be easily verified. It is only necessary to count the vibrations which take place in a given time with different amplitudes. The numbers will be found to be exactly the same. This will be found to hold good even when some of the vibrations compared are so small that they can only be observed with a telescope.

By employing balls suspended by threads of different lengths, Galileo discovered the influence of length on the time of vibration. He ascertained that when the length of the thread increases, the time of vibration increases also; not, however, in proportion to the length simply, but to its square root.

113. Cycloidal Pendulum.—It is obvious from § 64 that the effective component of gravity upon a particle resting on a smooth inclined plane is proportional to the sine of the inclination. The acceleration of a particle so situated is in fact $g \sin \alpha$, if α denote the inclination of the plane. When a particle is guided along a smooth curve its acceleration is expressed by the same formula, α now denoting the inclination of the curve at any point to the horizon. This inclination varies from point to point of the curve, so that the acceleration $g \sin \alpha$ is no longer a constant quantity. The motion of a common pendulum corresponds to the motion of a particle which is guided to move in a circular arc; and if x denote distance from the lowest point, measured along the arc, and r the radius of the circle (or the length of the pendulum), the acceleration at any point is $g \sin \frac{x}{r}$.

This is sensibly proportional to x so long as x is a small fraction of r; but in general it is not proportional to x, and hence the vibrations are not in general isochronous.

To obtain strictly isochronous vibrations we must substitute for the circular arc a curve which possesses the property of having an inclination whose sine is simply proportional to distance measured along the curve from the lowest point. The curve which possesses this property is the cycloid. It is the curve which is traced by a point in the circumference of a circle which rolls along a straight line. The cycloidal pendulum is constructed by suspending an ivory ball or some other small heavy body by a thread between two cheeks (Fig. 47), on which the thread winds as the ball swings to

either side. The cheeks must themselves be the two halves of a cycloid whose length is double that of the thread, so that each cheek has the same length as the thread. It can be demonstrated[1] that under these circumstances the path of the ball will be a cycloid identical with that to which the cheeks belong. Neglecting friction and the rigidity of the thread, the acceleration in this case is proportional to distance measured along the cycloid from its lowest point, and hence the time of vibration will be strictly the same for large as for small amplitudes. It will, in fact, be the same as that of a simple pendulum having the same length as the cycloidal pendulum and vibrating in a small arc.

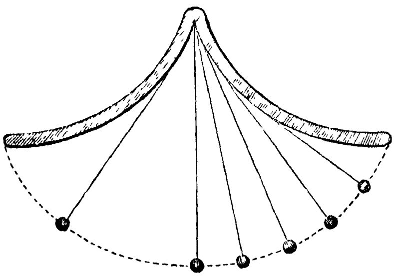


Fig 47.—Cycloidal Pendulum

Attempts have been made to adapt the cycloidal pendulum to clocks, but it has been found that, owing to the greater amount of friction, its rate was less regular than that of the common pendulum. It may be remarked, that the spring by which pendulums are often suspended has the effect of guiding the pendulum bob in a curve which is approximately cycloidal, and thus of diminishing the irregularity of rate resulting from differences of amplitude.

114. Moment of Inertia.—Just as the mass of a body is the measure of the force requisite for producing unit acceleration when the movement is one of pure translation; so the *moment of inertia* of a rigid body turning about a fixed axis is the measure of the couple requisite for producing unit acceleration of angular velocity. We suppose angle to be measured by $\frac{\text{arc}}{\text{radius}}$, so that the angle turned by the body is equal to the arc described by any point of it divided by the distance of this point from the axis; and the angular velocity of the body will be the velocity of any point divided by its distance from the axis. The moment of inertia of the body round the axis is numerically equal to the couple which would produce unit change of angular velocity in the body in unit time. We shall now show how to express the moment of inertia in terms of the masses of the particles of the body and their distances from the axis.

[1] Since the evolute of the cycloid is an equal cycloid.

Let m denote the mass of any particle, r its distance from the axis, and ϕ the angular acceleration. Then $r\phi$ is the acceleration of the particle m, and the force which would produce this acceleration by acting directly on the particle along the line of its motion is $mr\phi$ The moment of this force round the axis would be $mr^2\phi$ since its arm is r. The aggregate of all such moments as this for all the particles of the body is evidently equal to the couple which actually produces the acceleration of the body. Using the sign Σ to denote "the sum of such terms as," and observing that ϕ is the same for the whole body, we have

$$\text{Applied couple} = \Sigma\,(mr^2\phi) = \phi\,\Sigma\,(mr^2). \qquad (1)$$

When ϕ is unity, the applied couple will be equal to $\Sigma\,(mr^2)$, which is therefore, by the foregoing definition, the moment of inertia of the body round the axis.

115. Moments of Inertia Round Parallel Axes.—The moment of inertia round an axis through the centre of mass is always less than that round any parallel axis.

For if r denote the distance of the particle m from an axis not passing through the centre of mass, and x and y its distances from two mutually rectangular planes through this axis, we have $r^2 = x^2 + y^2$.

Now let two planes parallel to these be drawn through the centre of mass; let ξ and η be the distances of m from them, and ρ its distance from their line of intersection, which will clearly be parallel to the given axis. Also let a and b be the distances respectively between the two pairs of parallel planes, so that a^2+b^2 will be the square of the distance between the two parallel axes, which distance we will denote by h. Then we have

$$x = \xi \pm a$$
$$y = \eta \pm b$$
$$x^4 = a^2 + \xi^2 \pm 2a\,\xi, \qquad y^2 = b^2 + \eta^2 \pm 2b\,\eta$$

$$\Sigma\,(mr^2) = \Sigma\left\{m\,(a^2+b^2)\right\} + \Sigma\left\{m\,(\xi^2+\eta^2)\right\}$$
$$\pm\, 2a\,\Sigma\,(m\xi) \pm 2b\,\Sigma\,(m\eta)$$
$$= h^2\,\Sigma m + \Sigma\,(m\rho^2) \pm 2a\,\bar{\xi}\,\Sigma m \pm 2b\,\bar{\eta}\,\Sigma m.$$

where $\bar{\xi}$ and $\bar{\eta}$ are the values of ξ and η for the centre of mass. But these values are both zero, since the centre of mass lies on both the planes from which ξ and η are measured. We have therefore

$$\Sigma\,(mr^2) = h^2\,\Sigma m + \Sigma\,(m\rho^2), \qquad (2)$$

that is to say, the moment of inertia round the given axis exceeds the moment of inertia round the parallel axis through the centre of

mass by the product of the whole mass into the square of the distance between the axes.

116. Application to Compound Pendulum.—The application of this principle to the compound pendulum leads to some results of great interest and importance.

Let M be the mass of a compound pendulum, that is, a rigid body free to oscillate about a fixed horizontal axis. Let h, as in the preceding section, denote the distance of the centre of mass from this axis; let θ denote the inclination of h to the vertical, and ϕ the angular acceleration.

Then, since the forces of gravity on the body are equivalent to a single force Mg, acting vertically downwards at the centre of mass, and therefore having an arm $h \sin \theta$ with respect to the axis, the moment of the applied forces round the axis is $\mathrm{M}gh \sin \theta$; and this must, by § 114, be equal to $\phi \Sigma (mr^2)$. We have therefore

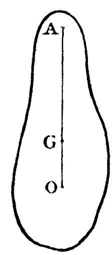
$$\frac{\Sigma (mr^2)}{\mathrm{M}h} = \frac{g \sin \theta}{\phi}. \tag{3}$$

If the whole mass were collected at one point at distance l from the axis, this equation would become

$$\frac{\mathrm{M}l^2}{\mathrm{M}l} = l = \frac{g \sin \theta}{\phi}; \tag{4}$$

and the angular motion would be the same as in the actual case if l had the value

$$l = \frac{\Sigma mr^2}{\mathrm{M}h}. \tag{5}$$



l is evidently the length of the equivalent simple pendulum.

117. Convertibility of Centres.—Again, if we introduce a length k such that $\mathrm{M}k^2$ is equal to $\Sigma (m\rho^2)$, that is, to the moment of inertia round a parallel axis through the centre of mass, we have

Fig. 48.

$$\Sigma (mr^2) = \Sigma (m\rho^2) + h^2 \Sigma m = \mathrm{M}k^2 + \mathrm{M}h^2,$$

and equation (5) becomes

$$l = \frac{k^2 + h^2}{h} = \frac{k^2}{h} + h, \tag{6}$$

$$\text{or } k^2 = (l - h)\, h. \tag{7}$$

In the annexed figure (Fig. 48) which represents a vertical section through the centre of mass, let G be the centre of mass, A the "centre

of suspension," that is, the point in which the axis cuts the plane of the figure, and O the "centre of oscillation," that is, the point at which the mass might be collected without altering the movement. Then, by definition, we have

$$l = AO,\ h = AG,\ \text{therefore } l - h = GO,$$

so that equation (7) signifies

$$k^2 = AG \,.\, GO. \qquad (8)$$

Since k^2 is the same for all parallel axes, this equation shows that when the body is made to vibrate about a parallel axis through O, the centre of oscillation will be the point A. That is to say; *the centres of suspension and oscillation are interchangeable, and the product of their distances from the centre of mass is* k^2.

118. If we take a new centre of suspension A′ in the plane of the figure, the new centre of oscillation O′ will lie in the production of A′G, and we must have

$$A'G \,.\, GO' = k^2 = AG \,.\, GO.$$

If A′G be equal to AG, GO′ will be equal to GO, and A′O′ to AO, so that the length of the equivalent simple pendulum will be unchanged. *A compound pendulum will* therefore *vibrate in the same time about all parallel axes which are equidistant from the centre of mass.*

When the product of two quantities is given, their sum is least when they are equal, and becomes continually greater as they depart further from equality. Hence the length of the equivalent simple pendulum AO or AG + GO is least when

$$AG = GO = k,$$

and increases continually as the distance of the centre of suspension from G is either increased from k to infinity or diminished from k to zero. Hence, when a body vibrates about an axis which passes very nearly through its centre of gravity, its oscillations are exceedingly slow.

119. **Kater's Pendulum.**—The principle of the convertibility of centres, established in § 117, was discovered by Huygens, and affords the most convenient practical method of constructing a pendulum of known length. In Kater's pendulum there are two parallel knife-edges about either of which the pendulum can be made to vibrate, and one of them can be adjusted to any distance

from the other. The pendulum is swung first upon one of these edges and then upon the other, and, if any difference is detected in the times of vibration, it is corrected by moving the adjustable edge. When the difference has been completely destroyed, the distance between the two edges is the length of the equivalent simple pendulum. It is necessary, in any arrangement of this kind, that the two knife-edges should be in a plane passing through the centre of gravity; also that they should be on opposite sides of the centre of gravity, and at unequal distances from it.

120. Determination of the Value of g.—Returning to the formula for the simple pendulum $T=\pi\sqrt{\frac{l}{g}}$, we easily deduce from it $g=\frac{\pi^2 l}{T^2}$, whence it follows that the value of g can be determined by making a pendulum vibrate and measuring T and l. T is determined by counting the number of vibrations that take place in a given time; l can be calculated, when the pendulum is of regular form, by the aid of formulæ which are given in treatises on rigid dynamics, but its value is more easily obtained by Kater's method, described above, founded on the principle of the convertibility of the centres of suspension and oscillation.

It is from pendulum observations, taken in great numbers at different parts of the earth, that the approximate formula for the intensity of gravity which we have given at § 91 has been deduced. Local peculiarities prevent the possibility of laying down any general formula with precision; and the exact value of g for any place can only be ascertained by observations on the spot.

CHAPTER IX.

CONSERVATION OF ENERGY.

121. Definition of Kinetic Energy.—We have seen in § 93 that the work which must be done upon a mass of m grammes to give it a velocity of v centimetres per second is $\frac{1}{2}mv^2$ ergs. Though we have proved this only for the case of falling bodies, with gravity as the working force, the result is true universally, as is shown in advanced treatises on mathematical physics. It is true whether the motion be rectilinear or curvilinear, and whether the working force act in the line of motion or at an angle with it.

If the velocity of a mass increases from v_1 to v_2, the work done upon it in the interval is $\frac{1}{2}m\ (v_2{}^2-v_1{}^2)$; in other words, is the increase of $\frac{1}{2}mv^2$.

On the other hand, if a force acts in such a manner as to oppose the motion of a moving mass, the force will do negative work, the amount of which will be equal to the decrease in the value of $\frac{1}{2}mv^2$.

For example, during any portion of the ascent of a projectile, the diminution in the value of $\frac{1}{2}mv^2$ is equal to gm multiplied by the increase of height; and during any portion of its descent the increase in $\frac{1}{2}mv^2$ is equal to gm multiplied by the decrease of height.

The work which must have been done upon a body to give it its actual motion, supposing it to have been initially at rest, is called the *energy of motion* or the *kinetic energy* of the body. It can be computed by multiplying *half the mass by the square of the velocity.*

122. Definition of Static or Potential Energy.—When a body of mass m is at a height s above the ground, which we will suppose level, gravity is ready to do the amount of work gms upon it by making it fall to the ground. A body in an elevated position may therefore be regarded as a reservoir of work. In like manner a wound-up clock, whether driven by weights or by a spring, has

work stored up in it. In all these cases there is force between parts of a system tending to produce relative motion, and there is room for such relative motion to take place. There is force ready to act, and space for it to act through. Also the force is always the same in the same relative position of the parts. Such a system possesses energy, which is usually called *potential.* We prefer to call it *statical,* inasmuch as its amount is computed on statical principles alone.[1] Statical energy depends jointly on mutual force and relative position. Its amount in any given position is the amount of work which would be done by the forces of the system in passing from this position to the standard position. When we are speaking of the energy of a heavy body in an elevated position above level ground, we naturally adopt as the standard position that in which the body is lying on the ground. When we speak of the energy of a wound-up clock, we adopt as the standard position that in which the clock has completely run down. Even when the standard position is not indicated, we can still speak definitely of the difference between the energies of two given positions of a system; just as we can speak definitely of the difference of levèl of two given points without any agreement as to the datum from which levels are to be reckoned.

123. Conservation of Mechanical Energy.—When a frictionless system is so constituted that its forces are always the same in the same positions of the system, the amount of work done by these forces during the passage from one position A to another position B will be independent of the path pursued, and will be equal to *minus* the work done by them in the passage from B to A. The earth and any heavy body at its surface constitute such a system; the force of the system is the mutual gravitation of these two bodies; and the work done by this mutual gravitation, when the body is moved by any path from a point A to a point B, is equal to the weight of the body multiplied by the height of A above B. When the system passes through any series of movements beginning with a given position and ending with the same position again, the algebraic total of work done by the forces of the system in this series of movements is zero. For instance, if a heavy body be carried by a roundabout path back to the point from whence it started, no work is done upon it by gravity upon the whole.

Every position of such a system has therefore a definite amount

[1] That is to say, the computation involves no reference to the laws of motion.

of statical energy, reckoned with respect to an arbitrary standard position. The work done by the forces of the system in passing from one position to another is (by definition) equal to the loss of static energy; but this loss is made up by an equal gain of kinetic energy. Conversely if kinetic energy is lost in passing from one position to another, the forces do negative work equal to this loss, and an equal amount of static energy is gained. The total energy of the system (including both static and kinetic) therefore remains unaltered.

An approximation to such a state of things is exhibited by a pendulum. In the two extreme positions it is at rest, and has therefore no kinetic energy; but its statical energy is then a maximum. In the lowest position its motion is most rapid; its kinetic energy is therefore a maximum, but its statical energy is zero. The difference of the statical energies of any two positions, will be the weight of the pendulum multiplied by the difference of levels of its centre of gravity, and this will also be the difference (in inverse order) between the kinetic energies of the pendulum in these two positions.

As the pendulum is continually setting the air in motion and thus doing external work, it gradually loses energy and at last comes to rest, unless it be supplied with energy from a clock or some other source. If a pendulum could be swung in a perfect vacuum, with an entire absence of friction, it would lose no energy, and would vibrate for an indefinite time without decrease of amplitude.

124. Illustration from Pile-driving.—An excellent illustration of transformations of energy is furnished by pile-driving. A large mass of iron called a *ram* is slowly hauled up to a height of several yards above the pile, and is then allowed to fall upon it. During the ascent, work must be supplied to overcome the force of gravity; and this work is represented by the statical energy of the ram in its highest position. While falling, it continually loses statical and gains kinetic energy; the amount of the latter which it possesses immediately before the blow being equal to the work which has been done in raising it. The effect of the blow is to drive the pile through a small distance against a resistance very much greater than the weight of the ram; the work thus done being nearly equal to the total energy which the ram possessed at any point of its descent. We say *nearly* equal, because a portion of the energy of the blow is spent in producing vibrations.

125. Hindrances to Availability of Energy.—There is almost

always some waste in utilizing energy. When water turns a mill-wheel, it runs away from the wheel with a velocity, the square of which multiplied by half the mass of the water represents energy which has run to waste.

Friction again often consumes a large amount of energy; and in this case we cannot (as in the preceding one) point to any palpable motion of a mass as representing the loss. Heat, however, is produced, and the energy which has disappeared as regarded from a gross mechanical point of view, has taken a molecular form. Heat is a form of molecular energy; and we know, from modern researches, what quantity of heat is equivalent to a given amount of mechanical work. In the steam-engine we have the converse process; mechanical work is done by means of heat, and heat is destroyed in the doing of it, so that the amount of heat given out by the engine is less than the amount supplied to it.

The sciences of electricity and magnetism reveal the existence of other forms of molecular energy; and it is possible in many ways to produce one form of energy at the expense of another; but in every case there is an exact equivalence between the quantity of one kind which comes into existence and the quantity of another kind which simultaneously disappears. Hence the problem of constructing a self-driven engine, which we have seen to be impossible in mechanics, is equally impossible when molecular forms of energy are called to the inventor's aid.

Energy may be transformed, and may be communicated from one system to another; but it cannot be increased or diminished in total amount. This great natural law is called the *principle of the conservation of energy*.

CHAPTER X.

ELASTICITY.

126. Elasticity and its Limits.—There is no such thing in nature as an absolutely rigid body. All bodies yield more or less to the action of force; and the property in virtue of which they tend to recover their original form and dimensions when these are forcibly changed, is called *elasticity*. Most solid bodies possess almost perfect elasticity for small deformations; that is to say, when distorted, extended, or compressed, within certain small limits, they will, on the removal of the constraint to which they have been subjected, instantly regain almost completely their original form and dimensions. These limits (which are called the limits of elasticity) are different for different substances; and when a body is distorted beyond these limits, it takes a *set*, the form to which it returns being intermediate between its original form and that into which it was distorted.

When a body is distorted within the limits of its elasticity, the force with which it reacts is directly proportional to the amount of distortion. For example, the force required to make the prongs of a tuning-fork approach each other by a tenth of an inch, is double of that required to produce an approach of a twentieth of an inch; and if a chain is lengthened a twentieth of an inch by a weight of 1 cwt., it will be lengthened a tenth of an inch by a weight of 2 cwt., the chain being supposed to be strong enough to experience no permanent set from this greater weight. Also, within the limits of elasticity, equal and opposite distortions, if small, are resisted by equal reactions. For example, the same force which suffices to make the prongs of a tuning-fork approach by a twentieth of an inch, will, if applied in the opposite direction, make them separate by the same amount.

127. Isochronism of Small Vibrations.—An important consequence of these laws is, that when a body receives a slight distortion within the limits of its elasticity, the vibrations which ensue when the constraint is removed are isochronous. This follows from § 111, inasmuch as the accelerations are proportional to the forces, and are therefore proportional at each instant to the deformation at that instant.

128. Stress, Strain, and Coefficients of Elasticity.—A body which, like indian-rubber, can be subjected to large deformations without receiving a permanent set, is said to have wide limits of elasticity.

A body which, like steel, opposes great resistance to deformation, is said to have large coefficients of elasticity.

Any change in the shape or size of a body produced by the application of force to the body is called a *strain;* and an action of force tending to produce a strain is called a *stress.*

When a wire of cross-section A is stretched with a force F, the longitudinal stress is $\frac{F}{A}$; this being the intensity of force per unit area with which the two portions of the wire separated by any cross-section are pulling each other. If the length of the wire when unstressed is L and when stressed $L+l$, the longitudinal strain is $\frac{l}{L}$. A stress is always expressed in units of force per unit of area. A strain is always expressed as the ratio of two magnitudes of the same kind (in the above example, two lengths), and is therefore independent of the units employed.

The quotient of a stress by the strain (of a given kind) which it produces, is called a *coefficient* or *modulus of elasticity.* In the above example, the quotient $\frac{FL}{Al}$ is called *Young's modulus* of elasticity.

As the wire, while it extends lengthwise, contracts laterally, there will be another coefficient of elasticity obtained by dividing the longitudinal stress by the lateral strain.

It is shown, in special treatises, that a solid substance may have 21 independent coefficients of elasticity; but that when the substance is *isotropic,* that is, has the same properties in all directions, the number reduces to 2.

129. Volume-elasticity.—The only coefficient of elasticity possessed by liquids and gases is elasticity of volume. When a body of volume V is reduced by the application of uniform normal pressure over its whole surface to volume $V-v$, the volume-strain is $\frac{v}{V}$, and if this

effect is produced by a pressure of p units of force per unit of area, the elasticity of volume is the quotient of the stress p by the strain $\frac{v}{V}$, or is $\frac{pV}{v}$. This is also called the *resistance to compression;* and its reciprocal $\frac{v}{pV}$ is called the *compressibility* of the substance. In dealing with gases, p must be understood as a pressure superadded to the original pressure of the gas.

Since a strain is a mere numerical quantity, independent of units, a coefficient of elasticity must be expressed, like a stress, in units of force per unit of area. In the C.G.S. system, stresses and coefficients of elasticity are expressed in dynes per square centimetre. The following are approximate values (thus expressed) of the two coefficients of elasticity above defined:—

	Young's Modulus.	Elasticity of Volume
Glass (flint),	60×10^{10}	40×10^{10}
Steel,	210×10^{10}	180×10^{10}
Iron (wrought),	190×10^{10}	140×10^{10}
Iron (cast),	130×10^{10}	96×10^{10}
Copper,	120×10^{10}	160×10^{10}
Mercury,		26×10^{10}
Water,		2×10^{10}
Alcohol,		$1{\cdot}2 \times 10^{10}$

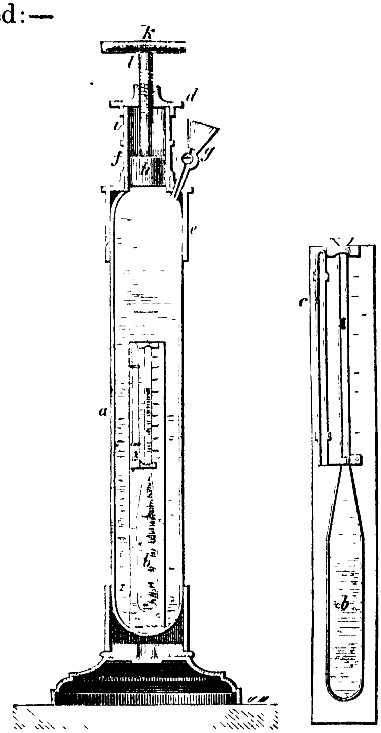


Fig. 49 —Œrsted's Piezometer.

130. Œrsted's Piezometer.—The compression of liquids has been observed by means of Œrsted's piezometer, which is represented in Fig. 49. The liquid whose compression is to be observed is contained in a glass vessel b, resembling a thermometer with a very large bulb and short tube. The tube is open above, and a globule of mercury at the top of the liquid column serves as an index. This apparatus is placed in a very strong glass vessel a full of water. When pressure is exerted by means of the piston klh, the index of mercury is seen to descend, showing a diminution of volume of the liquid, and showing moreover that this diminution of volume exceeds that of the containing vessel b. It might at first

sight appear that since this vessel is subjected to equal pressure within and without, its volume is unchanged; but in fact, its volume is altered to the same extent as that of a solid vessel of the same material; for the interior shells would react with a force precisely equivalent to that which is exerted by the contained liquid.

CHAPTER XI.

FRICTION.

131. Friction, Kinetical and Statical.—When two bodies are pressed together in such a manner that the direction of their mutual pressure is not normal to the surface of contact, the pressure can be resolved into two parts, one normal and the other tangential. The tangential component is called the *force of friction* between the two bodies. The friction is called *kinetical* or *statical* according as the bodies are or are not sliding one upon the other.

As regards kinetical friction, experiment shows that if the normal pressure between two given surfaces be changed, the tangential force changes almost exactly in the same proportion; in other words, the ratio of the force of friction to the normal pressure is nearly constant for two given surfaces. This ratio is called the *coefficient of kinetical friction* between the two surfaces, and is nearly independent of the velocity.

132. Statical Friction. Limiting Angle.—It is obvious that the statical friction between two given surfaces is zero when their mutual pressure is normal, and increases with the obliquity of the pressure if the normal component be preserved constant. The obliquity, however, cannot increase beyond a certain limit, depending on the nature of the bodies, and seldom amounting to so much as 45°. Beyond this limit sliding takes place. The limiting obliquity, that is, the greatest angle that the mutual force can make with the normal, is called the *limiting angle of friction* for the two surfaces; and the ratio of the tangential to the normal component when the mutual force acts at the limiting angle, is called the *coefficient of statical friction* for the two surfaces. The coefficient and limiting angle remain nearly constant when the normal force is varied.

The coefficient of statical friction is in almost every case greater

than the coefficient of kinetical friction; in other words, friction offers more resistance to the commencement of sliding than to the continuance of it.

A body which has small coefficients of friction with other bodies is called slippery.

133. Coefficient=tan θ. Inclined Plane.—If θ be the inclination of the mutual force P to the common normal, the tangential component will be P sin θ, the normal component P cos θ, and the ratio of the former to the latter will be tan θ. Hence *the coefficient of statical friction is equal to the tangent of the limiting angle of friction.*

When a heavy body rests on an inclined plane, the mutual pressure is vertical, and the angle θ is the same as the inclination of the plane. Hence if an inclined plane is gradually tilted till a body lying on it slides under the action of gravity, the inclination of the plane at which sliding begins is the limiting angle of friction between the body and the plane, and the tangent of this angle is the coefficient of statical friction.

Again, if the inclination of a plane be such that the motion of a body sliding down it under the action of gravity is neither accelerated nor retarded, the tangent of this inclination will be the coefficient of kinetical friction.

CHAPTER XII.

HYDROSTATICS.

134. Hydrodynamics.—We shall now treat of the laws of force as applied to fluids. This branch of the general science of dynamics is called *hydrodynamics* (ὕδωρ, water), and is divided into *hydrostatics* and *hydrokinetics*. Our discussions will be almost entirely confined to hydrostatics.

FLUIDS.—TRANSMISSION OF PRESSURE.

The name *fluid* comprehends both liquids and gases.

135. No Statical Friction in Fluids.—A fluid at rest cannot exert any tangential force against a surface in contact with it; its pressure at every point of such a surface is entirely normal. A slight tangential force is exerted by fluids in motion; and this fact is expressed by saying that all fluids are more or less *viscous*. An imaginary perfect fluid would be perfectly free from viscosity; its pressure against any surface would be entirely normal, whether the fluid were in motion or at rest.

136. Intensity of Pressure.—When pressure is uniform over an area, the total amount of the pressure, divided by the area, is called the *intensity of the pressure*. The C.G.S. unit of intensity of pressure is a pressure of a *dyne on each square centimetre* of surface. A rough unit of intensity frequently used is the pressure of a pound per square inch. This unit varies with the intensity of gravity, and has an average value of about 69,000 C.G.S. units. Another rough unit of intensity of pressure frequently employed is "an atmosphere"—that is to say, the average intensity of pressure of the atmosphere at the surface of the earth. This is about 1,000,000 C.G.S. units.

The single word "pressure" is used sometimes to denote "amount of pressure" (which can be expressed in dynes) and sometimes "intensity of pressure" (which can be expressed in dynes per square centimetre). The context usually serves to show which of these two meanings is intended.

137. Pressure the Same in all Directions.—The intensity of pressure at any point of a fluid is the same in all directions; it is the same whether the surface which receives the pressure faces upwards, downwards, horizontally, or obliquely.

This equality is a direct consequence of the absence of tangential force between two contiguous portions of a fluid.

For in order that a small triangular prism of the fluid (its ends being right sections) may be in equilibrium, the pressures on its three faces must balance each other. But when three forces balance each other, they are proportional to the sides of a triangle to which they are perpendicular;[1] hence the *amounts* of pressure on the three faces are proportional to the faces, in other words the *intensities* of these three pressures are equal. As we can take two of the faces perpendicular to any two given directions, this proves that the pressures in all directions at a point are of equal intensity.

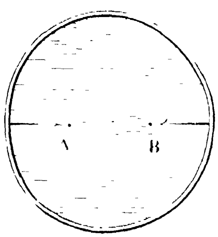


Fig. 50.

138. Pressure the Same at the Same Level.—In a fluid at rest, the pressure is the same at all points in the same horizontal plane. This appears from considering the equilibrium of a horizontal cylinder AB (Fig. 50), of small sectional area, its ends being right sections. The pressures on the sides are normal, and therefore give no component in the direction of the length; hence the pressures on the ends must be equal in amount; but they act on equal areas; therefore their intensities are equal.

A horizontal surface in a liquid at rest may therefore be called a "surface of equal pressure."

139. Difference of Pressure at Different Levels.—The increase of pressure with depth, in a fluid of uniform density, can be investigated as follows:—Consider the equilibrium of a vertical cylinder mm' (Fig. 51), its ends being right sections. The pressures on its

[1] This is an obvious consequence of the triangle of forces (art. 14); for if the sides of a triangle are parallel to three forces, we have only to turn the triangle through a right angle, and its sides will then be perpendicular to the forces.

sides are normal, and therefore horizontal. The only vertical forces acting upon it are its own weight and the pressures on its ends, of which it is to be observed that the pressure on the upper end acts downwards and that on the lower end upwards. The pressure on the lower end therefore exceeds that on the upper end by an amount equal to the weight of the cylinder. If a be the sectional area, w the weight of unit volume of the liquid, and h the length of the cylinder, the volume of the cylinder is ha, and its weight wha, which must be equal to $(p-p')\,a$ if p, p' are the intensities of pressure on the lower and upper ends respectively. We have therefore

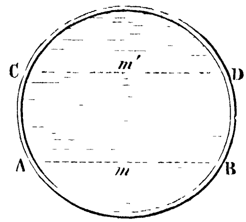


Fig 51.

$$p - p' = wh,$$

that is, *the increase of pressure in descending through a depth h is wh.*

The principles of this and the preceding section remain applicable whatever be the shape of the containing vessel, even if it be such as to render a circuitous route necessary in passing from one of two points compared to the other; for this route can always be made to consist of a succession of vertical and horizontal lines, and the preceding principles when applied to each of these lines separately, will give as the final result a difference of pressure wh for a difference of heights h.

If d denote the density of the liquid, in grammes per cub. cm., the weight of a cubic cm. will be gd dynes. The increase of pressure for an increase of depth h cm. is therefore ghd dynes per sq. cm. If there be no pressure at the surface of the liquid, this will be the actual pressure at the depth h.

140. **Free Surface.**—It follows from these principles that the free surface of a liquid at rest—that is, the surface in contact with the atmosphere—must be horizontal; since all points in this surface are at the same pressure. If the surface were not horizontal, but were higher at n than at n' (Fig. 52), the pressures at the two points m, m' vertically beneath them in any horizontal plane AB would be unequal, for they would be due to the weights

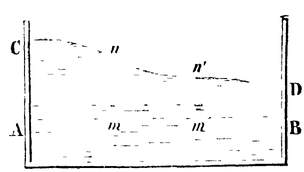


Fig 52.

of unequal columns *nm*, *n'm'*, and motion would ensue from *m* towards *m'*.

The same conclusion can be deduced from considering the equilibrium of a particle at the surface, as M (Fig. 53). If the tangent plane at M were not horizontal there would be a component of gravity tending to make the particle slide down; and this tendency would produce motion, since there is no friction to oppose it.

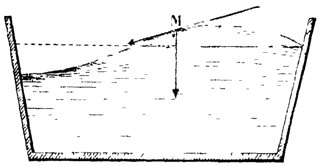


Fig 53.

141. Transmissibility of Pressure in Fluids.—Since the difference of the pressures at two points in a fluid can be determined by the foregoing principles, independently of any knowledge of the absolute intensity of either, it follows that when increase or diminution of pressure occurs at one point, an equal increase or diminution must occur throughout the whole fluid. *A fluid in a closed vessel perfectly transmits through its whole substance whatever pressure we apply to any part.* The changes in *amount* of pressure will be equal for all equal areas. For unequal areas they will be proportional to the areas.

Thus if the two vertical tubes in Fig. 54 have sectional areas which are as 1 to 16, a weight of 1 kilogram acting on the surface of the liquid in the smaller tube will be balanced by 16 kilograms acting on the surface of the liquid in the larger.

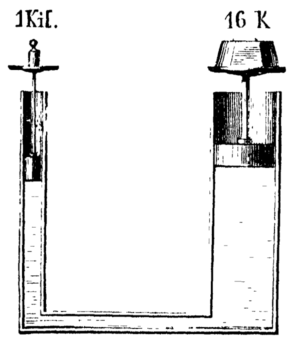


Fig. 54.—Principle of the Hydraulic Press.

This principle of the perfect transmission of pressure by fluids appears to have been first discovered and published by Stevinus; but it was rediscovered by Pascal a few years later, and having been made generally known by his writings is often called "Pascal's principle." In his celebrated treatise on the *Equilibrium of Liquids*, he says, "If a vessel full of water, closed on all sides, has two openings, the one a hundred times as large as the other, and if each be supplied with a piston which fits exactly, a man pushing the small piston will exert a force which will equilibrate that of a hundred men pushing the piston which is a hundred times as large.

and will overcome that of ninety-nine. And whatever may be the proportion of these openings, if the forces applied to the pistons are to each other as the openings, they will be in equilibrium."

142. **Hydraulic Press.**—This mode of multiplying force remained for a long time practically unavailable on account of the difficulty of making the pistons water-tight. The hydraulic press was first successfully made by Bramah, who invented the *cupped leather collar* illustrated in Fig. 166, § 264. Fig. 165 shows the arrangements of the press as a whole. Instead of pistons, *plungers* are employed; that is to say, solid cylinders of metal which can be pushed down into the liquid, or can be pushed up by the pressure of the liquid against their bases. The volume of liquid displaced by the advance of a plunger is evidently equal to that displaced by a piston of the same sectional area, and the above calculations for pistons apply to plungers as well. The plungers work through openings which are kept practically water-tight by means of the cup-leather arrangement. The cup-leather, which is shown both in plan and section in Fig. 166, consists of a leather ring bent so as to have a semicircular section. It is fitted into a hollow in the interior of the sides of the opening, so that water leaking up along the circumference of the plunger will fill the concavity of the leather, and, by pressing on it, will produce a packing which fits more tightly as the pressure on the plunger increases.

143. **Principle of Work Applicable.**—In Fig. 54, when the smaller piston advances and forces the other back, the volume of liquid driven out of the smaller tube is equal to the sectional area multiplied by the distance through which the piston advances. In like manner, the volume of liquid driven into the larger tube is equal to its sectional area multiplied by the distance that its piston is forced back. But these two volumes are equal, since the same volume of liquid that leaves one tube enters the other. The distances through which the two pistons move are therefore inversely as their sectional areas, and hence are inversely as the amounts of pressure applied to them. The *work done* in pushing forward the smaller piston is therefore equal to the work done by the liquid in pushing back the larger. This was remarked by Pascal, who says—

"It is, besides, worthy of admiration that in this new machine we find that constant rule which is met with in all the old ones such as the lever, wheel and axle, screw, &c., which is that the distance is increased in proportion to the force; for it is evident that

as one of these openings is a hundred times as large as the other, if the man who pushes the small piston drives it forward one inch, he will drive the large piston backward only one-hundredth part of that length."

144. Experiment on Upward Pressure.—The upward pressure exerted by a liquid against a horizontal surface facing downwards can be exhibited by the following experiment. Take a tube open at both ends (Fig. 55), and keeping the lower end covered with a piece of card, plunge it into water. The liquid will press the card against the bottom of the tube with a force which increases as it is plunged deeper. If water be now poured into the tube, the card will remain in its place as long as the level of the liquid is lower within the tube than without; but at the moment when equality of levels is attained it will become detached.

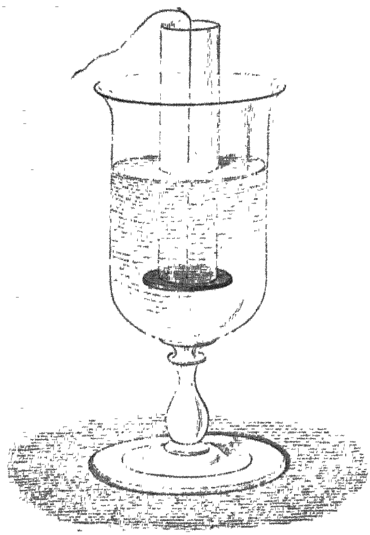


Fig. 55.—Upward Pressure.

145. Liquids in Superposition.—When one liquid rests on the top of another of different density, the foregoing principles lead to the result that the surface of demarcation must be horizontal. For the free surface of the upper liquid must, as we have seen, be horizontal. If now we take two small equal areas n and n' (Fig. 56) in a horizontal layer of the lower liquid, they must be subjected to equal pressures. But these pressures are measured by the weights of the liquid cylinders nrs, $n'tl$; and these latter cannot be equal unless the points r and t at the junction of the two liquids are at the same level. All points in the surface of demarcation are therefore in the same horizontal plane.

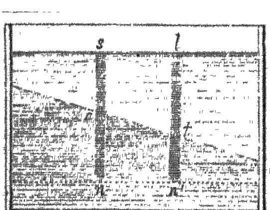


Fig 56.

The same reasoning can be extended downwards to any number of liquids of unequal densities, which rest one upon another, and shows that all the surfaces of demarcation between them must be horizontal.

An experiment in illustration of this result is represented in Fig. 57. Mercury, water, and oil are poured into a glass jar. The mercury, being the heaviest, goes to the bottom; the oil, being the lightest, floats at the top; and the surfaces of contact of the liquids are seen to be horizontal.

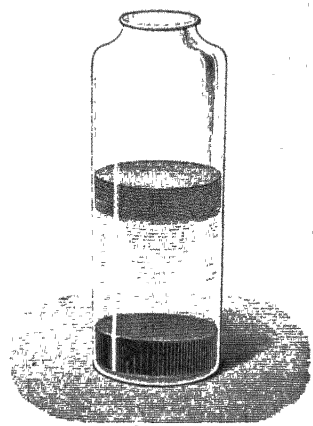


Fig 57
Phial of the Four Elements

Even when liquids are employed which gradually mix with one another, as water and alcohol, or fresh water and salt water, so that there is no definite surface of demarcation, but a gradual increase of density with depth, it still remains true that the density at all points in a horizontal plane is the same.

146. Two Liquids in Bent Tube.—If we pour mercury into a bent tube open at both ends (Fig. 58), and then pour water into one of the arms, the heights of the two liquids above the surface of junction will be very unequal, as shown in the figure. The general rule for the equilibrium of any two liquids in these circumstances is that *their heights above the surface of junction must be inversely as their densities*, since they correspond to equal pressures.

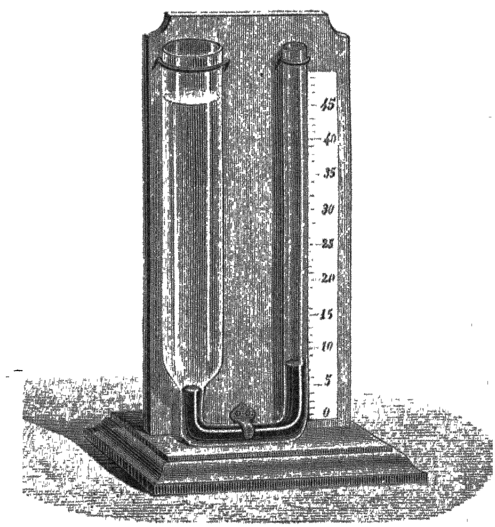


Fig. 58 —Equilibrium of Two Fluids in Communicating Vessels

147. Experiment of Pascal's Vases.—Since the amount of pressure on a horizontal area A at the depth h in a liquid is whA, where w denotes the weight of unit volume of the liquid, it follows that the pressure on the bottom of a vessel containing liquid is not affected by the breadth or narrowness of the upper part of the

vessel, provided the height of the free surface of the liquid be given. Pascal verified this fact by an experiment which is frequently exhibited in courses of physics. The apparatus employed (Fig. 59) is a tripod supporting a ring, into which can be screwed three vessels of different shapes, one widened upwards, another cylindrical, and the third tapering upwards. Beneath the ring is a movable disc

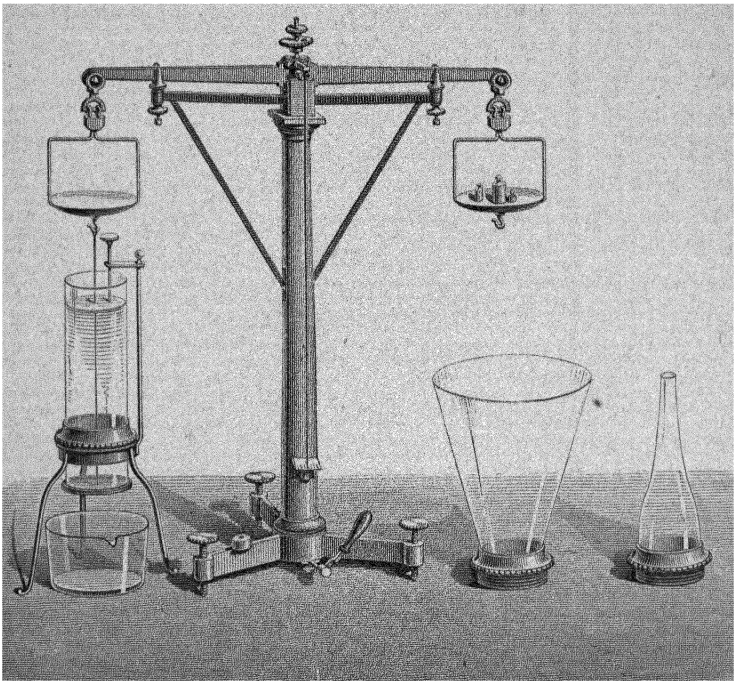


Fig. 59.—Experiment of Pascal's Vases.

supported by a string attached to one of the scales of a balance. Weights are placed in the other scale in order to keep the disc pressed against the ring. Let the cylindrical vase be mounted on the tripod, and filled up with water to such a level that the pressure is just sufficient to detach the disc from the ring. An indicator, shown in the figure, is used to mark the level at which this takes place. Let the experiment be now repeated with the two other vases, and the disc will be detached when the water has reached the same level as before.

In the case of the cylindrical vessel, the pressure on the bottom is evidently equal to the weight of the liquid. Hence in all three

cases the pressure on the bottom of the vessel is equal to the weight of a cylindrical column of the liquid, having the bottom as its base, and having the same height as the liquid in the vessel.

°148. **Resultant Pressure on Vessel.**—The pressure exerted by the bottom of the vessel upon the stand on which it rests, consists of the weight of the vessel itself, together with the resultant pressure of the contained liquid against it. The actual pressure of the liquid against any portion of the vessel is normal to this portion, and if we resolve it into two components, one vertical and the other horizontal, only the vertical component need be attended to, in computing the resultant; for the horizontal components will always destroy one another. At such points as n, n' (Fig. 60) the vertical component is downwards; at s and s' it is upwards; at r and r' there is no vertical component; and at AB the whole pressure is vertical. It can be demonstrated mathematically that the resultant pressure is always equal to the total weight of the contained liquid; a conclusion which can also be deduced from the consideration that the pressure exerted by the vessel upon the stand on which it rests must be equal to its own weight together with that of its contents.

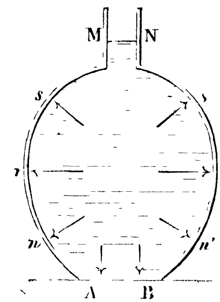


Fig 60 —Total Pressure

Some cases in which the proof above indicated becomes especially obvious, are represented in Fig. 61. In the cylindrical vessel ABDC, it is evident that the only pressure transmitted to the stand is that exerted upon the bottom, which is equal to the weight of the liquid. In the case of the vessel which is wider at the top, the stand is subjected to the weight of the liquid column ABSK, which presses on the bottom AB, together with the columns GHKC, RLDS, pressing on GH and RL; the sum of which weights composes the total weight of liquid contained in the vessel. Finally, in the third case, the pressure on the bottom AB, which is equal to the weight of a liquid column ABSK, must be diminished by the

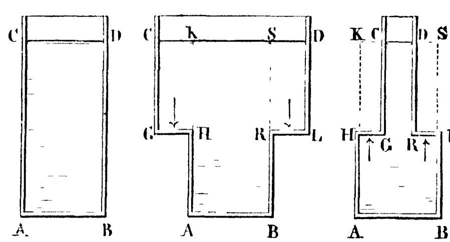


Fig 61 —Hydrostatic Paradox.

upward pressures on HG and RL. These last being represented by liquid columns HGCK, RLSD, there is only left to be transmitted to the stand a pressure equal to the weight of the water in the vessel.

§ 149. **Back Pressure in Discharging Vessel.**—The same analysis which shows that the resultant vertical pressure of a liquid against the containing vessel is equal to the weight of the liquid, shows also that the horizontal components of the pressures destroy one another. This conclusion is in accordance with everyday experience. However susceptible a vessel may be of horizontal displacement, it is not found to acquire any tendency to horizontal motion by being filled with a liquid.

When a system of forces are in equilibrium, the removal of one of them destroys the equilibrium, and causes the resultant of the system to be a force equal and opposite to the force removed. Accordingly if we remove an element of one side of the containing vessel, leaving a hole through which the liquid can flow out, the remaining pressure against this side will be insufficient to preserve equilibrium, and there will be an excess of pressure in the opposite direction.

This conclusion can be directly verified by the experiment represented in Fig. 62. A tall floating vessel of water is fitted with a horizontal discharge-pipe on one side near its base. The vessel is to be filled with water, and the discharge-pipe opened while the vessel is at rest. As the water flows out, the vessel will be observed to acquire a velocity, at first very slow, but continually increasing, in the opposite direction to that of the issuing stream.

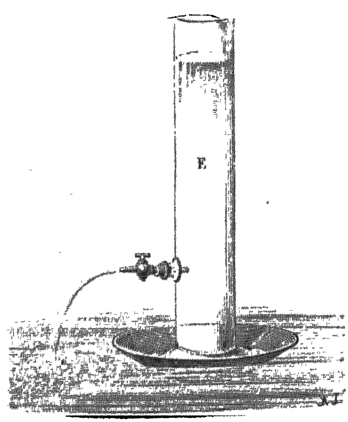


Fig. 62 Backward Movement of Discharging Vessel.

This experiment may also be regarded as an illustration of the law of action and reaction, which asserts that momentum cannot be imparted to any body without equal and opposite momentum being imparted to some other body. The water in escaping from the vessel acquires horizontal momentum in one direction, and the vessel with its remaining contents acquires horizontal momentum in the opposite direction.

The movements of the vessel in this experiment are slow. More marked effects of the same kind can be obtained by means of the hydraulic tourniquet (Fig. 63), which when made on a larger scale is called Barker's mill. It consists of a vessel of water free to rotate about a vertical axis, and having at its lower end bent arms through which the water is discharged horizontally, the direction of discharge being nearly at right angles to a line joining the discharging orifice to the axis. The unbalanced pressures at the bends of the tube, opposite to the openings, cause the apparatus to revolve in the opposite direction to the issuing liquid.

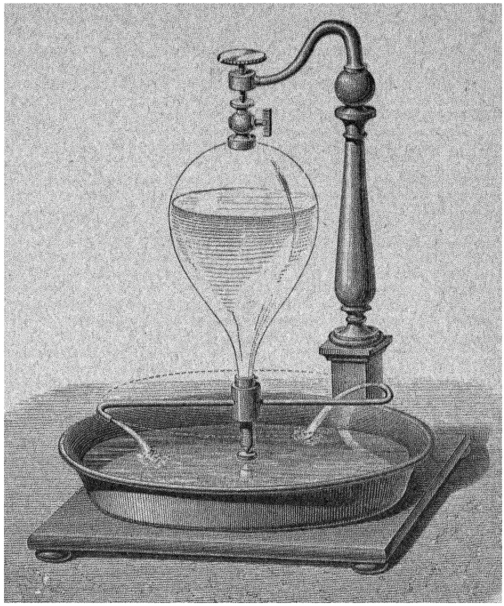


Fig 63 - Hydraulic Tourniquet.

150. Total and Resultant Pressures. Centre of Pressure.—The intensity of pressure on an area which is not horizontal is greatest on those parts which are deepest, and the average intensity can be shown to be equal to the actual intensity at the centre of gravity of the area. Hence if A denote the area, h the depth of its centre of gravity, and w the weight of unit volume of the liquid, the total pressure will be $w\,Ah$. Strictly speaking, this is the pressure due to the weight of the liquid, the transmitted atmospheric pressure being left out of account.

In attaching numerical values to w, A, and h, the same unit of length must be used throughout. For example, if h be expressed in feet, A must be expressed in square feet, and w must stand for the weight of a cubic foot of the liquid.

When we employ the centimetre as the unit of length, the value

of w will be sensibly 1 gramme if the liquid be water, so that the amount of pressure in grammes will be simply the product of the depth of the centre of gravity in centimetres by the area in square centimetres. For any other liquid, the pressure will be found by multiplying this product by the specific gravity of the liquid.

These rules for computing total pressure hold for areas of all forms, whether plane or curved; but the investigation of the total pressure on an area which is not plane is a mere mathematical exercise of no practical importance; for as the elementary pressures in this case are not parallel, their sum (which is the total pressure) is not the same thing as their resultant.

For a plane area, in whatever position, the elementary pressures, being everywhere normal to its plane, are parallel and give a resultant equal to their sum; and it is often a matter of interest to determine that point in the area through which the resultant passes. This point is called the *Centre of Pressure.* It is not coincident with the centre of gravity of the area unless the pressure be of equal intensity over the whole area. When the area is not horizontal, the pressure is most intense at those parts of it which are deepest, and the centre of pressure is accordingly lower down than the centre of gravity. For a horizontal area the two centres are coincident, and they are also sensibly coincident for any plane area whose dimensions are very small in comparison with its depth in the liquid, for the pressure over such an area is sensibly uniform.

151. **Construction for Centre of Pressure.**—If at every point of a plane area immersed in a liquid, a normal be drawn, equal to the depth of the point, the normals will represent the intensity of pressure at the respective points, and the volume of the solid constituted by all the normals will represent the total pressure. That normal which passes through the centre of gravity of this solid will be the line of action of the resultant, and will therefore pass through the centre of pressure.

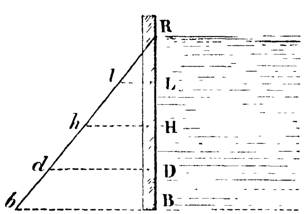


Fig. 64.—Centre of Pressure

Thus, if RB (Fig. 64) be a rectangular surface (which we may suppose to be the surface of a flood-gate or of the side of a dam), its lower side B being at the bottom of the water and its upper side R at the top, the pressure is zero at R and goes on increasing uniformly to B. The normals B*b*, D*d*, H*h*, L*l*, equal to the depths of a

series of points in the line BR will have their extremities b, d, h, l, in one straight line. To find the centre of pressure, we must find the centre of gravity of the triangle RBb and draw a normal through it. As the centre of gravity of a triangle is at one-third of its height, the centre of pressure will be at one-third of the height of BR. It will lie on the line joining the middle points of the upper and lower sides of the rectangle, and will be at one-third of the length of this line from its lower end.

The total pressure will be equal to the weight of a quantity of the liquid whose volume is equal to that of the triangular prism constituted by the aggregate of the normals, of which prism the triangle RBb is a right section. It is not difficult to show that the volume of this prism is equal to the product of the area of the rectangle by the depth of the centre of gravity of the rectangle, in accordance with the rule above given.

When the plane area has the form of a triangle, with one corner in the surface of the liquid and the opposite side horizontal, the solid constituted by the normals drawn as above directed will be a pyramid, and as the centre of gravity of a pyramid is three-fourths of the way down a line from the vertex to the centre of gravity of the base, the centre of pressure will be three-fourths of the way down the line which joins the vertex of the triangle to the middle point of the opposite side.

When the plane area is a triangle with one of its sides in the surface of the liquid, it can be shown that the centre of pressure is at the middle point of the line which joins the bisection of this side to the opposite corner. In fact, if the triangle be divided into narrow strips of equal width, by lines parallel to the side which is in the surface, the amounts of pressure on two strips equidistant respectively from the top and bottom are equal, for the areas of the two strips are directly as their distances from the bottom, and the intensities of pressure upon them are directly as their distances from the top.

152. Whirling Vessel. D'Alembert's Principle.—If an open vessel of liquid is rapidly rotated round a vertical axis, the surface of the liquid assumes a concave form, as represented in Fig. 65, where the dotted line is the axis of rotation. When the rotation has been going on at a uniform rate for a sufficient time, the liquid mass rotates bodily as if its particles were rigidly connected together, and when this state of things has been attained the form of the

surface is that of a paraboloid of revolution, so that the section represented in the figure is a parabola.

The usual mode of discussing the case is to treat it as one of statical equilibrium under the joint action of gravity and a fictitious force called centrifugal force, the latter force being, for each particle, equal and opposite to that which would produce the actual acceleration of the particle. This so-called centrifugal force is therefore to be regarded as a force directed radially outwards from the axis; and by compounding the centrifugal force of each particle with its weight we shall obtain what we are to treat as the resultant force on that particle. The form of the surface will then be determined by the condition that *at every point of the surface the normal must coincide with this resultant force;* just as in a liquid at rest, the normals must coincide with the direction of gravity.

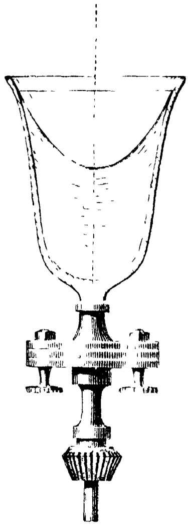


Fig. 65.—Rotating Vessel of Liquid

To show that the curve is a parabola; let P be any point upon it, PG the normal (terminated by the axis), and PN the perpendicular on the axis. The resultant of gravity and centrifugal force at P must be along GP. The intensity of gravity is g, and the intensity of centrifugal force is $(2\pi/T)^2$ NP, as indicated in equation (5), page 60. These, by the parallelogram of forces, must be as GN to NP. Hence we find GN $= g\ (T/2\pi)^2$, which is independent of the position of P. That is, the subnormal is constant; hence the curve is a parabola.

The plan here adopted of introducing fictitious forces equal and opposite to those which if directly applied to each particle of a system would produce the actual accelerations, and then applying the conditions of statical equilibrium, is one of very frequent application, and will always lead to correct results. This is known as D'Alembert's Principle.

152A. Conical Pendulum.—A proof similar to the above shows that, when a heavy particle suspended by a weightless thread revolves in a horizontal circle (so that the thread describes a cone), the vertical height of the point of suspension above the heavy particle is $g\ (T/2\pi)^2$, and is therefore independent of the length of the thread. An arrangement of this kind is called a *conical pendulum.*

CHAPTER XIII.

PRINCIPLE OF ARCHIMEDES.

153. Pressure of Liquids on Bodies Immersed.—When a body is immersed in a liquid, the different points of its surface are subjected to pressures which obey the rules laid down in the preceding chapter. As these pressures increase with the depth, those which tend to raise the body exceed those which tend to sink it, so that the resultant effect is a force in the direction opposite to that of gravity.

By resolving the pressure on each element into horizontal and vertical components, it can be shown that this resultant upward force is exactly equal to the weight of the liquid displaced by the body.

The reasoning is particularly simple in the case of a right cylinder (Fig. 66) plunged vertically in a liquid. It is evident, in the first place, that if we consider any point on the sides of the cylinder, the normal pressure on that point is horizontal and is destroyed by the equal and contrary pressure at the point diametrically opposite; hence, the horizontal pressures destroy each other. As regards the vertical pressures on the ends, one of them, that on the upper end AB, is in a downward direction, and equal to the weight of the liquid column ABNN; the other, that on the lower end CD, is in an upward direction, and equal to the weight of the liquid column CNND; this latter pressure exceeds the former by the weight of the liquid cylinder ABDC, so that the resultant effect of the pressure is to raise the body with a force equal to the weight of the liquid displaced.

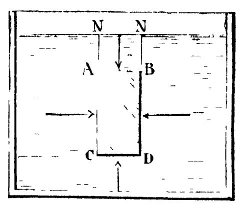


Fig 66 —Principle of Archimedes

By a synthetic process of reasoning, we may, without having recourse to the analysis of the different pressures, show that this conclusion is perfectly general. Suppose we have a liquid mass in equilibrium, and that we consider specially the portion M (Fig. 67); this portion is likewise in equilibrium. If we suppose it to become solid, without any change in its weight or volume, equilibrium will still subsist. Now this is a heavy mass, and as it does not fall, we must conclude that the effect of the pressures on its surface is to produce a resultant upward pressure exactly equal to its weight, and acting in a line which passes through its centre of gravity. If we now suppose M replaced by a body exactly occupying its place, the exterior pressures will remain the same, and their resultant effect will therefore be the same.

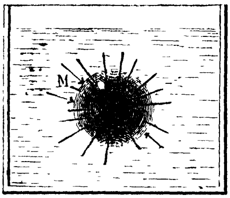


Fig. 67.—Principle of Archimedes.

The name *centre of buoyancy* is given to the centre of gravity of the liquid displaced,—that is, if the liquid be uniform, to the centre of gravity of the space occupied by the immersed body; and the above reasoning shows that the resultant pressure acts vertically upwards in a line which passes through this point. The results of the above explanations may thus be included in the following proposition: *Every body immersed in a liquid is subjected to a resultant pressure equal to the weight of the liquid displaced, and acting vertically upwards through the centre of buoyancy.*

This proposition constitutes the celebrated principle of Archimedes. The first part of it is often enunciated in the following form: *Every body immersed in a liquid loses a portion of its weight equal to the weight of the liquid displaced;* for when a body is immersed in a liquid, the force required to sustain it will evidently be diminished by a quantity equal to the upward pressure.

154. Experimental Demonstration of the Principle of Archimedes.— The following experimental demonstration of the principle of Archimedes is commonly exhibited in courses of physics:—

From one of the scales of a hydrostatic balance (Fig. 68) is suspended a hollow cylinder of brass, and below this a solid cylinder, whose volume is equal to the interior volume of the hollow cylinder; these are balanced by weights in the other scale. A vessel of water is then placed below the cylinders, in such a position that the lower cylinder shall be immersed in it. The equilibrium is immediately

destroyed, and the upward pressure of the water causes the scale with the weights to descend. If we now pour water into the hollow cylinder, equilibrium will gradually be re-established; and the beam

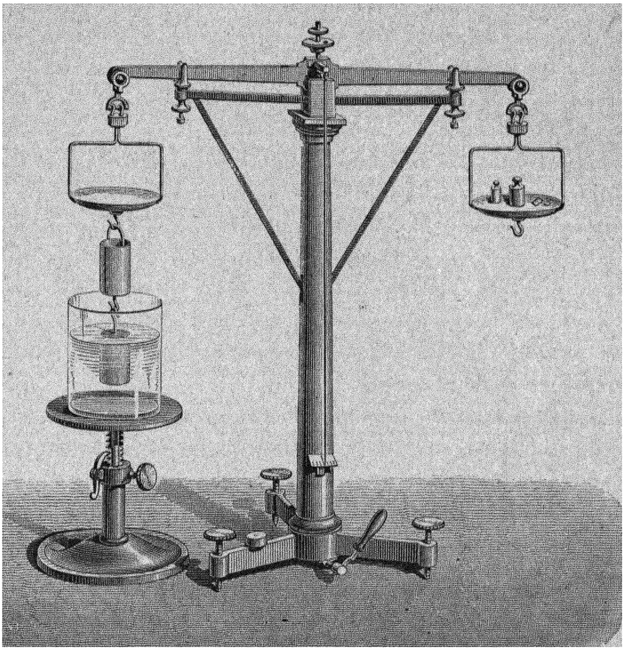


Fig. 68.—Experimental Verification of Principle of Archimedes

will be observed to resume its horizontal position when the hollow cylinder is full of water, the other cylinder being at the same time completely immersed. The upward pressure upon this latter is thus equal to the weight of the water added, that is, to the weight of the liquid displaced.

155. **Body Immersed in a Liquid.**—It follows from the principle of Archimedes that when a body is immersed in a liquid, it is subjected to two forces: one equal to its weight and applied at its centre of gravity, tending to make the body descend; the other equal to the weight of the displaced liquid, applied at the centre of buoyancy, and tending to make it rise. There are thus three different cases to be considered:

(1.) The weight of the body may exceed the weight of the liquid displaced, or, in other words, the mean density of the body may be

greater than that of the liquid; in this case, the body sinks in the liquid, as, for instance, a piece of lead dropped into water.

(2.) The weight of the body may be less than that of the liquid displaced; in this case the body will not remain submerged unless forcibly held down, but will rise partly out of the liquid, until the weight of the liquid displaced is equal to its own weight. This is what happens, for instance, if we immerse a piece of cork in water and leave it to itself.

(3.) The weight of the body may be equal to the weight of the liquid displaced; in this case, the two opposite forces being equal, the body takes a suitable position and remains in equilibrium.

These three cases are exemplified in the three following experiments (Fig. 69):—

(1.) An egg is placed in a vessel of water; it sinks to the bottom

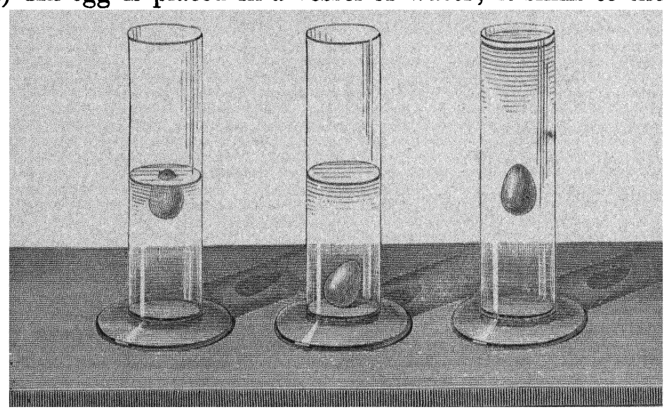


Fig 69 —Egg Plunged in Fresh and Salt Water.

of the vessel, its mean density being a little greater than that of the liquid.

(2.) Instead of fresh water, salt water is employed; the egg floats at the surface of the liquid, which is a little denser than it.

(3.) Fresh water is carefully poured on the salt water; a mixture of the two liquids takes place where they are in contact; and if the egg is put in the upper part, it will be seen to descend, and, after a few oscillations, remain at rest at such a depth that it displaces its own weight of the liquid. In speaking of the liquid displaced in this case, we must imagine each horizontal layer of liquid surrounding the egg to be produced through the space which the egg occupies; and by the centre of buoyancy we must understand the centre of

gravity of the portion of liquid which would thus take the place of the egg. We may remark that, in this position the egg is in stable equilibrium; for, if it rises, the upward pressure diminishing, its weight tends to make it descend again; if, on the contrary, it sinks, the pressure increases and tends to make it reascend.

156. Cartesian Diver.—The experiment of the *Cartesian diver*, which is described in old treatises on physics, shows each of the different cases that can present themselves when a body is immersed. The diver (Fig. 70) consists of a hollow ball, at the bottom of which is a small opening O; a little porcelain figure is attached to the ball, and the whole floats upon water contained in a glass vessel, the mouth of which is closed by a strip of caoutchouc or a bladder. If we press with the hand on the bladder, the air is compressed, and the pressure, transmitted through the different horizontal layers, condenses the air in the ball, and causes the entrance of a portion of the liquid by the opening O; the floating body becomes heavier, and in consequence of this increase of weight the diver descends. When we cease to press upon the bladder, the pressure becomes what it was before, some water flows out and the diver ascends. It must be observed, however, that as the diver continues to descend, more and more water enters the ball, in consequence of the increase of pressure, so that if the depth of the water exceeded a certain limit, the diver would not be able to rise again from the bottom.

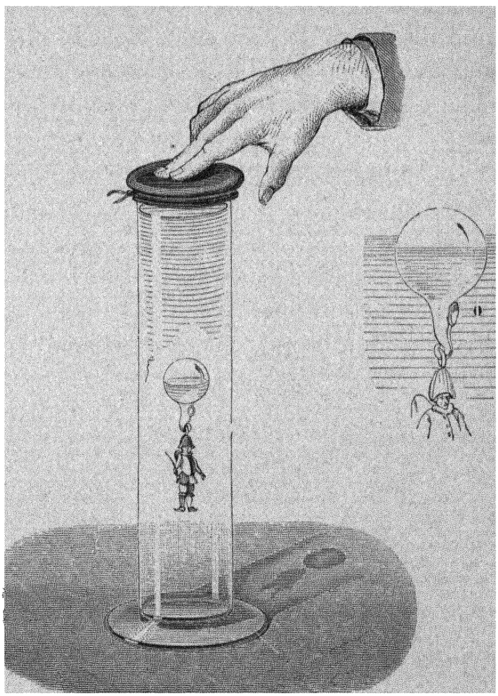


Fig 70.—Cartesian Diver.

If we suppose that at a certain moment the weight of the diver becomes exactly equal to the weight of an equal volume of the liquid, there will be equilibrium; but, unlike the equilibrium in the experiment (3) of last section, this will evidently be *unstable*, for a slight movement either upwards or downwards will alter the resultant force so as to produce further movement in the same direction. As a consequence of this instability, if the diver is sent down below a certain depth he will not be able to rise again.

157. Relative Positions of the Centre of Gravity and Centre of Buoyancy.—In order that a floating body either wholly or partially immersed in a liquid, may be in equilibrium, it is necessary that its weight be equal to the weight of the liquid displaced.

This condition is however not sufficient; we require, in addition, that the action of the upward pressure should be exactly opposite to that of the weight; that is, that the centre of gravity and the centre of buoyancy be in the same vertical line; for if this were not the case, the two contrary forces would compose a couple, the effect of which would evidently be to cause the body to turn.

In the case of a body completely immersed, it is further necessary for stable equilibrium that *the centre of gravity should be below* the centre of buoyancy; in fact we see, by Fig. 71, that in any other

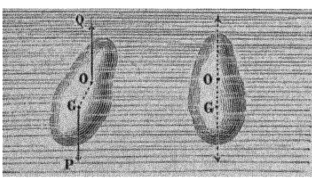


Fig. 71.

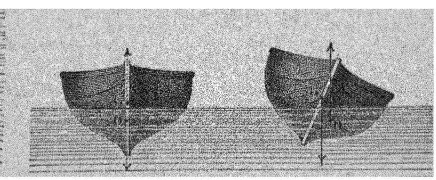


Fig. 72.

Relative Positions of Centre of Gravity and Centre of Pressure.

position than that of equilibrium, the effect of the two forces applied at the two points G and O would be to turn the body, so as to bring the centre of gravity lower, relatively to the centre of buoyancy. But this is not the case when the body is only partially immersed, as most frequently happens. In this case it may indeed happen that, with stable equilibrium, the centre of gravity is below the centre of pressure; but this is not necessary, and in the majority of instances is not the case. Let Fig. 72 represent the lower part of a floating body—a boat, for instance. The centre of pressure is at O, the centre of gravity at G, considerably above; if the body

is displaced, and takes the position shown in the figure, it will be seen that the effect of the two forces acting at O and at G is to restore the body to its former position. This difference from what takes place when the body is completely immersed, depends upon the fact that, in the case of the floating body, the figure of the liquid displaced changes with the position of the body, and the centre of buoyancy moves towards the side on which the body is more deeply immersed. It will depend upon the form of the body whether this lateral movement of the centre of buoyancy is sufficient to carry it beyond the vertical through the centre of gravity. The two equal forces which act on the body will evidently turn it to or from the original position of equilibrium, according as the new centre of buoyancy lies beyond or falls short of this vertical.[1]

158. Advantage of Lowering the Centre of Gravity.—Although stable equilibrium may subsist with the centre of gravity above the centre of buoyancy, yet for a body of given external form the stability is always increased by lowering the centre of gravity; as we thus lengthen the arm of the couple which tends to right the body when displaced. It is on this principle that the use of ballast depends.

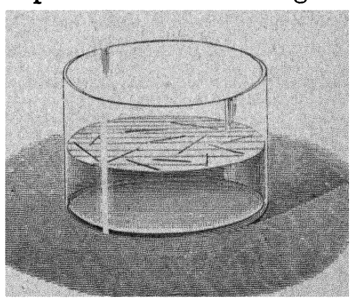


Fig 73.—Steel Needles Floating on Water

159. Phenomena in Apparent Contradiction to the Principle of Archimedes.—The principle of Archimedes seems at first sight to be contradicted by some well-known facts. Thus, for instance, if small needles are placed carefully on the surface of water, they will remain there in equilibrium (Fig. 73). It is on a similar principle

[1] If a vertical through the new centre of buoyancy be drawn upwards to meet that line in the body which in the position of equilibrium was a vertical through the centre of gravity, the point of intersection is called the *metacentre*. Evidently when the forces tend to restore the body to the position of equilibrium, the metacentre is above the centre of gravity; when they tend to increase the displacement, it is below. In ships the distance between these two points is usually nearly the same for all amounts of heeling, and this distance is a measure of the stability of the ship

We have defined the metacentre as the intersection of two lines. When these lines lie in different planes, and do not intersect each other, there is no metacentre. This indeed is the case for most of the displacements to which a floating body of irregular shape can be subjected. There are in general only two directions of heeling to which metacentres correspond, and these two directions are at right angles to each other.

that several insects *walk* on water (Fig. 74), and that a great number of bodies of various natures, provided they be *very minute*, can, if we may so say, be placed on the surface of a liquid without penetrating into its interior. These curious facts depend on the circumstance that the small bodies in question are not wetted by the liquid, and hence, in virtue of principles which will be explained in connection with capillarity (Chap. xvi.), depressions are formed around them on the liquid surface, as represented in Fig. 75. The curvature of the liquid surface in the neighbourhood of the body is very distinctly shown by observing the shadow cast by the floating body, when it is illumined by the sun; it is seen to be bordered by luminous bands, which are owing to the refraction of the rays of light in the portion of the liquid bounded by a curved surface.

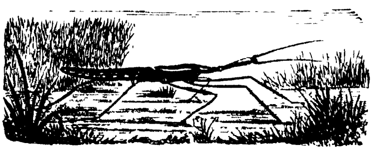


Fig. 74 —Insect Walking on Water

The existence of the depression about the floating body enables us to bring the condition of equilibrium in this special case under the general enunciation of the principle of Archimedes. Let M (Fig. 75) be the body, CD the region of the depression, and AB the corresponding portion of any horizontal layer; since the pressure at each point of AB must be the same as in other parts of the same horizontal layer, the total weight above AB is the same as if M did not exist and the cavity were filled with the liquid itself.

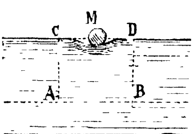


Fig. 75.

We may thus say in this case also that the weight of the floating body is equal to the weight of the *liquid displaced*, understanding by these words the liquid which would occupy the whole of the depression due to the presence of the body.

CHAPTER XIV.

DENSITY AND ITS DETERMINATION.

160. Definitions.—By the *absolute density* of a substance is meant the mass of unit volume of it. By the *relative density* is meant the ratio of its absolute density to that of some standard substance, or, what amounts to the same thing, the ratio of the mass of any volume of the substance in question to the mass of an equal volume of the standard substance. Since equal masses gravitate equally, the comparison of masses can be effected by weighing, and the relative density of a substance is the ratio of its weight to that of an equal volume of the standard substance. Water at a specified temperature and under atmospheric pressure is usually taken as the standard substance, and the density of a substance relative to water is usually called the *specific gravity* of the substance.

Let V denote the volume of a substance, M its mass, and D its absolute density; then by definition, we have $M=VD$.

If s denote the specific gravity of a substance, and d the absolute density of water in the standard condition, then $D=sd$ and $M=Vsd$.

When masses are expressed in lbs. and volumes in cubic feet, the value of d is about 62·4, since a cubic foot of cold water weighs about 62·4 lbs.[1]

In the C.G.S. system, the value of d is sensibly unity, since a cubic centimetre of water, at a temperature which is nearly that of the maximum density of water, weighs exactly a gramme.[2]

The gramme is defined, not by reference to water, but by a standard kilogramme of platinum, which is preserved in Paris, and

[1] In round numbers, a cubic foot of water weighs 1000 oz., which is 62·5 lbs.

[2] According to the best determination yet published, the mass of a cubic centimetre of pure water at 4° is 1·000013, at 3° is 1·000004, and at 2° is ·999982.

of which several very carefully made copies are preserved in other places. In the above statements (as in all very accurate statements of weights), the weighings are supposed to be made in vacuo; for the masses of two bodies are not accurately proportional to their apparent gravitations in air, unless the two bodies happen to have the same density.

161. Ambiguity of the word "Weight."—Properly speaking, "the weight of a body" means the force with which the body gravitates towards the earth. This force, as we have seen, differs slightly according to the place of observation. If m denote the mass of the body, and g the intensity of gravity at the place, the weight of the body is mg. When the body is carried from one place to another without gain or loss of material, m will remain constant and g will vary; hence the weight mg will vary, and in the same ratio as g.

But the employment of gravitation units of force instead of absolute units, obscures this fact. The unit of measurement varies in the same ratio as the thing to be measured, and hence the numerical value remains unaltered. A body weighs the same number of pounds or grammes at one place as at another, because the weights of the pound and gramme are themselves proportional to g. Expressed in absolute units, the weight of unit mass is g, and the weight of a mass m is mg. The latter is m times the former; hence when the weight of unit mass is employed as the unit of weight, the same number m which denotes the mass of a body also denotes its weight. What are usually called standard weights—that is, standard pieces of metal used for weighing—are really standards of mass; and when the result of a weighing is stated in terms of these standards, (as it usually is,) the "weight," as thus stated, is really the *mass* of the body weighed. The standard "weights" which we use in our balances are really standard masses. In discussions relating to density, weights are most conveniently expressed in gravitation measure, and hence the words mass and weight can be used almost indiscriminately.

162. Determination of Density from Weight and Volume.—The absolute density of a substance can be directly determined by weighing a measured volume of it. Thus if v cubic centimetres of it weigh m grammes, its density (in grammes per cubic centimetre) is $\frac{m}{v}$. This method can be easily applied to solids of regular geometrical forms; since their volumes can be computed from their

linear measurements. It can also be applied to liquids, by employing a vessel of known content. The bottle usually employed for this purpose is a bottle of thin glass fitted with a perforated stopper, so that it can be filled and stoppered without leaving a space for air. The difference between its weights when full and empty is the weight of the liquid which fills it; and the quotient of this by the volume occupied (which can be determined once for all by weighing the bottle when filled with water) is the density of the liquid.

The advantage of employing a perforated stopper is that it enables us to ensure constancy of volume. If a wide-mouthed flask were employed, without a stopper, it would be difficult to pronounce when the flask was exactly full. This source of inaccuracy would be diminished by making the mouth narrower: but when it is very narrow, the filling and emptying of the flask are difficult, and there is danger of forcing in bubbles of air with the liquid. When a perforated stopper is employed, the flask is first filled, then the stopper is inserted and some of the liquid is thus forced up through the perforation, overflowing at the top. When the stopper has been pushed home, all the liquid outside is carefully wiped off, and the liquid which remains is as much as just fills the stoppered flask including the perforation in the stopper.

In accurate work, the temperature must be observed, and due allowance made for its effect upon volume.

o **163. Specific Gravity Flask for Solids.**—The volume and density of a solid body of irregular shape, or consisting of a quantity of small pieces, can be determined by putting it into such a bottle (Fig. 76), and weighing the water which it displaces. The most convenient way of doing this is to observe (1) the weight of the solid; (2) the weight of the bottle full of water; (3) the weight of the bottle when it contains the solid, together with as much water as will fill it up. If the

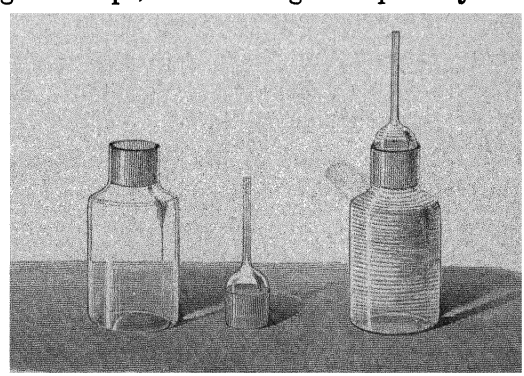


Fig 76.—Specific Gravity Flask for Solids

third of these results be subtracted from the sum of the first two, the remainder will be the weight of the water displaced; which, when expressed in grammes, is the same as the volume of the body expressed in cubic centimetres. The weight of the body, divided by this remainder, is the density of the body.

164. Method by Weighing in Water.—The methods of determining density which we are now about to describe depend upon the principle of Archimedes.

One of the commonest ways of determining the density of a solid body is to weigh it first in air and then in water (Fig. 77) the

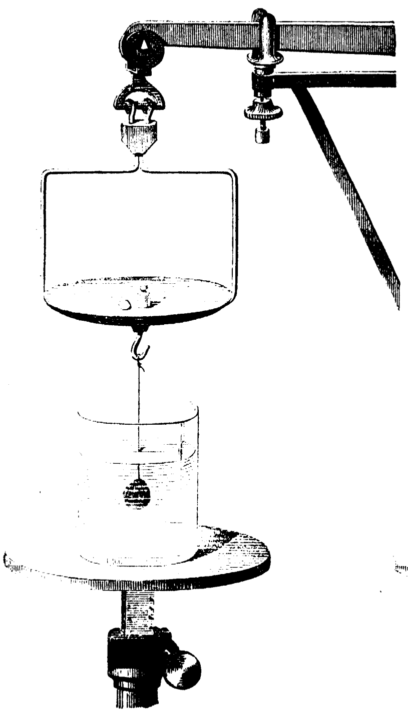


Fig. 77.—Specific Gravity of Solids.

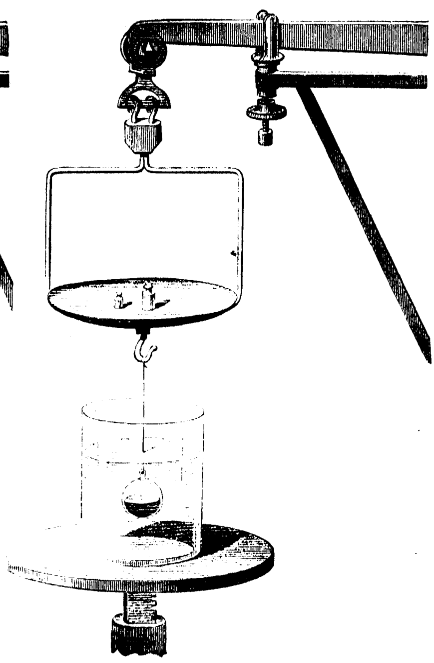


Fig. 78.—Specific Gravity of Liquids.

counterpoising weights being in air. Since the loss of weight due to its immersion in water is equal to the weight of the same volume of water, we have only to *divide the weight in air by this loss of weight.* We shall thus obtain the relative density of the body as compared with water—in other words, the specific gravity of the body.

Thus, from the observations

Weight in air,	125 *gm.*
Weight in water,	100 „
Loss of weight,	25 „

we deduce

$$\frac{125}{25} = 5 = \text{density.}$$

A very fine and strong thread or fibre should be employed for suspending the body, so that the volume of liquid displaced by this thread may be as small as possible.

165. Weighing in Water, with a Sinker.—If the body is lighter than water, we may employ a sinker—that is, a piece of some heavy material attached to it, and heavy enough to make it sink. It is not necessary to know the weight of the sinker in air, but we must observe its weight in water. Call this s Let w be the weight of the body in air, and w' the weight of the body and sinker together in water. Then w' will be less than s. The body has an apparent upward gravitation in water equal to $s-w'$, showing that the resultant pressure upon it exceeds its weight by this amount. Hence the weight of the liquid displaced is $w+s-w'$, and the specific gravity of the body is $\frac{w}{w+s-w'}$

If any other liquid than water be employed in the methods described in this and the preceding section, the result obtained will be the relative density as compared with that liquid. The result must therefore be multiplied by the density of the liquid, in order to obtain the absolute density

166. Density of Liquid Inferred from Loss of Weight.—The densities of liquids are often determined by observing the loss of weight of a solid immersed in them, and dividing by the known volume of the solid or by its loss of weight in water.

Thus, from the observations

Weight in air,	200 *gm*
Weight in liquid,	120 „
Weight in water,	110 „

we deduce

Loss in liquid, 80. Loss in water, 90.

$$\text{Density of liquid, } \frac{80}{90} = \frac{8}{9}.$$

A glass ball (sometimes weighted with mercury, as in Fig. 78) is the solid most frequently employed for such observations.

167. Measurement of Volumes of Solids by Loss of Weight.—The volume of a solid body, especially if of irregular shape, can usually be determined with more accuracy by weighing it in a liquid than by any other method. If it weigh w grammes in air, and w' grammes in water, its volume is $w-w'$ cubic centimetres, since it displaces $w-w'$ grammes of water. The mean diameter of a wire can be very accurately determined by an observation of this kind for volume, combined with a direct measurement of length. The volume divided by the length will be the mean sectional area, which is equal to πr^2, where r is the radius.

168. Hydrometers.—The name hydrometer is given to a class of instruments used for determining the densities of liquids by observing either the depths to which they sink in the liquids or the

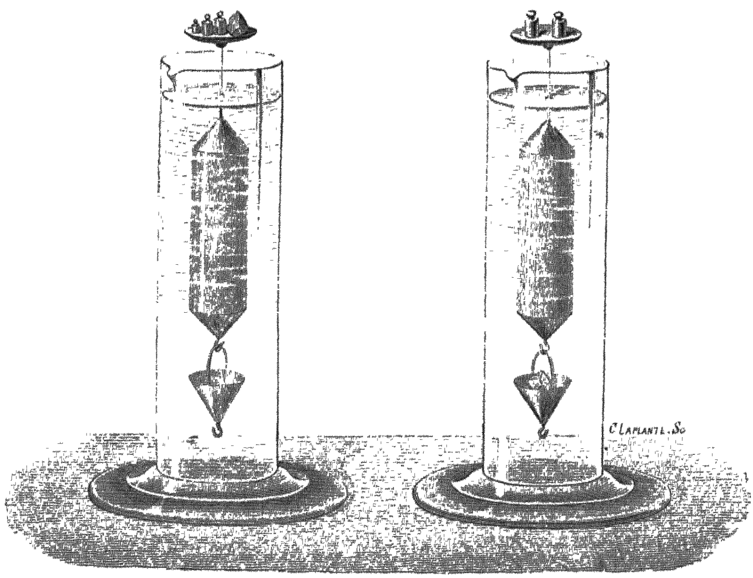


Fig. 79.—Nicholson's Hydrometer.

weights required to be attached to them to make them sink to a given depth. According as they are to be used in the latter or the former of these two ways, they are called hydrometers of constant or of variable immersion. The name areometer (from ἀραιός, rare) is used as synonymous with hydrometer, being probably borrowed from the French name of these instruments, *aréomètre*. The hydro-

meters of constant immersion most generally known are those of Nicholson and Fahrenheit.

169. **Nicholson's Hydrometer.**—This instrument, which is represented in Fig. 79, consists of a hollow cylinder of metal with conical ends, terminated above by a very thin rod bearing a small dish, and carrying at its lower end a kind of basket. This latter is of such weight that when the instrument is immersed in water a weight of 100 grammes must be placed in the dish above in order to sink the apparatus as far as a certain mark on the rod. By the principle of Archimedes, the weight of the instrument, together with the 100 grammes which it carries, is equal to the weight of the water displaced. Now, let the instrument be placed in another liquid, and the weights in the dish above be altered until they are just sufficient to make the instrument sink to the mark on the rod. If the weights in the dish be called w, and the weight of the instrument itself W, the weight of liquid displaced is now $W + w$, whereas the weight of the same volume of water was $W + 100$, hence the specific gravity of the liquid is $\frac{W + w}{W + 100}$.

This instrument can also be used either for weighing small solid bodies or for finding their specific gravities. To find the weight of a body (which we shall suppose to weigh less than 100 grammes), it must be placed in the dish at the top, together with weights just sufficient to make the instrument sink in water as far as the mark. Obviously these weights are the difference between the weight of the body and 100 grammes.

To find the specific gravity of a solid, we first ascertain its weight by the method just described; we then transfer it from the dish above to the basket below, so that it shall be under water during the observation, and observe what additional weights must now be placed in the dish. These additional weights represent the weight of the water displaced by the solid; and the weight of the solid itself divided by this weight is the specific gravity required.

170. **Fahrenheit's Hydrometer.**—This instrument, which is represented in Fig. 80, is generally constructed of glass, and differs from Nicholson's in having at its lower extremity a ball weighted with mercury instead of the basket. It resembles it in having a dish at the top, in which weights are to be placed sufficient to sink the instrument to a definite mark on the stem.

Hydrometers of constant immersion, though still described in text-books, have quite gone out of use for practical work.

171. **Hydrometers of Variable Immersion.**—These instruments are usually of the forms represented at A, B, C, Fig. 81. The lower end is weighted with mercury in order to make the instrument sink to a convenient depth and preserve an upright position. The stem is cylindrical, and is graduated, the divisions being frequently marked

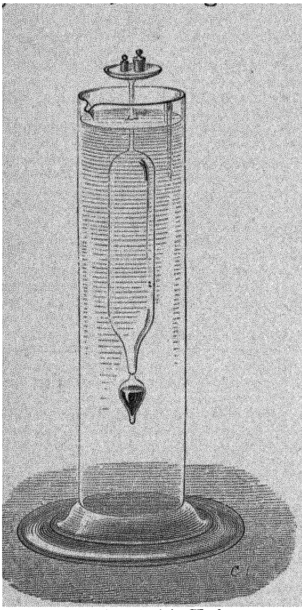


Fig. 80.—Fahrenheit's Hydrometer.

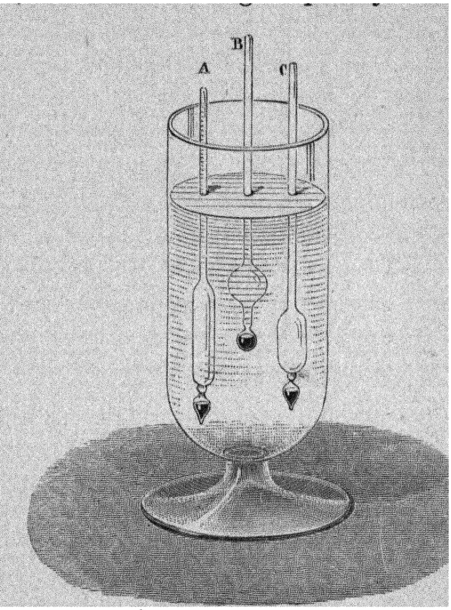


Fig [illegible] Hydrometers.

upon a piece of paper inclosed within the stem, which must in this case be of glass. It is evident that the instrument will sink the deeper the less is the specific gravity of the liquid, since the weight of the liquid displaced must be equal to that of the instrument. Hence if any uniform system of graduation be adopted, so that all the instruments give the same readings in liquids of the same densities, the density of a liquid can be obtained by a mere immersion of the hydrometer—an operation not indeed very precise, but very easy of execution. These instruments have thus come into general use for commercial purposes and in the excise.

172. **General Theory of Hydrometers of Variable Immersion.**—Let V be the volume of a hydrometer which is immersed when the instrument floats freely in a liquid whose density is d, then Vd repre-

sents the weight of liquid displaced, which by the principle of Archimedes is the same as the weight of the hydrometer itself. If V', d' be the corresponding values for another liquid, we have therefore

$$Vd = V'd', \text{ or } d : d' \therefore V' . V,$$

that is, the density varies inversely as the volume immersed. Let d_1, d_2, d_3...be a series of densities, and V_1, V_2, V_3...the corresponding volumes immersed, then we have

$$d_1, d_2, d_3 \quad \text{proportional to } \frac{1}{V_1}, \frac{1}{V_2}, \frac{1}{V_3} \cdots$$

$$\text{and } V_1, V_2, V_3 \quad \text{proportional to } \frac{1}{d_1}, \frac{1}{d_2}, \frac{1}{d_3} \cdots$$

Hence, if we wish the divisions to indicate equal differences of density, we must place them so that the corresponding volumes immersed form a harmonical progression. This implies that the distances between the divisions must diminish as the densities increase.

The following investigation shows how the density of a liquid may be computed from observations made with a hydrometer graduated with equal divisions. It is necessary first to know the divisions to which the instrument sinks in two liquids of known densities. Let these divisions be numbered n_1, n_2, reckoning from the top downwards, and let the corresponding densities be d_1, d_2. Now if we take for our unit of volume one of the equal parts on the stem, and if we take c to denote the volume which is immersed when the instrument sinks to the division marked zero, it is obvious that when the instrument sinks to the nth division (reckoned downwards on the stem from zèro) the volume immersed is $c-n$, and if the corresponding density be called d, then $(c-n)\ d$ is the weight of the hydrometer. We have therefore

$$(c-n_1)\ d_1 = (c-n_2)\ d_2, \text{ whence } c = \frac{n_1 d_1 - n_2 d_2}{d_1 - d_2}.$$

This value of c can be computed once for all.

Then the density D corresponding to any other division N can be found from the equation

$$(c-N)\ D = (c-n_1)\ d_1 \text{ which gives } D = \frac{c-n_1}{c-N} d_1.$$

173. **Beaumé's Hydrometers.**—In these instruments the divisions are equidistant. There are two distinct modes of graduation, according as the instrument is to be used for determining densities greater or less than that of water. In the former case the instrument is

called a salimeter, and is so constructed that when immersed in pure water of the temperature 12° Cent. it sinks nearly to the top of the stem, and the point thus determined is the zero of the scale. It is then immersed in a solution of 15 parts of salt to 85 of water, the density of which is about 1·116, and the point to which it sinks is marked 15. The interval is divided into 15 equal parts, and the graduation is continued to the bottom of the stem, the length of which varies accord ing to circumstances; it generally terminates at the degree 66, which corresponds to sulphuric acid, whose density is commonly the greatest that it is required to determine. Referring to the formulæ of last section, we have here

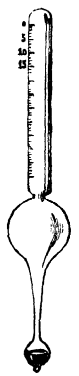


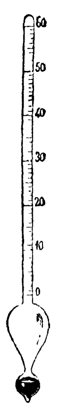
Fig. 82. Baumé's Salimeter.

$$n_1=o,\ d_1=1,\ n_2=15,\ d_2=1{\cdot}116;$$

whence

$$c=\frac{15\times 1{\cdot}116}{{\cdot}116}=144,\ D=\frac{144}{144-N}.$$

When the instrument is intended for liquids lighter than water, it is called an alcoholimeter. In this case the point to which it sinks in water is near the bottom of the stem, and is marked 10; the zero of the scale is the point to which it sinks in a solution of 10 parts of salt to 90 of water, the density of which is about 1·085, the divisions in this case being numbered upward from zero.



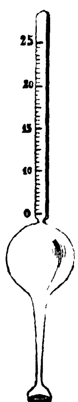


Fig. 83. Fig 84. Baumé's Alcoholimeters

In order to adapt the formulæ of last section to the case of graduations numbered upwards, it is merely necessary to reverse the signs of n_1, n_2, and N; that is we must put

$$c=\frac{n_2d_2-n_1d_1}{d_1-d_2},\ D=\frac{c+n_1}{c+N}d_1;$$

and as we have now $n_1=10$, $d_1=1$, $n_2=0$, $d_2=1{\cdot}085$ the formulæ give[1]

$$c=\frac{10}{{\cdot}085}=118,\ D=\frac{128}{118+N}.$$

§ 174. **Twaddell's Hydrometer.**—In this instrument the divisions are

[1] On comparing the two formulæ for D in this section with the tables in the Appendix to Miller's *Chemical Physics*, I find that as regards the salimeter they agree to two places of decimals and very nearly to three. As regards the alcoholimeter, the table in *Miller* implies that c is about 136, which would make the density corresponding to the zero of the scale about 1·074.

placed not as in Beaumé's, at equal distances, but at distances corresponding to equal differences of density. In fact the specific gravity of a liquid is found by multiplying the reading by 5, cutting off three decimal places, and prefixing unity. Thus the degree 1 indicates specific gravity 1·005, 2 indicates 1·010, &c.

175. Gay-Lussac's Centesimal Alcoholimeter.—When a hydrometer is to be used for a special purpose, it may be convenient to adopt a mode of graduation different in principle from any that we have described above, and adapted to give a direct indication of the proportion in which two ingredients are mixed in the fluid to be examined. It may indicate, for example, the quantity of salt in sea-water, or the quantity of alcohol in a spirit consisting of alcohol and water. Where there are three or more ingredients of different specific gravities the method fails. Gay-Lussac's alcoholimeter is graduated to indicate, at the temperature of 15° Cent., the percentage of pure alcohol in a specimen of spirit. At the top of the stem is 100, the point to which the instrument sinks in pure alcohol, and at the bottom is 0, to which it sinks in water. The position of the intermediate degrees must be determined empirically, by placing the instrument in mixtures of alcohol and water in known proportions, at the temperature of 15°. The law of density, as depending on the proportion of alcohol present, is complicated by the fact that, when alcohol is mixed with water, a diminution of volume (accompanied by rise of temperature) takes place.

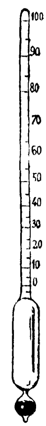


Fig 85. Centesimal Alcoholimeter.

176. Specific Gravity of Mixtures.—When two or more substances are mixed without either shrinkage or expansion (that is, when the volume of the mixture is equal to the sum of the volumes of the components), the density of the mixture can easily be expressed in terms of the quantities and densities of the components.

First, let the volumes $v_1, v_2, v_3 \ldots$ of the components be given, together with their densities $d_1, d_2, d_3 \ldots$

Then their masses (or weights) are $v_1d_1, v_2d_2, v_3d_3 \ldots$

The mass of the mixture is the sum of these masses, and its volume is the sum of the volumes $v_1, v_2, v_3 \ldots$; hence its density is

$$\frac{v_1d_1 + v_2d_2 + \ldots}{v_1 + v_2 + \ldots}.$$

Secondly, let the weights or masses $m_1, m_2, m_3 \ldots$ of the components be given, together with their densities $d_1, d_2, d_3 \ldots$

Then their volumes are $\frac{m_1}{d_1}, \frac{m_2}{d_2}, \frac{m_3}{d_3} \dots$

The volume of the mixture is the sum of these volumes, and its mass is $m_1 + m_2 + m_3 + \dots$; hence its density is

$$\frac{m_1 + m_2 + \dots}{\frac{m_1}{d_1} + \frac{m_2}{d_2} + \dots}$$

o **177. Graphical Method of Graduation.**—When the points on the stem which correspond to some five or six known densities, nearly equidifferent, have been determined, the intermediate graduations can be inserted with tolerable accuracy by the graphical method of interpolation, a method which has many applications in physics besides that which we are now considering. Suppose A and B (Fig. 86) to represent the extreme points, and I, K, L, R intermediate points, all of which correspond to known densities. Erect ordinates (that is to say, perpendiculars) at these points, proportional to the respective densities, or (which will serve our purpose equally well) erect ordinates II′, KK′, LL′, RR′, BC proportional to the excesses of the densities at I, K, L, R, B above the density at A. Any scale of equal parts can be employed for

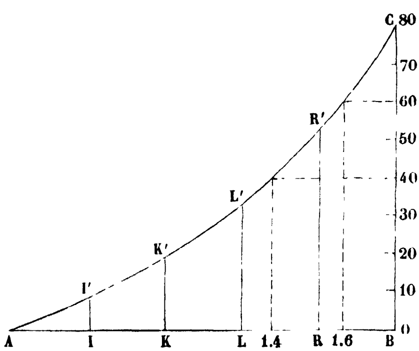


Fig 86 —Graphical Method of Graduation.

laying off the ordinates, but it is convenient to adopt a scale which will make the greatest ordinate BC not much greater nor much less than the base-line AB. In the figure, the density at B is supposed to be 1·80, the density at A being 1. The difference of density is therefore ·80, as indicated by the figures 80 on the scale of equal parts. Having erected the ordinates, we must draw through their extremities the curve AI′K′L′R′C, making it as free from sudden bends as possible, as it is upon the regularity of this curve that the accuracy of the interpolation depends. Then to find the point on the stem AB at which any other density is to be marked—say 1·60, we must draw through the 60th division, on the line of equal parts, a horizontal line to meet the curve, and, through the point thus found on the curve,

draw an ordinate. This ordinate will meet the base-line AB in the required point, which is accordingly marked 1·6 in the figure. The curve also affords the means of solving the converse problem, that is, of finding the density corresponding to any given point on the stem. At the given point in AB, which represents the stem, we must draw an ordinate, and through the point where this meets the curve we must draw a horizontal line to meet the scale of equal parts. The point thus determined on the scale of equal parts indicates the density required, or rather the excess of this density above the density of A.

CHAPTER XV.

VESSELS IN COMMUNICATION—LEVELS.

ᴰ **178. Liquids tend to Find their own Level.**—When a liquid is contained in vessels communicating with each other, and is in equilibrium, it stands at the same height in the different parts of the system, so that the free surfaces all lie in the same horizontal plane. This is obvious from the considerations pointed out in §§ 138, 139, being merely a particular case of the more general law that points of a liquid at rest which are at the same pressure are at the same level.

In the apparatus represented in Fig. 87, the liquid is seen to stand at the same height in the principal vessel and in the variously shaped tubes communicating with it. If one of these tubes is cut off at a height less than that of the liquid in the principal vessel, and is made to terminate in a narrow mouth the liquid will be seen to spout up nearly to the level of that in the principal vessel.

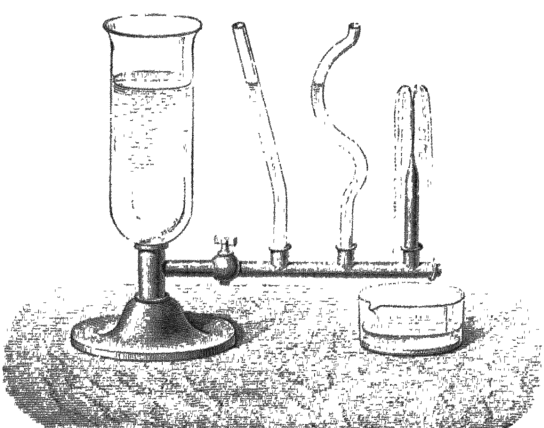


Fig. 87.—Vessels in Communication.

This tendency of liquids to find their own level is utilized for the water-supply of towns. Water will find its way from a reservoir through pipes of any length, provided that all parts of them are below the level of the water in the reservoir. It is necessary how-

ever to distinguish between the conditions of statical equilibrium and the conditions of flow. If no water were allowed to escape from the pipes in a town, their extremities might be carried to the height of the reservoir and they would still be kept full. But in practice there is a continual abstraction of energy, partly in the shape of the kinetic energy of the water which issues from taps, often with considerable velocity, and partly in the shape of work done against friction in the pipes. When there is a continual drawing off from various points of a main, the height to which the water will rise in the houses which it supplies is least in those which are most distant from the reservoir.

179. Water-level.—The instrument called the water-level is another illustration of the same principle. It consists of a metal tube *bb*, bent at right angles at its extremities. These carry two glass tubes

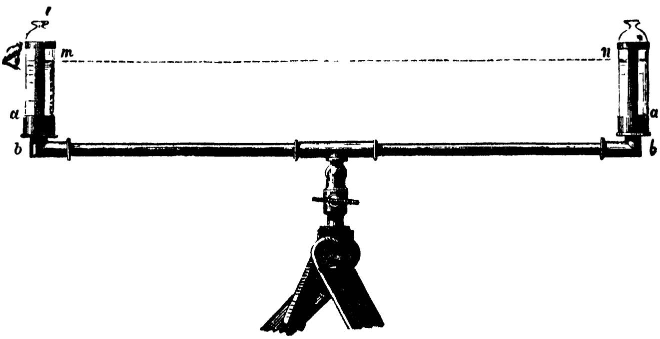


Fig. 88.—Water-level.

aa, very narrow at the top, and of the same diameter. The tube rests on a tripod stand, at the top of which is a joint that enables the observer to turn the apparatus and set it in any direction. The tube is placed in a position *nearly* horizontal, and water, generally coloured a little, is poured in until it stands at about three-fourths of the height of each of the glass tubes.

By the principle of equilibrium in vessels communicating with each other, the surfaces of the liquid in the two branches are in the same horizontal plane, so that if the line of the observer's sight just grazes the two surfaces it will be horizontal.

This is the principle of the operation called *levelling*, the object of which is to determine the difference of vertical height, or *difference of level*, between two given points. Suppose A and B to be the two points (Fig. 89). At each of these points is fixed a levelling-staff,

that is, an upright rod divided into parts of equal length, on which slides a small square board whose centre serves as a mark for the observer.

The level being placed at an intermediate station, the observer directs the line of sight towards each levelling-staff, and the mark is raised or lowered till the line of sight passes through its centre. The marks on the two staves are in this way brought to the same level. The staff in the rear is then carried in advance of the other, the level is again placed between the two, and another observation taken. In this way, by noting the division of the staff at which the sliding mark stands in each case, the difference of levels of two distant stations can be deduced from observations at a number of intermediate points.

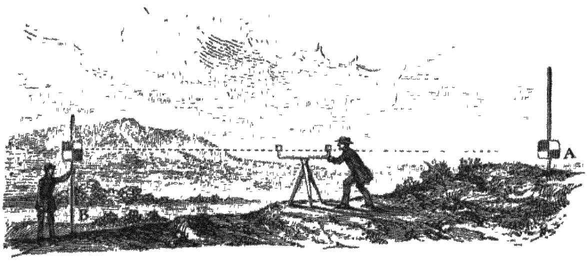


Fig. 89.—Levelling.

For more accurate work, a telescope with attached spirit-level (§ 181) is used, and the levelling staff has divisions upon it which are read off through the telescope.

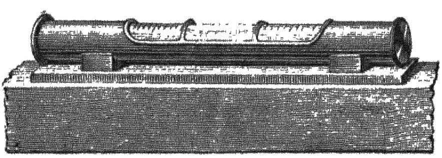


Fig. 90 —Spirit-level.

180. **Spirit-level.**—The spirit-level is composed of a glass tube slightly curved, containing a liquid, which is generally alcohol, and which fills the whole extent of the tube, except a small space occupied by an air-bubble. This tube is inclosed in a mounting which is firmly supported on a stand.

Suppose the tube to have been so constructed that a vertical section of its upper surface is an arc of a circle, and suppose the instrument placed upon a horizontal plane (Fig. 91). The air-bubble will take up a position MN at the highest part of the tube, such that the arcs MA and NB are equal. Hence it follows that if the level

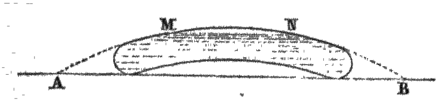


Fig. 91.

be reversed end for end, the bubble will occupy the same position in the tube, the point N coming to M, and *vice versa*. This will not be the case if AB is inclined to the horizon (Fig. 92), for then the bubble will always stand nearest to that end of the tube which is highest, and will therefore change its place in the tube when the latter is reversed. The test, then, of the horizontality of the line on which the spirit-level rests is, that after this operation of reversal the bubble should remain between the same marks on the tube. The maker marks upon the tube two points equidistant from the centre, the distance between them being equal to the usual length of the bubble; and the instrument ought to be so adjusted that when the line on which it stands is horizontal, the ends of the bubble are at these marks.

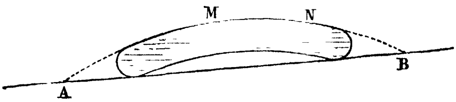


Fig 92.

In order that a plane surface may be horizontal, we must have two lines in it horizontal. This result may be attained in the following manner:—The body whose surface is to be levelled is made to rest on three levelling-screws which form the three vertices of an isosceles triangle; the level is first placed parallel to the base of the triangle, and, by means of one of the screws, the bubble is brought between the reference-marks. The instrument is then placed perpendicularly to its first position, and the bubble is brought between the marks by means of the third screw; this second operation cannot disturb the result of the first, since the plane has only been turned about a horizontal line as hinge.

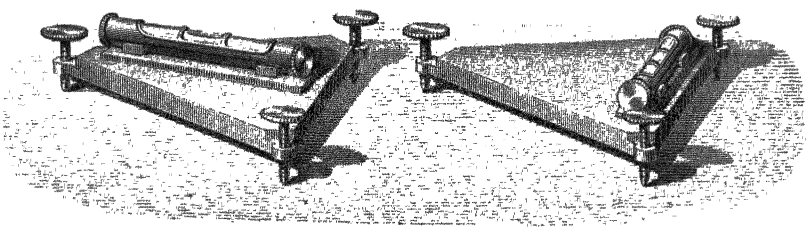


Fig 93.—Testing the Horizontality of a Surface.

181. Telescope with Attached Level.—In order to apply the spirit-level to land-surveying, an apparatus such as that represented in

Fig. 94 is employed. Upon a frame AA, movable about a vertical axis B, are placed a spirit-level *nn*, and a telescope LL, in positions parallel to each other. The telescope is furnished at its focus with two fine wires crossing one another, whose point of intersection determines the line of sight with great precision. The apparatus, which is provided with levelling-screws H, rests on a tripod stand, and the observer is able, by turning it about its axis, to command the different points of the horizon. By a process of adjustment which need not here be described, it is known that when the bubble is between the marks the line of sight is horizontal. By furnishing the instrument with a graduated horizontal circle P, we may obtain the azimuths of the points observed, and thus map out contour lines.

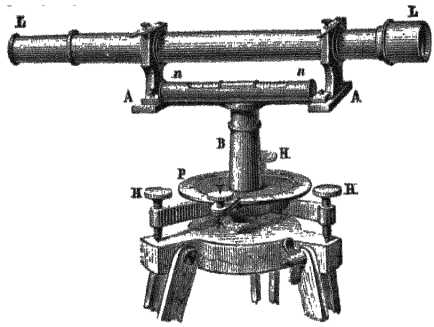


Fig 94.—Spirit-level with Telescope.

Divisions are sometimes placed on each side of the reference-marks of the bubble, for measuring small deviations from horizontality. It is, in fact, easy to see, by reference to Fig. 91, that by tilting the level through any small angle, the bubble is displaced by a quantity proportional to this angle, at least when the curvature of the instrument is that of a circle.

For determining the angular value corresponding to each division

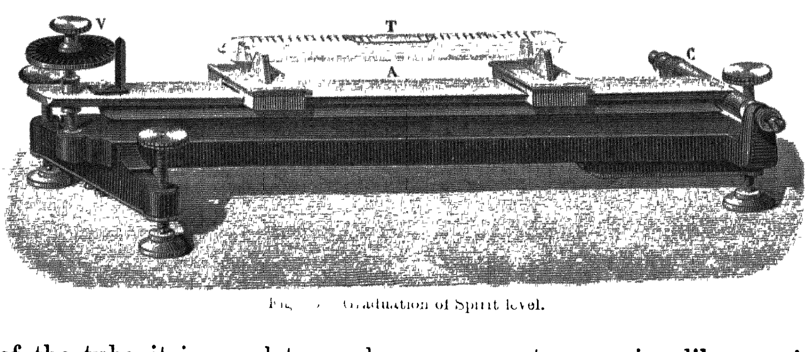


Fig. 95.—Graduation of Spirit-level.

of the tube, it is usual to employ an apparatus opening like a pair of compasses by a hinge C (Fig. 95), on one of the legs of which rests, by two V-shaped supports, the tube T of the level. The com-

pass is opened by means of a micrometer screw V, of very regular action; and as the distance of the screw from the hinge is known, as well as the distance between the threads of the screw, it is easy to calculate beforehand the value of the divisions on the micrometer head. The levelling-screws of the instrument serve to bring the bubble between its reference-marks, so that the micrometer screw is only used to determine the value of the divisions on the tube.

CHAPTER XVI.

CAPILLARITY.

182. Capillarity—General Phenomena.—The laws which we have thus far stated respecting the levels of liquid surfaces are subject to remarkable exceptions when the vessels in which the liquids are contained are very narrow, or, as they are called, capillary (*capillus*, a hair); and also in the case of vessels of any size, when we consider the portion of the liquid which is in close proximity to the sides.

1. *Free Surface.*—The surface of a liquid is not horizontal in the neighbourhood of the sides of the vessel, but presents a very decided curvature. When the liquid wets the vessel, as in the case of water in a glass vessel (Fig. 96), the surface is concave; on the contrary

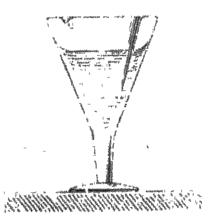


Fig. 96.

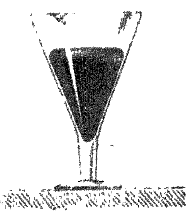


Fig. 97.

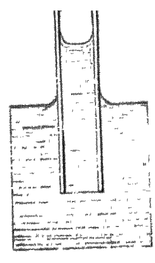


Fig. 98.

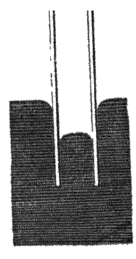


Fig. 99.

when the liquid does not wet the vessel, as in the case of mercury in a glass vessel (Fig. 97), the surface is, generally speaking, convex.

2. *Capillary Elevation and Depression.*—If a very narrow tube of glass be plunged in water, or any other liquid that will wet it (Fig. 98), it will be observed that the level of the liquid, instead of remaining at the same height inside and outside of the tube, stands perceptibly higher in the tube; a *capillary ascension* takes place, which varies in amount according to the nature of the liquid and

the diameter of the tube. It will also be seen that the liquid column thus raised terminates in a concave surface. If a glass tube be dipped in mercury, which does not wet it, it will be seen, by bringing the tube to the side of the vessel, that the mercury is depressed in its interior, and that it terminates in a convex surface (Fig. 99).

3. *Capillary Vessels in Communication with Others.*—If we take two bent tubes (Fig. 100), each having one branch of a considerable diameter and the other extremely narrow, and pour into one of them a liquid which wets it, and into the other mercury, the liquid will be observed in the former case to stand higher in the capillary than in the principal branch, and in the latter case to stand lower; the free surfaces being at the same time concave in the case of the liquid which wets the tubes, and convex in the case of the mercury.

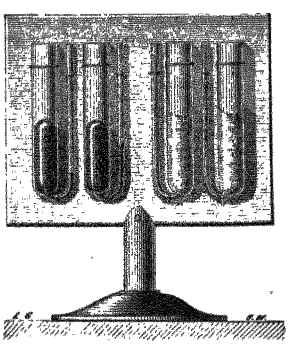


Fig. 100.

183. Circumstances which influence Capillary Elevation and Depression.—In wetted tubes the elevation depends upon the nature of the liquid; thus, at the temperature of 18° Cent., water rises $29{\cdot}79^{mm}$ in a tube 1 millimetre in diameter, alcohol rises $12{\cdot}18^{mm}$, nitric acid $22{\cdot}57^{mm}$, essence of lavender $4{\cdot}28^{mm}$, &c. The nature of the tube is almost entirely immaterial, provided the precaution be first taken of wetting it with the liquid to be employed in the experiment, so as to leave a film of the liquid adhering to the sides of the tube.

Capillary depression, on the other hand, depends both on the nature of the liquid and on that of the tube. Both ascension and depression diminish as the temperature increases; for example, the elevation of water, which in a tube of a certain diameter is equal to 132^{mm} at 0° Cent., is only 106^{mm} at 100°.

184. Law of Diameters.—*Capillary elevations and depressions, when all other circumstances are the same, are inversely proportional to the diameters of the tubes.* As this law is a consequence of the mathematical theories which are generally accepted as explaining capillary phenomena, its verification has been regarded as of great importance.

The experiments of Gay-Lussac, which confirmed this law, have been repeated, with slight modifications, by several observers. The

method employed consists essentially in measuring the capillary elevation of a liquid by means of a cathetometer (Fig. 101). The telescope *ll* is directed first to the top *n* of the column in the tube, and then to the end of a pointer *b*, which touches the surface of the

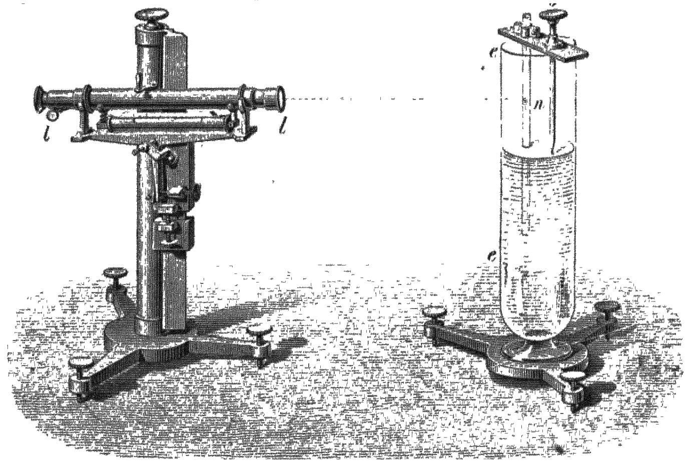


Fig. 101.—Verification of Law of Diameters.

liquid at a point where it is horizontal. In observing the depression of mercury, since the opacity of the metal prevents us from seeing the tube, we must bring the tube close to the side of the vessel *e*.

The diameter of the tube can be measured directly by observing its section through a microscope, or we may proceed by the method employed by Gay-Lussac. He weighed the quantity of mercury which filled a known length l of the tube; this weight w is that of a cylinder of mercury whose radius x is determined by the equation $13{\cdot}59\ \pi x^2 l = w$, where x and l are in centimetres, and w in grammes.

The result of these different experiments is, that in the case of wetted tubes the law is exactly fulfilled, provided that they be previously washed with the greatest care, so as to remove all foreign matters, and that the liquid on which the experiment is to be performed be first passed through them. When the liquid does not wet the tube, various causes combine to affect the form of the surface in which the liquid column terminates; and we cannot infer the depression from knowing the diameter, unless we also take into consideration some element connected with the form of the terminal surface, such as the length of the sagitta, or the angle made with the sides

of the tube by the extremities of the curved surface, which is called the *angle of contact.*

185. Fundamental Laws of Capillary Phenomena.—Capillary phenomena, as they take place alike in air and in vacuo, cannot be attributed to the action of the atmosphere. They depend upon molecular actions which take place between the particles of the liquid itself, and between the liquid and the solid containing it, the actions in question being purely superficial—that is to say, being confined to an extremely thin layer forming the external boundary of the liquid, and to an extremely thin superficial layer of the solid in contact with the liquid. For example, it is found in the case of glass tubes, that the amount of capillary elevation or depression is not at all affected by the thickness of the sides of the tube. The following are some of the principles which govern capillary phenomena.

1. For a given liquid in contact with a given solid, with a definite intimateness of contact (this last element being dependent upon the cleanness of the surface, upon whether the surface of the solid has been recently washed by the liquid, and perhaps upon some other particulars), there is (at any specified temperature) a definite angle of contact, which is independent of the directions of the surfaces with regard to the vertical.

2. Every liquid behaves as if a thin film, forming its external layer, were in a state of tension, and exerting a constant effort to contract. This tension, or contractile force, is exhibited over the whole of the free surface (that is, the surface which is exposed to air); but wherever the liquid is in contact with a solid, its existence is masked by other molecular actions. It is uniform in all directions in the free surface, and at all points in this surface, being dependent only on the nature and temperature of the liquid. Its intensity for several specified liquids is given in tabular form further on (§ 192) upon the authority of Van der Mensbrugghe. Tension of this kind must of course be stated in units of force per linear unit, because by doubling the width of a band we double the force required to keep it stretched. Mensbrugghe considers that such tension really exists in the superficial layer; but the majority of authors (and we think with more justice) regard it rather as a convenient fiction, which accurately represents the effects of the real cause. Two of the most eminent writers on the cause of capillary phenomena are Laplace and Dr. Thomas Young. The subject presents difficulties which have not yet been fully surmounted.

186. Application to Elevation in Tubes.—The law of diameters is a direct consequence of the two preceding principles; for if α denote the external angle of contact (which is acute in the case of mercury against glass), T the tension per unit length, and r the radius of the tube, then $2\pi r\mathrm{T}$ will be the whole amount of force exerted at the margin of the surface; and as this force is exerted in a direction making an angle α with the vertical, its vertical component will be $2\pi r\mathrm{T}\cos\alpha$, which is exerted in pulling the tube upwards and the liquid downwards.

If w be the weight of unit volume of the liquid, then $\pi r^2 w$ is the weight of as much as would occupy unit length of the tube; and if h denote the height of a column whose weight is equal to the force tending to depress the liquid, we have

$$\pi r^2 hw = 2\pi r\mathrm{T}\cos\alpha;$$

whence $h = \frac{2\mathrm{T}\cos\alpha}{r\,.\,w}$, which, when the other elements are given, varies inversely as r, the radius of the tube.

Having regard to the fact that the surface is not of the same height in the centre as at the edges, it is obvious that h denotes the mean height.

If α be obtuse, h will be negative—that is to say, there will be elevation instead of depression. In the case of water against a tube which has been well wetted with that liquid, α is 180°—that is to say, the tube is tangential to the surface. For this case the formula for h gives

$$\text{elevation} = \frac{2\mathrm{T}}{rw}.$$

Again, for two parallel vertical plates at distance u, the vertical force of capillarity for a unit of length is $2\mathrm{T}\cos\alpha$, which must be equal to whu, being the weight of a sheet of liquid of height h, thickness u, and length unity. We have therefore

$$h = \frac{2\mathrm{T}\cos\alpha}{uw},$$

which agrees with the expression for the depression or elevation in a circular tube whose radius is equal to the distance between these parallel plates.

The surface tension always tends to reduce the surface to the smallest area which can be inclosed by its actual boundary; and therefore always produces a normal force directed from the convex to the concave side of the superficial film. Hence, wherever there is

capillary elevation the free surface must be concave; wherever there is depression it must be convex.

187. It follows from a well-known proposition in statics (Todhunter's *Statics*, § 194), that if a *cylindrical* film be stretched with a uniform tension T (so that the force tending to pull the film asunder across any short line drawn on the film, is T times the length of the line), the resultant normal pressure (which the film exerts, for example, against the surface of a solid internal cylinder over which it is stretched) is T divided by the radius of the cylinder.

It can be proved that a film of any form, stretched with uniform tension T, exerts at each point a normal pressure equal to the sum of the pressures which would be exerted by two overlapping cylindrical films, whose axes are at right angles to one another, and whose cross sections are circles of curvature of normal sections at the point. That is to say, if P be the normal force per unit area, and r, r' the radii of curvature in two mutually perpendicular normal sections at the point, then

$$\mathrm{P} = \mathrm{T}\left(\frac{1}{r} + \frac{1}{r'}\right).$$

At any point on a curved surface, the normal sections of greatest and least curvature are mutually perpendicular, and are called the principal normal sections at the point. If the corresponding radii of curvature be R, R′, we have

$$\mathrm{P} = \mathrm{T}\left(\frac{1}{\mathrm{R}} + \frac{1}{\mathrm{R}'}\right); \tag{1}$$

or *the normal force per unit area is equal to the tension per unit length multiplied by the sum of the principal curvatures.*

In the case of capillary depressions and elevations, the superficial film at the free surface is to be regarded as pressing the liquid inwards, or pulling it outwards, according as this surface is convex or concave, with a force P given by the above formula. The value of P at any point of the free surface is equal to the pressure due to the height of a column of liquid extending from that point to the level of the general horizontal surface. It is therefore greatest at the edges of the elevated or depressed column in a tube, and least in the centre; and the curvature, as measured by $\frac{1}{\mathrm{R}} + \frac{1}{\mathrm{R}'}$, must vary in the same proportion. If the tube is so large that there is no sensible elevation or depression in the centre of the column, the centre of the free surface must be sensibly plane.

188. Another consequence of the formula is, that in circumstances

where there can be no normal pressure towards either side of the surface,

$$\frac{1}{R} + \frac{1}{R'} = 0; \tag{2}$$

which implies that either the surface is plane, in which case each of the two terms is separately equal to zero, or else

$$R = -R'; \tag{3}$$

that is, the principal radii of curvature are equal, and lie on opposite sides of the surface. The formulæ (2), (3) apply to a film of soapy water attached to a loop of wire. If the loop be in one plane, the film will be in the same plane. If the loop be not in one plane, the film cannot be in one plane, and will in fact assume that form which gives the least area consistent with having the loop for its boundary. At every point it will be observed to be, if we may so say, concave towards both sides, and convex towards both sides, the concavity being precisely equal to the convexity—that is to say, equation (3) is satisfied at every point of the film.

In this case both sides of the film are exposed to atmospheric pressure. In the case of a common soap-bubble the outside is exposed to atmospheric pressure, and the inside to a pressure somewhat greater, the difference of the pressures being balanced by the tendency of the film to contract. Formula (1) becomes for either the outer or inner surface of a spherical bubble

$$P = \frac{2T}{R};$$

but this result must be doubled, because there are two free surfaces; hence the excess of pressure of the inclosed above the external air is $\frac{4T}{R}$, R denoting the radius of the bubble.

The value of T for soapy water is about 1 grain per linear inch; hence, if we divide 4 by the radius of the bubble expressed in inches, we shall obtain the excess of internal over external pressure *in grains per square inch.*

The value of T for any liquid may be obtained by observing the amount of elevation or depression in a tube of given diameter, and employing the formula

$$T = \frac{whr}{2\cos\alpha}, \tag{4}$$

which follows immediately from the formula for h in § 186.

189. It is this uniform surface tension, of which we have been

speaking, which causes a drop of a liquid falling through the air either to assume the spherical form, or to oscillate about the spherical form. The phenomena of drops can be imitated on an enlarged scale, under circumstances which permit us to observe the actual motions, by a method devised by Professor Plateau of Ghent. Olive-oil is intermediate in density between water and alcohol. Let a mixture of alcohol and water be prepared, having precisely the density of olive-oil, and let about a cubic inch of the latter be gently introduced into it with the aid of a funnel or pipette. It will assume a spherical form, and if forced out of this form and then left free, will slowly oscillate about it; for example, if it has been compelled to assume the form of a prolate spheroid, it will pass to the oblate form, will then become prolate again, and so on alternately, becoming however more nearly spherical every time, because its movements are hindered by friction, until at last it comes to rest as a sphere.

§ 190. Capillarity furnishes no exception to the principle that the pressure in a liquid is the same at all points at the same depth. When the free surface within a tube is convex, and is consequently depressed below the plane surface of the external liquid, the pressure becomes suddenly greater on passing downwards through the superficial layer, by the amount due to the curvature. Below this it increases regularly by the amount due to the depth of liquid passed through. The pressure at any point vertically under the convex meniscus[1] may be computed, either by taking the depth of the point below the general free surface, and adding atmospheric pressure to the pressure due to this depth, according to the ordinary principles of hydrostatics, or by taking the depth of the point below that point of the meniscus which is vertically over it, adding the pressure due to the curvature at this point, and also adding atmospheric pressure.

When the free surface of the liquid within a tube is concave, the pressure suddenly diminishes on passing downwards through the superficial layer, by the amount due to the curvature as given by formula (1); that is to say, the pressure at a very small depth is less than atmospheric pressure by this amount. Below this depth it goes on increasing according to the usual law, and becomes equal to

[1] The convex or concave surface of the liquid in a tube is usually denoted by the name *meniscus* (μηνίσκος, a crescent), which denotes a form approximately resembling that of a watch-glass.

atmospheric pressure at that depth which corresponds with the level of the plane external surface. The pressure at any point in the liquid within the tube can therefore be obtained either by subtracting from atmospheric pressure the pressure due to the elevation of the point above the general surface, or by adding to atmospheric pressure the pressure due to the depth below that point of the meniscus which is on the same vertical, and subtracting the pressure due to the curvature at this point.

These rules imply, as has been already remarked, that the curvature is different at different points of the meniscus, being greatest where the elevation or depression is greatest, namely at the edges of the meniscus; and least at the point of least elevation or depression, which in a cylindrical tube is the middle point.

The principles just stated apply to all cases of capillary elevation and depression.

They enable us to calculate the force with which two parallel vertical plates, partially immersed in a liquid which wets them, are urged towards each other by capillary action. The pressure in the portion of liquid elevated between them is less than atmospheric, and therefore is insufficient to balance the atmospheric pressure which is exerted on the outer faces of the plates. The average pressure in the elevated portion of liquid is equal to the actual pressure at the centre of gravity of the elevated area, and is less than atmospheric pressure by the pressure of a column of liquid whose height is the elevation of this centre of gravity.

Even if the liquid be one which does not wet the plates, they will still be urged towards each other by capillary action; for the inner faces of the plates are exposed to merely atmospheric pressure over the area of depression, while the corresponding portions of the external faces are exposed to atmospheric pressure increased by the weight of a portion of the liquid.

These principles explain the apparent attraction exhibited by bodies floating on a liquid which either wets them both or wets neither of them. When the two bodies are near each other they behave somewhat like parallel plates, the elevation or depression of the liquid between them being greater than on their remote sides.

If two floating bodies, one of which is wetted and the other unwetted by the liquid, come near together, the elevation and depression of the liquid will be less on the near than on the remote sides, and apparent repulsion will be exhibited.

In all cases of capillary elevation or depression, the solid is pulled downwards or upwards with a force equal to that by which the liquid is raised or depressed. In applying the principle of Archimedes to a solid partially immersed in a liquid, it is therefore necessary (as we have seen in § 159), when the solid produces capillary depression, to reckon the void space thus created as part of the displacement; and when the solid produces capillary elevation, the fluid raised above the general level must be reckoned as *negative* displacement, tending to *increase* the apparent weight of the solid.

191. Thus far all the effects of capillary action which we have mentioned are connected with the curvature of the superficial film, and depend upon the principle that a convex surface increases and a concave surface diminishes the pressure in the interior of the liquid. But there is good reason for maintaining that whatever be the form of the free surface there is always pressure in the interior due to the molecular action at this surface, and that the pressure due to the curvature of the surface is to be added to or subtracted from a definite amount of pressure which is independent of the curvature and depends only on the nature and condition of the liquid. This indeed follows at once from the fact that capillary elevation can take place in vacuo. As far as the principles of the preceding paragraphs are concerned, we should have, at points within the elevated column, a pressure less than that existing in the vacuum. This, however, cannot be; we cannot conceive of negative pressure existing in the interior of a liquid, and we are driven to conclude that the elevation is owing to the excess of the pressure caused by the plane surface in the containing vessel above the pressure caused by the concave surface in the capillary tube.

There are some other facts which seem only explicable on the same general principle of interior pressure due to surface action,—facts which attracted the notice of some of the earliest writers on pneumatics, namely, that siphons will work in vacuo, and that a column of mercury at least 75 inches in length can be sustained—as if by atmospheric pressure—in a barometer tube, the mercury being boiled and completely filling the tube.

192. We have now to notice certain phenomena which depend on the difference in the surface tensions of different liquids, or of the same liquid in different states.

Let a thin layer of oil be spread over the upper surface of a thin sheet of brass, and let a lamp be placed underneath. The oil will be

observed to run away from the spot directly over the flame, even though this spot be somewhat lower than the rest of the sheet. This effect is attributable to the excess of surface tension in the cold oil above the hot.

In like manner, if a drop of alcohol be introduced into a thin layer of water spread over a nearly horizontal surface, it will be drawn away in all directions by the surrounding water, leaving a nearly dry spot in the space which it occupied. In this experiment the water should be coloured in order to distinguish it from the alcohol.

Again, let a very small fragment of camphor be placed on the surface of hot water. It will be observed to rush to and fro, with frequent rotations on its own axis, sometimes in one direction and sometimes in the opposite. These effects, which have been a frequent subject of discussion, are now known to be due to the diminution of the surface tension of the water by the camphor which it takes up. Superficial currents are thus created, radiating from the fragment of camphor in all directions; and as the camphor dissolves more quickly in some parts than in others, the currents which are formed are not equal in all directions, and those which are most powerful prevail over the others and give motion to the fragment.

The values of T, the apparent surface tension, for several liquids, are given in the following table, on the authority of Van der Mensbrugghe, in milligrammes (or thousandth parts of a gramme) per millimetre of length. They can be reduced to grains per inch of length by multiplying them by ·392; for example, the surface tension of distilled water is $7{\cdot}3 \times {\cdot}392 = 2{\cdot}86$ grains per inch.

Distilled water at 20° Cent.,	7·3	Solution of Marseilles soap, 1 part of soap to 40 of water,	2·83
Sulphuric ether,	1·88	Solution of saponine,	4·67
Absolute alcohol,	2·5	Saturated solution of carbonate of soda,	4·28
Olive-oil,	3·5	Water impregnated with camphor,	4·5
Mercury,	49·1		
Bisulphide of carbon,	3·57		

193. Endosmose.—Capillary phenomena have undoubtedly some connection with a very important property discovered by Dutrochet, and called by him *endosmose.*

The *endosmometer* invented by him to illustrate this phenomenon consists of a reservoir v (Fig. 102) closed below by a membrane ba, and terminating above in a tube of considerable length. This reservoir is filled, suppose, with a solution of gum in water, and is kept

immersed in water. At the end of some time the level of the liquid in the tube will be observed to have risen to *n*, suppose, and at the same time traces of gum will be found in the water in which the reservoir is immersed. Hence we conclude that the two liquids have penetrated through the membrane, but in different proportions; and this is what is called endosmose.

If instead of a solution of gum we employed water containing albumen, sugar, or gelatine in solution, a similar result would ensue. The membrane may be replaced by a slab of wood or of porous clay. Physiologists have justly attached very great importance to this discovery of Dutrochet. It explains, in fact, the interchange of liquids which is continually taking place in the tissues and vessels of the animal system, as well as the absorption of water by the spongioles of roots, and several similar phenomena.

As regards the power of passing through porous diaphragms, Graham has divided substances into two classes—*crystalloids* and *colloids* (κόλλη, glue). The former are susceptible of crystallization, form solutions free from viscosity, are sapid, and possess great powers of diffusion through porous septa. The latter, including gum, starch, albumen, &c., are characterized by a remarkable sluggishness and indisposition both to diffusion and to crystallization, and when pure are nearly tasteless.

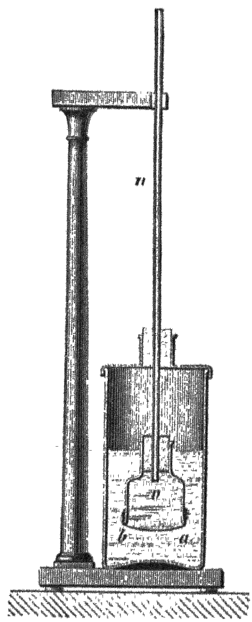


Fig. 102.—Endosmometer.

Diffusion also takes place through colloidal diaphragms which are not porous, the diaphragm in this case acting as a solvent, and giving out the dissolved material on the other side. In the important process of modern chemistry called *dialysis*, saline ingredients are separated from organic substances with which they are blended, by interposing a colloidal diaphragm (De La Rue's parchment paper) between the mixture and pure water. The organic matters, being colloidal, remain behind, while the salts pass through, and can be obtained in a nearly pure state by evaporating the water.

Gases are also capable of diffusion through diaphragms, whether

porous or colloidal, the rate of diffusion being in the former case inversely as the square root of the density of the gas. Hydrogen diffuses so rapidly through unglazed earthenware as to afford opportunity for very striking experiments; and it shows its power of traversing colloids by rapidly escaping through the sides of india-rubber tubes, or through films of soapy water.

CHAPTER XVII.

THE BAROMETER.

194. Expansibility of Gases.—Gaseous bodies possess a number of properties in common with liquids; like them, they transmit pressures entire and in all directions, according to the principle of Pascal; but they differ essentially from liquids in the permanent repulsive force exerted between their molecules, in virtue of which a mass of gas always tends to expand.

This property, called the expansibility of gases, is commonly illustrated by the following experiment:—

A bladder, nearly empty of air, and tied at the neck, is placed under the receiver of an air-pump. At first the air which it contains and the external air oppose each other by their mutual pressure, and are in equilibrium. But if we proceed to exhaust the receiver, and thus diminish the external pressure, the bladder gradually becomes inflated, and thus manifests the tendency of the gas which it contains to occupy a greater volume.

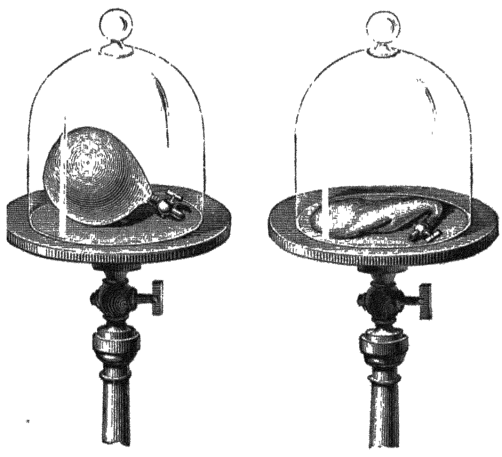


Fig. 103.—Expansibility of Gases.

However large a vessel may be, it can always be filled by *any quantity whatever* of a gas, which will always exert pressure against

the sides. In consequence of this property, the name of *elastic fluids* is often given to gases.

195. Air has Weight.—The opinion was long held that the air was without weight; or, to speak more precisely, it never occurred to any of the philosophers who preceded Galileo to attribute any influence in natural phenomena to the weight of the air. And as this influence is really of the first importance, and comes into play in many of the commonest phenomena, it very naturally happened that the discovery of the weight of air formed the commencement of the modern revival of physical science.

It appears, however, that Aristotle conceived the idea of the possibility of air having weight, and, in order to convince himself on this point, he weighed a skin inflated and collapsed. As he obtained the same weight in both cases, he relinquished the idea which he had for the moment entertained. In fact, the experiment, as he performed it, could only give a negative result; for if the weight of the skin was increased, on the one hand, by the introduction of a fresh quantity of air, it was diminished, on the other, by the corresponding increase in the upward pressure of the air displaced. In order to draw a certain conclusion, the experiment should be performed with a vessel which could receive within it air of different degrees of density, without changing its own volume.

Galileo is said to have devised the experiment of weighing a globe filled alternately with ordinary air and with compressed air. As the weight is greater in the latter case, Galileo should have drawn the inference that air is heavy. It does not appear, however, that the importance of this conclusion made much impression on him, for he did not give it any of those developments which might have been expected to present themselves to a mind like his.

Otto Guericke, the illustrious inventor of the air-pump, in 1650 performed the following experiment, which is decisive:—

A globe of glass (Fig. 104), furnished with a stop-cock, and of a sufficient capacity (about twelve litres), is exhausted of air. It is then suspended from one of the scales of a balance, and a weight sufficient to produce equilibrium is placed in the other scale. The stop-cock is then opened, the air rushes into the globe, and the beam is observed gradually to incline, so that an additional weight is required in the other scale, in order to re-establish equilibrium. If the capacity of the globe is 12 litres, about 15·5 grammes will be

needed, which gives 1·3 gramme as the approximate weight of a litre (or 1000 cubic centimetres) of air.[1]

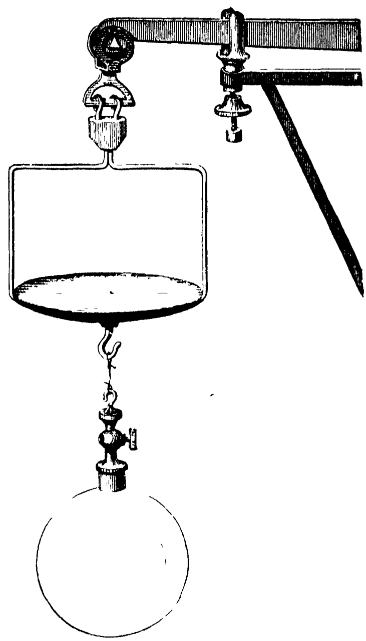


Fig 104 —Weight of Air.

If, in performing this experiment, we take particular precautions to insure its precision, as we shall explain in the book on Heat, it will be found that, at the temperature of freezing water, and under the pressure of one atmosphere, a litre of perfectly dry air weighs 1·293 gramme.[2] Under these circumstances, the ratio of the weight of a volume of air to that of an equal volume of water is $\frac{1{\cdot}293}{1000}=\frac{1}{773}$. Air is thus 773 times lighter than water.

By repeating this experiment with other gases, we may determine their weight as compared with that of air, and the absolute weight of a litre of each of them. Thus it is found that a litre of oxygen weighs 1·43 gramme, a litre of carbonic acid 1·97 gramme, a litre of hydrogen 0·089 gramme, &c.

[1] A cubic foot of air in ordinary circumstances weighs about an ounce and a quarter.

[2] In strictness, the weight in grammes of a litre of air under the pressure of 760 millimetres of mercury is different in different localities, being proportional to the intensity of gravity—not because the force of gravity on the litre of air is different, for though this is true, it does not affect the numerical value of the weight when stated in grammes, but because the pressure of 760 millimetres of mercury varies as the intensity of gravity, so that more air is compressed into the space of a litre as gravity increases. (§ 201, 6.)

The *weight in grammes* is another name for the *mass*. The force of gravity on a litre of air under the pressure of 760 millimetres is proportional to the square of the intensity of gravity.

This is an excellent example of the ambiguity of the word *weight*, which sometimes denotes a mass, sometimes a force; and though the distinction is of no practical importance so long as we confine our attention to one locality, it cannot be neglected when different localities are compared.

Regnault's determination of the weight of a litre of dry air at 0° Cent. under the pressure of 760 millimetres at Paris is 1·293187 gramme. Gravity at Paris is to gravity at Greenwich as 3456 to 3457. The corresponding number for Greenwich is therefore 1·293561.

196. Atmospheric Pressure.—The atmosphere encircles the earth with a layer some 50 or 100 miles in thickness; this heavy fluid mass exerts on the surface of all bodies a pressure entirely analogous both in nature and origin to that sustained by a body wholly immersed in a liquid. It is subject to the fundamental laws mentioned in §§ 137–139. The pressure should therefore diminish as we ascend from the surface of the earth, but should have the same value for all points in the same horizontal layer, provided that the air is in a state of equilibrium. On account of the great compressibility of gas, the lower layers are much more dense than the upper ones; but the density, like the pressure, is constant in value for the

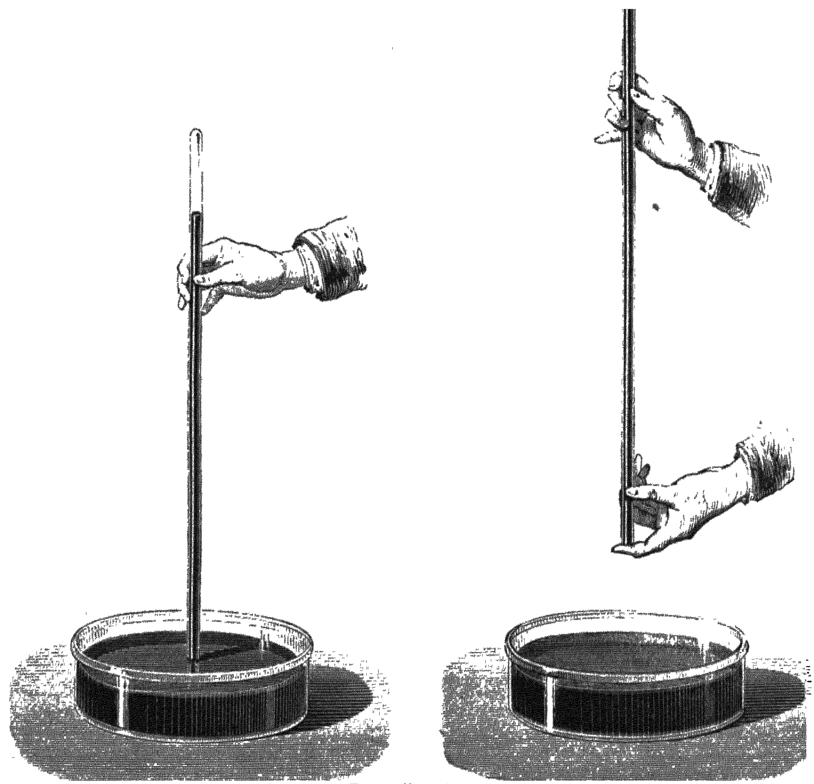


Fig. 165.—Torricellian Experiment.

same horizontal layer, throughout any portion of air in a state of equilibrium. Whenever there is an inequality either of density or pressure at a given level, wind must ensue.

We owe to Torricelli an experiment which plainly shows the pressure of the atmosphere, and enables us to estimate its intensity with great precision. This experiment, which was performed in 1643, one year after the death of Galileo, at a time when the weight and pressure of the air were scarcely even suspected, has immortalized the name of its author, and has exercised a most important influence upon the progress of natural philosophy.

197. Torricellian Experiment.—A glass tube (Fig. 105) about a quarter or a third of an inch in diameter, and about a yard in length, is completely filled with mercury; the extremity is then stopped with the finger, and the tube is inverted in a vessel containing mercury. If the finger is now removed, the mercury will descend in the tube, and after a few oscillations will remain stationary at a height which varies according to circumstances, but which is generally about 76 centimetres, or nearly 30 inches.[1]

The column of mercury is maintained at this height by the pressure of the atmosphere upon the surface of the mercury in the vessel. In fact, the pressure at the level ABCD (Fig. 106) must be the same within as without the tube; so that the column of mercury BE exerts a pressure equal to that of the atmosphere.

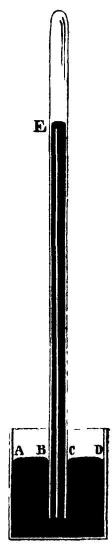


Fig 106

Accordingly, we conclude from this experiment of Torricelli that *every surface exposed to the atmosphere sustains a normal pressure equal, on an average, to the weight of a column of mercury whose base is this surface, and whose height is 30 inches.*

It is evident that if we performed a similar experiment with water, whose density is to that of mercury as 1 : 13·59, the height of the column sustained would be 13·59 times as much; that is, $30 \times 13{\cdot}59$ inches, or about 34 feet. This is the maximum height to which water can be raised in a pump; as was observed by Galileo.

In general the heights of columns of different liquids equal in weight to a column of air on the same base, are inversely proportional to their densities.

198. Pressure of one Atmosphere.—What is usually adopted in accurate physical discussions as the standard "atmosphere" of pressure is the pressure due to a height of 76 centimetres of pure mercury at the temperature zero Centigrade, gravity being supposed to have

[1] 76 centimetres are 29·922 inches.

the same intensity which it has at Paris. The density of mercury at this temperature is 13·596; hence, when expressed in gravitation measure, this pressure is $76 \times 13·596 = 1033·3$ grammes per square centimetre.[1] To reduce this to absolute measure, we must multiply by the value of g (the intensity of gravity) at Paris, which is 980·94; and the result is 1013600, which is the intensity of pressure in dynes per square centimetre. In some recent works, the round number a million dynes per square centimetre has been adopted as the standard atmosphere.

199. Pascal's Experiments.—It is supposed, though without any decisive proof, that Torricelli derived from Galileo the definite conception of atmospheric pressure.[2] However this may be, when the experiment of the Italian philosopher became known in France in 1644, no one was capable of giving the correct explanation of it, and the famous doctrine that "nature abhors a vacuum," by which the rising of water in a pump was accounted for, was generally accepted. Pascal was the first to prove incontestably the falsity of this old doctrine, and to introduce a more rational belief. For this purpose, he proposed or executed a series of ingenious experiments, and discussed minutely all the phenomena which were attributed to nature's abhorrence of a vacuum, showing that they were necessary consequences of the pressure of the atmosphere.

We may cite in particular the observation, made at his suggestion, that the height of the mercurial column decreases in proportion as we ascend. This beautiful and decisive experiment, which is repeated as often as heights are measured by the barometer, and which leaves no doubt as to the nature of the force which sustains the mercurial column, was performed for the first time at Clermont, and on the top of the mountain Puy-de-Dôme, on the 19th September, 1648.

200. The Barometer.—By fixing the Torricellian tube in a perman-

[1] This is about 14·7 pounds per square inch.

[2] In the fountains of the Grand-duke of Tuscany some pumps were required to raise water from a depth of from 40 to 50 feet. When these were worked, it was found that they would not draw. Galileo determined the height to which the water rose in their tubes, and found it to be about 32 feet; and as he had observed and proved that air has weight, he readily conceived that it was the weight of a column of the atmosphere which maintained the water at this height in the pumps. No very useful results, however, were expected from this discovery, until, at a later date, Torricelli adopted and greatly extended it. Desiring to repeat the experiment in a more convenient form, he conceived the idea of substituting for water a liquid that is 14 times as heavy, namely, mercury, rightly imagining that a column of one-fourteenth of the length would balance the force which sustained 32 feet of water (Biot, *Biographie Universelle*, article "Torricelli").—*D.*

ent position, we obtain a means of measuring the amount of the atmospheric pressure at any moment; and this pressure may be expressed by the height of the column of mercury which it supports. Such an instrument is called a *barometer.* In order that its indications may be accurate, several precautions must be observed. In the first place, the liquid used in different barometers must be identical; for the height of the column supported naturally depends upon the density of the liquid employed, and if this varies, the observations made with different instruments will not be comparable.

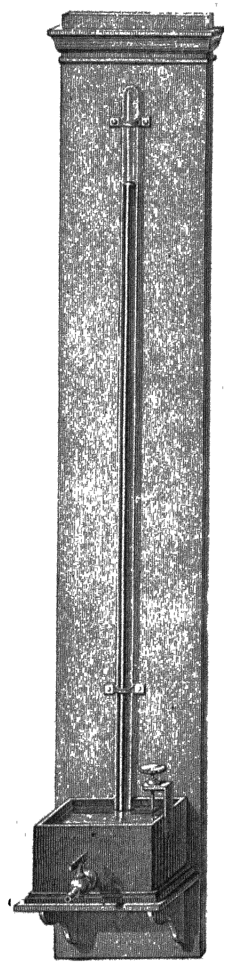


Fig. 107.—Barometer in its simplest form.

The mercury employed is chemically pure, being generally made so by washing with a dilute acid and by subsequent distillation. The barometric tube is filled nearly full, and is then placed upon a sloping furnace, and heated till the mercury boils. The object of this process is to expel the air and moisture which may be contained in the mercurial column, and which, without this precaution, would gradually ascend into the vacuum above, and cause a downward pressure of uncertain amount, which would prevent the mercury from rising to the proper height.

The next step is to fill up the tube with pure mercury, taking care not to introduce any bubble of air. The tube is then inverted in a cistern likewise containing pure mercury recently boiled, and is firmly fixed in a vertical position, as shown in Fig. 107.

We have thus a fixed barometer; and in order to ascertain the atmospheric pressure at any moment, it is only necessary to measure the height of the top of the column of mercury above the surface of the mercury in the cistern. One method of doing this is to employ an iron rod, working in a screw, and fixed vertically above the surface of the mercury in the dish. The extremities of this rod are pointed, and the lower extremity being brought down to touch the surface of the liquid below, the distance of the upper extremity from the top of the column of mercury is measured. Adding to this the

length of the rod, which has previously been determined once for all, we have the barometric height. This measurement may be effected with great precision by means of the cathetometer.

201. Cathetometer.—This instrument, which is so frequently employed in physics to measure the vertical distance between two points, was invented by Dulong and Petit.

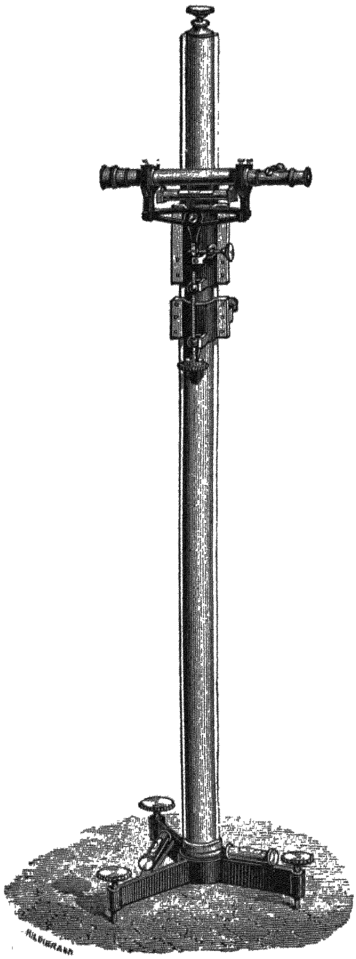


Fig. 108.—Cathetometer.

It consists essentially (Fig. 108) of a vertical scale divided usually into half millimetres. This scale forms part of a brass cylinder capable of turning very easily about a strong steel axis. This axis is fixed on a pedestal provided with three levelling screws, and with two spirit-levels at right angles to each other. Along the scale moves a sliding frame carrying a telescope furnished with cross-wires, that is, with two very fine threads, usually spider lines, in the focus of the eye-piece, whose point of intersection serves to determine the line of vision. By means of a clamp and slow-motion screw, the telescope can be fixed with great precision at any required height. The telescope is also provided with a spirit-level and adjusting screw. When the apparatus is in correct adjustment, the line of vision of the telescope is horizontal, and the graduated scale is vertical. If then we wish to measure the difference of level between two points, we have only to sight them successively, and measure the distance passed over on the scale, which is done by means of a vernier attached to the sliding frame.

202. Fortin's Barometer.—The barometer just described is intended to be fixed; when portability is required, the construction devised by Fortin (Fig. 109) is usually employed. It is also frequently em-

ployed for fixed barometers. The cistern, which is formed of a tube of boxwood, surmounted by a tube of glass, is closed below by a piece of leather, which can be raised or lowered by means of a screw. This screw works in the bottom of a brass case, which incloses the cistern except at the middle, where it is cut away in front and at the back, so as to leave the surface of the mercury open to view. The barometric tube is encased in a tube of brass with two slits at opposite sides (Fig. 110); and it is on this tube that the divisions are engraved, the zero point from which they are reckoned being the lower extremity of an ivory point fixed in the covering of the cistern. The temperature of the mercury, which is required for one of the corrections mentioned in next section, is given by a thermometer with its bulb resting against the tube. A cylindrical sliding piece (shown in Fig. 110) furnished with a vernier,[1] moves along the tube and enables us to determine the height with great precision. Its lower edge is the zero of the vernier. The way in which the barometric tube is fixed upon the cistern is worth notice. In the centre of the upper surface of the copper casing there is an opening, from which rises a short tube of the same metal, lined with a tube of boxwood. The barometric tube is pushed inside, and fitted in with a piece of chamois leather, which prevents the mercury from issuing, but does not exclude the air, which, passing through the pores of the leather, penetrates into the cistern, and so transmits its pressure.

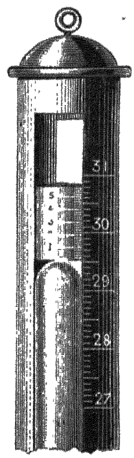


Fig 110.
Upper portion of Barometer.

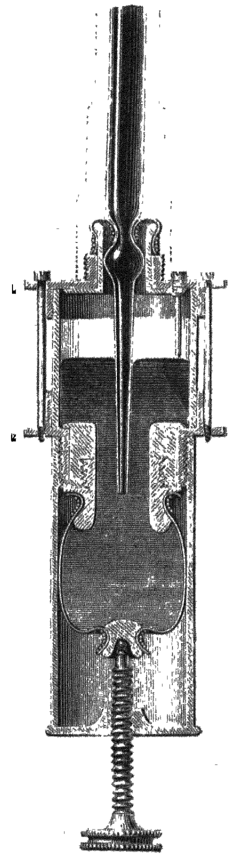


Fig 109
Cistern of Fortin's Barometer

Before taking an observation, the surface of the mercury is ad-

[1] The vernier is an instrument very largely employed for measuring the fractions of a unit of length on any scale. Suppose we have a scale divided into inches, and another scale containing nine inches divided into ten equal parts. If now we make the end of this

justed, by means of the lower screw, to touch the ivory point. The observer knows when this condition is fulfilled by seeing the extremity of the point touch its image in the mercury. The sliding piece which carries the vernier is then raised or lowered, until its base is seen to be tangential to the upper surface of the mercurial column, as shown in Fig. 110. In making this adjustment, the back of the instrument should be turned towards a good light, in order that the observer may be certain of the position in which the light is just cut off at the summit of the convexity.

When the instrument is to be carried from place to place, precautions must be taken to prevent the mercury from bumping against the top of the tube and breaking it. The screw at the bottom is to be turned until the mercury reaches the top of the tube, and the instrument is then to be inverted and carried upside down.

We may here remark that the goodness of the vacuum in a barometer, can be tested by the sound of the mercury when it strikes the top of the tube, which it can be made to do either by screwing

latter scale, which is called the vernier, coincide with one of the divisions in the scale of inches, as each division of the vernier is $\frac{9}{10}$ of an inch, it is evident that the first division on the scale will be $\frac{1}{10}$ of an inch beyond the first division on the vernier, the second on the scale $\frac{2}{10}$ beyond the second on the vernier, and so on until the ninth on the scale, which

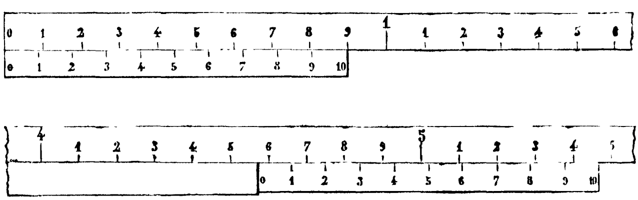


Fig. 111.—Vernier.

will exactly coincide with the tenth on the vernier. Suppose next that in measuring any length we find that its extremity lies between the degrees 5 and 6 on the scale; we bring the zero of the vernier opposite the extremity of the length to be measured, and observe what division on the vernier coincides with one of the divisions on the scale. We see in the figure that it is the seventh, and thus we conclude that the fraction required is $\frac{7}{10}$ of an inch.

If the vernier consisted of 19 inches divided into 20 equal parts, it would read to the $\frac{1}{20}$ of an inch; but there is a limit to the precision that can thus be obtained. An exact coincidence of a division on the vernier with one on the scale seldom or never takes place, and we merely take the division which approaches nearest to this coincidence; so that when the difference between the degrees on the vernier and those on the scale is very small, there may be so much uncertainty in this selection as to nullify the theoretical precision of the instrument. Verniers are also employed to measure angles; when a circle is divided into half degrees, a vernier is used which gives $\frac{1}{30}$ of a division on the circle, that is, $\frac{1}{30}$ of a half degree, or one minute.—*D.*

up or by inclining the instrument to one side. If the vacuum is good, a metallic clink will be heard, and unless the contact be made very gently, the tube will be broken by the sharpness of the collision. If any air be present, it acts as a cushion.

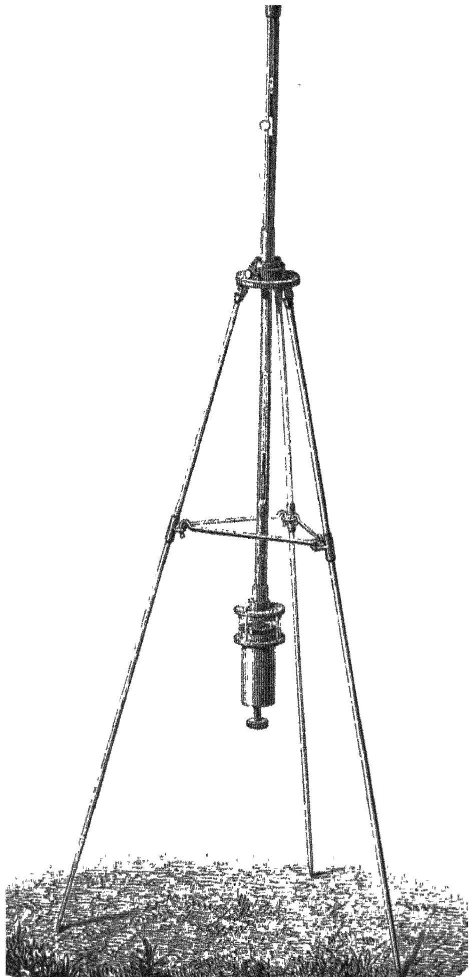


Fig. 112.—Barometer with Tripod Stand.

In making observations in the field, a barometer is usually suspended from a tripod stand (Fig. 112) by gimbals[1], so that it always takes a vertical position.

203. Float Adjustment.—In some barometers the ivory point for indicating the proper level of the mercury in the cistern is replaced by a float. F (Fig. 113) is a small ivory piston, having the float attached to its foot, and moving freely up and down between the two ivory guides I. A horizontal line (interrupted by the piston) is engraved on the two guides, and another is engraved on the piston, at such a height that the three lines form one straight line when the surface of the mercury in the cistern stands at the zero point of the scale.

204. Barometric Corrections.—In order that barometric heights

[1] A kind of universal joint, in common use on board ship for the suspension of compasses, lamps, &c. It is seen in Fig. 112, at the top of the tripod stand.

may be comparable as measures of atmospheric pressure, certain corrections must be applied.

1. *Correction for Temperature.* As mercury expands with heat, it follows that a column of warm mercury exerts less pressure than a column of the same height at a lower temperature; and it is usual to reduce the actual height of the column to the height of a column at the temperature of freezing water which would exert the same pressure.

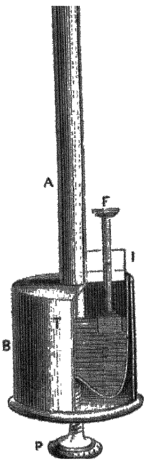


Fig 113.
Float Adjustment.

Let h be the observed height at temperature $t°$ Centigrade, and h_o the height reduced to freezing-point. Then, if m be the coefficient of expansion of mercury per degree Cent., we have

$$h_o\,(1+m\,t)=h, \text{ whence } h_o=h-h\,m\,t \text{ nearly.}$$

The value of m is $\frac{1}{5550}=\cdot00018018$. For temperatures Fahrenheit, we have

$$h_o\left\{1+m\,(t-32)\right\}=h,\ h_o=h-h\,m\,(t-32),$$

where m denotes $\frac{1}{9990}=\cdot0001001$.

But temperature also affects the length of the divisions on the scale by which the height of the mercurial column is measured. If these divisions be true inches at 0° Cent., then at $t°$ the length of n divisions will be $n\,(1+l\,t)$ inches, l denoting the coefficient of linear expansion of the scale, the value of which for brass, the usual material, is ·00001878. If then the observed height h amounts to n divisions of the scale, we have

$$h_o\,(1+m\,t)=h=n\,(1+l\,t);$$

whence

$$h_o=\frac{n\,(1+l\,t)}{1+m\,t}=n-n\,t\,(m-l), \text{ nearly;}$$

that is to say, if n be the height read off on the scale, it must be diminished by the correction $n\,t\,(m-l)$, t denoting the temperature of the mercury in degrees Centigrade. The value of $m-l$ is ·0001614.

For temperatures Fahrenheit, assuming the scale to be of the correct length at 32° Fahr., the formula for the correction (which is still subtractive), is $n\,(t-32)\,(m-l)$, where $m-l$ has the value ·00008967.[1]

[1] The correction for temperature is usually made by the help of tables, which give its amount for all ordinary temperatures and heights. These tables, when intended for

2. *Correction for Capillarity.*—In the preceding chapter we have seen that mercury in a glass tube undergoes a capillary depression; whence it follows that the observed barometric height is too small, and that we must add to it the amount of this depression. In all tubes of internal diameter less than about $\frac{3}{4}$ of an inch this correction is sensible; and its amount, for which no simple formula can be given, has been computed, from theoretical considerations, for various sizes of tube, by several eminent mathematicians, and recorded in tables, from which that given below is abridged. These values are applicable on the assumption that the meniscus which forms the summit of the mercurial column is decidedly convex, as it always is when the mercury is rising. When the meniscus is too flat, the mercury must be lowered by the foot-screw, and then screwed up again.

It is found by experiment, that the amount of capillary depression is only half as great when the mercury has been boiled in the tube as when this precaution has been neglected.

For purposes of special accuracy, tables have been computed, giving the amount of capillary depression for different degrees of convexity, as determined by the sagitta (or height) of the meniscus, taken in conjunction with the diameter of the tube. Such tables, however, are seldom used in this country.[1]

English barometers, are generally constructed on the assumption that the scale is of the correct length not at 32° Fahr., but at 62° Fahr, which is (by act of Parliament) the temperature at which the British standard yard (preserved in the office of the Exchequer) is correct. On this supposition, the length of n divisions of the scale at temperature $t°$ Fahr., is

$$n\left\{1 + l\,(t - 62)\right\};$$

and by equating this expression to

$$h_0\left\{1 + m\,(t - 32)\right\}$$

we find

$$\begin{aligned} h_0 &= n\left\{1 - m\,(t - 32) + l\,(t - 62)\right\} \\ &= n\left\{1 - (m - l)\,t + (32m - 62l)\right\} \\ &= n\left\{1 - \cdot 00008967\,t + \cdot 00255654\right\}; \end{aligned}$$

which, omitting superfluous decimals, may conveniently be put in the form—

$$n - \frac{n}{1000}\,(\cdot 09\,t - 2\cdot 56).$$

The correction vanishes when

$$\cdot 09\,t - 2\cdot 56 = 0;$$

that is, when $t = \frac{256}{9} = 28\cdot 5$.

For all temperatures higher than this the correction is subtractive.

[1] The most complete collection of meteorological and physical tables, is that edited by Professor Guyot, and published under the auspices of the Smithsonian Institution, Washington.

TABLE OF CAPILLARY DEPRESSIONS IN UNBOILED TUBES.

(To be halved for Boiled Tubes.)

Diameter of tube in inches.	Depression in inches.	Diameter.	Depression.	Diameter.	Depression.
·10	·140	·20	·058	·40	·015
·11	·126	·22	·050	·42	·013
·12	·114	·24	·044	·44	·011
·13	·104	·26	·038	·46	·009
·14	·094	·28	·033	·48	·008
·15	·086	·30	·029	·50	·007
·16	·079	·32	·026	·55	·005
·17	·073	·34	·023	·60	·004
·18	·068	·36	·020	·65	·003
·19	·063	·38	·017	·70	·002

3. *Correction for Capacity.*—When there is no provision for adjusting the level of the mercury in the cistern to the zero point of the scale, another correction must be applied. It is called the correction for *capacity*. In barometers of this construction, which were formerly much more common than they are at present, there is a certain point in the scale at which the mercurial column stands when the mercury in the cistern is at the correct level. This is called the neutral point. If A be the interior area of the tube, and C the area of the cistern (exclusive of the space occupied by the tube and its contents), when the mercury in the tube rises by the amount x, the mercury in the cistern falls by an amount $y=\frac{A}{C}x$; for the volume of the mercury which has passed from the cistern into the tube is $Cy=Ax$. The change of atmospheric pressure is correctly measured by $x+y=\left(1+\frac{A}{C}\right)x$; and if we now take x to denote the distance of the summit of the mercurial column from the neutral point, the corrected distance will be $\left(1+\frac{A}{C}\right)x$, and the correction to be applied to the observed reading will be $\frac{A}{C}x$, which is additive if the observed reading be above the neutral point, subtractive if below.

It is worthy of remark that the neutral point depends upon the volume of mercury. It will be altered if any mercury be lost or added; and as temperature affects the volume, a special temperature-correction must be applied to barometers of this class. The investigation will be found in a paper by Professor Swan in the *Philosophical Magazine* for 1861.

In some modern instruments the correction for capacity is avoided, by making the divisions on the scale less than true inches, in the

ratio $\frac{C}{A+C}$, and the effect of capillarity is at the same time compensated by lowering the zero point of the scale. Such instruments, if correctly made, simply require to be corrected for temperature.

4. *Index Errors.*—Under this name are included errors of graduation, and errors in the position of the zero of the graduations. An error of zero makes all readings too high or too low by the same amount. Errors of graduation (which are generally exceedingly small) are different for different parts of the scale.

Barometers intended for accurate observation are now usually examined at Kew Observatory before being sent out; and a table is furnished with each, showing its index error at every half inch of the scale, errors of capillarity and capacity (if any) being included as part of the index error. We may make a remark here once for all respecting the signs attached to errors and corrections. The sign of an error is always opposite to that of its correction. When a reading is too high the index error is one of excess, and is therefore positive; whereas the correction needed to make the reading true is subtractive, and is therefore negative.

5. *Reduction to Sea-level.*—In comparing barometric observations taken over an extensive district for meteorological purposes, it is usual to apply a correction for difference of level. Atmospheric pressure, as we have seen, diminishes as we ascend; and it is usual to add to the observed height the difference of pressure due to the elevation of the place above sea-level. The amount of this correction is proportional to the observed pressure. The law according to which it increases with the height will be discussed in the next chapter.

6. *Correction for Unequal Intensity of Gravity.*—When two barometers indicate the same height, at places where the intensity of gravity is different (for example, at the pole and the equator), the same mass of air is superincumbent over both; but the pressures are unequal, being proportional to the intensity of gravity as measured by the values of g (§ 91) at the two places.

If h be the height, in centimetres, of the mercurial column at the temperature 0° Cent., the absolute pressure, in dynes per square centimetre, will be $gh \times 13{\cdot}596$; since 13·596 is the density of mercury at this temperature.

o **205. Other kinds of Mercurial Barometer.**—The *Siphon Barometer*, which is represented in Fig. 114, consists of a bent tube, generally

of uniform bore, having two unequal legs. The longer leg, which must be more than 30 inches long, is closed, while the shorter leg is open. A sufficient quantity of mercury having been introduced to fill the longer leg, the instrument is set upright (after boiling to expel air), and the mercury takes such a position that the difference of levels in the two legs represents the pressure of the atmosphere.

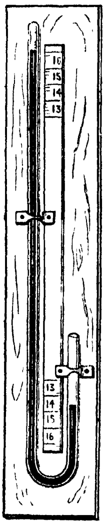


Fig. 114. Siphon Barometer

Supposing the tube to be of uniform section, the mercury will always fall as much in one leg as it rises in the other. Each end of the mercurial column therefore rises or falls through only half the height corresponding to the change of atmospheric pressure.

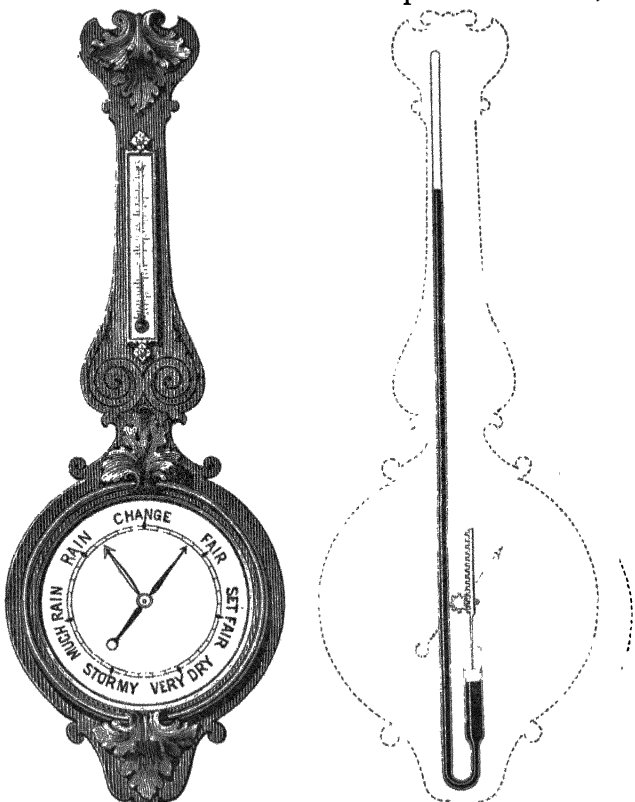
In the best siphon barometers there are two scales, one for each leg, as indicated in the figure, the divisions on one being reckoned upwards, and on the other downwards, from an intermediate zero point, so that the sum of the two readings is the difference of levels of the mercury in the two branches.

Inasmuch as capillarity tends to depress both extremities of the mercurial column, its effect is generally neglected in siphon barometers; but practically it causes great difficulty in obtaining accurate observations, for according as the mercury is rising or falling its extremity is more or less convex, and a great deal of tapping is usually required to make both ends of the column assume the same form, which is the condition necessary for annihilating the effect of capillary action.

Wheel Barometer.—The wheel barometer, which is in more general use than its merits deserve, consists of a siphon barometer, the two branches of which have usually the same diameter. On the surface of the mercury of the open branch floats a small piece of iron or glass suspended by a thread, the other extremity of which is fixed to a pulley, on which the thread is partly rolled. Another thread, rolled parallel to the first, supports a weight which balances the float. To the axis of the pulley is fixed a needle which moves on a dial. When the level of the mercury varies in either direction, the float follows its movement through the same distance; by the action of the counterpoise the pulley turns, and with it the needle, the extremity of which points to the figures on the dial, marking the barometric heights. The mounting of the dial is usually placed

in front of the tube, so as to conceal its presence. The wheel barometer is a very old invention, and was introduced by the celebrated Hooke in 1683. The pulley and strings are sometimes replaced by a rack and pinion, as represented in the figure (Fig. 115).

Besides the faults incidental to the siphon barometer, the wheel



Barometer.

barometer is encumbered in its movements by the friction of the additional apparatus. It is quite unsuitable for measuring the exact amount of atmospheric pressure, and is slow in indicating changes.

Marine Barometer.—The ordinary mercurial barometer cannot be used at sea on account of the violent oscillations which the mercury would experience from the motion of the vessel. In order to meet this difficulty, the tube is contracted in its middle portion nearly to

capillary dimensions, so that the motion of the mercury in either direction is hindered. An instrument thus constructed is called a marine barometer. When such an instrument is used on land it is always too slow in its indications.

206. **Aneroid Barometer** (α, νηρος).—This barometer depends upon the changes in the form of a thin metallic vessel partially exhausted

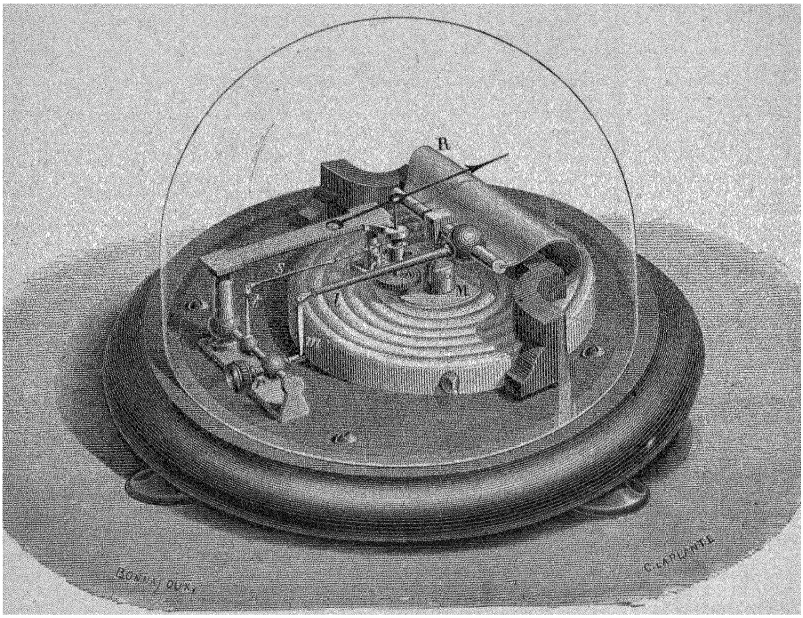


Fig 116.—Aneroid Barometer.

of air, as the atmospheric pressure varies. M. Vidie was the first to overcome the numerous difficulties which were presented in the construction of these instruments. We subjoin a figure of the model which he finally adopted.

The essential part is a cylindrical box partially exhausted of air, the upper surface of which is corrugated in order to make it yield more easily to external pressure. At the centre of the top of the box is a small metallic pillar M, connected with a powerful steel spring R. As the pressure varies, the top of the box rises or falls, transmitting its movement by two levers *l* and *m*, to a metallic axis *r*. This latter carries a third lever *t*, the extremity of which is attached to a chain *s* which turns a drum, the axis of which bears the index needle. A spiral spring keeps the chain constantly stretched, and thus makes the needle always take a position corre-

sponding to the shape of the box at the time. The graduation is performed empirically by comparison with a mercurial barometer. The aneroid barometer is very quick in indicating changes, and is much more portable than any form of mercurial barometer, being both lighter and less liable to injury. It is sometimes made small enough for the waistcoat pocket. It has the drawback of being affected by temperature to an extent which must be determined for each instrument separately, and of being liable to gradual changes which can only be checked by occasional comparison with a good mercurial barometer.

In the *metallic barometer*, which is a modification of the aneroid, the exhausted box is crescent-shaped, and the horns of the crescent separate or approach according as the external pressure diminishes or increases.

207. **Old Forms Revived.**—There are two ingenious modifications of the form of the barometer, which, after long neglect, have recently been revived for special purposes.

Counterpoised Barometer.—The invention of this instrument is attributed to Samuel Morland, who constructed it about the year 1680. It depends upon the following principle —If the barometric tube is suspended from one of the scales of a balance, there will be required to balance it in the other scale a weight equal to the weight of the tube and the mercury contained in it, minus the upward pressure due to the liquid displaced in the cistern.[1] If the atmospheric pressure increases, the mercury will rise in the tube, and consequently the weight of the floating body will increase, while the sinking of the mercury in the cistern will diminish the upward pressure due to the displacement. The beam will thus incline to

[1] A complete investigation based on the assumption of a constant upward pull at the top of the suspended tube shows that the sensitiveness of the instrument depends only on the internal section of the upper part of the tube and the external section of its lower part. Calling the former A and the latter B, it is necessary for stability that B be greater than A (which is not the case in the figure in the text) and the movement of the tube will be to that of the mercury in a standard barometer as A is to $B-A$. The directions of these movements will be opposite. If $B-A$ is very small compared with A, the instrument will be exceedingly sensitive; and as $B-A$ changes sign, by passing through zero, the equilibrium becomes unstable.

A curious result of the investigation is that the level of the mercury in the cistern remains constant.

In the instrument represented in the figure, stability is probably obtained by the weight of the arm which carries the pencil.

In King's barograph, B is made greater than A by fixing a hollow iron drum round the lower end of the tube.

the side of the barometric tube, and the reverse movement would occur if the pressure diminished. For the balance may be substituted, as in Fig. 117, a lever carrying a counterpoise; the variations f pressure will be indicated by the movements of this lever.

Such an instrument may very well be used as a *barograph* or recording barometer; for this purpose we have only to attach to the lever an arm with a pencil, which is constantly in contact with a sheet of paper moved uniformly by clock-work. The result will be a continuous trace, whose form corresponds to the variations of pressure. It is very easy to determine, either by calculation or by comparison with a standard barometer the pressure corresponding to a given position of the pencil on the paper; and thus, if the paper is ruled with twenty-four equidistant lines, corresponding to the twenty-four hours of the day, we can see at a glance what was the pressure at any given time. An arrangement of this kind has been adopted by the Abbé Secchi for the meteorograph of the observatory at Rome. The first successful employment of this kind of barograph appears to be due to Mr. Alfred King, a gas engineer of Liverpool, who invented and constructed such an instrument in 1853, for the use of the Liverpool Observatory, and subsequently designed a larger one, which is still in use, furnishing a very perfect record, magnified five-and-a-half times.

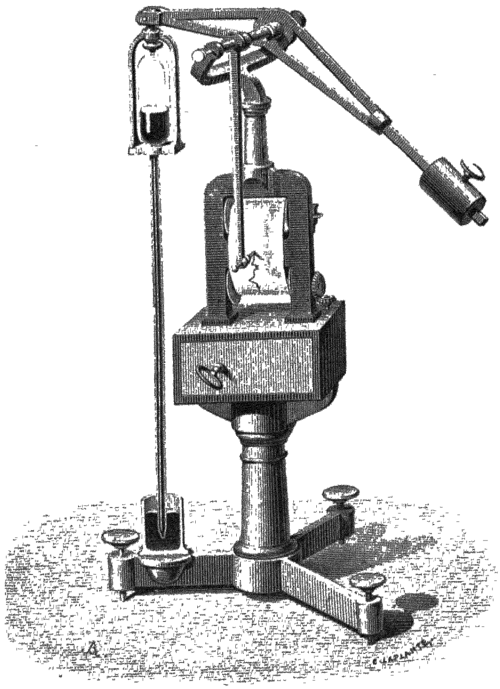


Fig 117.—Counterpoised Barometer.

Fahrenheit's Barometer.—Fahrenheit's barometer consists of a tube bent several times, the lower portions of which contain mercury; the upper portions are filled with water, or any other liquid, usually

coloured. It is evident that the atmospheric pressure is balanced by the sum of the differences of level of the columns of mercury, diminished by the sum of the corresponding differences for the columns of water; whence it follows that, by employing a considerable number of tubes, we may greatly reduce the height of the barometric column. This circumstance renders the instrument interesting as a scientific curiosity, but at the same time diminishes its sensitiveness, and renders it unfit for purposes of precision. It is therefore never used for the measurement of atmospheric pressure; but an instrument upon the same principle has recently been employed for the measurement of very high pressures, as will be explained in Chap. xix.

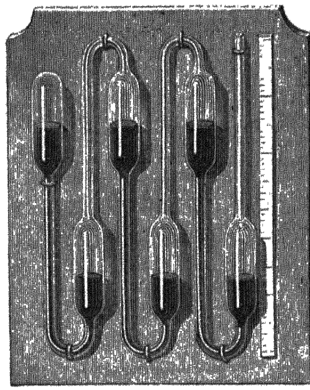


Fig. [illegible] Barometer

208. Photographic Registration.—Since the year 1847 various meteorological instruments at the Royal Observatory, Greenwich, have been made to yield continuous traces of their indications by the aid of photography, and the method is now generally employed at meteorological observatories in this country. The Greenwich system is fully described in the *Greenwich Magnetical and Meteorological Observations* for 1847, pp. lxiii.–xc. (published in 1849).

The general principle adopted for all the instruments is the same. The photographic paper is wrapped round a glass cylinder, and the axis of the cylinder is made parallel to the direction of the movement which is to be registered. The cylinder is turned by clockwork, with uniform velocity. The spot of light (for the magnets and barometer), or the boundary of the line of light (for the thermometers), moves, with the movements which are to be registered, backwards and forwards in the direction of the axis of the cylinder, while the cylinder itself is turned round. Consequently (as in Morin's machine, Chap. vii.), when the paper is unwrapped from its cylindrical form, there is traced upon it a curve of which the abscissa is proportional to the time, while the ordinate is proportional to the movement which is the subject of measure.

The barometer employed in connection with this system is a large siphon barometer, the bore of the upper and lower extremities of its arms being about 1·1 inch. A glass float in the quicksilver of the

lower extremity is partially supported by a counterpoise acting on a light lever (which turns on delicate pivots), so that the wire supporting the float is constantly stretched, leaving a definite part of the weight of the float to be supported by the quicksilver. This lever is lengthened to carry a vertical plate of opaque mica with a small aperture, whose distance from the fulcrum is eight times the distance of the point of attachment of the float-wire, and whose movement, therefore (§ 205), is four times the movement of the column of a cistern barometer. Through this hole the light of a lamp, collected by a cylindrical lens, shines upon the photographic paper.

Every part of the cylinder, except that on which the spot of light falls, is covered with a case of blackened zinc, having a slit parallel to the axis of the cylinder; and by means of a second lamp shining through a small fixed aperture, and a second cylindrical lens, a base line is traced upon the paper, which serves for reference in subsequent measurements.

The whole apparatus, or any other apparatus which serves to give a continuous trace of barometric indications, is called a *barograph;* and the names *thermograph, magnetograph, anemograph,* &c., are similarly applied to other instruments for automatic registration. Such registration is now employed at a great number of observatories; and curves thus obtained are regularly published in the Quarterly Reports of the Meteorological Office.

208A. **Glycerine Barometer.**—The fluctuations of the barometer will be rendered more visible by employing a liquid much lighter than mercury. To be suitable for the purpose, the liquid must not be liable either to evaporate or to increase in volume by exposure to the air, and the pressure of its vapour at atmospheric temperatures must not be considerable. Glycerine has been found to meet these requirements well, and as its density at 0° C. is 1·27, as compared with 13·596 for mercury, it magnifies the fluctuations about 10·7 times.

CHAPTER XVIII.

VARIATIONS OF THE BAROMETER.

209. **Measurement of Heights by the Barometer.**—As the height of the barometric column diminishes when we ascend in the atmosphere, it is natural to seek in this phenomenon a means of measuring heights. The problem would be extremely simple, if the air had everywhere the same density as at the surface of the earth. In fact, the density of the air at sea-level being about 10,500 times less than that of mercury, it follows that, on the hypothesis of uniform density, the mercurial column would fall an inch for every 10,500 inches, or 875 feet that we ascend. This result, however, is far from being in exact accordance with fact, inasmuch as the density of the air diminishes very rapidly as we ascend, on account of its great compressibility.

210. **Imaginary Homogeneous Atmosphere.**—If the atmosphere were of uniform and constant density, its height would be approximately obtained by multiplying 30 inches by 10,500, which gives 26,250 feet, or about 5 miles.

More accurately, if we denote by H the height (in centimetres) of the atmosphere at a given time and place, on the assumption that the density throughout is the same as the observed density D (in grammes per cubic centimetre) at the base, and if we denote by P the observed pressure at the base (in dynes per square centimetre), we must employ the general formula for liquid pressure (§ 139)

$$P = g\,HD, \text{ which gives } H = \frac{P}{gD}. \qquad (1)$$

The height H, computed on this imaginary assumption, is usually called the *height of the homogeneous atmosphere*, corresponding to the pressure P, density D, and intensity of gravity g. It is sometimes called the *pressure-height*. The *pressure-height* at any point

in a liquid or gas is the height of a column of fluid, having the same density as at the point, which would produce, by its weight, the actual pressure at the point. This element frequently makes its appearance in physical and engineering problems.

The expression for H contains P in the numerator and D in the denominator; and by Boyle's law, which we shall discuss in the ensuing chapter, these two elements vary in the same proportion, when the temperature is constant. Hence H is not affected by changes of pressure, but has the same value at all points in the air at which the temperature and the value of g are the same.

211. Geometric Law of Decrease.—The change of pressure as we ascend or descend *for a short distance* in the actual atmosphere, is sensibly the same as it would be in this imaginary "homogeneous atmosphere;" hence an ascent of 1 centimetre takes off $\frac{1}{\mathrm{H}}$ of the total pressure, just as an ascent of one foot from the bottom of an ocean 60,000 feet deep takes off $\frac{1}{60000}$ of the pressure.

Since H is the same at all heights in any portion of the air which is at uniform temperature, it follows that in ascending by successive steps of 1 centimetre in air at uniform temperature, each step takes off the same fraction $\frac{1}{\mathrm{H}}$ of the current pressure. The pressures therefore form a geometrical progression whose ratio is $1-\frac{1}{\mathrm{H}}$. *In an atmosphere of uniform temperature,* neglecting the variation of g with height, *the densities and pressures diminish in geometrical progression as the heights increase in arithmetical progression.*

212. Computation of Pressure-height.—For perfectly dry air at 0° Cent., we have the data (§§ 195, 198),

$$\mathrm{D} = \cdot 0012932 \text{ when } \mathrm{P} = 1013600;$$

which give

$$\frac{\mathrm{P}}{\mathrm{D}} = 783800000 \text{ nearly.}$$

Taking g as 981, we have

$$\mathrm{H} = \frac{783800000}{981} = 799000 \text{ centimetres nearly.}$$

This is very nearly 8 kilometres, or about 5 miles. At the temperature $t°$ Cent., we shall have

$$\mathrm{H} = 799000\ (1 + \cdot 00366\ t). \qquad (2)$$

Hence in air at the the temperature 0° Cent., the pressure diminishes by 1 per cent. for an ascent of about 7990 centimetres or, say, 80 metres. At 20° Cent., the number will be 86 instead of 80.

213. Formula for determining Heights by the Barometer.—To obtain an accurate rule for computing the difference of levels of two stations from observations of the barometer, we must employ the integral calculus.

Denote height above a fixed level by x, and pressure by p. Then we have

$$\frac{dx}{\mathrm{H}} = -\frac{dp}{p};$$

and if p_1, p_2 are the pressures at the heights x_1, x_2, we deduce by integration

$$x_2 - x_1 = \mathrm{H}\,(\log_e p_1 - \log_e p_2).$$

Adopting the value of H from (2), and remembering that Napierian logarithms are equal to common logarithms multiplied by 2·3026, we finally obtain

$$x_2 - x_1 = 1840000\,(1 + \cdot 00366\,t)\,(\log p_1 - \log p_2)$$

as the expression for the difference of levels, in centimetres. It is usual to put for t the arithmetical mean of the temperatures at the two stations.

The determination of heights by means of atmospheric pressure, whether the pressure be observed directly by the barometer, or indirectly by the boiling-point thermometer (which will be described in Part II.), is called *hypsometry* (ὑψος, height).

As a rough rule, it may be stated that, in ordinary circumstances, the barometer falls an inch in ascending 900 feet.

214. Diurnal Oscillation of the Barometer.—In these latitudes, the mercurial column is in a continual state of irregular oscillation; but in the tropics it rises and falls with great regularity according to the hour of the day, attaining two maxima in the twenty-four hours.

It generally rises from 4 A.M. to 10 A.M., when it attains its first maximum; it then falls till 4 P.M., when it attains its first minimum; a second maximum is observed at 10 P.M., and a second minimum at 4 A.M. The hours of maxima and minima are called the tropical hours (τρεπω, to turn), and vary a little with the season of the year. The difference between the highest maximum and lowest minimum is called the diurnal[1] *range*, and the half of this is called the *ampli-*

[1] The epithets *annual* and *diurnal*, when prefixed to the words *variation*, *range*, *amplitude*, denote the *period* of the variation in question; that is, the time of a complete oscillation. Diurnal variation does not denote variation from one day to another, but the variation which goes through its cycle of values in one day of twenty-four hours. Annual

tude of the diurnal oscillation. The amount of the former does not exceed about a tenth of an inch.

The character of this diurnal oscillation is represented in Fig. 119. The vertical lines correspond to the hours of the day; lengths have been measured upwards upon them proportional to the barometric heights at the respective hours, diminished by a constant quantity; and the points thus determined have been connected by a continuous curve. It will be observed that the two lower curves, one of which relates to Cumana, a town of Venezuela, situated in about 10° north latitude, show strongly marked oscillations corresponding to the maxima and minima. In our own country, the regular diurnal oscillation is masked by irregular fluctuations, so that a single day's observations give no clue to its existence. Nevertheless, on taking observations at regular hours for a number of consecutive days, and comparing the mean heights for the different hours, some indications of the law will be found. A month's observations will be sufficient for an approximate indication of the law; but observations extending over some years will be required, to establish with anything like precision the hours of maxima and the amplitude of the oscillation.

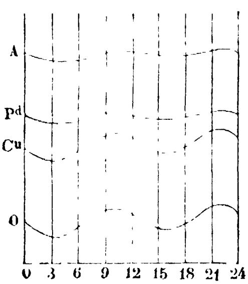


Fig. 119
Curves of Diurnal Variation.

The two upper curves represent the diurnal variation of the barometer at Padua (lat. 45° 24′) and Abo (lat. 60° 56′), the data having been extracted from Kaemtz's *Meteorology*. We see, by inspection of the figure, that the oscillation in question becomes less strongly marked as the latitude increases. The range at Abo is less than half a millimetre. At about the 70th degree of north latitude it becomes insensible; and in approaching still nearer to the pole, it appears from observations, which however need further confirmation, that the oscillation is reversed; that is to say, that the maxima here are contemporaneous with the minima in lower latitudes.

There can be little doubt that the diurnal oscillation of the barometer is in some way attributable to the heat received from the sun, which produces expansion of the air, both directly, as a mere

range denotes the range that occurs within a year. This rule is universally observed by writers of high scientific authority.

A table, exhibiting the values of an element for each month in the year, is a table of annual (not monthly) variation; or it may be more particularly described as a table of variations from month to month.

consequence of heating, and indirectly, by promoting evaporation; but the precise nature of the connection between this cause and the diurnal barometric oscillation has not as yet been satisfactorily established.

○ **215. Irregular Variations of the Barometer.**—The height of the barometer, at least in the temperate zones, depends on the state of the atmosphere; and its variations often serve to predict the changes of weather with more or less certainty. In this country the barometer generally falls for rain or S.W. wind, and rises for fine weather or N.E. wind.

Barometers for popular use have generally the words—

Set fair.	Fair.	Change.	Rain.	Much rain.	Stormy.

marked at the respective heights

30·5	30	29·5	29	28·5	28 inches.

These words must not, however, be understood as absolute predictions. A low barometer rising is generally a sign of fine, and a high barometer falling of wet weather. Moreover, it is to be borne in mind that the barometer stands about a tenth of an inch lower for every hundred feet that we ascend above sea-level.

The connection between a low or falling barometer and wet weather is to be found in the fact that moist air is specifically lighter than dry, even at the same temperature, and still more when, as usually happens, moist air is warmer than dry.

Change of wind usually begins in the upper regions of the air and gradually extends downwards to the ground; hence the barometer, being affected by the weight of the whole superincumbent atmosphere, gives early warning.

○ **216. Weather Charts. Isobaric Lines.**—The probable weather can be predicted with much greater certainty if the height of the barometer at surrounding places is also known. The weather forecasts issued daily from the Meteorological Office in London are based on reports received twice a day from about sixty stations scattered over the west of Europe, from the north of Norway to Lisbon, and from the west of Ireland to Berlin. The reading of the barometer reduced to sea-level at each place is recorded on a chart, and curves called *isobaric lines* or *isobars* are drawn through places at which the pressure has given values, proceeding usually by steps

of a tenth of an inch. Curves called *isothermal lines* or *isotherms* are also drawn through places of equal temperature. The strength and direction of wind, and the state of weather and of sea are also entered. The charts are compared with those of the previous day, and from the changes in progress the ensuing weather can be inferred with a fair probability of success.

The isobars furnish the most important aid in these forecasts; for from their form and distribution the direction and strength of the wind in each district can be inferred, and to a certain extent the state of the weather generally. As a rule the wind blows from places of higher to places of lower pressure, but not in the most direct line. It deviates more than 45° to the right of the direct line in the northern hemisphere, and to the left in the southern. This is known as Buys Ballot's law, and is a consequence of the earth's rotation.[1]

Very frequently a number of isobars form closed curves, encircling an area of low pressure, to which, in accordance with the above law, the wind blows spirally inwards, in the direction of watch-hands in the southern hemisphere, and against watch-hands in the northern. This state of things is called a *cyclone*. Cyclones usually approach the British Islands from the Atlantic, travelling in a north-easterly direction with a velocity of from ten to twenty miles an hour; sometimes disappearing within a day of their formation, and sometimes lasting for several days. They are the commonest type of distribution of pressure in western Europe, and are usually accompanied by unsettled weather.

The opposite state of things,—that is, a centre of maximum pressure from which the wind blows out spirally with watch-hands in the northern and against watch-hands in the southern hemisphere is called an *anticyclone*. It is usually associated with light winds and fine weather, and is favourable to frost in winter. Anticyclones usually move and change slowly.

The names *cyclone* and *anticyclone* are frequently applied to the distributions of pressure above indicated without taking account of the wind.

The strength of wind generally bears some proportion to the

[1] The influence of the earth's rotation in modifying the direction of winds is discussed in a paper "On the General Circulation and Distribution of the Atmosphere," by the editor of this work, in the *Philosophical Magazine* for September, 1871. Some of the results are stated in the last chapter of Part II. of the present work.

steepness of the barometric gradient, in other words to the closeness of the isobars. Violent storms of wind are usually cyclones, and it was to these that the name was first applied. The phenomenon reaches its extreme form in the tornadoes of tropical regions. The persistence of a cyclone can be explained by the fact that the centrifugal force of the spirally moving air tends to increase the original central depression.

The frontispiece of this volume is a chart of pressure and wind for the United States of America at 4·35 P.M. Washington time on the 15th of January, 1877, when a great storm was raging. The figures marked against the isobars are the pressures to tenths of an inch. They exhibit a very steep gradient on the north-west side of the central depression—a tenth of an inch for about forty-three nautical miles. The direction of the wind is shown by arrows, and the number of feathers on each arrow multiplied by five gives the velocity of the wind in miles per hour. It will be seen that the strongest winds are in or near the region of steepest gradient, and that the directions of the winds are for the most part in accordance with Buys Ballot's law.

CHAPTER XIX.

BOYLE'S (OR MARIOTTE'S) LAW.[1]

217. **Boyle's Law.**—All gases exhibit a continual tendency to expand, and thus exert pressure against the vessels in which they are confined. The intensity of this pressure depends upon the volume which they occupy, increasing as this volume diminishes. By a number of careful experiments upon this point, Boyle and Mariotte independently established the law that this pressure varies inversely as the volume, provided that the temperature remain constant. As the density also varies inversely as the volume, we may express the law in other words by saying that at the same temperature the density varies directly as the pressure.

If V and V′ be the volumes of the same quantity of gas, P and P′, D and D′, the corresponding pressures and densities, Boyle's law will be expressed by either of the equations

$$\frac{P}{P'} = \frac{V'}{V}, \quad \frac{P}{P'} = \frac{D}{D'}.$$

218. **Boyle's Tube.**—The correctness of this law may be verified by means of the following apparatus, which was employed by both the experimenters above named. It consists (Fig. 120) of a bent tube with branches of unequal length; the long branch is open, and the short branch closed. The tube is fastened to a board provided with two scales, one by the side of each branch. The

[1] Boyle, in his *Defence of the Doctrine touching the Spring and Weight of the Air against the Objections of Franciscus Linus*, appended to *New Experiments, Physico-mechanical*, &c. (second edition, 4to, Oxford, 1662), describes the two kinds of apparatus represented in Figs. 120, 121 as having been employed by him, and gives in tabular form the lengths of tube occupied by a body of air at various pressures. These observed lengths he compares with the theoretical lengths computed on the assumption that volume varies reciprocally as pressure, and points out that they agree within the limits of experimental error.

Mariotte's treatise, *De la Nature de l'Air*, is stated in the *Biographie Universelle* to have been published in 1679. (See Preface to Tait's *Thermodynamics*, p. iv.)

graduation of both scales begins from the same horizontal line through 0, 0. Mercury is first poured in at the extremity of the long branch, and by inclining the apparatus to either side, and cautiously adding more of the liquid if required, the mercury can be made to stand at the same level in both branches, and at the zero of both scales. Thus we have, in the short branch, a quantity of air separated from the external air, and at the same pressure. Mercury is then poured into the long branch, so as to reduce the volume of this inclosed air by one-half; it will then be found that the difference of level of the mercury in the two branches is equal to the height of the barometer at the time of the experiment; the compressed air therefore exerts a pressure equal to that of two atmospheres. If more mercury be poured in so as to reduce the volume of the air to one-third or one-fourth of the original volume, it will be found that the difference of level is respectively two or three times the height of the barometer; that is, that the compressed air exerts a pressure equal respectively to that of three or four atmospheres. This experiment therefore shows that if the volume of the gas becomes two, three, or four times as small, the pressure becomes two, three, or four times as great. This is the principle expressed in Boyle's law.

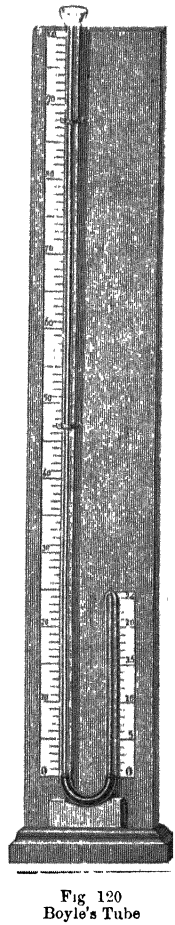


Fig 120
Boyle's Tube

The law may also be verified in the case where the gas expands, and where its pressure consequently diminishes. For this purpose a barometric tube (Fig. 121), partially filled with mercury, is inverted in a tall vessel, containing mercury also, and is held in such a position that the level of the liquid is the same in the tube and in the vessel. The volume occupied by the gas is marked, and the tube is raised; the gas expands, its pressure diminishes, and, in virtue of the excess of the atmospheric pressure, a column of mercury *ab* rises in the tube, such that its height, added to the pressure of the expanded air, is equal to the atmospheric pressure. It will then be seen that if the volume of air becomes double what it was before, the height of the column raised is one-half that of the barometer; that is, the

expanded air exerts a pressure equal to half that of the atmosphere If the volume is trebled, the height of the column is two-thirds that of the barometer; that is, the pressure of the expanded air is one-third that of the atmosphere, a result in accordance with Boyle's law.

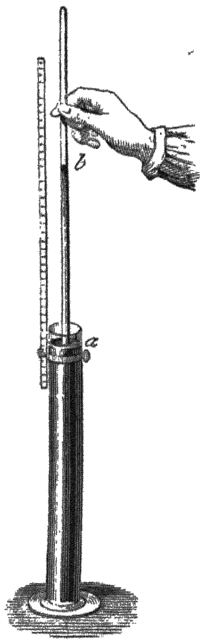


Fig. 121.—Proof of Boyle's Law for Expanding Air.

219. Despretz's Experiments.—The simplicity of Boyle's law, taken in conjunction with its apparent agreement with facts, led to its general acceptance as a rigorous truth of nature, until in 1825 Despretz published an account of experiments, showing that different gases are unequally compressible. He inverted in a cistern of mercury several cylindrical tubes of equal height, and filled them with different gases. The whole apparatus was then inclosed in a strong glass vessel filled with water, and having a screw piston as in Œrsted's piesometer (§ 130). On pressure being applied, the mercury rose to unequal heights in the different tubes, carbonic acid for example being more reduced in volume than air. These experiments proved that even supposing Boyle's law to be true for one of the gases employed, it could not be rigorously true for more than one.

In 1829 Dulong and Arago undertook a laborious series of experiments with the view of testing the accuracy of the law as applied to air; and the results which they obtained, even when the pressure was increased to twenty-seven atmospheres, agreed so nearly with it as to confirm them in the conviction that, for air at least, it was rigorously true. When re-examined, in the light of later researches, the results obtained by Dulong and Arago seem to point to a different conclusion.

220. Unequal Compressibility of Different Gases.—The unequal compressibility of different gases, which was first established by Despretz's experiments above described, is now usually exhibited by the aid of an apparatus designed by Pouillet (Fig. 122). A is a cast-iron reservoir, containing mercury surmounted by oil. In this latter liquid dips a bronze plunger P, the upper part of which has a thread cut upon it, and works in a nut, so that the plunger can be screwed up or down by means of the lever L. The reservoir A communicates

by an iron tube with another cast-iron vessel, into which are firmly fastened two tubes TT about six feet in length and $\frac{1}{10}$th of an inch in internal diameter, very carefully calibrated. Equal volumes of two gases, perfectly dry, are introduced into these tubes through their upper ends, which are then hermetically sealed. The plunger is then made to descend, and a gradually increasing pressure is exerted, the volumes occupied by the gases are measured, and it is ascertained that no two gases follow precisely the same law of compression. The difference, however, is almost insensible when the gases employed are those which are very difficult to liquefy, as air, oxygen, hydrogen, nitrogen, nitric oxide, and marsh-gas. But when we compare any one of these with one of the more liquefiable gases, such as carbonic acid, cyanogen, or ammonia, the difference is rapidly and distinctly manifested. Thus, under a pressure of twenty-five atmospheres, carbonic acid occupies a volume which is only $\frac{4}{5}$ths of that occupied by air.

§ 221. **Regnault's Experiments.**—Boyle's law, therefore, is not to be considered as rigorously exact; but it is so nearly exact that to demonstrate its inaccuracy for one of the more permanent gases, and still more to determine the law of deviation for each gas, very precise methods of measurement are necessary. In ordinary experiments on compression, and even in the elaborate investigations of Dulong and Arago, a definite portion of gas is taken and successively diminished in volume by the application of continually increasing pressure. In experiments of this kind, as the pressure increases, the volume under measurement becomes smaller, and the precision with which it can be measured consequently diminishes.

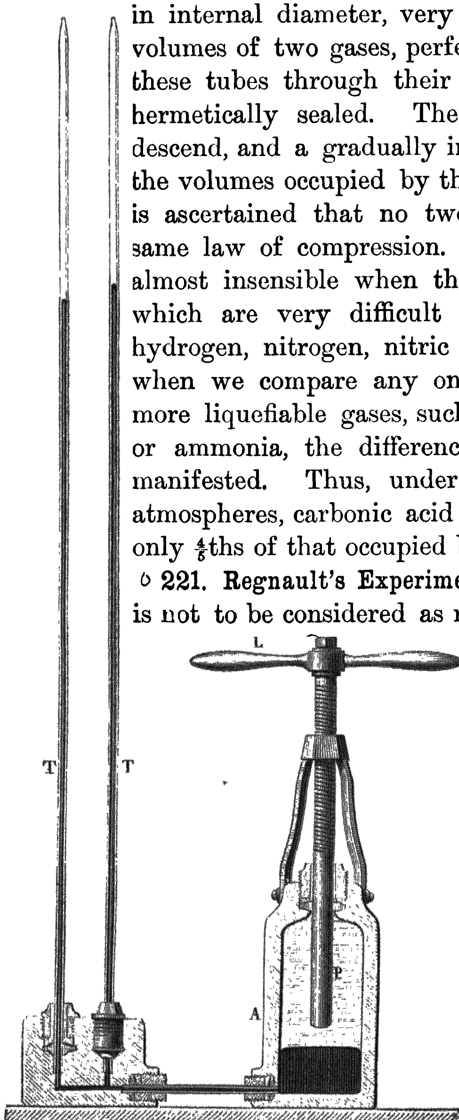


Fig. 122.—Pouillet's Apparatus for showing Unequal Compressibility of Different Gases

Regnault adopted the plan of operating in all cases upon the same volume of gas, which being initially at different pressures, was always reduced to one-half. The pressure was observed before and after this operation, and, if Boyle's law were true, its value should be found to be doubled. In this way the same precision of measurement is obtained at high as at low pressures.

A general view of Regnault's apparatus is given in Fig. 123. There is an iron reservoir containing mercury, furnished at the top with a force-pump for water. The lower part of this reservoir communicates with a cylinder which is also of iron, and in which are two openings to admit tubes. Communication between the reservoir and the cylinder can be established or interrupted by means of a stop-cock R, of very exact workmanship. Into one of the openings is fitted the lowest of a series of glass tubes A, which are placed end to end, and firmly joined to each other by metal fittings, so as to form a vertical column of about twenty-five metres in height.

The height of the mercurial column in this long manometric tube could be exactly

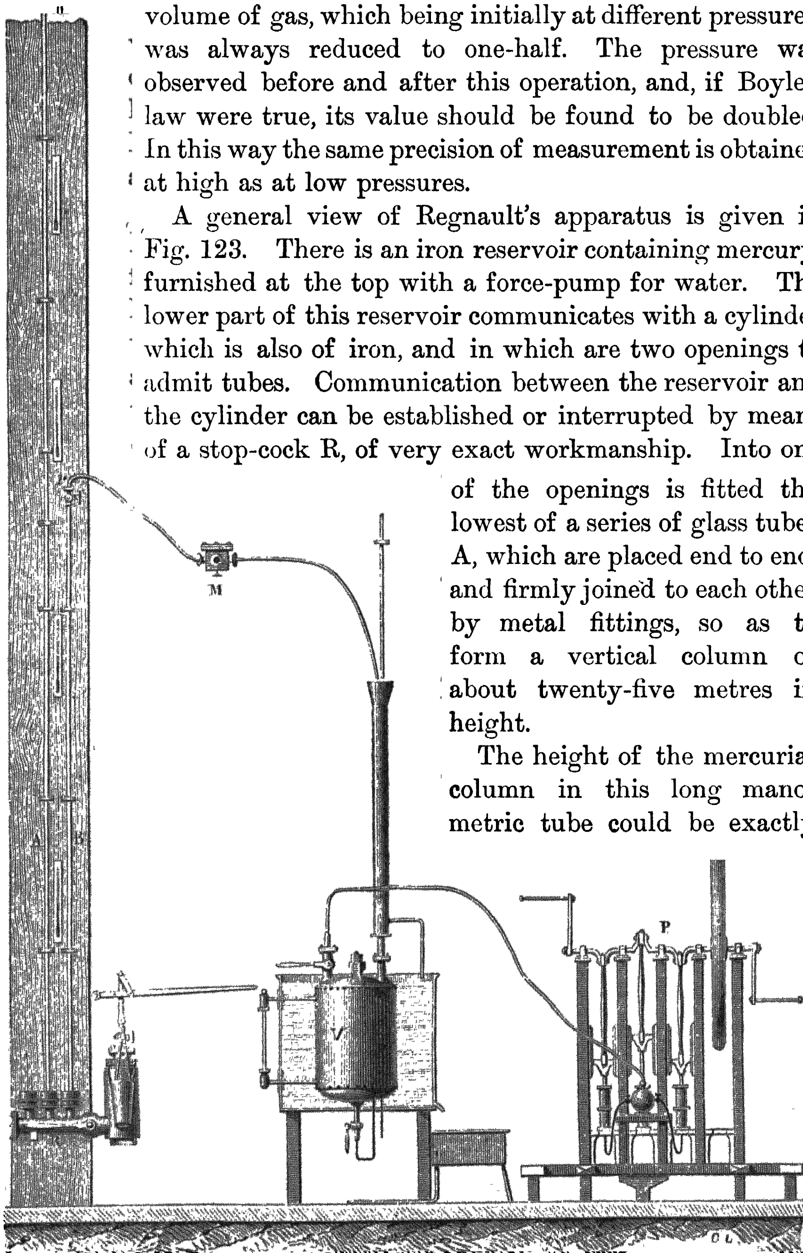


Fig. 123.—Regnault's Apparatus for Testing Boyle's Law.

determined by means of reference marks placed at distances of about ·95 of a metre, and by the graduation on the tubes forming the upper part of the column. The mean temperature of the mercurial column was given by thermometers placed at different heights. Into the second opening in the cylinder fits the lower extremity of the tube B, which is divided into millimetres, and also gauged with great accuracy. This tube has at its upper end a stop-cock *r* which can open communication with the reservoir V, into which the gas to be operated on is forced and compressed by means of the pump P.

An outer tube, which is not shown in the figure, envelops the tube B, and, being kept full of water, which is continually renewed, enables the operator to maintain the tube at a temperature sensibly constant, which is indicated by a very delicate thermometer. Before fixing the tube in its place, the point corresponding to the middle of its volume is carefully ascertained, and after the tube has been permanently fixed, the distance of this point from the nearest of the reference marks is observed.[1]

After these explanatory remarks we may describe the mode of conducting the experiments. The gas to be operated on, after being first thoroughly dried, was introduced at the upper part of the tube B, the stop-cock of the pump being kept open, so as to enable the gas to expel the mercury and occupy the entire length of the tube. The force-pump was then brought into play, and the gas was reduced to about half of its former volume; the pressure in both cases being ascertained by observing the height of the mercury in the long tube above the nearest mark. It is important to remark that it is not at all necessary to operate always upon exactly the same initial volume, and reduce it exactly to one-half, which would be a very tedious operation; these two conditions are approximately fulfilled, and the graduation of the tube enables the observer always to ascertain the actual volumes.

222. **Results.**—The general result of the investigations of Regnault

[1] Regnault's apparatus was fixed in a small square tower of about fifteen metres in height, forming part of the buildings of the Collége de France, and which had formerly been built by Savart for experiments in hydraulics. The tower could therefore contain only the lower part of the manometric column; the upper part rose above the platform at the top of the tower, resting against a sort of mast which could be ascended by the observer. The readings inside the tower could be made by means of a cathetometer, but this was impossible in the upper portion of the column, and for this reason the tubes forming this portion were graduated.—*D.*

is, that Boyle's law does not exactly represent the compressibility even of air, hydrogen, or nitrogen, which, with carbonic acid, were the gases operated on by him. He found that for all the gases on which he operated, except hydrogen, the product VP of the volume and pressure, instead of remaining constant, as it would if Boyle's law were exact, diminished as the compression was increased. This diminution is particularly rapid in the cases of the more liquefiable gases, such as carbonic acid, at least when the experiments are conducted at ordinary atmospheric temperatures. The lower the temperature, the greater is the departure from Boyle's law in the case of these gases. For hydrogen, he found the departure from Boyle's law to be in the opposite direction;—the product VP increased as the gas was more compressed.

223. Manometers or Pressure-gauges.—Manometers or pressure-gauges are instruments for measuring the elastic force of a gas or vapour contained in the interior of a closed space. This elastic force is generally expressed in units called atmospheres (§ 198), and is often measured by means of a column of mercury.

When one end of the column of mercury is open to the air, as in Regnault's experiments above described, the gauge is called an open mercurial gauge.

The open mercurial pressure-gauge is often used in the arts to measure pressures which are not very considerable. Fig. 124 represents one of its simplest forms. The apparatus consists of a box, generally of iron, at the top of which is an opening closed by a screw stopper, which is traversed by the tube *b*, open at both ends, and dipping into the mercury in the box. The air or vapour whose elastic force is to be measured enters by the tube *a*, and presses upon the mercury. It is evident that if the level of the liquid in the box is the same as in the tube, the pressure in the box must be exactly equal to that of the atmosphere. If the mercury in the tube rises above that in the box, the pressure of the air in the box must exceed that of the atmosphere by a pressure corresponding to the height of the column raised. The pressures are generally marked in atmospheres upon a scale beside the tube.

224. Multiple Branch Manometer.—When the pressures to be measured are considerable, as in the boiler of a high-pressure steam-engine, the above instrument, if employed at all, must be of a length corresponding to the pressure. If, for instance, the pressure in question is eight atmospheres, the length of the tube must be at least

8×30 inches $= 20$ feet. Such an arrangement is inconvenient even for stationary machines, and is entirely inapplicable to movable machines.

Without departing from the principle of the open mercurial pressure-gauge, namely, the balancing of the pressure to be observed against the weight of a liquid increased by one atmosphere, we may reduce the length of the instrument by an artifice already employed by Fahrenheit in his barometer (§ 207).

The apparatus for this purpose consists of an iron tube ABCD

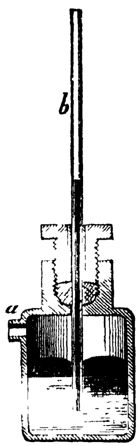


Fig 124 —Open Mercurial Manometer.

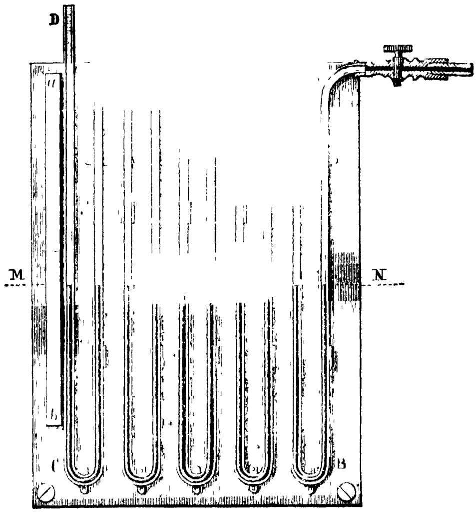


Fig 125 —Multiple Branch Manometer.

(Fig. 125) bent back upon itself several times. The extremity A communicates with the boiler by a stop-cock, and the last branch CD is of glass, with a scale by its side.

The first step is to fill the tube with mercury as far as the level MN. At this height are holes by which the mercury escapes when it reaches them, and which are afterwards hermetically sealed. The upper portions are filled with water through openings which are also stopped after the tube has been filled. If the mercury in the first tube, which is in communication with the reservoir of gas, falls through a distance h, it will alternately rise and fall through the same distance in the other tubes. The difference of pressure between the two ends of the gauge is represented by the weight of a column of mercury of height $10h$ diminished by the weight of a column of water of height $8h$. Reduced to mercury, the difference of pressure is therefore $10h - \frac{8h}{13\cdot5} = 9\cdot4h$.

225. Compressed-air Manometer.—This instrument, which may assume different forms, sometimes consists, as in Fig. 126, of a bent tube AB closed at one end a, and containing within the space Aa a quantity of air, which is cut off from external communication by a column of mercury. The apparatus has been so constructed, that when the pressure on B is equal to that of the atmosphere, the mercury stands at the same height in both branches; so that, under these circumstances, the inclosed air is exactly at atmospheric pressure. But if the pressure increases, the mercury is forced into the left branch, so that the air in that branch is compressed, until equilibrium is established. The pressure exerted by the gas at B is then equal to the pressure of the compressed air, together with that of a column of mercury equal to the difference of level of the liquid in the two branches. This pressure is usually expressed in atmospheres on the scale ab.

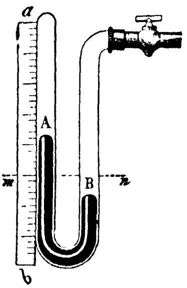


Fig. 126.—Compressed-air Manometer.

The graduation of this scale is effected empirically in practice, by placing the manometer in communication with a reservoir of compressed air whose pressure is given by an open mercurial gauge, or by a standard manometer of any kind.

If the tube AB be supposed cylindrical, the graduation can be calculated by an application of Boyle's law.

Let l be the length of the tube occupied by the inclosed air when its pressure is equal to that of one atmosphere; at the point to which the level of the mercury rises is marked the number 1. It is required to find what point the end of the liquid column should reach when a pressure of n atmospheres is exerted at B. Let x be the height of this point above 1; then the volume of the air, which was originally l, has become $l-x$, and its pressure is therefore equal to $\mathrm{H}\frac{l}{l-x}$, H being the mean height of the barometer. This pressure, together with that due to the difference of level $2x$, is equivalent to n atmospheres. We have thus the equation—

$$\mathrm{H}\frac{l}{l-x}+2x=n\mathrm{H},$$

whence

$$2x^2-(n\mathrm{H}+2l)x+(n-1)\mathrm{H}l=0.$$

$$x=\frac{n\mathrm{H}+2l\pm\sqrt{(n\mathrm{H}+2l)^2-8(n-1)\mathrm{H}l}}{4}.$$

We thus find two values of x; but that given by taking the positive

sign of the radical is inadmissible; for if we put $n=1$, we ought to have $x=o$, which will not be the case unless the sign of the radical is negative.

By giving n the successive values $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, &c., in this expression for x, we find the points on the scale corresponding to pressures of one atmosphere and a half, two atmospheres, &c.

As the pressure increases, the distance traversed by the mercury for an increment of pressure equal to one atmosphere becomes continually less, and the sensibility of the instrument accordingly decreases. This inconvenience is partly avoided by the arrangement shown in Fig. 127. The branch containing the air is made tapering so that, as the mercury rises, equal changes of volume correspond to increasing lengths.

226. Metallic Manometers.—The fragility of glass tubes, and the fact that they are liable to become opaque by the mercury clinging

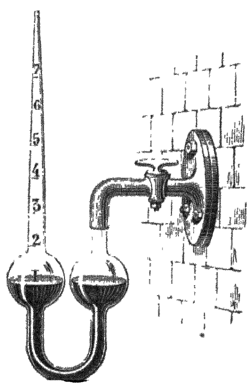


Fig 127.—Compressed air Manometer

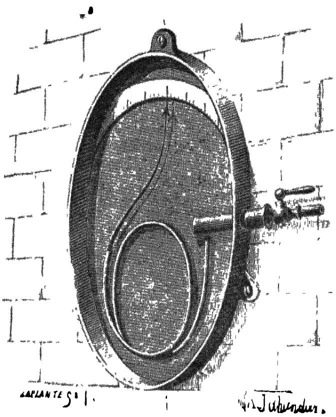


Fig 128.—Bourdon's Pressure gauge.

to their sides, are serious drawbacks to their use, especially in machines in motion. Accordingly, metallic manometers are often employed, their indications depending upon changes of form effected by the pressure of gas on its containing vessel. We shall here mention only Bourdon's gauge (Fig. 128). It consists essentially of a copper tube of elliptic section, which is bent through about 540°, as represented in the figure. One of the extremities communicates by a stop-cock with the reservoir of steam or compressed gas; to the other extremity is attached a steel needle which traverses a scale. When the pressure is the same within and without the tube the end of the needle stands at the mark 1; but if the pressure within the

tube increases, the curvature diminishes, the free extremity of the tube moves away from the fixed extremity, and the needle traverses the scale.

227. **Mixture of Gases.**—When gases of different densities are inclosed in the same space, experiment shows that, even under the most unfavourable circumstances, an intimate mixture takes place, so that each gas becomes uniformly diffused through the entire space. This fact has been shown by a decisive experiment due to Berthollet. He took two globes (Fig. 129) which could be screwed together, and placed them in a cellar. The lower globe was filled with carbonic acid, the upper globe with hydrogen. Communication was established between them, and after some time it was ascertained that the gases had become uniformly mixed; the proportions being the same in both globes. Gaseous diffusion is a comparatively rapid process. The diffusion of liquids, when not assisted by gravity, is, on the other hand, exceedingly slow.

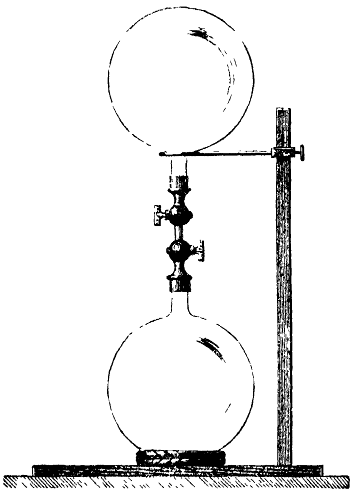


Fig. 129.—Mixture of Gases.

If several gases are inclosed in the same space, each of them exerts the same pressure as if the others were absent, in other words, the pressure exerted by the mixture is equal to the sum of the pressures which each would exert separately. This is known as "Dalton's law for gaseous mixtures." The separate pressures can easily be calculated by Boyle's law, when the original pressure and volume of each gas are known.

For example, let V and P, V′ and P′, V″ and P″ be the volumes and pressures of the gases which are made to pass into a vessel of volume U. The first gas exerts, when in this vessel, a pressure equal to $\frac{VP}{U}$, the second a pressure equal to $\frac{V'P'}{U}$, the third a pressure equal to $\frac{V''P''}{U}$, and so on, so that the total pressure M is equal to $\frac{VP}{U} + \frac{V'P'}{U} + \frac{V''P''}{U}$, whence $MU = VP + V'P' + V''P''$.

This law can easily be verified by passing different volumes of

gas into a graduated glass jar inverted over mercury, after having first measured their volumes and pressures. It may be observed that Boyle's law is merely a particular case of this. It is what this law becomes when applied to a mixture of two portions of the same gas.

228. Absorption of Gases by Liquids and Solids.—All gases are to a greater or less extent soluble in water. This property is of considerable importance in the economy of nature; thus the life of aquatic animals and plants is sustained by the oxygen of the air which the water holds in solution. The *volume* of a given gas that can be dissolved in water at a given temperature is generally found to be approximately the same at all pressures,[1] and the ratio of this volume to that of the water which dissolves it is called the *coefficient of solubility, or of absorption.* At the temperature 0° Cent., the coefficient of solubility for carbonic acid is 1, for oxygen ·04, and for ammonia 1150.

If a mixture of two or more gases be placed in contact with water, each gas will be dissolved to the same extent as if it were the only gas present.

Other liquids as well as water possess the power of absorbing gases, according to the same laws, but with coefficients of solubility which are different for each liquid.

Increase of temperature diminishes the coefficient of solubility, which is reduced to zero when the liquid boils.

Some solids, especially charcoal, possess the power of absorbing gases. Boxwood charcoal absorbs about nine times its volume of oxygen, and about ninety times its volume of ammonia. When saturated with one gas, if put into a different gas, it gives up a portion of that which it first absorbed, and takes up in its place a quantity of the second. Finely-divided platinum condenses on the surface of its particles a large quantity of many gases, amounting in the case of oxygen to many times its own volume. If a jet of hydrogen gas be allowed to fall, in air, upon a ball of spongy platinum, the gas combines rapidly, in the pores of the metal, with the oxygen of the air, giving out an amount of heat which renders the platinum incandescent and usually sets fire to the jet of hydrogen.

Most solids have in ordinary circumstances a film of air adhering

[1] Hence the *mass* of gas absorbed is directly as the pressure.

to their surfaces. Hence iron filings, if carefully sprinkled on water, will not be wetted, but will float on the surface, and hence also the power which many insects have of running on the surface of water without wetting their feet. The film of air in these cases prevents wetting, and hence, by the principles of capillarity, produces increased buoyancy.

228A. **Boyle's Law at Higher Pressures.**—More recent experiments, by two different observers, Cailletet and Amagat, show that the law of departure observed by Regnault, in the case of gases other than hydrogen, does not continue to hold, but undergoes a reversal, when the pressure is sufficiently increased.

For nitrogen, according to Cailletet, the product VP diminishes as the pressure increases up to about 60m. of mercury, at which point it attains its minimum value. It then begins to increase, attaining at 182m. a much larger value than it had at one atmosphere. According to Amagat, the minimum value of VP occurs at 50m. of mercury for nitrogen, at 100m. for oxygen, at 65m. for air, and at 50m. for carbonic oxide. The following are samples of the values found by Cailletet for nitrogen:—

P in metres of Mercury	VP.	P in metres of Mercury.	VP.
39·359	8184	89·231	8323
49·271	8022	124·122	8857
59·462	7900	174·100	9191
64·366	7951	181·985	9330

Cailletet's results were obtained by employing a simple column of mercury open to the air at the top, and contained in a steel tube, which was, in the first instance, fixed vertically against the side of a hill, and in later experiments was supported in the form of a steep spiral let into the circumference of a wooden cylinder in an artesian well. The gas to be compressed was contained in a glass vessel which may be described as an inverted piezometer, the pressure of the mercury being exerted against its external surface, as well as against the lower surface of the gas within it. The upper part of the piezometer consisted of a narrow tube, while the lower part was wide, so that enough gas could be employed to occupy a large portion of the narrow tube when compressed. In order to show the volume to which the gas was reduced, the interior of the piezometer was covered by chemical means with a thin deposit of gold, which was dissolved when the mercury came in contact with it.

NOTE.—For a fuller abstract, and references to original authorities, see *Jamin et Bouty*, tom. i. p. 213.

CHAPTER XX.

AIR-PUMP.

229. **Air-pump.**—The air-pump was invented by Otto Guericke about 1650, and has since undergone some improvements in detail which have not altered the essential parts of its construction.

Fig. 130 represents the pattern most commonly adopted in France. It contains a glass or metal cylinder called the barrel, in which a piston works. This piston has an opening through it which is closed at the lower end by a valve S opening upwards. The barrel

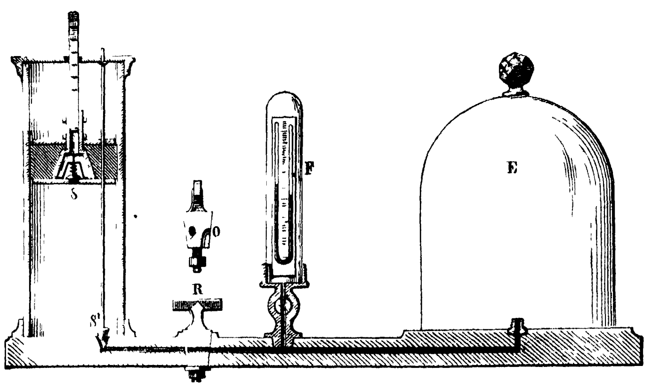


Fig. 130.—Air-pump.

communicates with a passage leading to the centre of a brass surface carefully polished, which is called the *plate* of the air-pump. The entrance to the passage is closed by a conical stopper S′, at the extremity of a metal rod which passes through the piston-head and works in it tightly, so as to be carried up and down with the motion of the piston. A catch at the upper part of the rod confines its motion within very narrow limits, and only permits the stopper to rise a small distance above the opening.

Suppose now that the piston is at the bottom of the cylinder, and is raised. The valve S′ is opened, and air from the receiver E rushes into the cylinder. On lowering the piston, the valve S′ closes its opening, the air which has entered the cylinder cannot return into the receiver, and, on being compressed, raises the valve S in the piston, and escapes into the air outside. On raising the piston again, a portion of the air remaining in the receiver will pass into the cylinder, whence it will escape on pushing down the piston, and so on.

We see, then, that if this motion be continued, a fresh portion of the air in the receiver will be removed at each successive stroke. But as the quantity of air removed at each stroke is only a fraction of the quantity which was in the receiver at the beginning of the stroke, we can never produce a perfect vacuum, though we might approach as near to it as we pleased if this were the only obstacle.

230. Theoretical Rate of Exhaustion.—It is easy to calculate the quantity of air left in the receiver after a given number of strokes of the piston. Let V be the volume of the barrel, V′ that of the receiver, and M the mass of air in the receiver at first. On raising the piston, the air which occupied the volume V′ occupies a volume $V' + V$; of the air thus expanded the volume V is removed, and the volume V′ left, being $\frac{V'}{V'+V}$ of the whole quantity or mass M. The quantity remaining after the second stroke is $\frac{V'}{V'+V}$ of that after the first, or is $\left(\frac{V'}{V'+V}\right)^2 M$; and after n strokes $\left(\frac{V'}{V'+V}\right)^n M$. Hence the density and (by Boyle's law) the pressure are each reduced by n strokes to $\left(\frac{V'}{V'+V}\right)^n$ of their original values.

This calculation gives the theoretical rate of exhaustion for a perfect pump. Ordinary pumps come nearly up to this standard during the earlier part of the process of exhaustion; but as further progress is made, the imperfections of the apparatus become more sensible, and set a limit to the exhaustion attainable.

231. Mercurial Gauges.—To enable the operator to observe the progress of the exhaustion, the instrument is usually provided with a mercurial gauge. Sometimes, as in Fig. 130, this consists of a short siphon-barometer, the difference of levels between its two columns being the measure of the pressure in the receiver. Another plan is to have a straight tube open at both ends, and more than 30

inches long; its upper end being connected with the receiver, while its lower end dips into a cistern of mercury. As exhaustion proceeds, the mercury rises in this tube, and its height above the mercury in the cistern measures the difference between the pressure in the receiver and that in the external air.

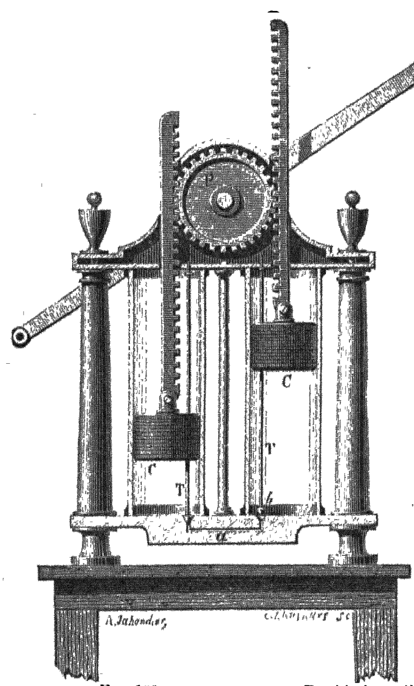
232. Admission Stop-cock.—After the receiver has been exhausted of air, if it were required to raise it from the plate, a very considerable force would be necessary, amounting to as many times fifteen pounds as the base of the receiver contained square inches. This difficulty is obviated by having an admission stop-cock R, which is shown in section above. It is perforated by a straight channel, which, when the machine is being worked, forms part of the communicating passage. At 90° from the extremities of this channel is another opening O, forming the mouth of a bent passage, leading to the external air. When we wish to admit the air into the receiver, we have only to turn the stop-cock so as to bring the opening O to the side next the receiver; if, on the contrary, we turn it towards the pump-barrel, all communication between the pump and the receiver is stopped, the risk of air entering is diminished, and the vacuum remains good for a greater length of time. This precaution is taken when we wish to leave bodies in a vacuum for a considerable time. Another method is to employ a separate plate, which can be detached so as to leave the machine available for other purposes.

233. Double-barrelled Air-pump.—The machine just described has only a single pump-barrel; air-pumps of this kind are sometimes employed, and are usually worked by a lever like a pump-handle. With this arrangement, it is evident that no air is expelled in the down-stroke; and that the piston, after having expelled the air from the barrel in the up-stroke, must descend idle in order to prepare for the next stroke.

Double-barrelled pumps are more frequently used. An idea of their general arrangement may be formed from Figs. 131, 132, and 133. Fig. 133 gives the machine in perspective, Fig. 131 is a section through the axes of the pump-barrels, and Fig. 132 shows the manner in which communication is established between the receiver and the two barrels. It will be observed that the two passages from the barrels unite in a single passage to the centre of the plate *p*.

Two racks carrying the pistons CC work with the pinion P. This pinion is turned by a double-handed lever, which is moved alter-

nately in opposite directions. In this arrangement, when one piston ascends the other descends, and consequently in each single stroke the air of the receiver passes into one or other pump-barrel. A vacuum is thus produced by half the number of strokes which would be required with a single-barrelled pump. It has besides another advantage, as compared with the single-barrelled pump above described. In that pump the force required to raise the piston increases as the exhaustion proceeds, and when it is nearly completed there is the resistance of almost an atmosphere to be overcome. In the



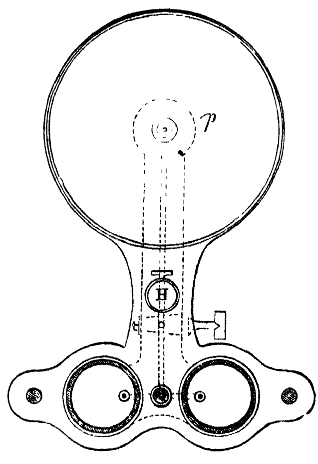


Fig. 131. Double-barrelled Air-pump. Fig. 132.

double-barrelled pump, with the same construction of barrel, the force opposing the ascent of one piston is precisely equal, at the beginning of each stroke, to that which assists the descent of the other. This equality, however, exists only at the beginning of the stroke; for the air below the descending piston is compressed, and its tension increases till it becomes equal to that of the atmosphere and raises the piston valve. During the remainder of the stroke, the resistance to the ascent of the other piston is entirely uncompensated, and up to this point the compensation has been gradually diminishing. But the more nearly we approach to a perfect vacuum, the later in the stroke does this compensation occur.

The pump, accordingly, becomes easier to work as the exhaustion proceeds.

234. Single-barrelled Pumps with Double Action.—We do not, however, require two pump-barrels in order to obtain double action,

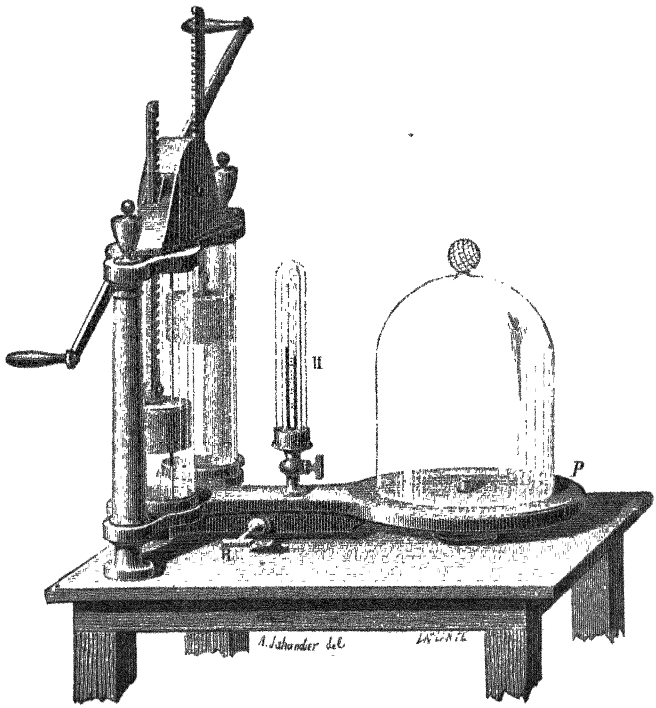


Fig. 133.—Air-pump.

as the same effect may be obtained with a single barrel. An arrangement for this purpose was long ago suggested by Delahire for water-pumps; but the principle has only lately been applied to the construction of air-pumps.

Fig. 134 represents the single barrel of the double-acting pump of Bianchi. It will be seen that the piston-valve opens into the hollow piston-rod; a second valve, also opening upwards, is placed at the top of the pump-barrel. Two other openings, one above, the other below, serve to establish communication, by means of a bent vertical tube, between the pump-barrel and the passage to the plate. These openings are closed alternately by two conical stoppers at the two extremities of a metal rod passing through the piston, and carried with it in its vertical movement by means of friction. When the

piston ascends, as in the figure, the upper opening is closed and the lower one is open; when the piston begins to descend, the opposite effect is immediately produced. Accordingly we see that, whichever be the direction in which the piston is moving, the receiver is being exhausted of air. In fact, when the piston ascends, air from the receiver will enter by the lower opening, and the air above the piston will be gradually compressed, and will finally escape by the valve above. In the descending movement, air will enter by the upper opening, and the compressed air beneath the piston will escape by the piston-valve. The movement of the piston is produced by a peculiar arrangement shown in Fig. 135, which gives a general view of the apparatus.

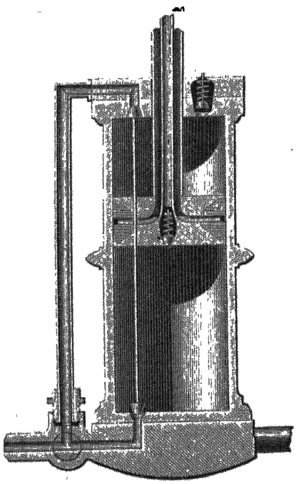


Fig 134.
Barrel of Bianchi's Air-pump.

The pump-barrel, which is composed entirely of cast-iron, oscillâtes about an axis passing through its base. On the top are guides in which the end of a crank travels. The pump is worked by turning a heavy fly-wheel of cast-iron, on the axis of which is a pinion which drives a toothed wheel on the axis of the crank. The end of the crank is attached to the extremity of the piston-rod. It is evident that on turning the fly-wheel the pump-barrel will oscillate from side to side, following the motions of the crank, and the piston will alternately ascend and descend in the barrel, the length of which should be equal to the diameter of the circle described by the end of the crank.

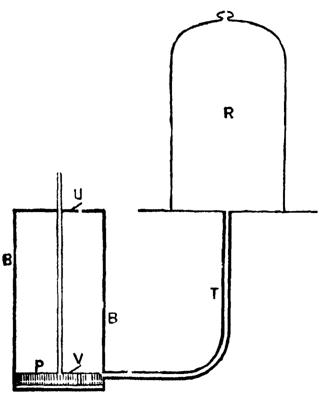


Fig. 136

o **235. English forms of Air-pump.**— Some of the drawbacks to the single-barrelled pump are obviated by inserting a valve, opening upwards, in the top of the barrel as at U, Fig. 136. The top of the piston is thus relieved from atmospheric pressure, and the operation of pumping does not become more laborious as

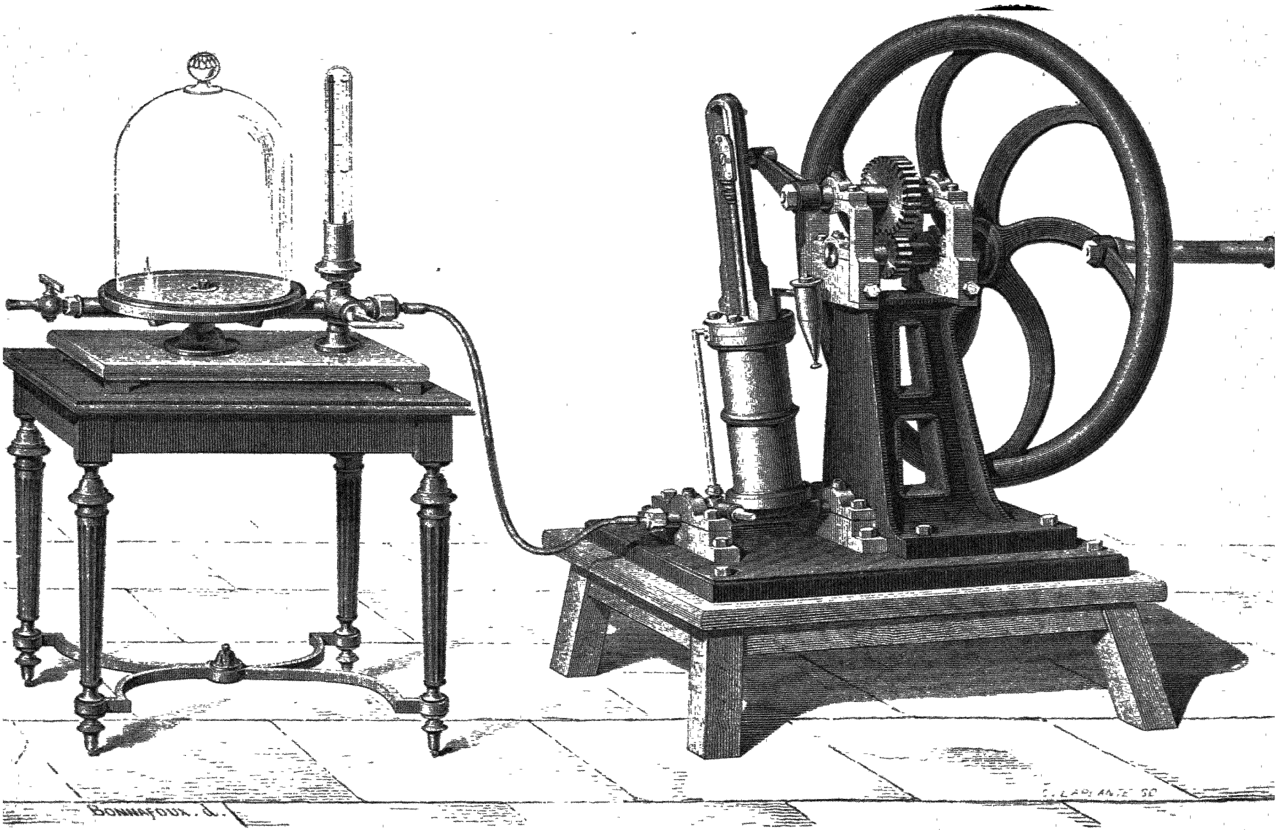


Fig. 135.—Bianchi's Air-pump.

the exhaustion proceeds, but less laborious, the difference being most marked when the receiver is small.

In the up-stroke, the piston-valve V keeps shut, and the air above the piston is pushed out of the barrel through the valve U. In the down-stroke, U is kept closed by the preponderance of atmospheric pressure outside, and V opens, allowing the air to pass up through it as the piston descends to the bottom of the barrel. When the exhaustion is far advanced, U does not open till the piston has nearly reached the top. This is a simple and good form of pump.

Another form very much in use in this country is the double-acting pump of Professor T. Tate, the working parts of which are shown in Fig. 137. CD is the barrel; A and B are two solid pistons rigidly connected by a rod, and moved by the piston-rod AH, which passes through a stuffing-box S. VV are valves in the two ends of the barrel, both opening outwards, and R is a passage leading from the middle of the cylinder to the receiver. The distance between the extreme faces of the pistons is about $\frac{3}{8}$ths of an inch less than half the length of the cylinder. The volume of air expelled at each single stroke is thus about half the volume of the cylinder.

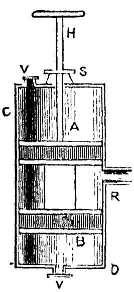


Fig. 137. Tate's Pump.

This figure and description are in accordance with the original account of the pump given by the inventor in the *Philosophical Magazine*. It is now usual to replace the two pistons by a single piston of great thickness, its two faces being as far apart as the extreme faces of the two pistons in the figure. It is also usual to make the barrel horizontal.

The valves of these pumps, and of most English pumps are "silk valves." They consist of a short and narrow slit in a thin plate of brass, with a flap of oiled silk secured at both ends to the plate, in such a position that its central portion covers the slit. When the pressure of the air is greater on the further side of the plate than on the side where the silk is, the flap is slightly lifted and the air gets through; but excess of pressure on the near side presses the flap down over the slit and makes it air-tight.

o **236. Various Experiments with the Air-pump.**—At the time when the air-pump was invented, several experiments were devised to show the effects of a vacuum, some of which have become classical, and are usually repeated in courses of experimental physics.

Burst Bladder.—On the plate of an air-pump (Fig. 138) is

placed a glass cylinder open at the bottom, and having a piece of bladder or thin indian-rubber tightly stretched over the top. As the exhaustion proceeds, this bends inwards in consequence of the atmospheric pressure above it, and finally bursts with a loud report.

Magdeburg Hemispheres.—We take two hemispheres (Fig. 139), which can be exactly fitted on each other; their exact adjustment is further assisted by a projecting internal rim, which is smeared with lard. The apparatus is exhausted of air through the medium of the stop-cock attached to one of the hemispheres; and when a vacuum has been produced, it will be found that a considerable force is required to separate the two parts, this force increasing with the size of the hemispheres.

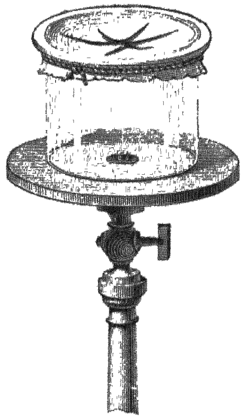


Fig 138
Burst Bladder.

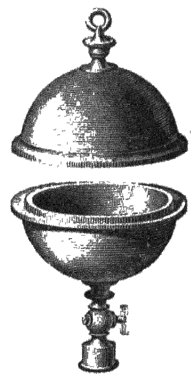


Fig 139.
Magdeburg Hemisphere

This resistance to separation is due to the normal exterior pressure of the air on every point of the surface, a pressure which is counterbalanced by only a very feeble pressure from the interior. In order to estimate the resultant effect of these different pressures, let us suppose that one hemisphere is vertically over the other, and that the external surface is cut into a series of steps,—that is to say, of alternate vertical and horizontal elements. It is evident that the pressure urging either hemisphere towards the other will be simply the sum of the pressures upon its horizontal elements; and this sum is identical with the pressure which would be exerted upon a circular area equal to the common base of the hemispheres. For example, if this area is 10 square inches, and the external pressure exceeds the internal by 14 lbs. to the inch, the hemispheres will be pressed together with a force of 140 lbs.

Fountain in Vacuo.—The apparatus for this experiment consists of a bell-shaped vessel of glass (Fig. 140), the base of which is pierced by a tube fitted with a stop-cock which enables us to exhaust the vessel of air. If, after a vacuum has been produced, we place the

lower end of the tube in a vessel of water, and open the stop-cock, the liquid, being pressed externally by the atmosphere, mounts up the tube and ascends in a jet into the interior of the vessel. This experiment is often made in the opposite manner. Under the receiver of the air-pump is placed a vial partly filled with water, and having its cork pierced by a tube open at both ends, the lower end being beneath the surface of the water. As the exhaustion proceeds, the air in the vial, by its excess of pressure, acts upon the liquid and makes it issue in a jet.

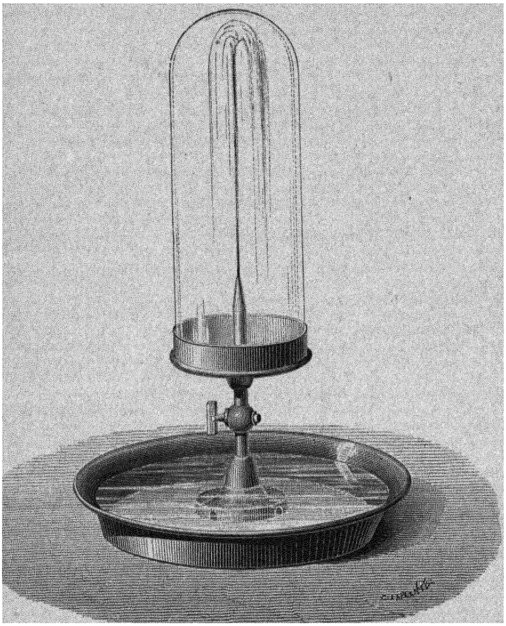


Fig. 140.—Fountain in Vacuo.

237. Limit to the Action of the Air-pump.—We have said above (§ 230) that the air-pump does not continue the process of rarefaction indefinitely, but that at a certain stage its effect ceases, and the pressure of the air in the receiver undergoes no further diminution. If the pump is very badly made, this pressure is considerable; but even with the most perfect machines it is always sensible. A pump such as we have described may be considered good if it reduces the pressure of the air in the receiver to a tenth of an inch of mercury. A fiftieth of an inch is perhaps the lowest limit.

LEAKAGE.—This limit to the action of the machine is due to various causes. In the first place, there is frequently leakage at different parts of the apparatus; and although at the beginning of the operation the quantity of air which thus enters is small in comparison with that which is pumped out, still, as the exhaustion proceeds, the air enters faster, on account of the diminished internal pressure, and at the same time the quantity expelled at each stroke becomes less,

so that at length a point is reached at which the inflow and outflow are equal.

In order to prevent leakage as far as possible, the plate of the pump and the base of the receiver must be truly plane so as to fit accurately; the base of the receiver must be ground (that is roughened) and must be well greased before pressing it down on the plate. The piston must also be well lubricated with oil.

SPACE UNTRAVERSED BY PISTON.—Another reason of imperfect exhaustion is that, after all possible precautions, a space is still left between the bottom of the pump-barrel and the lower surface of the piston when the latter is at the end of its downward stroke. It is evident that at this moment the air contained in this *untraversed space* is of the same tension as the atmosphere. On raising the piston, this air is indeed rarefied; but it still preserves a certain tension, and it is evident that when the air in the receiver has been brought to this stage of rarefaction, the machine will cease to produce any effect.

If v is the volume of this space, and V the volume of the pump-barrel, the air, which at volume v has a pressure H equal to that of the atmosphere, will have, at volume V, a pressure $H\frac{v}{V}$. This gives the limit to the action of the machine as deduced from the consideration of the untraversed space.

AIR GIVEN OUT BY OIL.—Finally, perhaps the most important cause, and the most difficult to remedy, is the absorption of air by the oil used for lubricating the pistons. This oil is poured on the top of the piston, but the pressure of the external air forces it between the piston and the barrel, whence it falls in greater or less quantity to the bottom of the barrel, where it absorbs air, and partially yields it up at the moment when the piston begins to rise, thus evidently tending to derange the working of the machine. It has been attempted to get rid of untraversed space by employing a kind of piston of mercury. This has also the advantage of fitting the barrel more accurately, and thus preventing the entrance of air. The use of oil is at the same time avoided, and we thus escape the injurious effects mentioned above. We proceed to describe two machines founded upon this principle.

238. Kravogl's Air-pump.—This contains a hollow glass cylinder AB (Fig. 141) tapering at the upper end, and surmounted by a kind of funnel. The piston is of the same shape as the cylinder, and is

covered with a layer of mercury, whose depth over the point of the piston is about $\frac{1}{50}$th of an inch when the piston is at the bottom of its stroke, but is nearly an inch when the piston rises and fills the

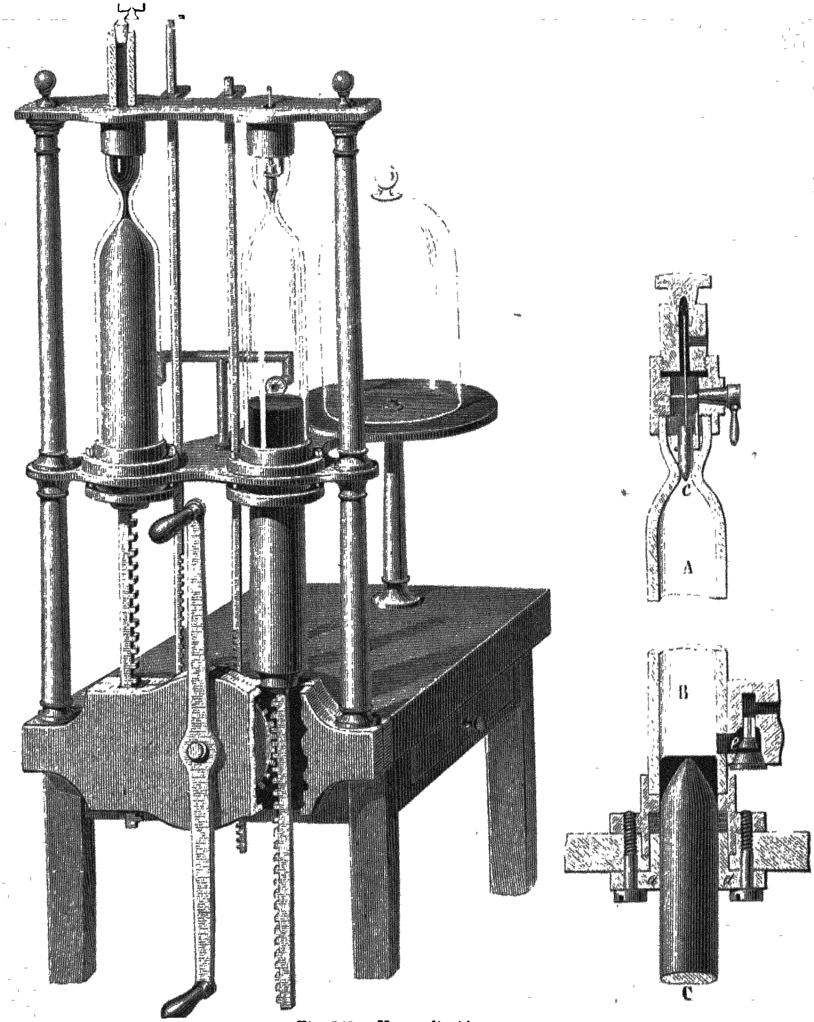


Fig. 141.—Kravogl's Air-pump.

funnel-shaped cavity in which the pump-barrel terminates. A small interval, filled by the liquid, is left between the barrel and the piston; but at the bottom of the barrel the piston passes through a leather box carefully made, so as to be perfectly air-tight.

The air from the receiver enters through the lateral opening *e*, and

is driven before the mercury into the funnel above. With the air passes a certain quantity of mercury, which is detained by a steel valve *c* at the narrowest part of the funnel. This valve rises automatically when the surface of the mercury is at a distance of about half an inch from the funnel, and falls back into its former position when the piston is at the end of its upward stroke. In the downward stroke, when the mercury is again half an inch from the funnel. the valve opens again and allows a portion of the mercury to pass.

The effect of this arrangement is easily understood; there is no "untraversed space," the presence of the mercury above and around the piston causes a very complete fit, and excludes the external air; and hence the machine, when well made, is very effective.

When this is the case, and when the mercury used in the apparatus is perfectly dry, a vacuum of about $\frac{1}{250}$th of an inch can be obtained. The dryness of the mercury is a very important condition, for at ordinary temperatures the elastic force of the vapour of water has a very sensible value. If we wish to employ the full powers of the machine, we must have, between the vessel to be exhausted of air and the pump-barrel, a desiccating apparatus.

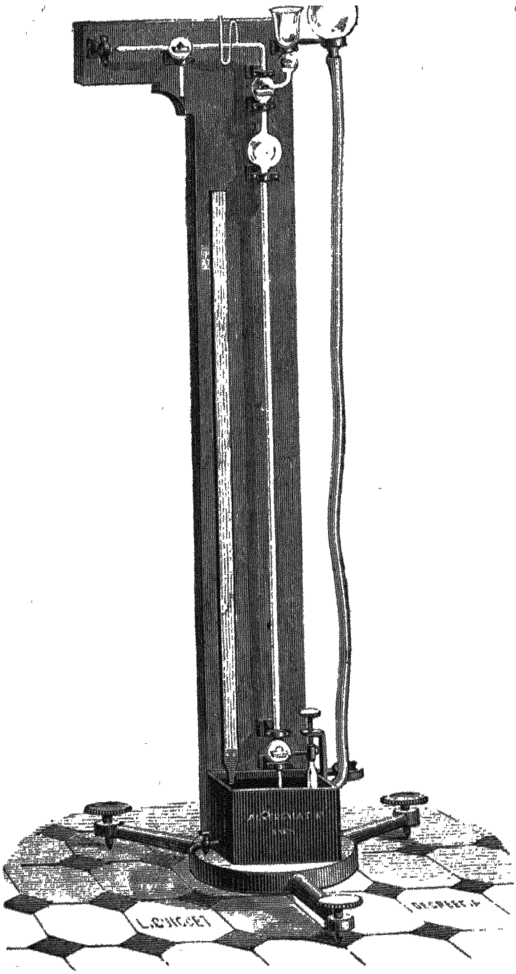
The arrangement of the valve *e* is peculiar. It is of a conical form, so as, in its lowest position, to permit the passage of air coming from the receiver. Its ascent is produced by the pressure of the mercury, which forces it against the conical extremity of the passage, and the liquid is thus prevented from escaping.

The figure represents a double-barrelled machine analogous to the ordinary air-pump. Besides the pinion working with the racks of the pistons, there is a second smaller pinion, not shown in the figure, which governs the movements of the valves *c*. All the parts of this machine, as the stop-cocks, valves, pipes, &c., must be of steel, to avoid the action which the mercury would have upon any other metal.

239. **Geissler's Machine.**—Geissler, of Bonn, invented a mercurial air-pump, in which the vacuum is produced by communication of the receiver with a Torricellian vacuum. Fig. 142 represents this machine as constructed by Alvergniat. It consists of a vertical tube, serving as a barometric tube, and communicating at the bottom, by means of a caoutchouc tube, with a globe which serves as the cistern.

At the top of the tube is a three-way stop-cock, by which communication can be established either with the receiver to the left, or

with a funnel to the right, which latter has an ordinary stop-cock at the bottom. By means of another stop-cock on the left, communication with the receiver can be opened or closed. These stop-cocks are made entirely of glass. The machine works in the following manner; communication being established with the funnel, the globe which serves as cistern is raised, and placed, as shown in the figure, at a higher level than the stop-cock of the funnel. By the law of equilibrium in communicating vessels, the mercury fills the barometric tube, the neck of the funnel, and part of the funnel itself. If the communication between the funnel and tube be now stopped, and the globe lowered, a Torricellian vacuum is produced in the upper part of the vertical tube.



Communication is now opened with the receiver; the air rushes into the vacuum, and the column of mercury falls a little. Communication is now stopped between the tube and receiver, and opened between the tube and the funnel, the simple stop-cock of the funnel being, however, left shut. If at this moment the globe is replaced in the position shown in the figure, the air tends to escape by the funnel, and it is easy to allow it to do so. Thus, a part of the air of the receiver has been removed,

and the apparatus is in the same position as at the beginning. The operation described is equivalent to a stroke of the piston in the ordinary machine, and this process must be repeated till the receiver is exhausted.

As the only mechanical parts of this machine are glass stop-cocks, which are now executed with great perfection, it is capable of giving very good results. With dry mercury a vacuum of $\frac{1}{250}$th of an inch may very easily be obtained. The working of the machine, however, is inconvenient, and becomes exceedingly laborious when the receiver is large. It is therefore employed directly only for producing a vacuum in very small vessels; when the spaces to be exhausted of air are at all large, the operation is begun with the ordinary machine, and the mercurial air-pump is only employed to render the vacuum thus obtained more perfect.

240. **Sprengel's Air-pump.**—This instrument, which may be regarded as an improvement upon Geissler's, is represented in its simplest form in Fig. 143. *cd* is a glass tube longer than a barometer tube, down which mercury is allowed to fall from the funnel A. Its lower end dips into the glass vessel B, into which it is fixed by means of a cork. This vessel has a spout at its side, a few millimetres higher than the lower end of the tube. The first portions of mercury which run down will consequently close the tube, and prevent the possibility of air entering it from below. The upper part of *cd* branches off at *x* into a lateral tube communicating with the receiver R, which it is required to exhaust. A convenient height for the whole instrument is 6 feet. The funnel A is supported by a ring as shown in the figure, or by a board with a hole cut in it. The tube *cd* consists of two parts, connected by a piece of india-rubber tubing, which can be compressed by a clamp so as to keep the tube closed when desired. As soon as the mercury is allowed to run down, the exhaustion begins, and the whole length of the tube, from *x* to *d*, is seen to be filled with cylinders of mercury separated by cylinders of air, all moving downwards. Air and mercury escape through the spout of the bulb B, which is above the basin H, where the mercury is collected. This has to be poured back from time to time into the funnel A, to pass through the tube again and again until the exhaustion is completed.

As the exhaustion is progressing, it will be noticed that the inclosed air between the mercury cylinders becomes less and less, until the lower part of *cd* presents the aspect of a continuous column of mer-

cury about 30 inches high. Towards this stage of the operation a considerable noise begins to be heard, similar to that of a shaken water-hammer, and common to all liquids shaken in a vacuum. The operation may be considered completed when the column of mercury does not inclose any air, and when a drop of mercury falls upon the top of this column without inclosing the slightest air-bubble. The height of this column now corresponds exactly with the height of the column of mercury in a barometer; or, what is the same, it represents a barometer whose vacuum is the receiver R and connecting tube.

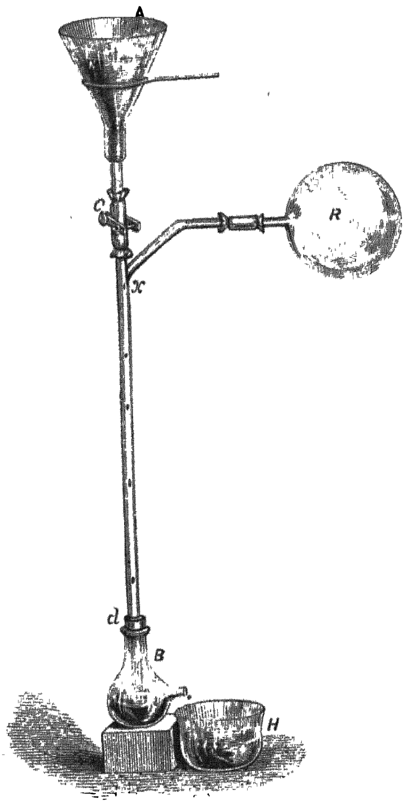


Fig 143 Sprengel's Air-pump.

Dr. Sprengel recommends the employment of an auxiliary air-pump of the ordinary kind, to commence the exhaustion, when time is an object, as without this from 20 to 30 minutes are required to exhaust a receiver of the capacity of half a litre. As, however, the employment of the auxiliary pump involves additional connections and increased leakage, it should be avoided when the best possible exhaustion is desired. The fall tube must not exceed about a tenth of an inch in diameter, and special precautions must be employed to make the india-rubber connections air-tight. (See *Chemical Journal* for 1865, p. 9.)

By this instrument air has been reduced to $\frac{1}{1300000}$th of atmospheric density, and the average exhaustion attainable by its use is about one-millionth, which is equivalent to ·00003 of an inch of mercury.

241. **Double Exhaustion.**—In the mercurial machines just described there is no "untraversed space," as the liquid completely expels all the air from the pump-barrel. These machines are of very recent

invention. Babinet long before introduced an arrangement for the purpose, not of getting rid of this space, but of exhausting it of air.

For this purpose, when the machine ceases to work with the ordinary arrangement, the communication of the receiver with one of the pump-barrels is shut off, and this barrel is employed to exhaust the air from the other. This change is effected by means of a stop-cock at the point of junction of the passages leading from the two barrels (Fig. 144). The stop-cock has a T-shaped aperture, the point of intersection of the two branches being in constant communication with the receiver. In a different plane from that of the T-shaped aperture is another aperture *mn*, which, by means of the tube *l*, establishes communication between the pump-barrel B and the communicating passage of the pump-barrel A. From this explanation it will be seen that if the stop-cock be turned as shown in the first figure, the two pump-barrels both communicate with the receiver, and the operation proceeds in the ordinary manner. But if the stop-cock be turned through a quarter of a revolution, as shown in the second figure, the pump-barrel B alone communicates with the receiver, while it is itself exhausted of air by the barrel A.

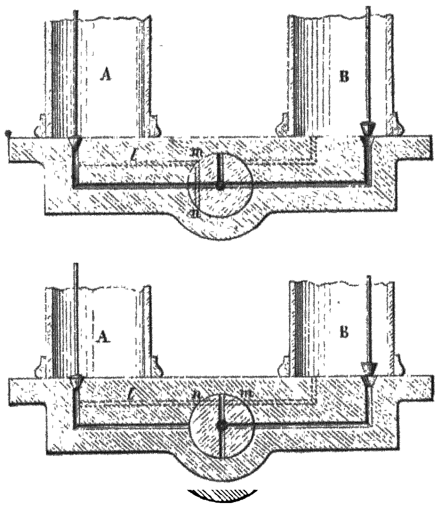


Fig 144 —Babinet's Doubly-exhausting Stop-cock.

It is easy to express by a formula the effect of this double exhaustion. Suppose the pump to have ceased, under the ordinary method of working, to produce any farther exhaustion, the air in the receiver has therefore reached a tension nearly equal to $H\frac{v}{V}$ (§ 237). At this moment the stop-cock is turned into its second position. When the piston B descends, the piston A rises, and the air of the "untraversed space" in B is drawn into A and rarefied. During the inverse operation, the air in A is prevented from returning to B, and thus the rarefied air from B, becoming still further rarefied, will draw a fresh quantity of air from the receiver. This air will then be driven

into A, where it will be compressed by the descending movement of the piston, and will find its way into the air outside.[1]

This double exhaustion will itself cease to work when air ceases to pass from the pump-barrel B into the pump-barrel A. Now when the piston in this latter is raised, the elastic force of the air which was contained in its "untraversed space" is equal to $H\frac{v}{V}$, for, on the last opening of the valve, the air in this space escaped into the atmosphere. On the other hand, when the piston in B is at the end of its upward stroke, the tension of the air is the same as in the receiver. Let this be denoted by x. When the piston in B descends, the air is compressed into the "untraversed space" and the passage leading to A. Let the volume of this passage be l. Then the tension will increase, and become $x\frac{V+l}{v+l}$. When the machine ceases to produce any farther effect, this tension cannot be greater than that in the pump-barrel A, which is $H\frac{v}{V}$; we have thus, to determine the limit to the action of the pump, the equation

$$x\frac{V+l}{v+l} = H\frac{v}{V}, \text{ whence}$$

$$x = H.\frac{v}{V}.\frac{v+l}{V+l}.$$

242. Air-pump with Free Piston.—We shall describe one more air-pump (Fig. 145), constructed by Deleuil, and founded upon an interesting principle. We know that gases possess a remarkable power of adhesion for solids, so that a body placed in the atmosphere may be considered as covered with a very thin coat of air, forming, so to speak, a permanent envelope. On account of this circumstance, gases find very great difficulty in moving in very narrow spaces. This is the principle of the "air-pump with free piston."

The piston P (Fig. 146), which is composed entirely of metal, is of considerable length; and on its outer surface is a series of parallel circular grooves very close together. It does not touch the pump-barrel at any point; but the distance between the two is very small, about ·001 of an inch. This free piston is surrounded by a cushion of air, which forms its only stuffing, and is sufficient to enable the machine to work in the ordinary manner, notwithstanding the per-

[1] It will be observed that during the process of double exhaustion the piston of B behaves like a solid piston; its valve never opens, because the pressure below it is always less than atmospheric.

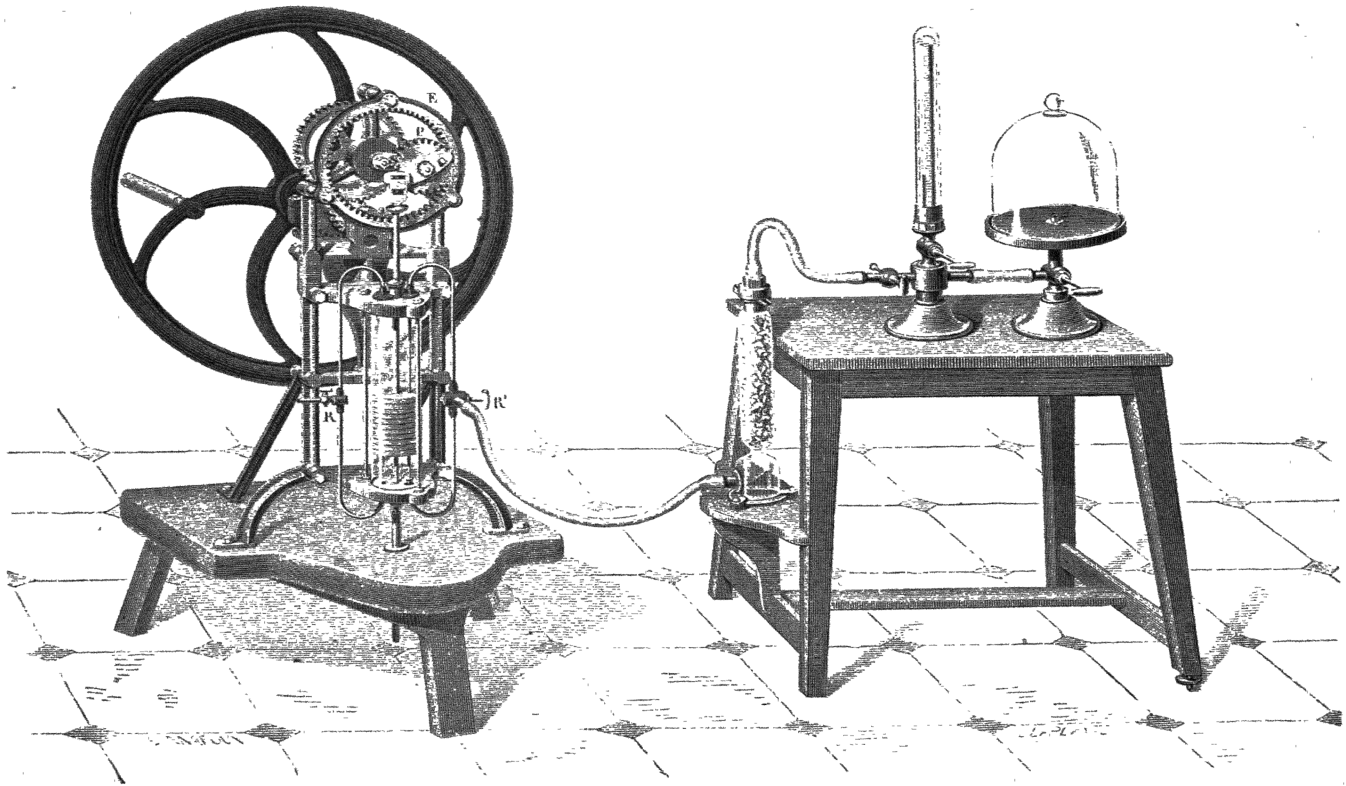


Fig. 145.—Deleuil's Air-pump

manent communication between the upper and lower surfaces of the piston. This machine gives a vacuum about as good as is obtainable by ordinary pumps, and it has the important advantages of not requiring oil, and of having less friction. It consequently wears better, and is less liable to the development of heat, which is a frequent source of annoyance in air-pumps. It is single-barrelled with double action, like Bianchi's. The two openings S and S′ are to admit air from the receiver; they are closed and opened alternately by conical stoppers at the end of the rod T, which passes through the piston, and is carried with it by friction in its movement. They communicate with tubes which unite, at R′, with a tube leading from the receiver. A and A′ are valves for the expulsion of the air, which escapes by tubes uniting at R. The alternate movement of the piston is produced by what is called Delahire's gearing. This depends on the principle, that *when a circle rolls without sliding in the interior of another circle of double the diameter, any point on the circumference of the rolling circle describes a diameter of the fixed circle.* In order to utilize this property, the end of the piston-rod is jointed to the extremity of a piece of metal which is rigidly attached to the pinion P, the joint being exactly opposite the circumference of the pinion. This latter is driven by a fly-wheel with suitable gearing, and works with the fixed wheel E, which is toothed on the inside. Thus the piston will freely, and without any lateral effort, describe a vertical line, the length of the stroke being equal to the diameter of the fixed wheel.

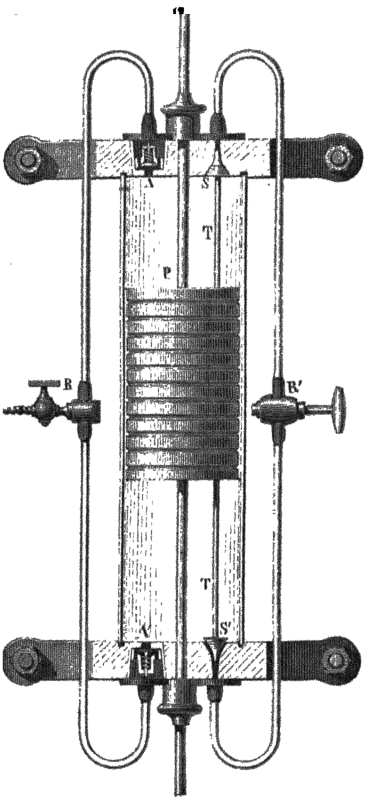


Fig. 146
Piston and Barrel of Deleuil's Air-pump.

243. Compressing Pump.—It can easily be seen from the descrip-

tion of the air-pump, that if the expulsion-valves were connected with a tube communicating with a reservoir, the air removed by the pump would be forced into this reservoir. This communication is established in the instrument just described. If, therefore, R′ be made to communicate with the external air, this air will be continually drawn in at that point and forced out into the reservoir connected with R, so that the instrument will act as a compressing pump. The compressing-pump is thus seen to be the same instrument as the air-pump, the only difference being that the receiver is connected with the expulsion valves, instead of with the exhaustion-valves; it is thus, so to speak, the air-pump reversed. This fact can be very well seen in the structure of a small pump frequently employed in the laboratory, and represented in Fig. 147.

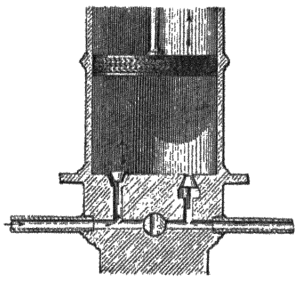


Fig. 147.—Barrel of Condensing Pump.

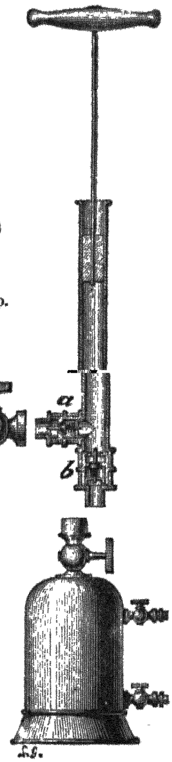


Fig. 148. Condensing Pump

At the bottom of the pump-barrel are two valves, communicating with two separate reservoirs, that on the left being an admission-valve, and that on the right an expulsion-valve.

When the piston is raised, rarefaction is produced in the reservoir to the left; and when it is pushed down, the air in the reservoir to the right is compressed.

In Fig. 148 is represented a compressing-pump often employed. At the bottom of the pump-barrel is a valve *b* opening downward; in a lateral tube is an admission-valve *a* opening inward. The position of these valves is shown in the figure. They are conical metal stoppers, fitted with a rod passing through a hole in a small plate behind, an arrangement which prevents the valve from overturning. The rod is surrounded by a small spiral spring, which keeps the valve pressed against the opening. If the lower part of the

pump-barrel be screwed upon a reservoir, at each upward stroke of the piston the barrel will be filled with air through the valve a, and at every downward stroke this air will be forced into the reservoir.

If the lateral tube be made to communicate with a bladder or gas-holder filled with any gas, this gas will be forced into the reservoir, and compressed.

244. Calculation of the Effect of the Instrument.—The density of the compressed air after a given number of strokes of the piston may easily be calculated. If v be the volume of the pump-barrel, and V that of the reservoir; at each stroke of the piston there is forced into the reservoir a volume of air equal to that of the pump-barrel; which gives a volume nv at the end of n strokes. The air in the reservoir, accordingly, which when at atmospheric pressure had density D, and occupied a volume $V + nv$, will, when the volume is reduced to V, have the density $D\frac{V + nv}{V}$, and the pressure will, by Boyle's law, be $\frac{V + nv}{V}$ atmospheres.

If this formula were rigorously applicable in all cases, there would be no limits to the pressure attainable, except those depending on the strength of the reservoir and the motive power available. But, in fact, the untraversed space left below the piston, when at the end of its downward stroke, sets a limit to the action of the instrument, just as in the common air-pump. For when the air in the barrel is reduced from the volume of the barrel v to that of the untraversed space v', its tension becomes $H\frac{v}{v'}$; and this air cannot pass into the reservoir unless the tension of the air in the reservoir is less than this quantity. This is accordingly the utmost limit of compression that can be attained.

We must, however, carefully distinguish between the effects of untraversed space in the air-pump and in the compression-pump. In the first of these instruments the object aimed at is to rarefy the air to as great a degree as possible, and untraversed space must consequently be regarded as a defect of the most serious importance.

The object of the condensing-pump, on the contrary, is to compress the air, not indefinitely, but up to a certain point. Thus, for instance, one pump is intended to give a compression of five atmospheres, another of ten, &c. In each of these cases the maker

provides that this limit shall be reached, and the untraversed space has no injurious effect beyond increasing the number of strokes required to produce the desired amount of condensation.

245. **Various Contrivances for producing Compression.**—In order to expedite the process of compression, several pumps such as we have

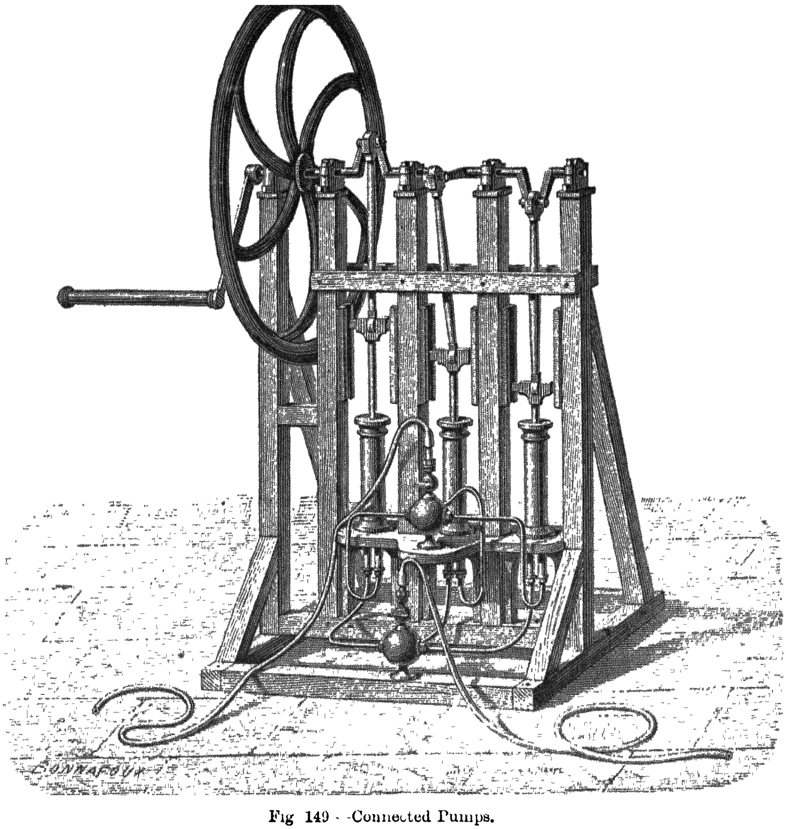


Fig 149 - -Connected Pumps.

described are combined, which may be done in various ways. Fig. 149 represents the system employed by Regnault in his investigations connected with Boyle's law and the elastic force of vapour. It consists of three pumps, the piston-rods of which are jointed to three cranks on a horizontal axle, by means of three connecting-rods. This axle, which carries a fly-wheel, is turned by means of one or two handles. The different admission-valves are in communication with a single reservoir in connection with the external air, and the com-

pressed gas is forced into another reservoir which is in communication with the experimental apparatus.

A serious obstacle to the working of these instruments is the heat generated by the compression of the air, which expands the different parts of the instrument unequally, and often renders the piston so tight that it can scarcely be driven. In some of these instruments which are employed in the arts, this inconvenience is lessened by keeping the lower valves covered with water, which has the additional advantage of getting rid of "untraversed space." In this way a pressure of forty atmospheres may easily be obtained with air. Air may also be compressed directly, without the intervention of pumps, when a sufficient height of water can be obtained. It is only necessary to lead the liquid in a tube to the bottom of a reservoir containing air. This air will be compressed until its pressure exceeds that of the atmosphere by the amount due to the height of the summit of the tube. It is by a contrivance of this kind that compressed air has been obtained for driving the boring-machines employed in the great Alpine tunnels.

246. Practical Applications of the Air-pump and of Compressed Air. —Besides the use made of the air-pump and the compression-pump in the laboratory, these instruments are variously employed in the arts.

The air-pump is employed by sugar-refiners to lower the boiling point of the syrup. Compression-pumps are used by soda-water manufacturers to force the carbonic acid into the reservoirs containing the water which is to be aerated. The small apparatus described above (Fig. 148) is sufficient for this purpose; it is only necessary to fill the side-vessel with carbonic acid, and to pour a certain quantity of water into the reservoir below. Compressed air has for several years been employed to assist in laying the foundations of bridges in rivers where the sandy nature of the soil requires very deep excavations. Large tubes called *caissons*, in connection with a condensing pump, are gradually let down into the river; the air by its pressure keeps out the water, and the workmen, who are admitted into the apparatus by a sort of lock, are thus enabled to walk on dry ground.

In pneumatic despatch tubes, which have recently been established in many places, a kind of train is employed, consisting of a piston preceded by boxes containing the despatches. By exhausting the air at the forward end of the tube, or forcing in compressed air at

the other end, the train is blown through the tube with great velocity.

The atmospheric railway, which was for a few years in existence, was worked upon the same principle an air-tight piston travelled through a fixed tube, and was connected by an ingenious arrangement with a train above.

Excavating machines driven by compressed air are coming into extensive use in mining operations. They have the advantage of assisting ventilation, inasmuch as the compressed air, which at each stroke of the machine escapes into the air of the mine, cools as it expands.

In the air-gun, the bullet is projected by a portion of compressed air which, on pulling the trigger, escapes into the barrel from a reservoir in which it has been artificially compressed.

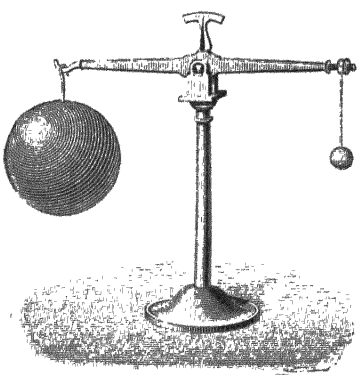
We may add that the large machines employed in iron-works for supplying air to the furnaces, are really compression-pumps.

CHAPTER XXI.

UPWARD PRESSURE OF THE AIR.

247. The Baroscope.—The principle of Archimedes, explained in Chap. XIII., applies to all fluids, whether liquid or gaseous. Hence the resultant of the whole pressure of the atmosphere on the surface of a body is equal to the weight of the air displaced. The force required to support a body in air, is less than the force required to support it in vacuo, by this amount. This principle is illustrated by the baroscope (Fig. 150).

This is a kind of balance, the beam of which supports two balls of very unequal sizes, which balance each other in the air. If the apparatus is placed under the receiver of an air-pump, after a few strokes of the piston the beam will be seen to incline towards the larger ball, and the inclination will increase as the exhaustion proceeds. The reason is that the air, before it was pumped out, produced an upward pressure, which was greater for the large than for the small ball, on account of its greater displacement; and this disturbing force is now removed.



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If after exhausting the air, carbonic acid, which is heavier than air, were admitted at atmospheric pressure, the large ball would be subjected to a greater increase of upward pressure than the small one, and the beam would incline to the side of the latter.

248. Balloons.—Suppose a body to be lighter than an equal volume of air, then this body will rise in the atmosphere. For example, if

we fill soap-bubbles with hydrogen (Fig. 151), and shake them off from the end of the tube at which they are formed, they will be seen, if sufficiently large, to ascend in the air. This curious experiment is due to the philosopher Cavallo, who announced it in 1782.[1]

The same principle applies to balloons, which essentially consist of an envelope inclosing a gas lighter than air. In conse-

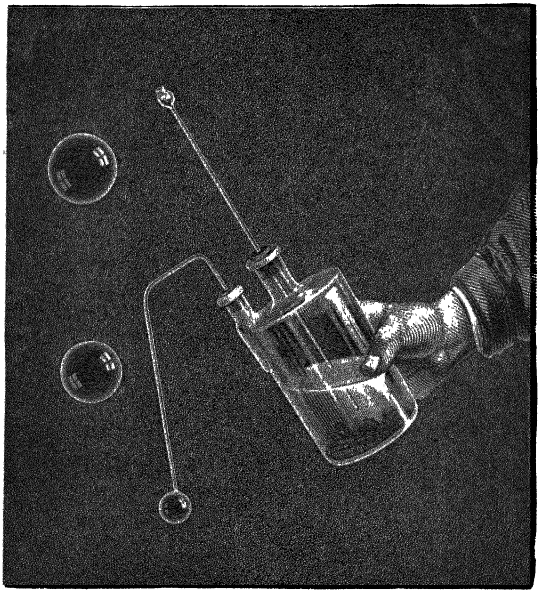

Fig 151 —Ascent of Soap-bubbles filled with Hydrogen

quence of this difference of density, we can always, by taking a sufficiently large volume, make the weight of the gas and containing envelope less than that of the air displaced. In this case the balloon will ascend.

The invention of balloons is due to the brothers Joseph and Stephen Montgolfier. The balloons made by them were globe-shaped, and constructed of paper, or of paper covered with cloth, the air inside being rarefied by the action of heat. It is curious to remark

[1] The first idea of a balloon must be attributed to Francisco de Lana, who, about 1670, proposed to exhaust the air in globes of copper of sufficient size and thinness to weigh less, under these conditions, than the air displaced. The experiment was not tried, and would certainly not have succeeded, for the pressure of the atmosphere would have caused the globes to collapse. The theory, however, was thoroughly understood by the author, who made an exact calculation of the amount of force tending to make the globes ascend. —*D.*

that in their first attempts they employed hydrogen gas, and showed that balloons filled with this gas could ascend. But as the hydrogen readily escaped through the paper, the flight of the balloons was short, and thus the use of hydrogen was abandoned, and hot air was alone employed.

The name *montgolfières* is still often applied to fire-balloons. They generally consist of a paper envelope with a wide opening below,

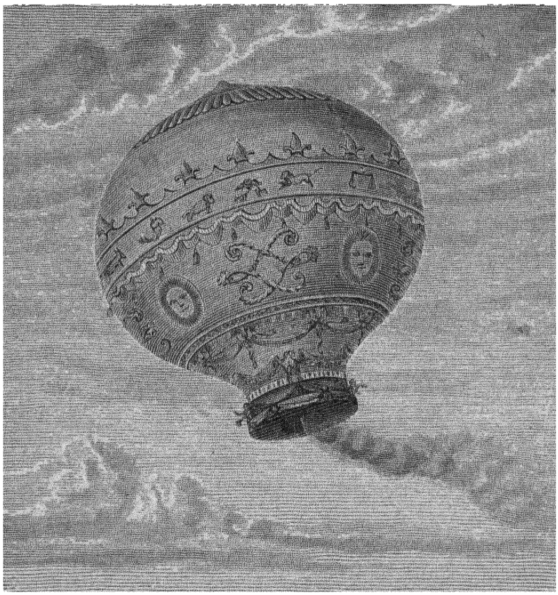

Fig. 152.—Fire balloon of Pilatre de Rozier

in the centre of which is a sponge held in a wire frame. The sponge is dipped in spirit and ignited, when the balloon is to be sent up.

The first public experiment of the ascent of a balloon was performed at Annonay on the 5th June, 1783. On October 21st of the same year, Pilatre de Rozier and the Marquis d'Arlandes achieved the first aerial voyage in a fire-balloon, represented in our figure.

Charles proposed to reintroduce the use of hydrogen by employing an envelope less permeable to the gas. This is usually made of silk varnished on both sides, or of two sheets of silk with a sheet of india-rubber between. Instead of hydrogen, coal-gas is now generally employed, on account of its cheapness and of the facility with which it can be procured.

▷ **249.**—The lifting power of a balloon is the difference between its weight and that of the air displaced. It is easy to compare the three modes of inflation in this respect.

A cubic metre of air weighs about	.... 1·300	kilogramme.
A cubic metre of hydrogen . . .	.... · 089	„
A cubic metre of coal-gas	.... about ·750	„
A cubic metre of air heated to 200° Cent.	.. ·750	„

We thus see that the lifting power per cubic metre with hydrogen is 1·211, and with coal-gas or hot air about ·500 kilogramme. If, for instance, the total weight to be raised is estimated at 1500 kilogrammes, the volume of a balloon filled with hydrogen capable of raising the weight will be $\frac{1500}{1 \cdot 210} = 1239$ cubic metres. If coal-gas were employed, the required volume would be $\frac{1500}{\cdot 550} = 2727$ cubic metres.

The car in which the aeronauts sit is usually made of wicker-work or whalebone. It is sustained by cords attached to a net-work (Fig. 153) covering the entire upper half of the balloon, so as to distribute the weight as evenly as possible. The balloon terminates below in a kind of neck opening freely into the air. At the top there is another opening in the inside, which is closed by a valve held to by a spring. Attached to the valve is a cord which passes through the interior of the balloon, and hangs above the car within reach of the hand of the aeronaut.

When the aeronaut wishes to descend, he opens the valve for a few moments and allows some of the gas to escape. An important part of the equipment consists of sand-bags for ballast, which are gradually emptied to check too rapid descent. In the figure is represented a contrivance called a parachute, by means of which the descent is sometimes effected. This is a kind of large umbrella with a hole at the top, from the circumference of which hang cords supporting a small car. When the parachute is left to itself, it opens out, and the resistance of the air, acting upon a large surface, moderates the rate of descent. The hole at the top is essential to safety, as it affords a regular passage for air which would otherwise escape from time to time from under the edge of the parachute, thus producing oscillations which might prove fatal to the aeronaut.

Balloons are not fully inflated at the commencement of the ascent; but the inclosed gas expands as the pressure diminishes outside. The lifting power thus remains nearly constant until

the balloon has risen so high as to be fully inflated. Suppose, for instance, that the atmospheric pressure is reduced by one-half, the volume of the balloon will then be doubled; it will thus displace a volume of air twice as great as before, but of only half the density, so that the buoyancy will remain the same. This conclusion, however, is not quite exact, because the solid parts of the balloon do not expand like the gas, and the weight of air displaced by them accordingly diminishes as the balloon rises. If the balloon continues to ascend after it is completely inflated, its lifting power diminishes rapidly, becoming zero when a stratum of air is reached in which the weight of the volume displaced is equal to that of the balloon itself. It is carried past this stratum in the first instance in virtue of the velocity which it has acquired, and finally comes to rest in it after a number of oscillations.

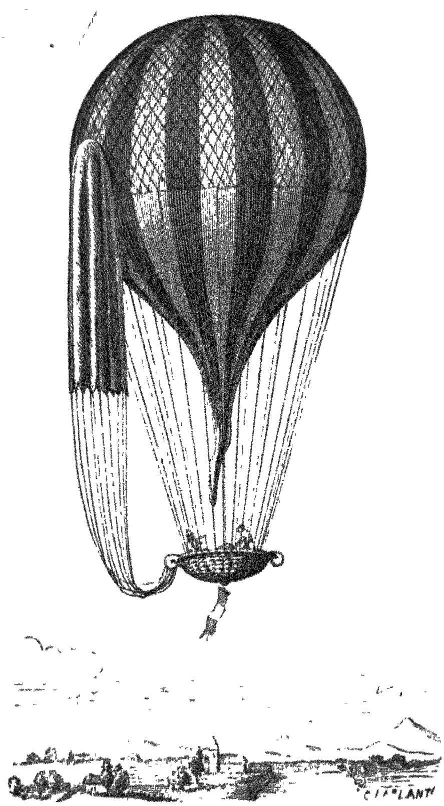

Fig. 153.—Balloon with Car and Parachute

250. Height Attainable.—The pressure of the air in the stratum of equilibrium can be calculated as follows:

Let V be the volume of gas which the balloon can contain when fully inflated.

v the volume, and w the weight, of the solid parts, including the aeronauts themselves.

δ the density of the gas at the standard pressure and temperature, and D the density of air under the same conditions.

Then if P denote the standard pressure, and p the pressure in the stratum of equilibrium, the density of the gas when this stratum

has been reached will be $\frac{p}{P}\delta$, and the density of the air will be $\frac{p}{P}D$. Equating the weight of the air displaced to that of the floating body, we have

$$\frac{p}{P}(V + v) D = \frac{p}{P} V\delta + w,$$

whence p can be determined.

251. **Effect of the Air upon the Weight of Bodies.**—The upward pressure of the air impairs the exactness of weighings obtained even with a perfectly true balance, tending, by the principle of the baroscope, to make the denser of two equal masses preponderate. The stamped weights used in weighing are, strictly speaking, standards of mass, and will equilibrate any equal masses in vacuo; but in air the equilibrium will be destroyed by the greater upward pressure of the air upon the larger and less dense body. When the specific gravities of the weights and of the body weighed are known, it is easy from the apparent weight to deduce the true weight (that is to say, the mass) of the body.

Let x be the real weight (or mass) of a body which balances a standard weight of w grammes when the weighing is made in air. Let d be the density of the body, δ that of the standard weight, and a the density of the air. Then the weight of air displaced by the body is $\frac{a}{d}x$, and the weight of air displaced by the standard weight is $\frac{a}{\delta}w$. Hence we have

$$x - \frac{a}{d}x = w - \frac{a}{\delta}w.$$

$$x = w\frac{1 - \frac{a}{\delta}}{1 - \frac{a}{d}} = w\left\{1 + a\left(\frac{1}{d} - \frac{1}{\delta}\right)\right\} \text{ nearly.}$$

Let us take, for instance, a piece of sulphur whose weight has been found to be 100 grammes, the weights being of copper, the density of which is 8·8. The density of sulphur is 2.

We have, by applying the formula,

$$x = 100\left\{1 + \frac{1}{770}\left(\frac{1}{2} - \frac{1}{8{\cdot}8}\right)\right\} = 100{\cdot}05 \text{ grammes.}$$

We see then that the difference is not altogether insensible. It varies in sign, as the formula shows, according as d or δ is the greater. When the density of the body to be weighed is less than

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that of the weights used, the real weight is greater than the apparent weight; if the contrary, the case is reversed. If the body to be weighed were of the same density as the weights used, the real and apparent weights would be equal. We may remark, that in determining the *ratio* of the weights of two bodies of the same density, by means of standard weights which are all of one material, we need not concern ourselves with the effect of the upward pressure of the air; as the correcting factor, which has the same value for both cases, will disappear in the quotient.

CHAPTER XXII.

PUMPS FOR LIQUIDS.

252. Machines for raising water have been known from very early ages, and the invention of the common pump is pretty generally ascribed to Ctesibius, teacher of the celebrated Hero of Alexandria; but the true theory of its action was not understood till the time of Galileo and Torricelli.

253. **Reason of the Rising of Water in Pumps.**—Suppose we take a tube with a piston at the bottom (Fig. 154), and immerse the lower end of it in water. The raising of the piston tends to produce a vacuum below it, and the atmospheric pressure, acting upon the external surface of the liquid, compels it to rise in the tube and follow the upward motion of the piston. This upward movement of the water would take place even if some air were interposed between the piston and the water; for on raising the piston, this air would be rarefied, and its pressure no longer balancing that of the atmosphere, this latter pressure would cause the liquid to ascend in a column whose weight, added to the pressure of the air below the piston, would be equal to the atmospheric pressure. This is the principle on which water rises in pumps. These instruments have a considerable variety of forms, of which we shall describe the most important types.

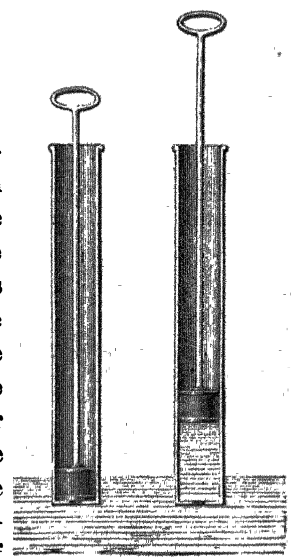


Fig. 154.—Principle of Suction pump.

254. **Suction-pump.**—The suction-pump (Fig. 155) consists of a

cylindrical pump-barrel traversed by a piston, and communicating by means of a smaller tube, called the suction-tube, with the water in the pump-well. At the junction of the pump-barrel and the tube is a valve opening upward, called the suction-valve, and in the piston is an opening closed by another valve, also opening upward.

Suppose now the suction-tube to be filled with air at the atmospheric pressure, and the water consequently to be at the same level inside the tube and in the well. Suppose the piston to be at the end of its downward stroke, and to be now raised. This motion tends to produce a vacuum below the piston, hence the air contained in the suction-tube will open the suction-valve, and rush into the pump-barrel. The elastic force of this air being thus diminished, the atmospheric pressure will cause the water to rise in the tube to a height such that the pressure due to this height, increased by the pressure of the air, inside, will exactly counterbalance the pressure of the atmosphere. If the piston now descends, the suction-valve closes, the water remains at the level to which it has been raised, and the air, being compressed in the barrel, opens the piston-valve and escapes. At the next stroke of the piston, the water will rise still further, and a fresh portion of air will escape.

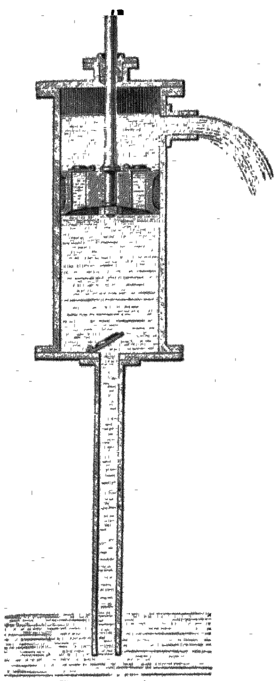


Fig. 155.—Suction-pump.

If, then, the length of the suction-tube is less than about 30 feet, the water will, after a certain number of strokes of the piston, be able to reach the suction-valve and rise into the pump-barrel. When this point has been reached the action changes. The piston in its downward stroke compresses the air, which escapes through it, but the water also passes through, so that the piston when at the bottom of the pump-barrel will have above it all the water which has previously risen into the barrel. If the piston be now raised, supposing the total height to which it is raised to be not more than 34 feet above the level of the water in the well, as should always be the case, the water will follow it in its upward movement, and will fill the

pump-barrel. In the downward stroke this water will pass up through the piston-valve, and in the following upward stroke it will be discharged at the spout. A fresh quantity of water will by this time have risen into the pump-barrel, and the same operations will be repeated.

We thus see that from the time when the water has entered the pump-barrel, at each upward stroke of the piston a volume of water is ejected equal to the contents of the pump-barrel.

In order that the water may be able to rise into the pump-barrel, the suction-valve must not be more than 34 feet above the level of the water in the well, otherwise the water would stop at a certain point of the tube, and could not be raised higher by any farther motion of the piston.

Moreover, in order that the working of the pump may be such as we have described, that is, that at each upward stroke of the piston a quantity of water may be removed equal to the volume of the pump-barrel, it is necessary that the piston when at the top of its stroke should not be more than 34 feet above the water in the well.

o 255. **Effect of untraversed space.**—If the piston does not descend to the bottom of the barrel, it is possible that the water may fall short of rising to the suction-valve, even though the total height reached by the piston be less than 34 feet. When the piston is at the end of its downward stroke, the air below it in the barrel is at atmospheric pressure; and when the limit of working has been reached, this air will expand during the upward stroke until it fills the barrel. Its pressure will now be the same as that of the air in the top of the suction-tube; and if this pressure be equivalent to h feet of water, the height to which water can be drawn up will be only $34-h$ feet.

Example. The suction-valve of a pump is at a height of 27 feet above the surface of the water, and the piston, the entire length of whose stroke is 7·8 inches, when at the lowest point is 3·1 inches from the fixed valve; find whether the water will be able to rise into the pump-barrel.

When the piston is at the end of its downward stroke, the air below it in the barrel is at the atmospheric pressure; when the piston is raised this air becomes rarefied, and its pressure, by Boyle's law, becomes $\frac{3·1}{10·9}$ that of the atmosphere; this pressure can therefore

balance a column of water whose height is $34 \times \frac{3 \cdot 1}{10 \cdot 9}$ feet, or 9·67 feet. Hence, the maximum height to which the water can attain is $34 - 9 \cdot 67$ feet $= 24 \cdot 33$ feet; and consequently, as the suction-tube is 27 feet long, the water will not rise into the pump-barrel, even supposing the pump to be perfectly free from leakage.

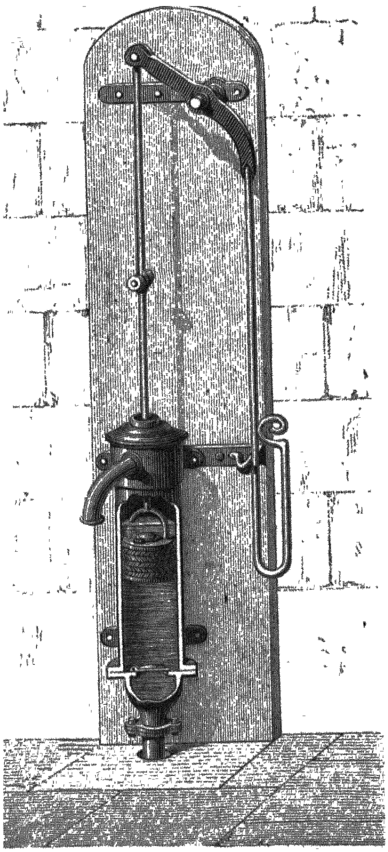
Practically, the pump-barrel should not be more than about 25 feet above the surface of the water in the well; but the spout may be more than 34 feet above the barrel, as the water after rising above the piston is simply pushed up by the latter, an operation which is independent of atmospheric pressure. Pumps in which the spout is at a great height above the barrel are commonly called *lift-pumps*, but they are not essentially different from the suction-pump.

256. Force necessary to raise the Piston.—The force which must be expended in order to raise the piston, is equal to the weight of a column of water, whose base is the section of the piston, and whose height is that to which the water is raised. Let S be the section of the piston, P the atmospheric pressure upon this area, h the height of the column of water which is above the piston in its present position, and h' the height of the column of water below it; then the upper surface of the piston is subjected to a pressure equal to $P + S\,h$; the lower face is subjected to a pressure in the opposite direction equal to $P - S\,h'$, and the entire downward pressure is represented by the difference between these two, that is, by $S\,(h + h')$.

The same conclusion would be arrived at even if the water had not yet reached the piston. In this case, let l be the height of the column of water raised; then the pressure below the piston is $P - S\,l$; the pressure above is simply the atmospheric pressure P, and, consequently, the difference of these pressures acts downward, and its value is $S\,l$.

257. Efficiency of Pumps.—From the results of last section it follows that the force required to raise the piston, multiplied by the height through which it is raised, is equal to the weight of water discharged multiplied by the height of the spout above the water in the well. This is an illustration of the principle of work (§ 49). As this result has been obtained from merely statical considerations, and on the hypothesis of no friction, it presents too favourable a view of the actual efficiency of the pump.

Besides the friction of the solid parts of the mechanism, there is work wasted in generating the velocity with which the fluid, as a whole, is discharged at the spout, and also in producing eddies and other internal motions of the fluid. These eddies are especially produced at the sudden enlargements and contractions of the passages through which the fluid flows. To these drawbacks must be added loss from leakage of water, and at the commencement of the operation from leakage of air, through the valves and at the circumference of the piston. In common household pumps, which are generally roughly made, the *efficiency* may be as small as ·25 or ·3; that is to say, the product of the weight of water raised, and



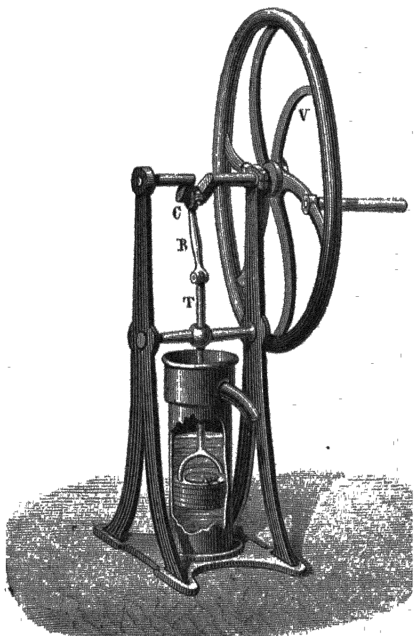


Fig. 156.

Suction-pump.

the height through which it is raised, may be only ·25 or ·3 of the work done in driving the pump.

In Figs. 156 and 157 are shown the means usually employed for working the piston. In the first figure the upward and downward

movement of the piston is effected by means of a lever. The second figure represents an arrangement often employed, in which the alternate motion of the piston is effected by means of a rotatory motion. For this purpose the piston-rod T is joined by means of the connecting-rod B to the crank C of an axle turned by a handle attached to the fly-wheel V.

258. Forcing-pump.—The forcing-pump consists of a pump-barrel dipping into water, and having at the bottom a valve opening upward. In communication with the pump-barrel is a side-tube, with a valve at the point of junction, opening from the barrel into the tube. A solid piston moves up and down the pump-barrel, and it is evident that when this piston is raised, water enters the barrel by the lower valve, and that when the piston descends, this water is forced into the side-tube. The greater the height of this tube, the greater will be the force required to push the piston down, for the resistance to be overcome is the pressure due to the column of water raised.

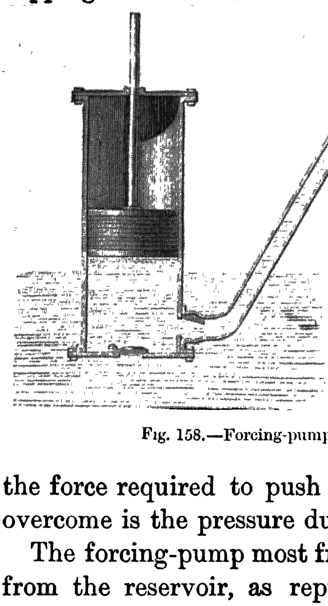


Fig. 158.—Forcing-pump.

The forcing-pump most frequently has a short suction-pipe leading from the reservoir, as represented in Fig. 159. In this case the water is raised from the reservoir into the barrel by atmospheric pressure during the up-stroke, and is forced from the barrel into the ascending pipe in the down-stroke.

259. Plunger.—When the height to which the water is to be forced is very considerable, the different parts of the pump must be very strongly made and fitted together, in order to resist the enormous pressure produced by the column of water, and to prevent leakage. In this case the ordinary piston stuffed with tow or leather washers cannot be used, but is replaced by a solid cylinder of metal called a *plunger*. Fig. 160 represents a section of a pump thus constructed. The plunger is of smaller section than the barrel, and passes through a stuffing-box in which it fits air-tight. The volume of water which enters the barrel at each up-stroke, and is expelled in the down-stroke, is the same as the volume of a length of the plunger equal

to the length of stroke; and the hydrostatic pressure to be overcome is proportional to the section of the plunger, not to that of the barrel. As the operation proceeds, air is set free from the water, and would eventually impede the working of the pump were it not permitted to escape. For this purpose the plunger is pierced with a narrow passage, which is opened from time to time to blow out the air.

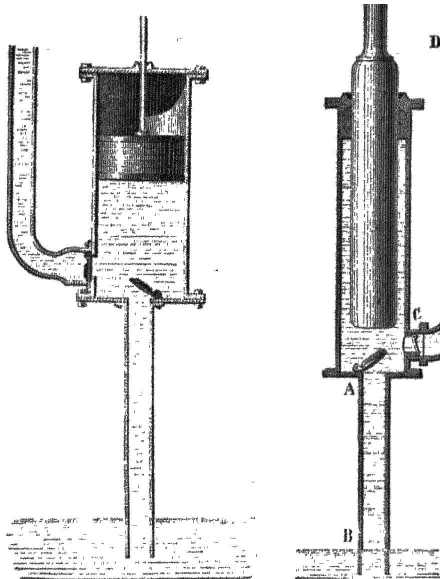


Fig 159 Fig 160.
Suction and Force Pump

The drainage of deep mines is usually effected by a series of pumps. The water is first raised by one pump to a reservoir, into which dips the suction-tube of a second pump, which sends the water up to a second reservoir, and so on. The piston-rods of the different pumps are all joined to a single rod called the *spear*, which receives its motion from a steam-engine.

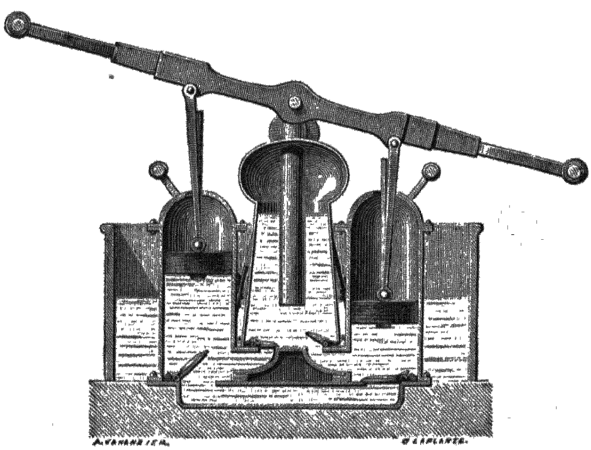


Fig. 161.—Fire-engine.

260. Fire-engine.—The ordinary fire-engine is formed by the union of two forcing-pumps which play into a common reservoir, containing in its upper portion (called the air-chamber) air compressed by the working of the engine. A tube dips into the water in this reservoir, and to the upper end of this tube is screwed the leather hose through which the water is discharged. The piston-rods are jointed to a lever, the ends of which are raised and depressed alternately, so that one piston is ascending while the other is descending. Water is thus continually being forced into the common reservoir except at the instant of reversing stroke, and as the compressed air in the air-chamber performs the part of a reservoir of work (nearly analogous to the fly-wheel), the discharge of water from the nozzle of the hose is very steady.

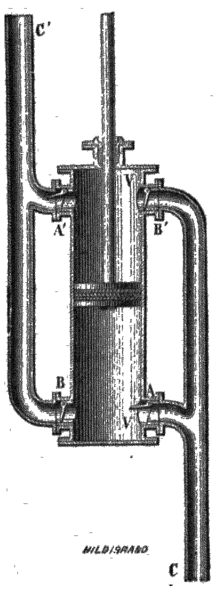


Fig. 162.
Double-action Pump

The engine is sometimes supplied with water by means of an attached cistern (as in Fig. 162) into which water is poured; but it is more usually furnished with a suction-pipe which renders it self-feeding.

261. Double-acting Pumps.—These pumps, the invention of which is due to Delahire, are often employed for household purposes. They consist of a pump-barrel VV (Fig. 162), with four openings in it, A, A′, B, B′. The openings A and B′ are in communication with the suction-tube C; A′ and B are in communication with the ejection-tube C′. The four openings are fitted with four valves opening all in the same direction, that is, from right to left, whence it follows that A and B′ act as suction-valves, and A′ and B as ejection-valves, and, consequently, in whichever direction the piston may be moving, the suction and ejection of water are taking place at the same time.

262. Centrifugal Pumps.—Centrifugal pumps, which have long been used as blowers for air, and have recently come into extensive use for purposes of drainage and irrigation, consist mainly of a flat casing or box of approximately circular outline, in which the fluid is made to revolve by a rotating propeller furnished with fans or blades. These extend from near the centre outwards to the circumference of the propeller, and are usually curved backwards. The

fluid between them, in virtue of the centrifugal force generated by its rotation, tends to move outwards, and is allowed to pass off through a large conduit which leaves the case tangentially.

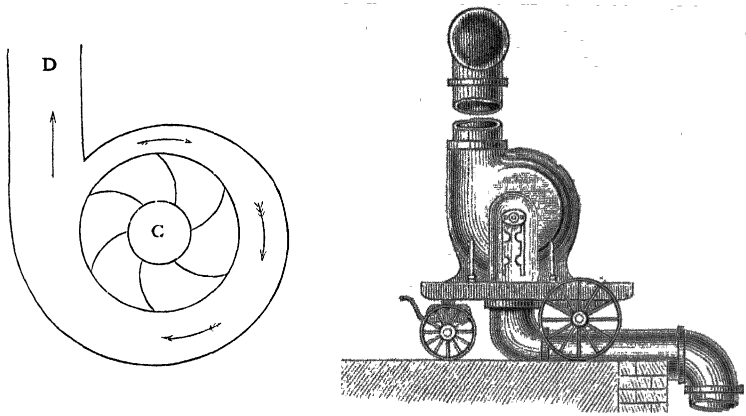


Fig. 163.—Centrifugal Pump.

The first part of Fig. 163 is a section of the propeller and casing, C being a central opening at which the fluid enters, and D the conduit through which it escapes. The second part of the figure represents a small pump as mounted for use. The largest class of

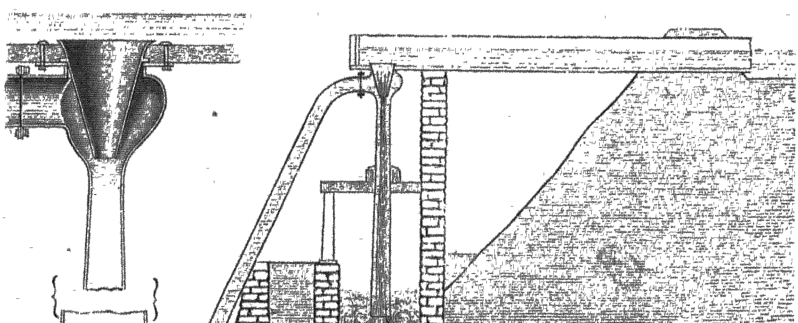


Fig 164 —Jet Pump

centrifugal pumps are usually immersed in the water to be pumped, and revolve horizontally.

263. Jet-pump.—The jet-pump is a contrivance by Professor

James Thomson for raising water by means of the descent of other water from above, the common outfall being at an intermediate level. Its action somewhat resembles that of the blast-pipe of the locomotive. The pipe corresponding to the locomotive chimney must have a narrow throat at the place where the jet enters, and must thence widen very gradually towards its outlet, which is immersed in the outfall water so as to prevent any admission of air during the pumping. The water is drawn up from the low level through a suction-pipe, terminating in a chamber surrounding the jet-nozzle.

Fig. 164 represents the pump in position, the jet-nozzle with its surroundings being also shown separately on a larger scale.

The action of the jet-pump is explained by the following considerations.

Suppose we have a horizontal pipe varying gradually in sectional area from one point to another, and completely filled by a liquid flowing steadily through it. Since the same quantity of liquid passes all cross-sections of the pipe, the velocity will vary inversely as the sectional area. Those portions of the liquid which are passing at any moment from the larger to the smaller parts of the pipe are being accelerated, and are therefore more strongly pushed behind than in front; while the opposite is the case with those which are passing from smaller to larger. Places of large sectional area are therefore places of small velocity and high pressure, and on the other hand, places of small area have high velocity and low pressure. Pressure, in such discussions as this, is most conveniently expressed by *pressure-height*, that is, by the height of an equivalent column of the liquid. Neglecting friction, it can be shown that if v_1, v_2 be the velocities at two points in the pipe, and h_1, h_2 the pressure-heights at these points,

$$v_2^2 - v_1^2 = 2g\,(h_1 - h_2),$$

g denoting the intensity of gravity. The change in pressure-height is therefore equal and opposite to the change in $\frac{v^2}{2g}$. This is for a horizontal pipe.

In an ascending or descending pipe, there is a further change of pressure-height, equal and opposite to the change of actual height.

Let H be the pressure-height at the free surfaces, that is, the height of a column of water which would balance atmospheric pressure;

k the difference of level between the jet-nozzle and the free surface above it.

l the difference of level between the jet-nozzle and the free surface of the water which is to be raised.

v the velocity with which the liquid rushes through the jet-nozzle,

then the pressure-height at the jet-nozzle may be taken as $H + k - \frac{v^2}{2g}$; and if this be less than $H - l$ the water will be sucked up. The condition of working is therefore that

$$H - l \text{ be greater than } H + k - \frac{v^2}{2g}, \text{ or}$$

$$\frac{v^2}{2g} \text{ greater than } k + l,$$

where it will be observed that $k + l$ is the difference of levels of the highest and lowest free surfaces.

264. Hydraulic Press.—The hydraulic press (Fig. 165) consists of a suction and force pump aa worked by means of a lever turning about an axis O. The water drawn from the reservoir BB is forced along

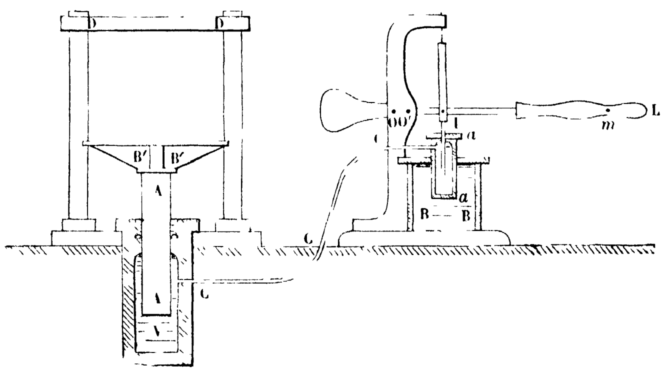


Fig. 165.—Bramah Press.

the tube CC into the cistern V. In the top of the cistern is an opening through which moves a heavy metal plunger AA. This carries on its upper end a large plate B′B′, upon which are placed the objects to be pressed. Suppose the plunger A to be in its lowest position when the pump begins to work. The cistern first begins to fill with water; then the pressure exerted by the plunger of the pump is transmitted, according to the principles laid down in § 141, to the bottom of the plunger A; which accordingly rises, and the objects to

be pressed, being intercepted between the plate and the top of a fixed frame, are subjected to the transmitted pressure. The amount of this pressure depends both on the ratio of the sections of the pistons, and on the length of the lever used to work the force-pump. Suppose, for instance, that the distance of the point m, where the hand is applied, from the point O, is equal to twelve times the distance IO, and suppose the force exerted to be equal to fifty pounds. By the principle of the lever this is equivalent to a force of 50×12 at the point I; and if the section of the piston A be at the same time 100 times that of the piston of the pump, the pressure transmitted to A will be $50 \times 12 \times 100 = 60{,}000$ pounds. These are the ordinary conditions of the press usually employed in workshops. By drawing out the pin which serves as an axis at O, and introducing it at O′, we can increase the mechanical advantage of the lever.

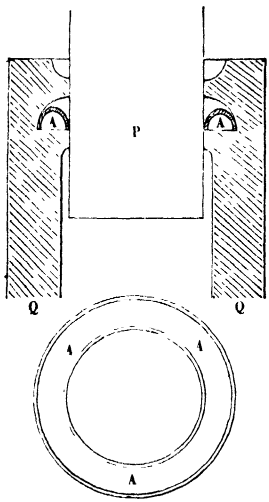


Fig. 166 —Cup-leather.

Two parts essential to the working of the hydraulic press are not represented in the figure. These are a safety-valve, which opens when the pressure attains the limit which is not to be exceeded; and, secondly, a tap in the tube C, which is opened when we wish to put an end to the action of the press. The water then runs off, and the piston A descends again to the bottom of the cistern.

The hydraulic press was clearly described by Pascal, and at a still earlier date by Stevinus, but for a long time remained practically useless; because as soon as the pressure began to be at all strong, the water escaped at the surface of the piston A. Bramah invented the *cupped leather collar*, which prevents the liquid from escaping, and thus enables us to utilize all the power of the machine. It consists of a leather ring AA (Fig. 166), bent so as to have a semicircular section. This is fitted into a hollow in the interior of the sides of the cistern, so that water passing between the piston and cylinder will fill the concavity of the cupped leather collar, and by pressing on it will produce a packing which fits more tightly as the pressure on the piston increases.

The hydraulic press is very extensively employed in the arts.

It is of great power, and may be constructed to give pressures of two or three hundred tons. It is the instrument generally employed in cases where very great force is required, as in testing anchors or raising very heavy weights. It was used for raising the sections of the Britannia tubular bridge, and for launching the *Great Eastern*.

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CHAPTER XXIII.

EFFLUX OF LIQUIDS.—TORRICELLI'S THEOREM.

265. If an opening is made in the side of a vessel containing water, the liquid escapes with a velocity which is greater as the surface of the liquid in the vessel is higher above the orifice, or to employ the usual phrase, as the *head* of liquid is greater. This point in the dynamics of liquids was made the subject of experiments by Torricelli, and the result arrived at by him was that the velocity of efflux is equal to that which would be acquired by a body falling freely from the upper surface of the liquid to the centre of the orifice. If h be this height, the velocity of efflux is given by the formula

$$v = \sqrt{2gh}.$$

This is called Torricelli's theorem. It supposes the orifice to be small compared with the horizontal section of the vessel, and to be exposed to the same atmospheric pressure as the upper surface of the liquid in the vessel.

It may be deduced from the principle of conservation of energy; for the escape of a mass m of liquid involves a loss mgh of energy of position, and must involve an equal gain of energy of motion. But the gain of energy of motion is $\frac{1}{2}mv^2$; hence we have

$$\tfrac{1}{2}mv^2 = mgh, \; v^2 = 2gh.$$

The form of the issuing jet will depend, to some extent, on the form of the orifice. If the orifice be a round hole with sharp edges, in a thin plate, the flow through it will not be in parallel lines, but the outer portions will converge towards the axis, producing a rapid narrowing of the jet. The section of the jet at which this convergence ceases and the flow becomes sensibly parallel, is called the *contracted vein* or *vena contracta*. The pressure within the jet at this part is atmospheric, whereas in the converging part it is greater

than atmospheric; and it is to the contracted vein that Torricelli's formula properly applies, v denoting the velocity at the contracted vein, and h the depth of its central point below the free surface of the liquid in the vessel.

266. Area of Contracted Vein. Froude's Case.—A force is equal to the momentum which it generates in the unit of time. Let A denote the area of an orifice through which a liquid issues horizontally, and a the area of the contracted vein. From the equality of action and reaction it follows that the resultant force which ejects the issuing stream is equal and opposite to the resultant horizontal force exerted on the vessel. The latter may be taken as a first approximation to be equal to the pressure which would be exerted on a plug closing the orifice, that is to ghA if the density of the liquid be taken as unity.

The horizontal momentum generated in the water in one second is the product of the velocity v and the mass ejected in one second. The volume ejected in one second is va. This is equal to the mass, since the density is unity, and hence the momentum is v^2a, that is, $2gha$. Equating this last expression for the momentum to the foregoing expression for the force, we have

$$2gha = gh\mathrm{A}$$
$$a = \tfrac{1}{2}\mathrm{A},$$

that is, the area of the contracted vein is half the area of the orifice.

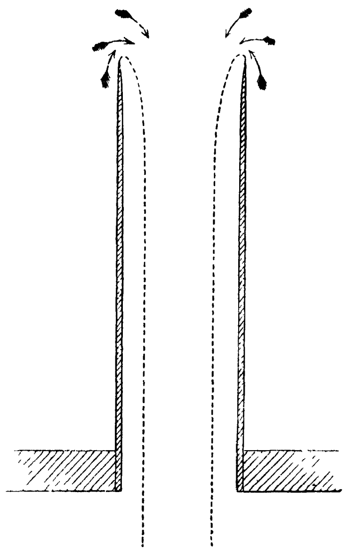


Fig. 167.

Mr. Froude has pointed out that this reasoning is strictly correct when the liquid is discharged through a cylindrical pipe projecting inwards into the vessel and terminating with a sharp edge (Fig. 167); and he has verified the result by accurate experiments in which the jet was discharged vertically downwards. The direction of flow in different parts of the jet is approximately indicated by the arrows and dotted lines in the figure; and, on a larger, scale by those in Fig. 168, in which the sections of the orifice and of the contracted vein are also indicated by the lines marked D and d. We may remark that since liquids press equally in all directions, there can

be no material difference between the velocities of a vertical and of a horizontal jet at the same depth below the free surface.

267. Contracted Vein for Orifice in Thin Plate.—When the liquid

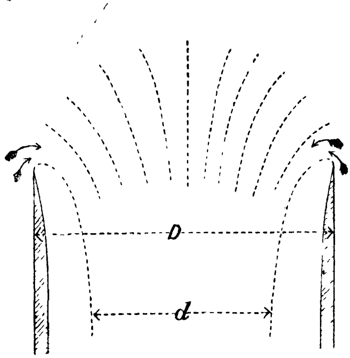


Fig. 168.

is simply discharged through a hole cut in the side of the vessel and bounded by a sharp edge, the direction of flow in different parts of the stream is shown by the arrows and dotted lines in Fig. 169. The pressure on the sides, in the neighbourhood of the orifice, is less than that due to the depth, because the curved form of the lines of flow implies (on the principles of centrifugal force) a smaller pressure on their concave than on their convex side. The pressure around the orifice is therefore less than it would be if the hole were plugged. The unbalanced horizontal pressure on the vessel (if we suppose the side containing the jet to be vertical) will therefore exceed the statical pressure on the plug ghA, since the removal of the plug not only removes the pressure on the plug but also a portion of the

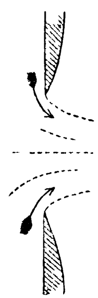


Fig. 169.

pressure on neighbouring parts. This unbalanced force, which is greater than ghA, is necessarily equal to the momentum generated per second in the liquid, which is still represented by the expression v^2a or $2gha$; hence $2gha$ is greater than ghA, or a is greater than $\frac{1}{2}A$. Reasoning similar to this applies to all ordinary forms of orifice. The peculiarity of the case investigated by Mr. Froude consists in the circumstance that the pressure on the parts of the vessel in the neighbourhood of the orifice is normal to the direction of the jet, and any changes in its amount which may be produced by unplugging the orifice have therefore no influence upon the pressures on the vessel in or opposite to the direction of the jet.[1]

268. Apparatus for Illustration.—In the preceding investigations,

[1] This section and the preceding one are based on two communications read before the Philosophical Society of Glasgow, February 23d and March 31st, 1876; one being an extract from a letter from Mr. Froude to Sir William Thomson, and the other a communication from Professor James Thomson, to whom we are indebted for the accompanying illustrations.

no account is taken of friction. When experiments are conducted on too small a scale, friction may materially diminish the velocity; and further, if the velocity be tested by the height or distance to which the jet will spout, the resistance of the air will diminish this height or distance, and thus make the velocity appear less than it really is.

Fig. 170 represents an apparatus frequently employed for illustrat-

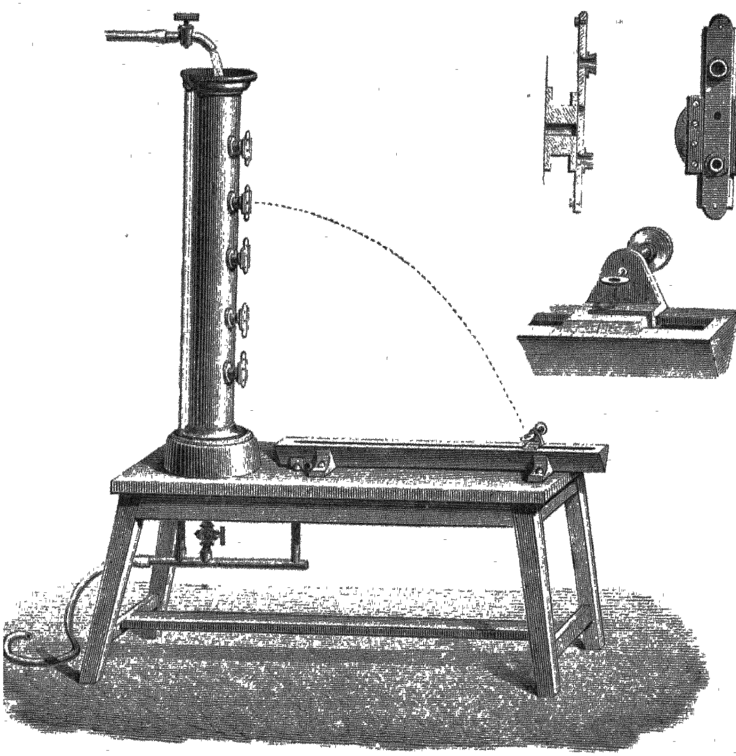


Fig. 170.—Apparatus for verifying Torricelli's Theorem.

ing some of the consequences of Torricelli's theorem. An upright cylindrical vessel is pierced on one side with a number of orifices in the same vertical line, which can be opened or closed at pleasure. A tap placed above the vessel supplies it with water, and, with the help of an overflow pipe, maintains the surface at a constant level, which is as much above the highest orifice as each orifice is above that next below it. The liquid which escapes is received in a trough, the edge of which is graduated. A travelling piece with an index

line engraved on it slides along the trough; it carries, as shown in one of the separate figures, a disc pierced with a circular hole, and capable of being turned in any direction about a horizontal axis passing through its centre. In this way the disc can always be placed in such a position that its plane shall be at right angles to the liquid jet, and that the jet shall pass freely and exactly through its centre. The index line then indicates the range of the jet with considerable precision. This range is reckoned from the vertical plane containing the orifices, and is measured on the horizontal plane passing through the centre of the disc. The distance of this latter plane below the lowest orifice is equal to that between any two consecutive orifices.

The jet, consisting as it does of a series of projectiles travelling in the same path, has the form of a parabola.

Let a be the range of the jet, b the height of the orifice above the centre of the ring, and v the velocity of discharge, which we assume to be horizontal. Then if t be the time occupied by a particle of the liquid in passing from the orifice to the ring, we have to express that a is the distance due to the horizontal velocity v in the time t, and that b is the vertical distance due to gravity acting for the same time. We have therefore

$$a = vt$$
$$b = \tfrac{1}{2}gt^2$$
$$\text{whence } t^2 = \frac{a^2}{v^2} = \frac{2b}{g}, \; v^2 = \frac{ga^2}{2b}.$$

But according to Torricelli's theorem, if h be the height of the surface of the water above the orifice, we have $v^2 = 2gh$; and comparing this with the above value of v^2 we deduce

$$\frac{a^2}{2b} = 2h, \; a^2 = 4bh.$$

One consequence of this last formula is, that if the values of b and h be interchanged, the value of a will remain unaltered. This amounts to saying that the highest orifice will give the same range as the lowest, the highest but one the same as the lowest but one, and so on; a result which can be very accurately verified.

If we describe a semicircle on the line $b+h$, the length of an ordinate erected at the point of junction of b and h is $\sqrt{bh}$, and since $a = \sqrt{4bh} = 2\sqrt{bh}$, it follows that the range is double of this ordinate. This is on the hypothesis of no friction. Practically it is less than double. The greatest ordinate of the semicircle is the central one, and accordingly the greatest range is given by the central orifice.

o **269. Efflux from Air-tight Space.**—When the air at the free surface of the liquid in a vessel is at a different pressure from the air into which the liquid is discharged, we must express this difference of pressures by an equivalent column of the liquid, and the velocity of efflux will be that due to the height of the surface above the orifice increased or diminished by this column. Efflux will cease altogether when the pressure on the free surface, together with that due to the height of the free surface above the orifice, is equal to the pressure outside the orifice; or if efflux continue under such circumstances it can only do so by the admission of bubbles of air. This explains the action of vent-pegs.

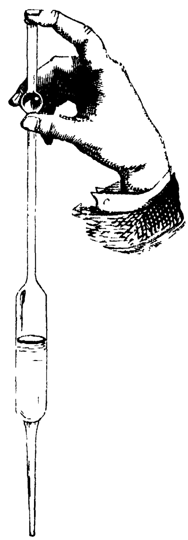


Fig 171.—Pipette.

Pipette.—This is a glass tube (Fig. 171) open at both ends, and terminating below in a small tapering spout. If water be introduced into the tube, either by aspiration or by direct immersion in water, and if the upper end be closed with the finger, the efflux of the liquid will cease almost instantly. On admitting the air above, the efflux will begin again, and can again be stopped at pleasure.

The Magic Funnel.—This funnel is double, as is shown in Fig. 172. Near the handle is a small opening by which the space between the two funnels communicates with the external air. Another opening connects this same space with the tube of the inner funnel. If the interval between the two funnels be filled with any liquid, this liquid will run out or will cease to flow according as the upper hole is open or closed. The opening and closing of the hole can be easily effected with the thumb of the hand holding the funnel without the knowledge of the spectator. This device has been known from very early times.

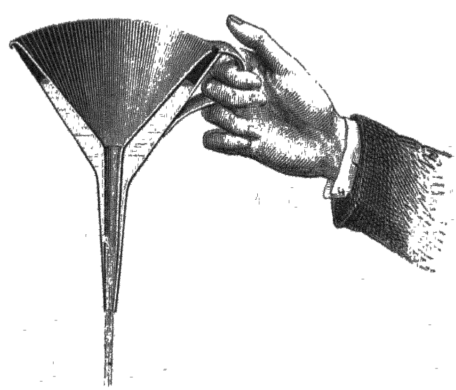


Fig. 172.—Magic Funnel.

The instrument may be used in a still more curious manner. For this purpose the space inside is secretly filled with highly-coloured wine, which is prevented from escaping by closing the opening above. Water is then poured into the central funnel, and escapes either by itself or mixed with wine, according as the thumb closes or opens the orifice for the admission of air. In the second case, the water being coloured with the wine, it will appear that wine alone is issuing from the funnel; thus the operator will appear to have the power of making either water or wine flow from the vessel at his pleasure.

The Inexhaustible Bottle.—The inexhaustible bottle (Fig. 173) is a toy of the same kind. It is an opaque bottle of sheet-iron or

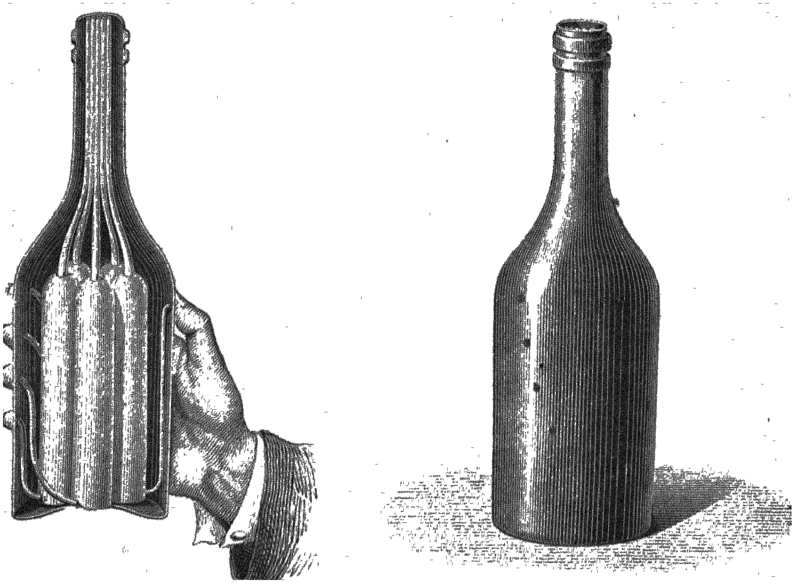


Fig 173.—Inexhaustible Bottle

gutta-percha, containing within it five small vials. These communicate with the exterior by five small holes, which can be closed by the five fingers of the hand. Each vial has also a small neck which passes up the large neck of the bottle. The five vials are filled with five different liquids, any one of which can be poured out at pleasure by uncovering the corresponding hole.

270. **Intermittent Fountain.**—The intermittent fountain is an apparatus analogous to the preceding, except that the interruptions in the efflux are produced automatically by the action of the instru-

ment, without the intervention of the operator. It consists of a globe V (Fig. 174), which can be closed air-tight by means of a stopper, and is in communication with efflux tubes *a*, which discharge into a basin B, having a small hole *o* in its bottom for permitting the water to escape into a lower basin C. A central tube *t*, open at both ends, extends nearly to the top of the globe, and nearly to the bottom of the basin B.

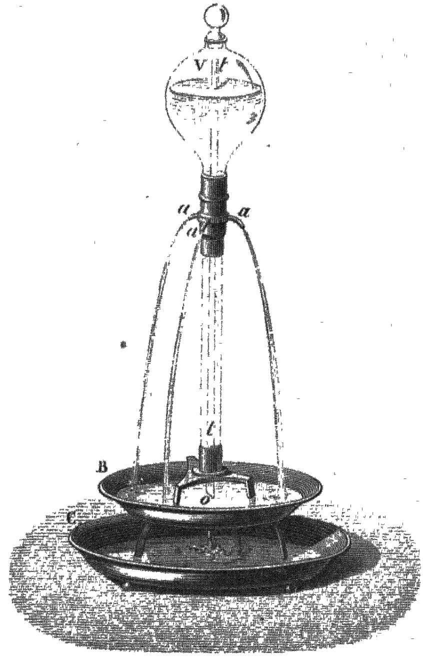


Fig 174 —Intermittent Fountain

Suppose the globe to be filled with water, the basins being empty. Then the water will flow from the efflux tubes *a*, while air will pass up through the central tube. As the water issues from the efflux tubes much faster than it escapes through the opening *o*, the level rises in the basin B till the lower end of the tube *t* is covered. The pressure of the air in the upper part of the globe then rapidly diminishes, and the efflux from the tubes *a* is stopped. But as the water continues to escape from the basin B through the opening *o*, the bottom of the tube *t* is again uncovered, the liquid again issues from the efflux tubes, and the same changes are repeated.

271. Siphon.—The siphon is an instrument in which a liquid, under the combined action of its own weight and atmospheric pressure, flows first up-hill and then down-hill, but always in such a way as to bring about a lowering of the centre of gravity of the whole liquid mass.

In its simplest form, it consists of a bent tube, one end of which is immersed in the liquid to be removed, while the other end either discharges into the air, at a lower level than the surface of the liquid in the vessel, as in Fig. 175, or dips into the liquid of a receiving vessel, the surface of this liquid being lower than that of the liquid in the discharging vessel.

We shall discuss the latter case, and shall denote the difference of levels of the two surfaces by h, while the height of a column of the liquid equivalent to atmospheric pressure will be denoted by H.

Let the siphon be full of liquid, and imagine a diaphragm to be drawn across it at any point, so as to prevent flow. Let this dia-

Fig. 175.—Siphon.

phragm be at a height x above the higher of the two free surfaces, and at a height y above the lower, so that we have

$$y - x = h.$$

The pressure on the side of the diaphragm next the higher free surface will be $\mathrm{H} - x$, (pressure being expressed in terms of the equivalent liquid column,) and the pressure on the other side of the diaphragm will be $\mathrm{H} - y$, which is less than the former by $y - x$, that is by h. The diaphragm therefore experiences a resultant force due to a depth h of the liquid, urging it from the higher to the lower free surface, and if the diaphragm be removed, the liquid will be propelled in this direction.

In practice, the two legs of the siphon are usually of unequal length, and the flow is from the shorter to the longer; but this is by no means essential, for by a sufficiently deep immersion of the long

leg, the direction of flow may be reversed. The direction of flow depends not on the lengths of the legs, but on the levels of the two free surfaces.

If the liquid in the discharging vessel falls below the end of the siphon, or if the siphon is lifted out of it, air enters, and the siphon is immediately emptied of liquid. If the liquid in the receiving vessel is removed, so that the discharging end of the siphon is surrounded by air, as in the figure, the flow will continue, unless air bubbles up the tube and breaks the liquid column. This interruption is especially liable to occur in large tubes. It can be prevented by bending the end of the siphon round, so as to discharge the liquid in an ascending direction. To adapt the foregoing investigation to the case of a siphon discharging into air, we have only to substitute the level of the discharging end for the level of the lower free surface, so that y will denote the depth of the discharging end below the diaphragm, and h its depth below the surface of the liquid which is to be drawn off.

As the ascent of the liquid in the siphon is due to atmospheric pressure on the upper free surface, it is necessary that the highest point of the siphon (if intended for water) should not be more than about 33 feet above this surface.

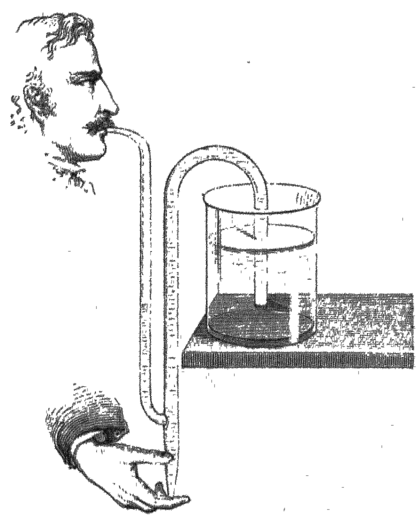


Fig. 176.—Starting the Siphon.

o 272. Starting the Siphon.—In order to make a siphon begin working, we must employ means to fill it with the liquid. This can sometimes be done by dipping it in the liquid, and then placing it in position while the ends are kept closed; or by inserting one end in the liquid which we wish to remove, and sucking at the other. It is usually more convenient to apply suction by means of a side tube, as in Fig. 176, this tube being sometimes provided with an enlargement to prevent the liquid from entering the mouth. One end of the siphon is inserted in the liquid which is to be removed, while the other end is stopped, and the operator applies suction at

the side tube till the liquid flows over. In siphons for commercial purposes, the suction is usually produced by a pump.

273. **Siphon for Sulphuric Acid.**—Fig. 177 represents a siphon used for transferring sulphuric acid from one vessel to another. The long branch is first filled with sulphuric acid. This is effected by means of two funnels (which can be plugged at pleasure) at the bend of the tube. One of these admits the liquid, and the other suffers the air to escape. The two funnels are then closed, and the tap at the lower end of the tube is opened so as to allow the liquid to escape. The air in the short branch follows the acid, and becomes rarefied; the acid behind it rises, and if it passes the bend, the siphon will be started, for each portion of the liquid which issues from the tube will draw an equal portion from the short to the long branch.

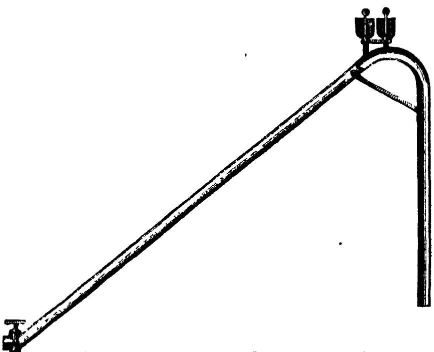


Fig. 177.—Siphon for Sulphuric Acid.

To insure the working of the sulphuric acid siphon, it is not sufficient to have the vertical height of the long branch greater than that of the short branch; it is farther necessary that it should exceed a certain limit, which depends upon the dimensions of the siphon in each particular case. In order to calculate this limit, we must remark that when the liquid begins to flow, its height diminishes in the long and increases in the short branch; if these two heights should become equal, there would be equilibrium. We see, then, that in order that the siphon may work, it is necessary that when the liquid rises to the bend of the tube, there should be in the long branch a column of liquid whose vertical height is at least equal to that of the short branch, which we shall denote by h, and the actual length of the short branch from the surface of the liquid in which it dips to the summit of the bend by h'. Then if α be the inclination of the long branch to the vertical, and L the length of the long branch, which we suppose barely sufficient, the length of the column of liquid remaining in the long branch will be $h \sec \alpha$. The air which at atmospheric pressure H occupied the length h', now under the pressure $H - h$ occupies a length $L - h \sec \alpha$; hence by Boyle's law, we have

$$Hh' = (H - h)\ (L - h \sec a), \text{ whence } L = h \sec a + \frac{Hh'}{H - h}.$$

In this formula H denotes the height of a column of sulphuric acid whose pressure equals that of the atmosphere.

274. Cup of Tantalus.—The siphon may be employed to produce the intermittent flow of a liquid. Suppose, for instance, that we have a cup (Fig. 178) in which is a bent tube rising to a height n, and with the short branch terminating near the bottom of the cup, while the long branch passes through the bottom. If liquid be poured into the cup, the level will gradually rise in the short branch of the bent tube, till it reaches the summit of the bend, when the siphon will begin to discharge the liquid. If the liquid then escapes by the siphon faster than it is poured into the vessel, the level of the liquid in the cup will gradually fall below the termination of the shorter branch. The siphon will then empty itself, and will not recommence its action till the liquid has again risen to the level of the bend.

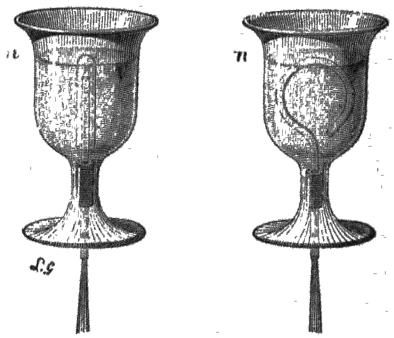


Fig. 178 —Vase of Tantalus.

The siphon may be concealed in the interior of the figure of a man whose mouth is just above the top of the siphon. If water be poured in very slowly, it will continually rise nearly to his lips and then descend again. Hence the name. Instead of a bent tube we may employ, as in the first figure, a straight tube covered by a bell-glass left open below; in this case the space between the tube and the bell takes the place of the shorter leg of the siphon.

It is to an action of this kind that natural intermittent springs are generally attributed. Suppose a reservoir (Fig. 179) to communicate with an outlet by a bent tube forming a siphon, and suppose it to be fed by a stream of water at a slower rate than the siphon is able to discharge it. When the water has reached the bend, the siphon will become charged, and the reservoir will be emptied; flow will then cease until it becomes charged again.

275. Mariotte's Bottle.—This is an apparatus often employed to obtain a uniform flow of water. Through the cork at the top of the bottle (Fig. 180) passes a straight vertical tube open at both ends, and

in one side of the bottle near the bottom is a second opening furnished with a horizontal efflux tube b at a lower level than the lower end of the vertical tube. Suppose that both the bottle and the vertical tube are in the first instance full of water, and that the efflux tube is then opened. The liquid flows out, and the vertical tube is rapidly emptied. Air then enters the bottle through the vertical tube, and bubbles up from its lower end a through the liquid to the upper part of the bottle. As soon as this process begins, the velocity of efflux, which up to this point has been rapidly diminishing (as is shown by the diminished range of the

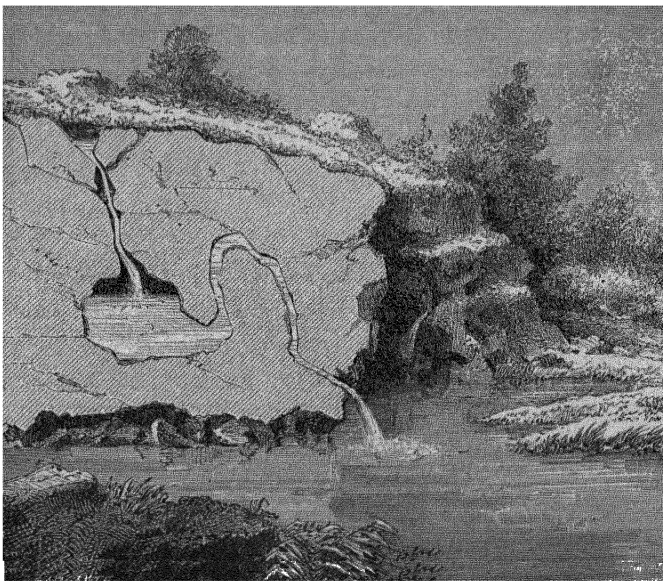


Fig. 179.—Intermittent Spring.

jet), becomes constant, and continues so till the level of the liquid has fallen to a, after which it again diminishes. During the time of constant flow, the velocity of efflux is that due to the height of a above b, and the air in the upper part of the bottle is at less than atmospheric pressure, the difference being measured by the height of the surface of the liquid above a. Strictly speaking, since the air enters not in a continuous stream but in bubbles, there must be slight oscillations of velocity, keeping time with the bubbles, but they are scarcely perceptible.

Instead of the vertical tube, we may have a second opening in the

side of the bottle, at a higher level than the first; as shown in Fig. 180. Air will enter through the pipe *a*, which is fitted in this upper opening, and the liquid will issue at the lower pipe *b*, with a constant velocity due to the height of *a* above *b*.

Mariotte's bottle is sometimes used in the laboratory to produce

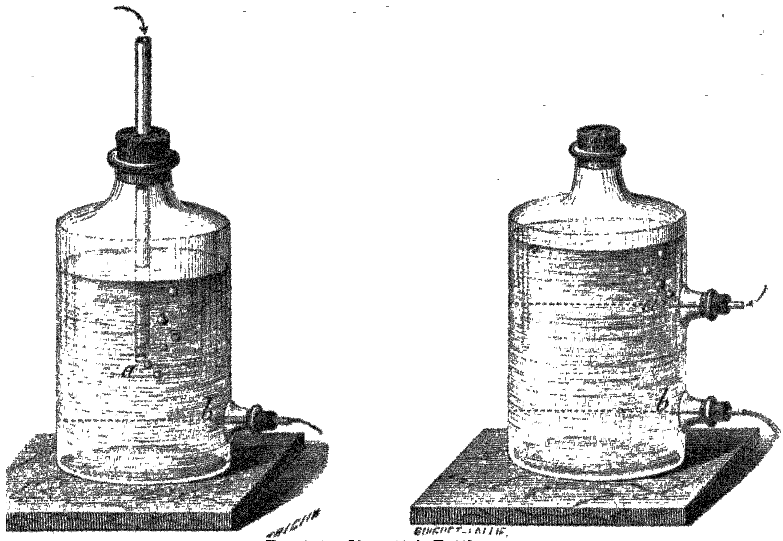


Fig. 180.—Mariotte's Bottle.

the uniform flow of a gas by employing the water which escapes to expel the gas. We may also draw in gas through the tube of Mariotte's bottle; in this case, the flow of the *water* is uniform, but the flow of the *gas* is continually accelerated, since the space occupied by it in the bottle increases uniformly, but the density of the gas in this space continually increases.

EXAMPLES.

Parallelogram of Velocities, and Parallelogram of Forces.

1. A ship sails through the water at the rate of 10 miles per hour, and a ball rolls across the deck in a direction perpendicular to the course, at the same rate. Find the velocity of the ball relative to the water.

2. The wind blows from a point intermediate between N. and E. The northerly component of its velocity is 5 miles per hour, and the easterly component is 12 miles per hour. Find the total velocity.

3. The wind is blowing due N.E. with a velocity of 10 miles an hour. Find the northerly and easterly components.

4. Two forces of 6 and 8 units act upon a body in lines which meet in a point and are at right angles. Find the magnitude of their resultant.

5 Two equal forces of 100 units act upon a body in lines which meet in a point and are at right angles. Find the magnitude of their resultant.

6. A force of 100 units acts at an inclination of 45° to the horizon. Resolve it into a horizontal and a vertical component.

7. Two equal forces act in lines which meet in a point, and the angle between their directions is 120°. Show that the resultant is equal to either of the forces.

8. A body is pulled north, south, east, and west by four strings whose directions meet in a point, and the forces of tension in the strings are equal to 10, 15, 20, and 32 lbs. weight respectively. Show that the resultant is equal to 13 lbs. weight.

9. Five equal forces act at a point, in one place. The angles between the first and second, between the second and third, between the third and fourth, and between the fourth and fifth, are each 60°. Find their resultant.

10. If θ be the angle between the directions of two forces P and Q acting at a point, and R be their resultant, show that

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta.$$

11. Show that the resultant of two equal forces P, acting at an angle θ, is $2P \cos \frac{1}{2}\theta$.

Parallel Forces, and Centre of Gravity.

10*. A straight rod 10 ft. long is supported at a point 3 ft. from one end. What weight hung from this end will be supported by 12 lbs. hung from the other, the weight of the rod being neglected?

11*. Weights of 15 and 20 lbs. are hung from the two ends of a straight rod 70 in. long. Find the point about which the rod will balance, its own weight being neglected.

12. A weight of 100 lbs. is slung from a pole which rests on the shoulders of two men, A and B. The distance between the points where the pole presses their shoulders is 10 ft., and the point where the weight is slung is 4 ft. from the point where the pole presses on A's shoulder. Find the weight borne by each, the weight of the pole being neglected.

13. A uniform straight lever 10 ft. long balances at a point 3 ft. from one end, when 12 lbs. are hung from this end and an unknown weight from the other. The lever itself weighs 8 lbs. Find the unknown weight.

14. A straight lever 6 ft. long weighs 10 lbs., and its centre of gravity is 4 ft. from one end. What weight at this end will support 20 lbs. at the other, when the lever is supported at 1 ft. distance from the latter?

15. Two equal weights of 10 lbs. each are hung one at each end of a straight lever 6 ft. long, which weighs 5 lbs.; and the lever, thus weighted, balances about a point 3 in. distant from the centre of its length. Find its centre of gravity.

16. A uniform lever 10 ft. long balances about a point 1 ft. from one end, when loaded at that end with 50 lbs. Find the weight of the lever.

17. A straight lever 10 ft. long, when unweighted, balances about a point 4 ft. from one end; but when loaded with 20 lbs. at this end and 4 lbs. at the other. it balances about a point 3 ft. from the end. Find the weight of the lever.

18. A lever is to be cut from a bar weighing 3 lbs. per ft What must be its length that it may balance about a point 2 ft. from one end, when weighted at this end with 50 lbs.? (The solution of this question involves a quadratic equation.)

19. A lever is supported at its centre of gravity, which is nearer to one end than to the other. A weight P at the shorter arm is balanced by 2 lbs. at the longer; and the same weight P at the longer arm is balanced by 18 lbs. at the shorter. Find P.

20. Weights of 2, 3, 4 and 5 lbs. are hung at points distant respectively 1, 2, 3 and 4 ft. from one end of a lever whose weight may be neglected. Find the point about which the lever thus weighted will balance. (This and the following questions are best solved by taking moments round the end of the lever. The sum of the moments of the four weights is equal to the moment of their resultant.)

21. Solve the preceding question, supposing the lever to be 5 ft. long, uniform, and weighing 2 lbs.

22. Find, in position and magnitude, the resultant of two parallel and oppositely directed forces of 10 and 12 units, their lines of action being 1 yard apart.

23. A straight lever without weight is acted on by four parallel forces at the following distances from one end:—

At 1 ft.,	a force of	2	units, acting	upwards.
At 2 ft.,	„	3	„ „	downwards.
At 3 ft.,	„	4	„ „	upwards.
At 4 ft.	.	5	„ „	downwards.

Where must the fulcrum be placed that the lever may be in equilibrium, and what will be the pressure against the fulcrum?

24. A straight lever, turning freely about an axis at one end, is acted on by four parallel forces, namely—

A downward force of	3 lbs.	at	1 ft.	from axis.
A downward force of	5	„	3 ft.	„
An upward force of	4	„	2 ft.	„
An upward force of	6	„	4 ft.	„

What must be the weight of the lever that it may be in equilibrium, its centre of gravity being 3 ft. from the axis?

25. In a pair of nut-crackers, the nut is placed one inch from the hinge, and the hand is applied at a distance of six inches from the hinge. How much pressure must be applied by the hand, if the nut requires a pressure of 13 lbs. to break it, and what will be the amount of the pressure on the hinges?

26. In the steelyard, if the horizontal distance between the fulcrum and the knife-edge which supports the body weighed be 3 in., and the movable weight be 7 lbs., how far must the latter be shifted for a difference of 1 lb. in the body weighed?

27. The head of a hammer weighs 20 lbs. and the handle 2 lbs. The distance between their respective centres of gravity is 24 inches. Find the distance of the centre of gravity of the hammer from that of the head.

28. One of the four triangles into which a square is divided by its diagonals is removed. Find the distance of the centre of gravity of the remainder from the intersection of the diagonals.

29. A square is divided into four equal squares and one of these is removed. Find the distance of the centre of gravity of the remaining portion from the centre of the original square.

30. Find the centre of gravity of a sphere 1 decimetre in radius, having in its interior a spherical excavation whose centre is at a distance of 5 centimetres from the centre of the large sphere and whose radius is 4 centimetres.

31. Weights P, Q, R, S are hung from the corners A, B, C, D of a uniform square plate whose weight is W Find the distances from the sides AB, AD of the point about which the plate will balance.

32. An isosceles triangle stands upon one side of a square as base, the altitude of the triangle being equal to a side of the square. Show that the distance of the centre of the whole figure from the opposite side of the square is $\frac{7}{9}$ of a side of the square.

33. A right cone stands upon one end of a right cylinder as base, the altitude of the cone being equal to the height of the cylinder. Show that the distance of the centre of the whole volume from the opposite end of the cylinder is $\frac{11}{16}$ of the height of the cylinder.

Work and Stability.

34. A body consists of three pieces, whose masses are as the numbers 1, 3, 9; and the centres of these masses are at heights of 2, 3, and 5 cm. above a certain level. Find the height of the centre of the whole mass above this level.

35. The body above-mentioned is moved into a new position, in which the heights of the centres of the three masses are 1, 3, and 7 cm. Find the new height of the centre of the whole mass.

36. Find the work done against gravity in moving the body from the first position into the second; employing as the unit of work the work done in raising the smallest of the three pieces through 1 cm.

37. Find the portions of this work done in moving each of the three pieces.

38. The dimensions of a rectangular block of stone of weight W are AB = a, AC = b, AD = c, and the edges AB, AC are initially horizontal. How much work is done against gravity in tilting the stone round the edge AB until it balances.

39. A chain of weight W and length l hangs freely by its upper end which is attached to a drum upon which the chain can be wound, the diameter of the drum being small compared with l. Compute the work done against gravity in winding up two-thirds of the chain.

40. Two equal and similar cylindrical vessels with their bases at the same level contain water to the respective heights h and H centimetres, the area of either base being a sq. cm. Find, in gramme-centimetres, the work done by gravity in equalizing the levels when the two vessels are connected.

41. Two forces acting at the ends of a rigid rod without weight equilibrate each other. Show that the equilibrium is stable if the forces are pulling outwards and unstable if they are pushing inwards.

42. Two equal weights hanging from the two ends of a string, which passes over a fixed pulley without friction, balance one another. Show that the equilibrium is neutral if the string is without weight, and is unstable if the string is heavy.

43. Show that a uniform hemisphere resting on a horizontal plane has two positions of stable equilibrium. Has it any positions of unstable equilibrium?

Inclined Plane, &c.

44. On an inclined plane whose height is $\frac{1}{8}$ of its length, what power acting parallel to the plane will sustain a weight of 112 lbs. resting on the plane without friction?

45. The height, base, and length of an inclined plane are as the numbers 3, 4, 5. What weight will be sustained on the plane without friction by a power of 100 lbs. acting (a) parallel to the base, (b) parallel to the plane?

46. Find the ratio of the power applied to the pressure produced in a screw-press without friction, the power being applied at the distance of 1 ft. from the axis of the screw, and the distance between the threads being $\frac{1}{8}$ in.

47. In the system of pulleys in which one cord passes round all the pulleys, its different portions being parallel, what power will sustain a weight of 2240 lbs. without friction, if the number of cords at the lower block be 6?

48. A balance has unequal arms, but the beam assumes the horizontal position when both scale-pans are empty. Show that if the two apparent weights of a body are observed when it is placed first in one pan and then in the other, the true weight will be found by multiplying these together and taking the square root.

Force, Mass, and Velocity.

The motion is supposed to be rectilinear.

49. A force of 1000 dynes acting on a certain mass for one second gives it a velocity of 20 cm. per sec. Find the mass in grammes.

50. A constant force acting on a mass of 12 gm. for one sec. gives it a velocity of 6 cm. per sec. Find the force in dynes.

51. A force of 490 dynes acts on a mass of 70 gm. for one sec. Find the velocity generated.

52. In the preceding example, if the time of action be increased to 5 sec., what will be the velocity generated?

In the following examples the unit of momentum referred to is the momentum of a gramme moving with a velocity of a centimetre per second.

53. What is the momentum of a mass of 15 gm. moving with a velocity of translation of 4 cm. per sec.?

54. What force, acting upon the mass for 1 sec., would produce this velocity?

55. What force, acting upon the mass for 10 sec., would produce the same velocity?

56. Find the force which, acting on an unknown mass for 12 sec., would produce a momentum of 84.

57. Two bodies initially at rest move towards each other in obedience to mutual attraction. Their masses are respectively 1 gm. and 100 gm. If the force of attraction be $\frac{1}{100}$ of a dyne, find the velocity acquired by each mass in 1 sec.

58 A gun is suspended by strings so that it can swing freely. Compare the velocity of discharge of the bullet with the velocity of recoil of the gun; the masses of the gun and bullet being given, and the mass of the powder being neglected.

59 A bullet fired vertically upwards, enters and becomes imbedded in a block of wood falling vertically overhead; and the block is brought to rest by the impact. If the velocities of the bullet and block immediately before collision were respectively 1500 and 100 ft. per sec., compare their masses.

Falling Bodies and Projectiles.

Assuming that a falling body acquires a velocity of 980 cm. per sec. by falling for 1 sec., find.—

60. The velocity acquired in $\frac{1}{10}$ of a second.

61. The distance passed over in $\frac{1}{10}$ sec.

62. The distance that a body must fall to acquire a velocity of 980 cm. per sec.

63. The time of rising to the highest point, when a body is thrown vertically upwards with a velocity of 6860 cm. per sec.

64 The height to which a body will rise, if thrown vertically upwards with a velocity of 490 cm. per sec.

65. The velocity with which a body must be thrown vertically upwards that it may rise to a height of 200 cm.

66. The velocity that a body will have after $\frac{3}{10}$ sec., if thrown vertically upwards with a velocity of 300 cm. per sec.

67. The point that the body in last question will have attained.

68. The velocity that a body will have after $2\frac{1}{2}$ secs., if thrown vertically upwards with a velocity of 800 cm. per sec.

69. The point that the body in last question will have reached.

Assuming that a falling body acquires a velocity of 32 ft. per sec. by falling for 1 sec., find:—

70. The velocity acquired in 12 sec.

71. The distance fallen in 12 sec.

72. The distance that a body must fall to acquire a velocity of 10 ft. per sec.

73. The time of rising to the highest point, when a body is thrown vertically upwards with a velocity of 160 ft. per sec.

74. The height to which a body will rise, if thrown vertically upwards with a velocity of 32 ft. per sec.

75. The velocity with which a body must be thrown vertically upwards that it may rise to a height of 25 ft.

76. The velocity that a body will have after 3 sec., if thrown vertically upwards with a velocity of 100 ft. per sec.

77. The height that the body in last question will have ascended.

78. The velocity that a body will have after $1\frac{1}{2}$ sec., if thrown vertically downwards with a velocity of 30 ft. per sec.

79. The distance that the body in last question will have described.

80. A body is thrown horizontally from the top of a tower 100 m. high with a velocity of 30 metres per sec. When and where will it strike the ground?

81. Two bodies are successively dropped from the same point, with an interval of $\frac{1}{5}$ of a second. When will the distance between them be one metre?

82. Show that if x and y are the horizontal and vertical co-ordinates of a projectile referred to the point of projection as origin, their values after time t are

$$x = Vt \cos \alpha, \; y = Vt \sin \alpha - \tfrac{1}{2} gt^2.$$

83. Show that the equation to the trajectory is

$$y = x \tan \alpha - \frac{g}{2V^2} \frac{x^2}{\cos^2 \alpha},$$

and that if V and α can be varied at pleasure, the projectile can in general be made to traverse any two given points in the same vertical plane with the point of projection.

Atwood's Machine.

Two weights are connected by a cord passing over a pulley as in Atwood's machine, friction being neglected, and also the masses of the pulley and cord; find :—

84. The acceleration when one weight is double of the other.

85. The acceleration when one weight is to the other as 20 to 21.

Taking g as 980, in terms of the cm. and sec., find :—

86. The velocity acquired in 10 sec., when one weight is to the other as 39 to 41.

87. The velocity acquired in moving through 50 cm., when the weights are as 19 to 21.

88. The distance through which the same weights must move that the velocity acquired may be double that in last question.

89. The distance through which two weights which are as 49 to 51 must move that they may acquire a velocity of 98 cm. per sec.

Energy and Work.

90. Express in ergs the kinetic energy of a mass of 50 gm. moving with a velocity of 60 cm. per sec.

91. Express in ergs the work done in raising a kilogram through a height of 1 metre, at a place where g is 981.

92. A mass of 123 gm. is at a height of 2000 cm. above a level floor. Find its energy of position estimated with respect to the floor as the standard level (g being 981).

93. A body is thrown vertically upwards at a place where g is 980. If the velocity of projection is 9800 cm. per sec. and the mass of the body is 22 gm., find the energy of the body's motion when it has ascended half way to its maximum height. Also find the work done against gravity in this part of the ascent.

94. The height of an inclined plane is 12 cm., and the length 24 cm. Find the work done by gravity upon a mass of 1 gm. in sliding down this plane (g being 980), and the velocity with which the body will reach the bottom if there be no friction.

95. If the plane in last question be not frictionless, and the velocity on reaching the bottom be 20 cm. per sec., find how much energy is consumed in friction.

96. Find the work expended in discharging a bullet whose mass is 30 gm. with a velocity of 40,000 cm. per sec.; and the number of such bullets that will be discharged with this velocity in a minute if the rate of working is 7460 million ergs per sec. (one horse-power).

97. One horse-power being defined as 550 foot-pounds per sec.; show that it is nearly equivalent to 8·8 cubic ft. of water lifted 1 ft. high per sec. (A cubic foot of water weighs $62\frac{1}{2}$ lbs. nearly. A foot-pound is the work done against gravity in lifting a pound through a height of 1 ft.)

98. How many cubic feet of water will be raised in one hour from a mine 200 ft. deep, if the rate of pumping be 15 horse-power?

Centrifugal Force.

99. What must be the radius of curvature, that the centrifugal force of a body travelling at 30 miles an hour may be one-tenth of the weight of the body; g being 981, and a mile an hour being 44·7 cm. per sec.?

100. A heavy particle moves freely along a frictionless tube which forms a vertical circle of radius a. Find the velocity which the particle will have at the lowest point, if it all but comes to rest at the highest. Also find its velocity at the lowest point if in passing the highest point it exerts no pressure against the tube. [Use the principle that what is lost in energy of position is gained in energy of motion.]

101. Show that the total intensity of centrifugal force due to the earth's rotation, at a place in latitude λ, is ω^2 R cos λ, ω denoting $\frac{2\pi}{T}$, and R the earth's radius; that the vertical component (tending to diminish gravity) is ω^2 R $\cos^2 \lambda$, and that the horizontal component (directed from the pole towards the equator) is ω^2 R cos λ sin λ.

Pendulum, and Moment of Inertia.

101*. The length of the seconds pendulum at Greenwich is 99·413 cm.; find the length of a pendulum which makes a single vibration in $1\frac{1}{2}$ sec.

102. The weight of a fly-wheel is M grammes, and the distance of the inside of the rim from the axis of revolution is R centims. Supposing this distance to be identical with k (§ 117), find the moment of inertia.

If a force of F dynes acts steadily upon the wheel at an arm of a centims., what will be the value of the angular velocity $\frac{2\pi}{T}$ after the lapse of t seconds from the commencement of motion?

103. For a uniform thin rod of length a, swinging about a point of suspension at one end, the moment of inertia is the mass of the rod multiplied by $\frac{1}{3}a^2$. Find the length of the equivalent simple pendulum; also the moment of inertia round a parallel axis through the centre of the rod.

104. At what point in its length must the rod in last question be suspended to give a minimum time of vibration: and at what point must it be suspended to give the same time of vibration as if suspended at one end?

105. Show that if P be the mass of the pulley in Atwood's machine, r its radius, and Pk^2 its moment of inertia, the value of C in § 100 will be $P\frac{k^2}{r^2}$ *plus* the mass of the string. [The mass of the friction-wheels is neglected.]

106. A body moves with constant velocity in a vertical circle, going once round per second; and its shadow is cast upon level ground by a vertical sun. Find the value of μ (§ 111) for the shadow, using the centimetre and second as units.

107. What is the value of μ for one of the prongs of a C tuning-fork which makes 512 complete vibrations per second?

Pressure of Liquids.

Find, in gravitation measure (grammes per sq. cm.), atmospheric pressure being neglected:—

108. The pressure at the depth of a kilometre in sea-water of density 1·025.

109. The pressure at the depth of 65 cm. in mercury of density 13·59.

110. The pressure at the depth of 2 cm. in mercury of density 13·59 surmounted by 3 cm. of water of unit density, and this again by $1\frac{1}{2}$ cm. of oil of density ·9.

Find, in centimetres of mercury of density 13·6, atmospheric pressure being included, and the barometer being supposed to stand at 76 cm.:—

111. The pressure at the depth of 10 metres in water of unit density.

112. The pressure at the depth of a mile in sea-water of density 1·026, a mile being 160933 cm.

Find, in dynes per square centimetre, taking g as 981:—

113. The pressure due to 1 cm. of mercury of density 13·596.

114. The pressure due to a foot of water of unit density, a foot being 30·48 cm.

115. The pressure due to the weight of a layer a metre thick, of air of density ·00129.

116. At what depth, in brine of density 1·1, is the pressure the same as at a depth of 33 feet in water of unit density?

117. At what depth, in oil of density ·9, is the pressure the same as at the depth of 10 inches in mercury of density 13·596?

118. With what value of g will the pressure of 3 cm. of mercury of density 13·596 be 4×10^4?

Find, in grammes weight, the amount of pressure (atmospheric pressure being neglected):—

119. On a triangular area of 9 sq. cm. immersed in naphtha of density ·848; the centre of gravity of the triangle being at the depth of 6 cm.

120. On a rectangular area 12 cm. long, and 9 cm broad, immersed in mercury of density 13·596; its highest and lowest corners being at depths of 3 cm. and 7 cm. respectively.

121. On a circular area of 10 cm. radius, immersed in alcohol of density ·791, the centre of the circle being at the depth of 4 cm.

122. On a triangle whose base is 5 cm. and altitude 6 cm., the base being at the uniform depth of 9 cm, and the vertex at the depth of 7 cm., in water of unit density.

123. On a sphere of radius r centimetres, completely immersed in a liquid of density d; the centre of the sphere being at the depth of h centimetres. [The amount of pressure in this case is not the resultant pressure.]

Density, and Principle of Archimedes.

Densities are to be expressed in grammes per cubic centimetre.

124. A rectangular block of stone measures $86 \times 37 \times 16$ cm., and weighs 120 kilogrammes. Find its density.

125. A specific-gravity bottle holds 100 gm. of water, and 180 gm. of sulphuric acid. Find the density of the acid.

126. A certain volume of mercury of density 13·6 weighs 216 gm., and the same volume of another liquid weighs 14·8 gm. Find the density of this liquid.

127. Find the mean section of a tube 16 cm. long, which holds 1 gm. of mercury of density 13·6.

128. A bottle filled with water, weighs 212 gm. Fifty grammes of filings are thrown in, and the water which flows over is removed, still leaving the bottle just filled. The bottle then weighs 254 gm. Find the density of the filings.

129. Find the density of a body which weighs 58 gm. in air, and 46 gm. in water of unit density.

130. Find the density of a body which weighs 63 gm. in air, and 35 gm. in a liquid of density ·85.

131. A glass ball loses 33 gm. when weighed in water, and loses 6 gm. more when weighed in a saline solution. Find the density of the solution.

132. A body, lighter than water, weighs 102 gm. in air; and when it is immersed in water by the aid of a sinker, the joint weight is 23 gm. The sinker alone weighs 50 gm. in water. Find the density of the body.

133. A piece of iron, when plunged in a vessel full of water, makes 10 grammes run over. When placed in a vessel full of mercury it floats, displacing 78 grammes of mercury. Required the weight, volume, and specific gravity of the iron.

134. Find the volume of a solid which weighs 357 gm. in air, and 253 gm. in water of unit density.

135. Find the volume of a solid which weighs 458 gm. in air, and 409 gm. in brine of density 1·2.

136. How much weight will a body whose volume is 47 cubic cm. lose, by weighing in a liquid whose density is 2·5?

137. Find the weights in air, in water, and in mercury, of a cubic cm. of gold of density 19·3.

138. A wire 1293 cm. long loses 508 gm. by weighing in water. Find its mean section, and mean radius.

139. A copper wire 2156 cm. long weighs 158 gm. in air, and 140 gm. in water. Find its volume, density, mean section, and mean radius.

140. What will be the weights, in air and in water, of an iron wire 1000 cm. long and a millimetre in diameter, its density being 7·7?

141. How much water will be displaced by 1000 c.c. of oak of density ·9, floating in equilibrium?

142. A ball, of density 20 and volume 3 c.c., is surmounted by a cylindrical stem, of density 2·5, of length 12 cm., and of cross section $\frac{1}{3}$ sq. cm. What length of the stem will be in air when the body floats in equilibrium in mercury of density 13·6?

143. A hollow closed cylinder, of mean density ·4 (including the hollow space), is weighted with a ball of volume 5, and mean density 2. What must be the volume of the cylinder, that exactly half of it may be immersed, when the body is left to itself in water?

144. A long cylindrical tube, constructed of flint glass of density 3, is closed at both ends, and is found to have the property of remaining at whatever depth it is placed in water. If the mass of the ends can be neglected, show that the ratio of the internal to the external radius is $\sqrt{\frac{2}{3}}$.

145. A glass bottle provided with a stopper of the same material weighs 120 gm. when empty. When it is immersed in water, its apparent weight is 10 gm., but when the stopper is loosened and the water let in, its apparent weight is 80 gm. Find the density of the glass and the capacity of the bottle.

146. A hydrometer sinks to a certain depth in a fluid of density ·8; and if 100 gm. be placed upon it, it sinks to the same depth in water. Find the weight of the hydrometer.

147. Find the mean density of a combination of 8 parts by volume of a substance of density 7, with 19 of a substance of density 3.

148. Find the mean density of a combination of 8 parts by weight of a substance of density 7, with 19 of a substance of density 3.

149. What volume of fir, of density ·5, must be joined to 3 c.c. of iron, of density 7·1, that the mean density of the whole may be unity?

150. What mass of fir, of density ·5, must be joined to 300 gm. of iron, of density 7·1, that the mean density of the whole may be unity?

151. Two parts by volume of a liquid of density ·8, are mixed with 7 of water, and the mixture shrinks in the ratio of 21 to 20. Find its density.

152. A piece of iron of density 7·5 floats in mercury of density 13·5, and is completely covered with water which rests on the top of the mercury. How much of the iron is immersed in the mercury?

153. Two liquids are mixed. The total volume is 3 litres, with a sp. gr. of 0·9. The sp. gr. of the first liquid is 1·3, of the second 0·7. Find their volumes.

154. What volume of platinum of density 21·5 must be attached to a litre of iron of density 7·5 that the system may float freely at all depths in mercury of density 13·5?

155. What must be the thickness of a hollow sphere of platinum with an external radius of 1 decim., that it may barely float in water?

156. A sphere of cork of density ·24, 3 cm. in radius, is weighted with a sphere of gold of density 19·3. What must be the radius of the latter that the system may barely float in alcohol of density ·8?

157. An alloy of gold and silver has density D. The density of gold is d, that of silver d'. Find the proportions by weight of the two metals in the alloy, supposing that neither expansion nor contraction occurs in its formation.

158. A mixture of gold, of density 19·3, with silver, of density 10·5, has the density 18. Assuming that the volume of the alloy is the sum of the volumes of its components, find how many parts of gold it contains for one of silver—(*a*) by volume; (*b*) by weight.

159. A body weighs gM dynes in air of density A, gm in water, and gx in vacuo. Find x in terms of M, m, and A.

Capillarity.

160. A horizontal disc of glass is held up by means of a film of water between it and a similar disc of the same or a larger size above it.

If R denote the radius of the lower disc,

d the distance between the discs, which is very small compared with R,

T the surface tension of water,

show that the weight of the lower disc together with that of the water between the discs is approximately equal to $\frac{2T\pi R^2}{d}$.

[The disc of water will be concave at the edge, and the radius of curvature of the concavity may be taken as $\frac{1}{2}d$.]

161. The surface-tension of water at 20° C. is 81 dynes per linear centim. How high will water be elevated by capillary action in a wetted tube whose diameter is half a millimetre?

162. How much will mercury be depressed by capillary action in a glass tube of half a millimetre diameter, the surface-tension of mercury at 20° C. being 418 dynes per cm., its density 13·54, and the cosine of the angle of contact ·703?

163. Show by the method of § 186 that the capillary elevation or depression will be the same in a square tube as in a circular tube whose diameter is equal to a side of the square.

164. Two equal discs in a vertical position have a film of water between them sustained by capillary action. Show that if the water at the lowest point is at atmospheric pressure, the water at the centre of the discs is at a pressure less than atmospheric by rg dynes per sq. cm., r being the common radius of the discs in cm.; and that the discs are pressed together with a force of $\pi r^3 g$ dynes.

Barometer, and Boyle's Law.

165. A bent tube, having one end open and the other closed, contains mercury which stands 20 cm. higher in the open than in the closed branch. Compare the pressure of the air in the closed branch with that of the external air; the barometer at the time standing at 75 cm.

166. The cross sections of the open and closed branches of a siphon barometer are as 6 to 1. What distance will the mercury move in the closed branch, when a normal barometer alters its reading by 1 inch?

167. If the section of the closed limb of a siphon barometer is to that of the open limb as a to b, show that a rise of 1 cm. in the mercury in the closed limb corresponds to a rise of $\frac{a+b}{b}$ cm. of the theoretical barometer.

168. Compute, in dynes per sq. cm., the pressure due to the weight of a column of mercury 76 cm. high at the equator, where g is 978, and at the pole, where g is 983.

169. The volumes of a given quantity of mercury at 0° C. and 100° C. are as 1 to 1·0182. Compute the height of a column of mercury at 100°, which will produce the same pressure as 76 cm. of mercury at 0°.

170. The volumes of a given mass of mercury, at 0° and 20°, are as 1 to 1·0036. Find the height reduced to 0°, when the actual height (in true centimetres), at a temperature of 20°, is 76·2.

171. In performing the Torricellian experiment a little air is left above the mercury. If this air expands a thousandfold, what difference will it make in the height of the column of mercury sustained when a normal barometer reads 76 cm.?

172. In performing the Torricellian experiment, an inch in length of the tube is occupied with air at atmospheric pressure, before the tube is inverted. After the inversion, this air expands till it occupies 15 inches, while a column of mercury 28 inches high is sustained below it. Find the true barometric height.

173. The mercury stands at the same level in the open and in the closed branch of a bent tube of uniform section, when the air confined at the closed end is at the pressure of 30 inches of mercury, which is the same as the pressure of the external air. Express, in atmospheres, the pressure which, acting on the surface of the mercury in the open branch, compresses the confined air to half its original

volume, and at the same time maintains a difference of 5 inches in the levels of the two mercurial columns.

174. At what pressure (expressed in atmospheres) will common air have the same density which hydrogen has at one atmosphere; their densities when compared at the same pressure being as 1276 to 88·4?

175. Two volumes of oxygen, of density ·00141, are mixed with three of nitrogen, of density ·00124. Find the density of the mixture—(*a*) if it occupies five volumes; (*b*) if it is reduced to four volumes.

176. The mass of a cub. cm. of air, at the temperature 0° C., and at the pressure of a million dynes to the square cm., is ·0012759 gramme. Find the mass of a cubic cm. of air at 0° C., under the pressure of 76 cm. of mercury—(*a*) at the pole, where g is 983·1; (*b*) at the equator, where g is 978·1; (*c*) at a place where g is 981.

177. Show that the density of air at a given temperature, and under the pressure of a given column of mercury, is greater at the pole than at the equator by about 1 part in 196; and that the gravitating force of a given volume of it is greater at the pole than at the equator by about 1 part in 98.

178. A cylindrical test-tube, 1 decim. long, is plunged, mouth downwards, into mercury. How deep must it be plunged that the volume of the inclosed air may be diminished by one-half?

179. The pressure indicated by a siphon barometer whose vacuum is defective is 750 mm., and when mercury is poured into the open branch till the barometric chamber is reduced to half its former volume, the pressure indicated is 740 mm. Deduce the true pressure.

180. An open manometer, formed of a bent tube of iron whose two branches are parallel and vertical, and of a glass tube of larger size, contains mercury at the same level in both branches, this level being higher than the junction of the iron with the glass tube. What must be the ratio of the sections of the two tubes, that the mercury may ascend half a metre in the glass tube when a pressure of 6 atmospheres is exerted in the opposite branch?

181. A curved tube has two vertical legs, one having a section of 1 sq. cm., the other of 10 sq. cm. Water is poured in, and stands at the same height in both legs. A piston, weighing 5 kilogrammes, is then allowed to descend, and press with its own weight upon the surface of the liquid in the larger leg. Find the elevation thus produced in the surface of the liquid in the smaller leg.

Pumps, &c.

182. The sectional area of the small plunger in a Bramah press is 1 sq. cm., and that of the larger 100 sq. cm. The lever handle gives a mechanical advantage of 6. What weight will the large plunger sustain when 1 cwt. is hung from the handle?

183. The diameter of the small plunger is half an inch; that of the larger 1 foot. The arms of the lever handle are 3 in. and 2 ft. Find the total mechanical advantage.

184. Find, in grammes weight, the force required to sustain the piston of a suction-pump without friction, if the radius of the piston be 15 cm., the depth

from it to the surface of the water in the well 600 cm., and the height of the column of water above it 50 cm. Show that the answer does not depend on the size of the pipe which leads down to the well.

185. Two vessels of water are connected by a siphon. A certain point P in its interior is 10 cm. and 30 cm. respectively above the levels of the liquid in the two vessels. The pressure of the atmosphere is 1000 grammes weight per sq. cm. Find the pressure which will exist at P—(a) if the end which dips in the upper vessel be plugged; (b) if the end which dips in the lower vessel be plugged.

186. If the receiver has double the volume of the barrel, find the density of the air remaining after 10 strokes, neglecting leakage, &c.

187. Air is forced into a vessel by a compression pump whose barrel has $\frac{1}{10}$th of the volume of the vessel. Compute the density of the air in the vessel after 20 strokes.

188. In the pump of Fig. 136 show that the excess of the pressure on the upper above that on the lower side of the piston, at the end of the first up-stroke, is $\frac{V}{V+V'}$ of an atmosphere [in the notation of § 230]; and hence that the first stroke is more laborious with a small than with a large receiver.

189. In Tate's pump show that the pressure to be overcome in the first stroke is nearly equal to an atmosphere during the greater part of the stroke; and that, when half the air has been expelled from the receiver, the pressure to be overcome varies, in different parts of the stroke, from half an atmosphere to an atmosphere.

ANSWERS TO EXAMPLES.

Ex. 1. 14·14. Ex. 2. 13. Ex. 3. 7·07 each. Ex. 4. 10. Ex. 5. 141·4. Ex. 6. 70·7 each. Ex. 7. Introduce a force equal and opposite to the resultant. Then we have three forces making angles of 120° with each other. Ex. 9. Equal to one of the forces.

Ex. 10*. 28. Ex. 11*. 40 in. from smaller weight. Ex. 12. 60 lbs. by A, 40 lbs. by B. Ex. 13. $2\frac{6}{7}$ lbs. Ex. 14. 2 lbs. Ex. 15. 15 in. from centre. Ex. 16. $12\frac{1}{2}$ lbs. Ex. 17. 32 lbs. Ex. 18. 10·4 ft. nearly. Ex. 19. 6 lbs. Ex. 20. $2\frac{6}{7}$ ft. from end. Ex. 21. $2\frac{13}{16}$. Ex. 22. 2 units acting at distance of 5 yards from the greater force. Ex. 23. 6 ft. from the end; pressure 2 units. Ex. 24. $4\frac{2}{3}$ lbs. Ex. 25. $2\frac{1}{6}$ lbs., $10\frac{5}{6}$ lbs. Ex. 26. $\frac{3}{7}$ in. Ex. 27. $2\frac{2}{11}$ in. Ex. 28. $\frac{1}{9}$ of side of square. Ex. 29. $\frac{1}{12}$ of diagonal of large square. Ex. 30. $\frac{40}{117}$ cm. from centre of large sphere. Ex. 31. Denoting side of square by a, distance from AB is $\frac{\frac{1}{2}W + R + S}{W + P + Q + R + S}\,a$, distance from AD is $\frac{\frac{1}{2}W + Q + R}{W + P + Q + R}\,a$.

Ex. 34. $4\frac{4}{13}$ cm. Ex. 35. $5\frac{8}{13}$ cm. Ex. 36. 17. Ex. 37. $-1, 0, +18$. Ex. 38. $\frac{1}{2}W\left(\sqrt{(b^2+c^2)}-c\right)$. Ex. 39. $\frac{4}{9}Wl$. Ex. 40. $\frac{a}{4}(H-h)^2$.

Ex. 44. 14 lbs. Ex. 45. (*a*) $133\frac{1}{3}$ lbs.; (*b*) $166\frac{2}{3}$ lbs. Ex. 46. 1 to 603 nearly. Ex. 47. $373\frac{1}{3}$.

Ex. 49. 50. Ex. 50. 72. Ex. 51. 7 cm. per sec. Ex. 52. 35. Ex. 53. 60. Ex. 54. 60 dynes. Ex. 55. 6 dynes. Ex. 56. 7 dynes. Ex. 57. Smaller mass $\frac{1}{100}$, larger $\frac{1}{10000}$ cm. per sec. Ex. 58. Inversely as masses of bullet and gun. Ex. 59. Mass of bullet is $\frac{1}{15}$ of mass of block.

Ex. 60. 98 cm. per sec. Ex. 61. 4·9 cm. Ex. 62. 490 cm. Ex. 63. 7 sec. Ex. 64. $122\frac{1}{2}$ cm. Ex. 65. 626 cm. per sec. Ex. 66. 6 cm. per sec. upwards. Ex. 67. 45·9 cm. above point of projection. Ex. 68. 1650 cm. per sec. downwards. Ex. 69. $1062\frac{1}{2}$ cm. below starting point. Ex. 70. 384 ft. per sec. Ex. 71. 2304 ft. Ex. 72. $1\frac{9}{16}$ ft. Ex. 73. 5 sec. Ex. 74. 16 ft. Ex. 75. 40 ft. per sec. Ex. 76. 4 ft. per sec. upwards. Ex. 77. 156 ft. Ex. 78. 78 ft. per sec. Ex. 79. 81 ft. Ex. 80. After 4·52 sec. At 135·6 m. from tower. Ex. 81. After ·41 sec. from dropping of second body.

Ex. 84. $\frac{1}{3}$ *g*. Ex. 85. $\frac{1}{41}$ *g*. Ex. 86. 245 cm. per sec. Ex. 87. 70 cm. per sec. Ex. 88. 200 cm. Ex. 89. 245 cm.

Ex. 90. 90,000 ergs. Ex. 91. 98,100,000 ergs. Ex. 92. 241,326,000 ergs. Ex. 93. 528,220,000 ergs each. Ex. 94. 11,760 ergs; $\sqrt{23520} = 153{\cdot}4$ cm. per sec. Ex. 95. 11,560 ergs. Ex. 96. 24×10^9 ergs in each discharge. Not quite 19 discharges per min. Ex. 98. 2376 nearly.

Ex. 99. 18330 cm. or about 600 ft. Ex. 100. $2\sqrt{ga}$, $\sqrt{5ga}$.

Ex. 101*. 223·679 cm. Ex. 102. MR^2, $\frac{Fat}{MR^2}$. Ex. 103. $\frac{2}{3}a$; mass of rod multiplied by $\frac{1}{12}a^2$. Ex. 104. At either of the two points distant $\frac{a}{2\sqrt{3}}$ from centre; at either of the two points distant $\frac{a}{6}$ from centre. Ex. 106. $(2\pi)^2 = 39{\cdot}48$. Ex. 107. $(1024\pi)^2 = 10350000$.

Ex. 108. 102500. Ex. 109. 883·35. Ex. 110. 31·53. Ex. 111. 149·5. Ex. 112. 12217. Ex. 113. 13338. Ex. 114. 29901. Ex. 115. 126·5. Ex. 116. 30. Ex. 117. 12 ft. 7 in. Ex. 118. 980·68. Ex. 119. 45·79. Ex. 120. 7342. Ex. 121. 994. Ex. 122. 125. Ex. 123. $4\pi r^2 hd$.

Ex. 124. 2·357. Ex. 125. 1·8. Ex. 126. ·932. Ex. 127. ·0046 sq. cm. Ex. 128. 6·25. Ex. 129. $4\frac{5}{8}$. Ex. 130. 1·9125. Ex. 131. $1\frac{2}{11}$. Ex. 132. $\frac{34}{43}$. Ex. 133. 10 cub. cm., 78 gm., 7·8. Ex. 134. 104. Ex. 135. $40{\cdot}8\dot{3}$. Ex. 136. 117·5. Ex. 137. 19·3, 18·3, 5·7. Ex. 138. ·393 sq. cm., ·354 cm. Ex. 139. 18, $8{\cdot}77\dot{7}$, ·00835 sq. cm., ·0516 cm. Ex. 140. 60·48, 52·62. Ex. 141. 900 c.c. Ex. 142. 5·56 cm. Ex. 143. 50 c.c. Ex. 145. 3, 70 c.c. Ex. 146. 400 gm. Ex. 147. $4\frac{5}{27} = 4{\cdot}185$. Ex. 148. $3\frac{96}{157} = 3{\cdot}6115$. Ex. 149. 36·6 c.c. Ex. 150. 257·7 gm. Ex. 151. 1·0033. Ex. 152. $\frac{18}{25}$ of the iron. Ex. 153. 1 lit. of first, 2 lit. of second. Ex. 154. $\frac{3}{4}$ of a litre. Ex. 155. $1 - \sqrt[3]{\frac{41}{43}}$ decim. $=$ ·158 cm. Ex. 156. $\sqrt[3]{\frac{15{\cdot}12}{18{\cdot}5}} =$ ·935 cm. Ex. 157. Gold : silver : : $\frac{1}{d'} - \frac{1}{D} : \frac{1}{D} - \frac{1}{d}$. Ex. 158. (*a*) 5·77, (*b*) 10·6. Ex. 159. $\frac{M - mA}{1 - A}$.

Ex. 161. 6·6 cm. nearly. Ex. 162. 1·77 cm.

Ex. 165. $\frac{19}{15}$. Ex. 166. $\frac{6}{7}$ in. Ex. 168. 1010564, 1015730. Ex. 169. 77·3832. Ex. 170. 75·93. Ex. 171. ·076. Ex. 172. 30 in. Ex. 173. $2\frac{1}{6}$. Ex. 174. ·0693. Ex. 175. (*a*) ·001308, (*b*) ·001635. Ex. 176. (*a*) ·0012961, (*b*) ·0012895, (*c*) ·0012933. Ex. 177. d varies as g, and therefore gd varies as g^2. Ex. 178. Its top must be $76-5=71$ cm. deep. Ex. 179. 760 m. Ex. 180. 33 to 5. Ex. 181. $454\frac{6}{11}$ cm.

Ex. 182. 30 tons. Ex. 183. 4608. Ex. 184. 459500 nearly. Ex. 185. (*a*) 970. (*b*) 990 gm. wt. per sq. cm. Ex. 186. $\frac{1}{58}$ of an atmosphere, nearly. Ex. 187. 3 atmospheres.

PART II.—HEAT.

HEAT.

CHAPTER I.

THERMOMETRY.

1. Heat—Cold.—The words *heat* and *cold* express sensations so well known as to need no explanation; but these sensations are modified by subjective causes, and do not furnish an invariable criterion of objective reality. In fact, we may often see one person suffer from heat while another complains of cold. Even for the same person the sensations of heat and cold are comparative. A temperature of 50° Fahr. suddenly occurring amid the heat of summer produces a very decided sensation of cold, whereas the same temperature in winter has exactly the opposite effect. We may mention an old experiment upon this subject, which is at once simple and instructive. If we plunge one hand into water at 32° Fahr., and the other into water at about 100°; and if after having left them some time in this position we immerse them simultaneously in water at 70°, they will experience very different sensations. The hand which was formerly in the cold water now experiences a sensation of heat; that which was in the hot water experiences a sensation of cold, though both are in the same medium. This plainly shows that the sensations of heat and cold are modified by the condition of the observer, and consequently cannot serve as a sure guide in the study of calorific phenomena. Recourse must therefore be had to some more constant standard of reference, and such a standard is furnished by the thermometer.

2. Temperature.—If several bodies heated to different degrees are placed in presence of each other, an interchange of heat takes place between them, by which they undergo modifications of opposite kinds; those that are hottest grow cooler, and those that are coldest grow warmer; and after a longer or shorter time these inverse phenomena cease to take place, and the bodies come to a state of mutual

equilibrium. They are then said to be at the same *temperature.* If a source of heat is now brought to act upon them, their temperature is said to *rise;* if they are left to themselves in a colder medium, they all grow cold, and their temperature is said to *fall.* *Two bodies are said to have the same temperature if when they are placed in contact no heat passes from the one to the other.* If when two bodies are placed in contact heat passes from one to the other, that which gives heat to the other is said to have the higher temperature. Heat always tends to pass from bodies of higher to those of lower temperature.

3. Expansion.—At the same time that bodies undergo these changes in temperature, which may be verified by the different impressions which they make upon our organs, they are subjected to other modifications which admit of direct measurement, and which serve as a means of estimating the changes of temperature themselves. These modifications are of different kinds, and we shall have occasion to speak of them all in the course of this work; but that which is especially used as the basis of thermometric measurement is change of volume. In general, when a body is heated, it increases in volume; and, on the other hand, when it is cooled its volume diminishes. The expansion of bodies under the action of heat may be illustrated by the following experiments.

1. *Solid Bodies.*—We take a ring through which a metal sphere

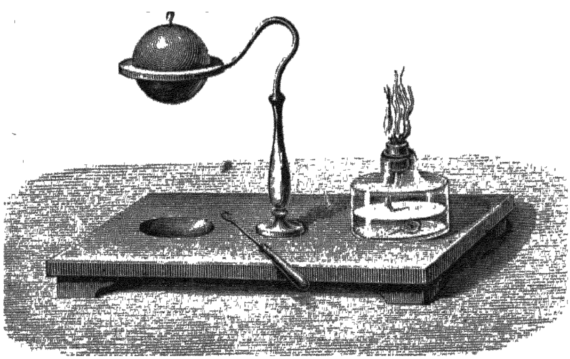


Fig. 1.—Gravesande's Ring.

just passes. This latter is heated by holding it over a spirit-lamp, and it is found that after this operation it will no longer pass through the ring. Its volume has increased. If it is now cooled by immersion in water, it resumes its former volume, and will again pass

through the ring. If, while the sphere was hot, we had heated the ring to about the same degree, the ball would still have been able to pass, their relative dimensions being unaltered. This little apparatus is called *Gravesande's Ring.*

2. *Liquids.*—A liquid, as water for instance, is introduced into the

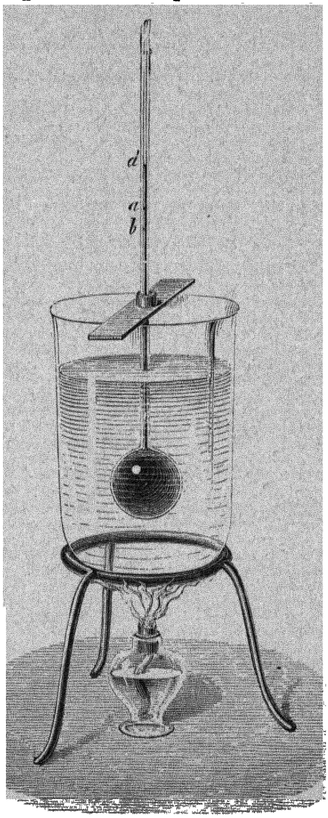


Fig. 2.—Expansion of Liquids.

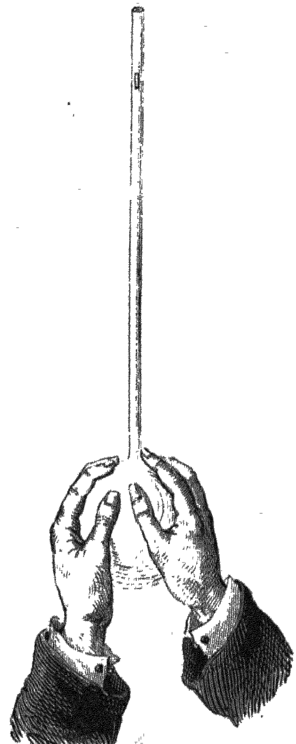


Fig. 3.—Expansion of Gases.

apparatus shown in Fig. 2, so as to fill at once the globe and a portion of the tube as far as a. The instrument is then immersed in a vessel containing hot water, and at first the extremity of the liquid column descends for an instant to b; but when the experiment has continued for some time, the liquid rises to a point a' at a considerable height above. This twofold phenomenon is easily explained. The globe, which receives the first impression of heat, increases in volume before any sensible change can take place in the temperature of the liquid. The liquid consequently is unable to fill the entire

capacity of the globe and tube up to the original mark, and thus the extremity of the liquid column is seen to fall. But the liquid, receiving in its turn the impression of heat, expands also, and as it passes the original mark, we may conclude that it not only expands, but expands more than the vessel which contains it.

3. *Gases.*—The globe in Fig. 3 contains air, which is separated from the external air by a small liquid index. We have only to warm the globe with the hands and the index will be seen to be pushed quickly upwards, thus showing that gases are exceedingly expansible.

4. General Idea of the Thermometer.—Since the volume of a body is changed by heat, we may specify its temperature by stating its volume. And the body will not only indicate its own temperature by this means, it will also exhibit the temperature of the bodies by which it is surrounded, and which are in equilibrium of temperature with it. Any body which gives quantitative indications of temperature may be called a thermometer.

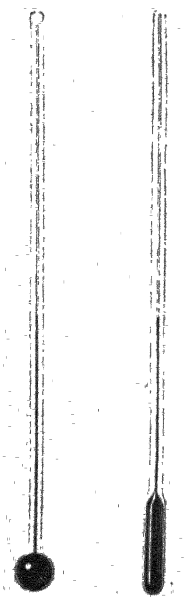


Fig. 4.—Mercurial Thermometers.

5. Choice of the Thermometric Substance.—As the expansions of different substances are not exactly proportional to one another, it is necessary to select some one substance or combination of substances to furnish a standard; and the standard usually adopted is the apparent expansion of mercury in a graduated glass vessel. The instrument which exhibits this expansion is called the mercurial thermometer. It consists essentially, as shown in Fig. 4, of a tube of very small diameter, called the *stem*, terminating in a reservoir which, whatever its shape, is usually called the *bulb*. The reservoir and a portion of the tube are filled with mercury. If the temperature varies, the level of the liquid will rise or fall in the tube, and the points at which it stands can be identified by means of a scale attached to or engraved on the tube.

6. Construction of the Mercurial Thermometer.—The construction of an accurate mercurial thermometer is an operation of great delicacy, and comprises the following processes.

1. *Choice of the Tube.*—The first object is to procure a tube of as uniform bore as possible. In order to test the uniformity of the

bore, a small column of mercury is introduced into the tube, and the length which it occupies in different parts of the tube is measured. If these lengths are not equal, the tube is not of uniform bore. When

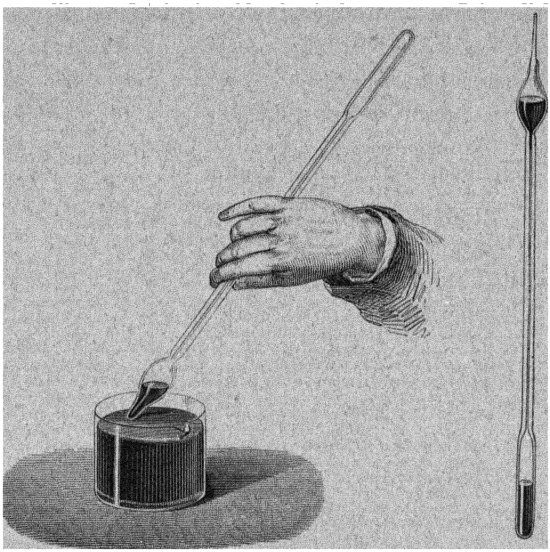


Fig. 5.—Introduction of the Mercury.

a thermometer of great precision is required, the tube is *calibrated;* that is, divided into parts of equal volume, by marking upon it the lengths occupied by the column in its different positions.

When a suitable tube has been obtained, a reservoir is either blown

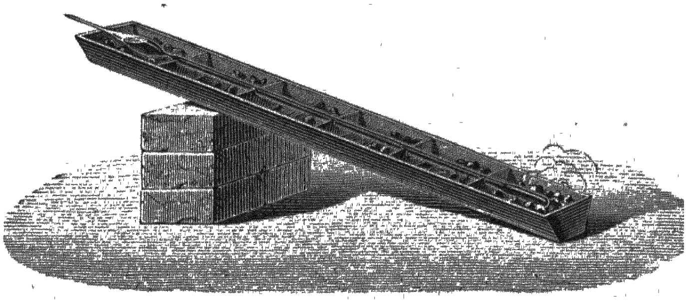


Fig. 6.—Furnace for heating Thermometers.

at one end or attached by melting, the former plan being usually preferable.

2. *Introduction of the Mercury.*—At the upper end of the tube a temporary bulb is blown, and drawn out to a point, at which there is

a small opening. This bulb, and also the permanent bulb, are gently heated, and the point is then immersed in a vessel containing mercury (Fig. 5). The air within the instrument, growing cold, diminishes in expansive force, so that a quantity of mercury is forced into the temporary bulb by the pressure of the atmosphere. The instrument is then set upright, and by alternate heating and cooling of the permanent bulb, a large portion of the mercury is caused to descend into it from the bulb at the top. The instrument is then laid in a sloping position on a special furnace (Fig. 6) till the mercury boils. The vapour of the boiling mercury drives out the air, and when the mercury cools it forms a continuous column, filling the permanent bulb and tube. If any bubbles of air are seen, the operation of boiling and cooling is repeated until they are expelled.

3. *Determination of the Fixed Points.*—The instrument, under these conditions, and with any scale of equal parts marked on the tube, would of course indicate variations of temperature, but these indications would be arbitrary, and two thermometers so constructed would in general give different indications.

In order to insure that the indications of different thermometers may be identical, it has been agreed to adopt two standard temperatures, which can easily be reproduced and maintained for a considerable time, and to denote them by fixed numbers. These two temperatures are the freezing-point and boiling-point of water; or to speak more strictly, the temperature of melting ice, and the temperature of the steam given off by water boiling under average atmospheric pressure. It has been observed that if the thermometer be surrounded with melting ice (or melting snow), the mercury, under whatever circumstances the experiment is performed, invariably stops at the same point, and remains stationary there as long as the melting continues. This then is a fixed temperature. On the Centigrade scale it is called zero, on Fahrenheit's scale 32°.

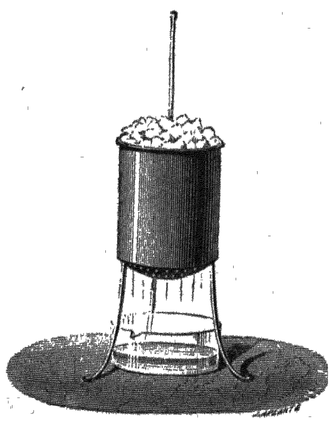


Fig. 7.—Determination of Freezing-point.

In order to mark this point on a thermometer, it is surrounded by melting ice, which is contained in a perforated vessel, so as to allow

the water produced by melting to escape. When the level of the mercury ceases to vary, a mark is made on the tube with a fine diamond at the extremity of the mercurial column. This is frequently called for brevity the *freezing-point.*

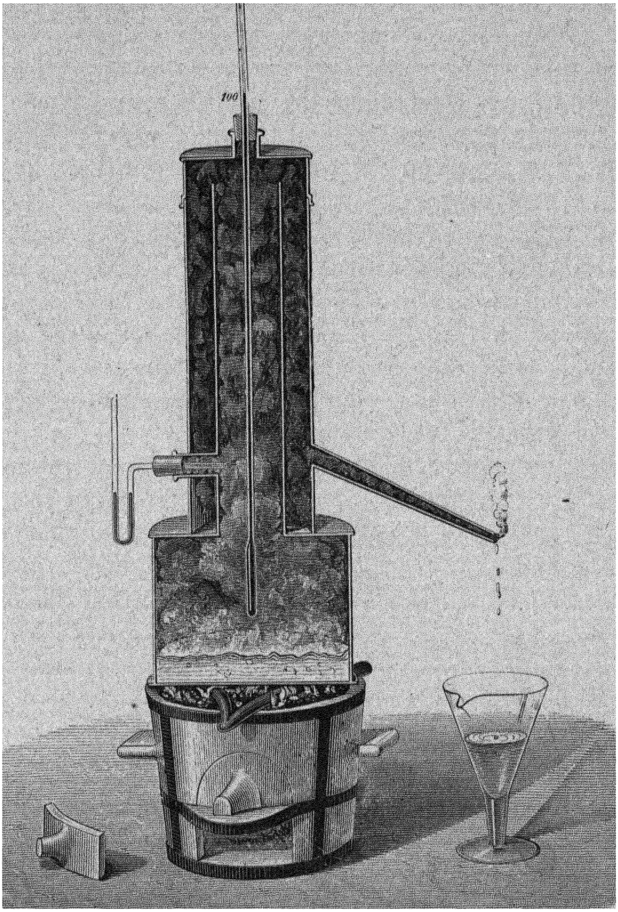


Fig. 8. Determination of Boiling point

It has also been observed that if water be made to boil in an open metallic vessel, under average atmospheric pressure (76 centimetres, or 29·922 inches), and if the thermometer be plunged into the steam, the mercury stands at the same point during the entire time of ebullition, provided that the external pressure does not change. This second fixed temperature is called 100° in the Centigrade scale (whence

the name), and 212° on Fahrenheit's scale. In order to mark this second point on the thermometer, an apparatus is employed which was devised by Gay-Lussac, and perfected by Regnault. It consists of a copper boiler (Fig. 8) containing water which is raised to ebullition by means of a furnace. The steam circulates through a double casing, and escapes by a tube near the bottom. The thermometer is fixed in the interior casing, and when the mercury has become stationary, a mark is made at the point at which it stops, which denotes what is commonly called for brevity the *boiling-point.*

A small manometric tube, open at both ends, serves to show, by the equality of level of the mercury in its two branches, that the ebullition is taking place at a pressure equal to that which prevails externally, and consequently that the steam is escaping with sufficient freedom. It frequently happens that the external pressure is not exactly 760 millimetres, in which case the boiling-point should be placed a little above or a little below the point at which the mercury remains stationary, according as the pressure is less or greater than this standard pressure. When the difference on either side is inconsiderable, the position of the boiling-point may be roughly calculated by the rule, that a difference of 27 millimetres in the pressure causes a difference of 1° in the temperature of the steam produced. We shall return to this point in Chap. ix.

It now only remains to divide the portion of the instrument between the freezing and boiling points into equal parts corresponding to single degrees, and to continue the division beyond the fixed points. Below the zero point are marked the numbers 1, 2, 3, &c. These temperatures are expressed with the sign —. Thus the temperature of 17° below zero is written —17°.

7. Adjustment of the Quantity of Mercury.—In order to avoid complicating the above explanation, we have omitted to consider an operation of great importance, which should precede those which we have just described. This is the determination of the volume which

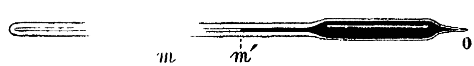


Fig. 9.

must be given to the reservoir, in order that the instrument may have the required range. When the reservoir is cylindrical, this is easily effected in the following manner. Suppose we wish the thermometer to indicate temperatures comprised between —20° and 130° Cent., so that the range is to be 150°; the reservoir is left open at O (Fig. 9),

and is filled through this opening, which is then hermetically sealed. The instrument is then immersed in two baths whose temperatures differ, say, by 50°, and the mercury rises through a distance $m\,m'$. This length, if the quantity of mercury in the reservoir be exactly sufficient, should be the third part of the length of the stem. The quantity of mercury in the reservoir is always taken too large at first, so that it has only to be reduced, and thus the space traversed by the liquid is at first too great. Suppose it to be equal to $\frac{3}{4}$ths of the length of the stem. The degrees will then be too long, in the ratio $\frac{3}{4} : \frac{1}{3} = \frac{9}{4}$; that is, the reservoir is $\frac{9}{4}$ of what it should be. We therefore measure off $\frac{4}{9}$ths of the length of the reservoir, beginning at the end next the stem; this distance is marked by a line, and the end O is then broken and the mercury suffered to escape. The glass is then melted down to the marked line, and the reservoir is thus brought to the proper dimensions. It only remains to regulate the quantity of mercury admitted, by making it fill the tube at the highest temperature which the instrument is intended to indicate.

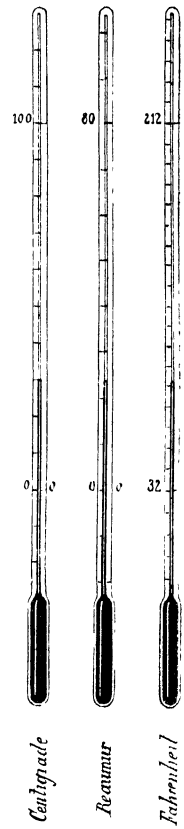


Fig. 10
Thermometric Scales

If the reservoir were spherical, which is a shape generally ill adapted for delicate thermometers, the foregoing process would be inapplicable, and it would be necessary to determine the proper size by trial.

8. **Thermometric Scales.**—In the *Centigrade* scale the freezing-point is marked 0°, and the boiling-point 100°. In *Réaumur's* scale, which is still popularly used on the Continent, the freezing-point is also marked 0°, but the boiling-point is marked 80°. Hence, 5 degrees on the former scale are equal to 4 on the latter, and the reduction of temperatures from one of these scales to the other can be effected by multiplying by $\frac{4}{5}$ or $\frac{5}{4}$.

For example, the temperature 75° Centigrade is the same as 60° Réaumur, since $75 \times \frac{4}{5} = 60$; and the temperature 36° Réaumur is the same as 45° Centigrade, since $36 \times \frac{5}{4} = 45$.

The relation between either of these scales and that of *Fahrenheit* is rather more complicated, inasmuch as Fahrenheit's zero is not at freezing-point, but at 32 of his degrees below it.

As regards intervals of temperature, 180 degrees Fahrenheit are equal to 100 Centigrade, or to 80 Réaumur, and hence, in lower terms, 9 degrees Fahrenheit are equal to 5 Centigrade, or to 4 Réaumur.

The conversion of temperatures themselves (as distinguished from intervals of temperature) will be best explained by a few examples.

Example 1. To find what temperatures on the other two scales are equivalent to the temperature 50° Fahrenheit.

Subtracting 32, we see that this temperature is 18 Fahrenheit degrees above freezing-point, and as this interval is equivalent to $18 \times \frac{5}{9}$, that is 10 Centigrade degrees, or to $18 \times \frac{4}{9}$, that is 8 Réaumur degrees, the equivalent temperatures are respectively 10° Centigrade and 8° Réaumur.

Example 2. To find the degree on Fahrenheit's scale, which is equivalent to the temperature 25° Centigrade.

An interval of 25 Centigrade degrees is equal to $25 \times \frac{9}{5}$, that is 45 Fahrenheit degrees, and the temperature in question is above freezing-point by this amount. The number denoting it on Fahrenheit's scale is therefore $32 + 45$, that is 77°.

The rules for the conversion of the three thermometric scales may be summed up in the following formulæ, in which F, C, and R denote equivalent temperatures expressed in degrees of the three scales:—

$$F = \tfrac{9}{5}C + 32 = \tfrac{9}{4}R + 32.$$
$$C = \tfrac{5}{4}R = \tfrac{5}{9}(F - 32).$$
$$R = \tfrac{4}{5}C = \tfrac{4}{9}(F - 32).$$

It is usual, in stating temperatures, to indicate the scale referred to by the abbreviations *Fahr.*, *Cent.*, *Réau.*, or more briefly by the initial letters F., C., R.

9. Displacement of the Zero Point.—A thermometer left to itself after being made, gradually undergoes a contraction of the bulb, leading to a uniform error of excess in its indications. This phenomenon is attributable to molecular change in the glass, which has, so to speak, been tempered in the construction of the instrument, and to atmospheric pressure on the exterior of the bulb, which is unresisted by the internal vacuum. The change is most rapid at first, and usually becomes insensible after a year or so, unless the thermometer is subjected to extreme temperatures. Its total amount is usually about half a degree. On account of this change it is advisable not to graduate a thermometer till some time after it has been sealed.

10. Sensibility of the Thermometer.—The power of the instrument to detect very small differences of temperature may be regarded as measured by the length of the degrees, which is proportional to the capacity of the bulb directly and to the section of the tube inversely (§ 24).

Quickness of action, on the other hand, requires that the bulb be small in at least one of its dimensions, so that no part of the mercury shall be far removed from the exterior, and also that the glass of the bulb be thin.

Quickness of action is important in measuring temperatures which vary rapidly. It should also be observed that, as the thermometer, in coming to the temperature of any body, necessarily causes an inverse change in the temperature of that body, it follows that when the mass of the body to be investigated is very small, the thermometer itself should be of extremely small dimensions, in order that it may not cause a sensible variation in the temperature which is to be observed.

11. Alcohol Thermometer.—In the construction of thermometers, other liquids may be introduced instead of mercury; and alcohol is very frequently employed for this purpose.

Alcohol has the disadvantage of being slower in its action than mercury, on account of its inferior conductivity; but it can be employed for lower temperatures than mercury, as the latter congeals at —39° Cent. (—38° Fahr.), whereas the former has never congealed at any temperature yet attained.

If an alcohol thermometer is so graduated as to make it agree with a mercurial thermometer (which is the usual practice), its degrees will not be of equal length, but will become longer as we ascend on the scale. If mercury is regarded as expanding equally at all temperatures, alcohol must be described as expanding more at high than at low temperatures.

12. Self-registering Thermometers.—It is often important for meteorological purposes to have the means of knowing the highest or the lowest temperature that occurs during a given interval. Instruments intended for this purpose are called maximum and minimum thermometers.

The oldest instrument of this class is *Six's* (Fig. 11), which is at once a maximum and a minimum thermometer. It has a large cylindrical bulb C filled with alcohol, which also occupies a portion of the tube. The remainder of the tube is partly filled with mercury,

which occupies a portion of the tube shaped like the letter U, one extremity of the mercurial column being in contact with the alcohol already mentioned, while the other extremity is in contact with a second column of alcohol; and beyond this there is a small space occupied only with air, so as to leave room for the expansion of the liquids. When the alcohol in the bulb expands, it pushes the mercurial column before it, and when it contracts the mercurial column follows it. The extreme points reached by the two ends of the mercurial column are registered by a pair of light steel indices c, d (shown on an enlarged scale at K), which are pushed before the ends of the column, and then are held in their places by springs, which are just strong enough to prevent slipping, so that the indices do not follow the mercury in its retreat. One of the indices d registers the maximum and the other c the minimum temperature which has occurred since the instrument was last set. The setting consists in bringing the indices into contact with the ends of the mercurial column, and is usually effected by means of a magnet. This instrument is now, on account of its complexity, little used. It possesses, however, the advantages of being equally quick (or slow) in its action for maximum and minimum temperatures, which is an important property when these temperatures are made the foundation for the computation of the mean temperature of the interval, and of being better able than most of the self-registering thermometers to bear slight jolts without disturbance of the indices.

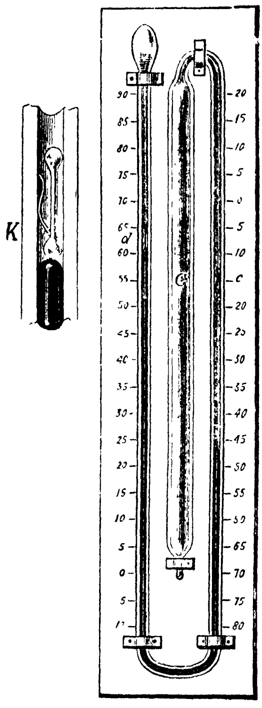


Fig. 11.—Six's Self-registering Thermometer.

Rutherford's self-registering thermometers are frequently mounted together on one frame, as in Fig. 12, but are nevertheless distinct instruments. His *minimum* thermometer, which is the only minimum thermometer in general use, has alcohol for its fluid, and is always placed with its tube horizontal, or nearly so. In the fluid column there is a small index n of glass or enamel, shaped like a dumb-bell.

When contraction occurs, the index, being wetted by the liquid, is drawn backwards by the contractile force of the liquid surface (see *Capillarity* in Part I.); but when expansion takes place the index remains stationary in the interior of the liquid. Hence the minimum temperature is indicated by the position of the

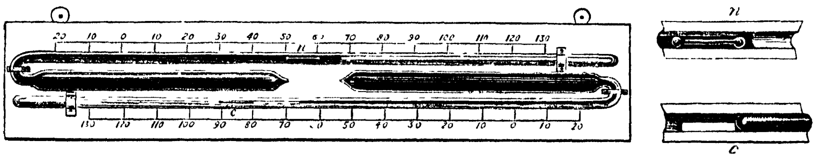


Fig 12—Rutherford's Maximum and Minimum Thermometers

forward end of the index. The instrument is set by inclining it so as to let the index slide down to the end of the liquid column.

The only way in which this instrument is liable to derangement is by a portion of the spirit evaporating from the column and becoming condensed in the end of the tube, which usually terminates in a small bulb. When the portion thus detached is large, or when the column of spirit becomes broken into detached portions by rough usage in travelling, "let the thermometer be taken in the hand by the end farthest from the bulb, raised above the head, and then forcibly swung down towards the feet; the object being, on the principle of centrifugal force, to send down the detached portion of spirit till it unites with the column. A few throws or swinging strokes will generally be sufficient; after which the thermometer should be placed in a slanting position, to allow the rest of the spirit still adhering to the sides of the tube to drain down to the column. But another method must be adopted if the portion of spirit in the top of the tube be small. Heat should then be applied slowly and cautiously to the end of the tube where the detached portion of spirit is lodged; this being turned into vapour by the heat will condense on the surface of the unbroken column of spirit. Care should be taken that the heat is not too quickly applied. . . . The best and safest way to apply the requisite amount of heat, is to bring the end of the tube slowly down towards a minute flame from a gas-burner; or if gas is not to be had, a piece of heated metal will serve instead."[1]

Rutherford's *maximum* thermometer is a mercurial thermometer, with the stem placed horizontally, and with a steel index c in the tube, outside the mercurial column. When expansion occurs, the

[1] Buchan's *Handy Book of Meteorology*, p. 62.

index, not being wetted by the liquid, is forced forwards by the contractile force of the liquid surface (see *Capillarity* in Part I.); but when contraction takes place, the index remains stationary outside the liquid. Hence the maximum temperature is indicated by the position of the backward end of the index The instrument is set by bringing the index into contact with the end of the liquid column, an operation which is usually effected by means of a magnet.

This thermometer is liable to get out of order after a few years' use, by chemical action upon the surface of the index, which causes it to become wetted by the mercury, and thus renders the instrument useless.

Phillips' maximum thermometer (invented by Professor Phillips, the eminent geologist, and made by Casella) is recommended for use in the official *Instructions for Taking Meteorological Observations,* drawn up by Sir Henry James for the use of the Royal Engineers. It is a mercurial thermometer not deprived of air. It has an exceed-

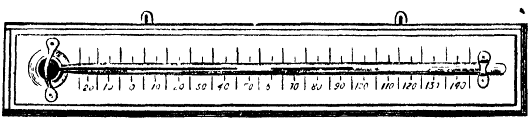


Fig 13 —Phillips' Maximum Thermometer

ingly fine bore, and the mercurial column is broken by the insertion of a small portion of air. The instrument is set by reducing this portion of air to the smallest dimensions which it can be made to assume, and is placed in a horizontal position. When the mercury expands, it pushes forwards this intervening air and the detached column of mercury beyond it; but when contraction takes place the intervening air expands, and the detached column remains unmoved.

The detached column is not easily shaken out of its place, and when the bore of the tube is made sufficiently narrow the instrument may even be used in a vertical position, a property which is often of great service.

In Negretti and Zambra's maximum thermometer (Fig. 14), which is employed at the Royal Observatory, Greenwich, there is an obstruction in the bent part of the tube, near the bulb, which barely leaves room for the mercury to pass when forced up by expansion, and is sufficient to prevent it from returning when the bulb cools

The objection chiefly urged against this thermometer is the extreme mobility of the detached column, which renders it very liable to

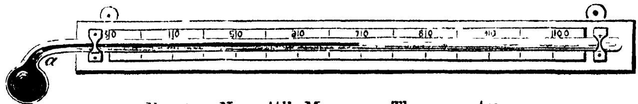


Fig 14.—Negretti's Maximum Thermometer

accidental displacement; but in the hands of a skilful observer this is of no moment. Dr. Balfour Stewart (*Elementary Treatise on Heat*, p. 20, 21), says:—"When used, the stem of this instrument ought to be inclined downwards. . . . It does not matter if the column past the obstruction go down to the bottom of the tube; for when the instrument is read, it is gently tilted up until this detached column flows back to the obstruction, where it is arrested, and the end of the column will then denote the maximum temperature. In resetting the instrument, it is necessary to shake the detached column past the obstruction in order to fill up the vacancy left by the contraction of the fluid after the maximum had been reached."

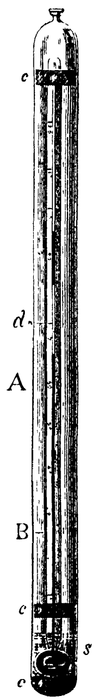


Fig 15 Thomson's Protected Thermometer

DEEP-SEA AND WELL THERMOMETERS.—Self-registering thermometers intended for observing at great depths in water should be inclosed in an outer case of glass hermetically sealed, the intervening space being occupied wholly or partly by air, so that the pressure outside may not be transmitted to the thermometer. A thermometer not thus protected gives too high a reading, because the compression of the bulb forces the liquid up the tube. The instrument represented in Fig. 15 was designed by Lord Kelvin for the Committee on Underground Temperature appointed by the British Association. A is the protecting case, B the Phillips' thermometer inclosed in it, and supported by three pieces of cork *ccc*. A small quantity of spirit *s* occupies the lower part of the case; *d* is the air-bubble characteristic of Phillips' thermometer, and serving to separate one portion of the mercurial column from the rest. In the figure this air-bubble is represented as expanded by the descent of the lower portion of mercury, while the upper portion remains suspended by adhesion. This instrument has been found to register correctly even under a pressure of 2½ tons to the square inch.

The use of the spirit *s* is to bring the bulb more quickly to the temperature of the surrounding medium.

Another instrument, designed, like the foregoing, for observations in wells and borings, is *Walferdin's maximum thermometer* (Fig. 16). Its tube terminates above in a fine point opening into a cavity of considerable size, which contains a sufficient quantity of mercury to cover the point when the instrument is inverted. The instrument is set by placing it in this inverted position and warming the bulb until the mercury in the stem reaches the point and becomes connected with the mercury in the cavity. The bulb is then cooled to a temperature lower than that which is to be observed; and during the operation of cooling, mercury enters the tube so as always to keep it full. The instrument is then lowered in the erect position into the bore where observations are to be made, and when the temperature of the mercury rises a portion of it overflows from the tube. To ascertain the maximum temperature which has been experienced, the instrument may be immersed in a bath of known temperature, less than that of the boring, and the amount of void space in the upper part of the tube will indicate the excess of the maximum temperature experienced above that of the bath.

Fig 16.

If the tube is not graduated, the maximum temperature can be ascertained by gradually raising the temperature of the bath till the tube is just full.

If the tube is graduated, the graduations can in strictness only indicate true degrees for some one standard temperature of setting, since the length of a true degree is proportional to the quantity of mercury in the bulb and tube; but a difference of a few degrees in the temperature of setting is immaterial, since 10° Cent. would only alter the length of a degree by about one six-hundredth part.

13. **Thermograph.**—A continuous automatic record of the indications of a thermometer can be obtained by means of photography, and this plan is now adopted at numerous observatories. The following description relates to the Royal Observatory, Greenwich. A sheet of sensitized paper is mounted on a vertical cylinder just behind the mercurial column, which is also vertical, and is protected from the action of light by a cover of blackened zinc, with the exception of a narrow vertical strip just behind the mercurial column. A strong beam of light from a lamp or gas flame is concentrated by a cylindric

lens, so that if the thermometer were empty of mercury a bright vertical line of light would be thrown on the paper. As this beam of light is intercepted by the mercury in the tube, which for this purpose is made broad and flat, only the portion of the paper above the top of the mercurial column receives the light, and is photographically affected. The cylinder is made to revolve slowly by clock-work, and if the mercury stood always at the same height, the boundary between the discoloured and the unaffected parts of the paper would be straight and horizontal, in consequence of the horizontal motion of the paper itself. In reality, the rising and falling of the mercury, combined with the horizontal motion of the paper, causes the line of separation to be curved or wavy, and the height of the curve above a certain datum-line is a measure of the temperature at each instant of the day.[1] The whole apparatus is called a *thermograph*, and apparatus of a similar character is employed for obtaining a continuous photographic record of the indications of the barometer[2] and magnetic instruments.

14. **Metallic Thermometers.**—Thermometers have sometimes been constructed of solid metals. Breguet's thermometer, for example (Fig. 17), consists of a helix carrying at its lower end a horizontal needle which traverses a dial. The helix is composed of three metallic strips, of silver, gold, and platinum, soldered together so as to form a single ribbon. The silver, which is the most expansible, is placed in the interior of the helix; the platinum, which is the least expansible, on the exterior; and the gold serves to connect them. When the temperature rises, the helix unwinds and produces a deflection of the needle; when the temperature falls, the helix winds up and deflects the needle in the opposite direction.

Fig 18 represents another dial-thermometer, in which the thermometric portion is a double strip composed of steel and brass, bent into the form of a nearly complete circle, as shown by the dotted lines in the figure. One extremity is fixed, the other is jointed to the

[1] Strictly speaking, the temperatures corresponding to the various points of the curve are not read off by reference to a single datum-line, but to a number of datum-lines which represent the shadows of a set of horizontal wires stretched across the tube of the thermometer at each degree, a broader wire being placed at the decades, and also at 32°, 52°, and 72°.

In order to give long degrees, the bulb of the thermometer is made very large—eight inches long, and ·4 of an inch in internal diameter.—(*Greenwich Observations*, 1847.)

[2] See *Photographic Registration* in Part I.

shorter arm of a lever, whose longer arm carries a toothed sector. This latter works into a pinion, to which the needle is attached.

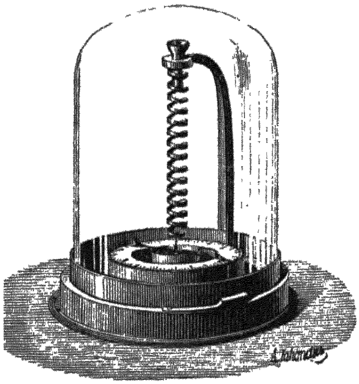


Fig 17.—Breguet's Thermometer.

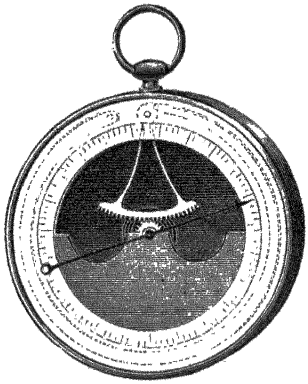


Fig. 18.—Metallic Thermometer.

Immisch's thermometer, which is extremely portable and convenient, contains a crescent-shaped thin metallic vessel (Fig. 19) filled with a highly expansible liquid. With rise of temperature, the horns A B of the crescent are further separated by the expansive force of the liquid, and the movement is transmitted to a hand which travels round a dial.

It may be remarked that dial-thermometers are very well adapted for indicating maximum and minimum temperatures, it being only necessary to place on opposite sides of the needle a pair of movable indices, which could be pushed in either direction according to the variations of temperature.

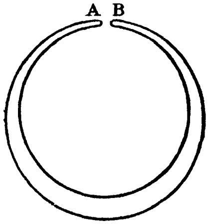


Fig. 19.

Generally speaking, metallic thermometers offer great facilities for automatic registration.

In Secchi's meteorograph, for example, the temperature is indicated and registered by the expansion of a long strip of brass (about 17 metres long) kept constantly stretched by a suitable weight; this expansion is rendered sensible by a system of levers connected with the tracing point. The thermograph of Hasler and Escher consists of a steel and a brass band connected together and rolled into the form of a spiral. The movable extremity of the spiral, by acting upon a projecting arm, produces rotation of a steel axis which carries the tracer.

15. **Pyrometers.**—Metallic thermometers can generally be employed for measuring higher temperatures than a mercurial thermometer could bear; but there is great difficulty in constructing any instrument to measure temperatures as high as those of furnaces. Instruments intended for this purpose are called pyrometers.

Wedgwood, the famous potter, invented an apparatus of this kind, consisting of a gauge for measuring the contraction experienced by a piece of baked clay when placed in a furnace; and Brongniart introduced into the porcelain manufactory at Sèvres the instrument represented in Fig. 20, consisting of an iron bar lying in a groove in a porcelain slab, with one end abutting against the bottom of the groove, and the other projecting through the side of the furnace, where it gave motion to an indicator.

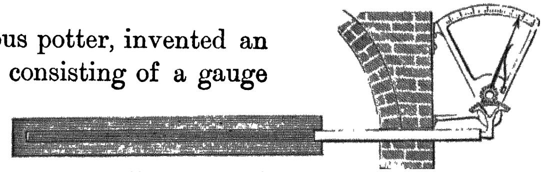


Fig 20 —Brongniart's Pyrometer.

Neither of these instruments has, however, been found to furnish consistent indications, and the only instrument that is now relied on for the measurement of very high temperatures is the air-thermometer.

Of late years much attention has been given to the measurement of temperature by an electrical method depending on the fact that the resistance of a metal to the passage of a current of electricity increases with the temperature. Platinum is the metal usually selected for this purpose, and the instrument employed is called the *platinum pyrometer*, or the *electrical pyrometer*.

16. **Differential Thermometer.**—Leslie of Edinburgh invented, in the beginning of the present century, the instrument shown in Fig. 21, for detecting small differences of temperature. A column of sulphuric acid, coloured red, stands in the two branches of a bent tube, the extremities of which terminate in two equal bulbs containing air. When both globes are at the same temperature, whatever that temperature may be, the liquid, if the instrument is in order, stands at the same height in both branches. This height is marked zero on both scales. When there is a difference of temperature between them, the expansion of the air in the warmer bulb produces a depression of the liquid on that side and an equal elevation on the other side.

The differential thermometer is an instrument of great sensibility, and enabled Leslie to conduct some important investigations on the subject of the radiation of heat. It is now, however, superseded by the thermo-pile invented by Melloni. This latter instrument will be

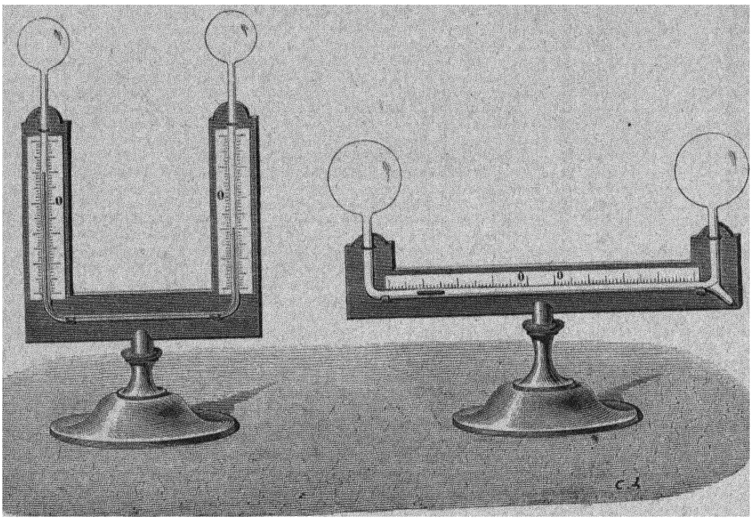


Fig. 21.—Leslie's Differential Thermometer. Fig. 22.—Rumford's Thermoscope.

described in another portion of this work. Rumford's thermoscope (Fig. 22) is analogous to Leslie's differential thermometer. It differs from it in having the horizontal part much longer, and the vertical branches shorter. In the horizontal tube is an alcohol index, which, when the two globes are at the same temperature, occupies exactly the middle.

CHAPTER II.

MATHEMATICS OF EXPANSION.

17. Expansion. Factor of Expansion.—When a body expands from volume V to volume $V+v$, the ratio $\left(\frac{v}{V}\right)$ is called the *expansion of volume* or the *cubical expansion* of the body.

In like manner if the length, breadth, or thickness of a body increases from L to $L+l$, the ratio $\left(\frac{l}{L}\right)$ is called the *linear expansion.*

The ratio $\left(\frac{V+v}{V}\right)$ will be called, in this treatise, the *factor of cubical expansion,* and the ratio $\left(\frac{L+l}{L}\right)$ the *factor of linear expansion.* In each case the factor of expansion is *unity plus the expansion.*

Similar definitions apply to expansion of area or superficial expansion; but it is seldom necessary to consider this element in thermal discussions.

18. Relation between Linear and Cubical Expansion.—If a cube, whose edge is the unit length, expands equally in all directions, the length of each edge will become $1+l$, where l is the linear expansion; and the volume of the cube will become $(1+l)^3$ or $1+3l+3l^2+l^3$.

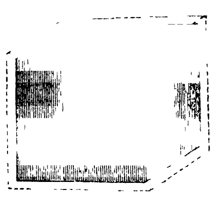


Fig. 23

In the case of the thermal expansion of solid bodies l is always very small, so that l^2 and l^3 can be neglected, and the expansion of volume is therefore $3l$; that is to say, the *cubical expansion is three times the linear expansion.* This is illustrated geometrically by Fig. 23, which represents a unit cube with a plate of thickness l and therefore of volume l applied to each of three faces; the total volume added being therefore $3l$.

Similar reasoning shows that *the superficial expansion* is *double the linear expansion.*

These results have been deduced from the supposition of equal expansion in all directions. If the expansions of the cube in the directions of three conterminous edges be denoted by a, b, c, the angles being supposed to remain right angles, the volume will become $(1+a)(1+b)(1+c)$ or $1+a+b+c+ab+ac+bc+abc$, which, when a, b and c are so small that their products can be neglected, becomes $1+a+b+c$; so that the expansion of volume is the sum of the expansions of length, breadth, and thickness.

19. Variation of Density.—Since the density of a body varies inversely as its volume, the density after expansion will be obtained by *dividing* the original density by the factor of expansion. In fact, if V, D denote the volume and density before, and V′, D′ after expansion, the mass of the body, which remains unchanged, is equal to V D, and also to V′ D′. We have therefore $\frac{D'}{D}=\frac{V}{V'}=\frac{1}{1+e}$, where e denotes the expansion of volume, and therefore $1+e$ the factor of expansion.

Since $\frac{1}{1+e}$ is $1-e+e^2-e^3+$&c., it is sensibly equal to $1-e$ when e is small. We have therefore $D'=D(1-e)$.

20. Real and Apparent Expansion.—When the volume of a liquid is specified by the number of divisions which it occupies in a graduated vessel, it is necessary to take into account the expansion of the vessel, if we wish to determine the true expansion of the liquid.

Let a denote the apparent expansion computed by disregarding the expansion of the vessel and attending only to the number of divisions occupied. Then if n be the number of divisions occupied before, and n' after expansion, we have

$$n'=n\ (1+a).$$

Let g denote the real expansion of the containing vessel; then if d be the volume of each division before, and d' after expansion, we have

$$d'=d\ (1+g).$$

Let m denote the real expansion of the liquid. Then if v denote the real volume of the liquid before, and v' after expansion, we have

$$v'=v\ (1+m).$$

But since the volume v consists of n parts each having the volume d, we have

$$v=nd,$$

and in like manner

$$v'=n'd'.$$

Substituting for n' and d' in this last equation, we have

$$v' = n(1+a)\,d(1+g) = v(1+a)(1+g).$$
$$\text{But } v' = v(1+m).$$

Hence we have

$$(1+a)(1+g) = 1+m;$$

that is, *the factor of real expansion of the liquid is the product of the factor of real expansion of the vessel and the factor of apparent expansion.* Multiplying out, we have

$$1+a+g+ag = 1+m,$$

and as the term ag, being the product of two small quantities, is usually negligible, we have sensibly

$$a+g=m;$$

that is, the expansion of the liquid is the sum of the expansion of the glass and the apparent expansion.

This investigation is applicable to the mercurial thermometer when the capacity of the bulb has been expressed in degrees of the stem.

Similar reasoning applies to the apparent expansion of a bar of one metal as measured by means of a graduated bar of a less expansible metal. The real expansion of the bar to be measured will be sensibly equal to the sum of the expansion of the measuring bar and the apparent expansion.

In adopting the mercurial thermometer as the standard of temperature (the tube being graduated into equal parts), we virtually adopt the apparent expansion of mercury in glass as our standard of *uniform* expansion.

21. Physical Meaning of the Degrees of the Mercurial Thermometer.—Since the stem of a mercurial thermometer is divided into degrees of equal capacity, we can express the capacity of the bulb in degrees. Let the capacity of the bulb together with as much of the stem as is below the freezing-point be N degrees, and let the interval from freezing to boiling point be n degrees; then $\frac{n}{N}$ is the apparent expansion of the mercury from freezing to boiling point. When the Centigrade scale is employed, this apparent expansion is $\frac{100}{N}$, and the apparent expansion from zero to $t°$ is $\frac{t}{N}$. Hence the apparent expansion from zero to $t°$ is $\frac{t}{100}$ of the apparent expansion from zero to 100°. This last statement constitutes the definition of the temperature $t°$ when the mercurial thermometer is regarded as the standard.

22. Comparability of Mercurial Thermometers.—If two mercurial thermometers, each of them constructed so as to have its degrees rigorously equal in capacity, agree in their indications at all temperatures, the above investigation shows that the apparent expansions of the mercury in the two instruments must be exactly proportional. But we have shown in § 20 that the apparent expansion a is equal to $m-g$, m denoting the real expansion of the mercury, and g that of the glass. Mercury, being a liquid and an elementary substance, can always be obtained in the same condition, so that m will have the same value in the two thermometers; but it is difficult to ensure that two specimens of glass shall be exactly alike; hence g has different values in different thermometers. The agreement of the two thermometers does not, however, require identity in the values of $m-g$, but only proportionality; in other words it requires that the fraction

$$\frac{m-g_1}{m-g_2}$$

(where g_1 and g_2 are the values of g for the two instruments) shall have the same value at all temperatures.

The average value of g is about $\frac{1}{7}$ of that of m. In other words mercury expands about 7 times as much as glass.

23. Steadiness of Zero in Spirit Thermometers.—It is obvious from § 21 that the volume of a degree can be computed by multiplying the capacity of the bulb by the number which denotes the apparent expansion for one degree. Alcohol expands about 6 times as much as mercury, and its apparent expansion in glass is about 7 times that of mercury. Hence with the same size of bulb, the degrees of an alcohol thermometer will be about 7 times as large as those of a mercurial thermometer, and a contraction of the bulb which produces a change of one degree in the reading of a mercurial thermometer, would only produce a change of one-seventh of a degree in the reading of an alcohol thermometer. This is the reason, or at all events one reason, why displacement of the zero point (§ 9) is insignificant in spirit thermometers.

24. Length of a Degree on the Stem.—Since the length of a degree upon the stem of a thermometer is equal to the volume of a degree divided by the sectional area of the tube, the formula for this length is $\frac{aC}{s}$, where a denotes the apparent expansion for one degree, C the capacity of the bulb with as much of the stem as is below zero, and

s the sectional area of the stem. The value of a for the mercurial Centigrade thermometer is about $\frac{1}{6480}$.

25. Weight Thermometer.—In the weight thermometer (Fig. 24) the apparent expansion of mercury is observed by comparing the weight of the mercury which passes the zero point with that of the mercury which remains below it. The tube is open, and its mouth is the zero point. The instrument is first filled with mercury at zero, and is then exposed to the temperature which it is required to measure. The mercury which overflows is caught and weighed, and the weight of the mercury which remains in the instrument is also determined—usually by subtracting the weight of the overflow from that of the original contents. The weight of the overflow, divided by the weight of what remains, is equal to the apparent expansion; for it is the same as the ratio of the volume of mercury above the zero point to the volume below it in an ordinary thermometer.

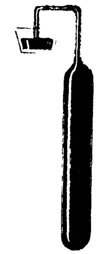


Fig 24
Weight Thermometer

In order to measure temperatures in degrees, with this thermometer, the apparent expansion from 0° to 100° C. must be determined once for all and put on record. One hundredth part of this must be divided into the apparent expansion observed at the unknown temperature $t°$, and the quotient will be t.

26. Expansion of Gases.—In the case of solids and liquids the expansions produced by heat are usually very small, so that it is not important to distinguish between the value of $\frac{v}{V}$ and the value of $\frac{v}{V+v}$ (§ 17). But in the case of gases much larger expansions occur, and it is essential to attend to the above distinction. By general agreement, the volume of a gas at zero (Centigrade) is taken as the standard with which the volume at any other temperature is to be compared. We shall denote the volume at zero by V_0, and the volume at temperature $t°$ by V_t. Then, if the pressure be the same at both temperatures, we shall write

$$V_t = V_0 (1 + \alpha t)$$

where α is called the mean coefficient of expansion between the temperatures 0° and $t°$. Experiment has shown that when temperatures are measured by the mercurial thermometer, graduated in the manner which we have already described, α is practically the same at all temperatures which lie within the range of the mercurial

thermometer. In other words, the expansions of gases are sensibly proportional to the apparent expansion of mercury in glass. Moreover, the coefficient a is not only the same for different temperatures, but it is also the same for different gases; its value being always very approximately

$$\cdot 00366 \text{ or } \frac{1}{273}.$$

By Boyle's law, the product of the volume and pressure of a gas remains constant when the temperature is constant. We have been supposing the pressure to remain constant, so that the product in question is proportional to the volume only. If the volume is kept constant the pressure will vary in proportion to $1+at$, so that we shall have

$$P_t = P_0 (1 + at),$$

P_0 and P_t denoting the pressures at 0° and t° respectively. If we remove all restriction, we have

$$(V P)_t = (V P)_0 (1 + at),$$

where $(V P)_0$, $(V P)_t$ denote the products of volume and pressure at 0° and t° respectively. Hence the value of the expression

$$\frac{V P}{1 + at}$$

will be the same for all values of V, P and t. Since the mass is unchanged, the density D varies inversely as the volume, and therefore

$$\frac{P}{D (1 + at)}$$

is also constant.

27. General Definition of Coefficient of Expansion.—If V_0 denote the volume of any substance at temperature 0° (Centigrade), V_t its volume under the same pressure at temperature t°, and $V_{t'}$ its volume at a higher temperature t'°, the *mean coefficient of expansion* a between the temperatures t and t' is defined by the equation

$$V_{t'} - V_t = V_0 \, a \, (t' - t),$$

and the *coefficient of expansion at the temperature* t° is the limit to which a approaches as t' approaches t; that is, in the language of the differential calculus, it is

$$\frac{1}{V_0} \frac{dV}{dt}.$$

If we make V_0 unity, the coefficient of expansion at temperature t will be simply

$$\frac{dV}{dt}.$$

CHAPTER III.

EXPANSION OF SOLIDS.

28. **Observations of Linear Expansion.**—Laplace and Lavoisier determined the linear expansion of a great number of solids by the following method.

The bar AB (Fig. 25) whose expansion is to be determined, has one end fixed at A, while the other can move freely, pushing before it the lever OB, which is movable about the point O, and carries a telescope whose line of sight is directed to a scale at some distance. A displacement BB′ corresponds to a considerably greater length CC′ on the scale, the ratio of the former to the latter being the same as that of OB to OC.

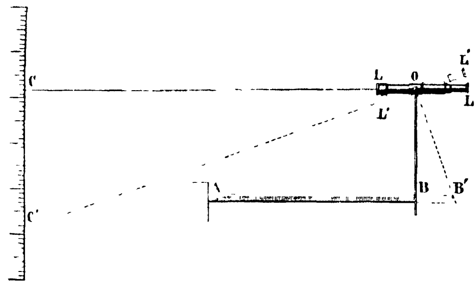


Fig 25
Principle of the Method of Laplace and Lavoisier

The apparatus employed by Laplace and Lavoisier is shown in Fig 26. The trough C, in which is laid the bar whose expansion is to be determined, is placed between four massive uprights of hewn stone N. One of the extremities of the bar rests against a fixed bar B′, firmly joined to two of the uprights; the other extremity, which rests upon a roller to give it greater freedom of movement, pushes the bar B, which produces the rotation of the axis aa'. This axis carries with it in its rotation the telescope LL′, which is directed to the scale. The first step is to surround the bar with melting ice, and take a reading through the telescope when the bar is at the temperature zero. The temperature of the trough is then raised, and read-

ings are taken, which, by comparison with the first, give the increase of length.

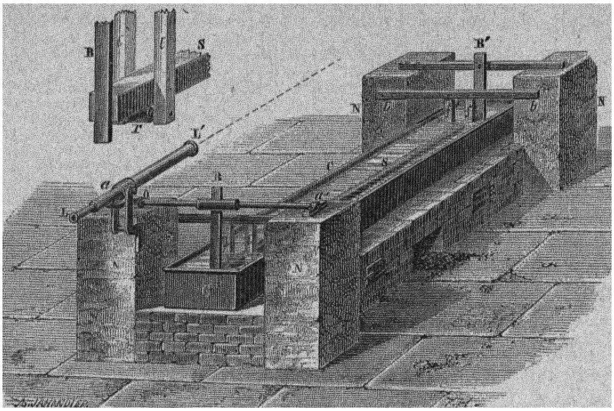


Fig. 26.—Apparatus of Laplace and Lavoisier.

The following table contains the most important results thus obtained:—

COEFFICIENTS OF LINEAR EXPANSION.

Gold, Paris standard, annealed,	0·000015153	Soft wrought iron, ...	0·000012204
„ „ unannealed,	0·000015515	Round iron, wire drawn, . .	0·000012350
Steel not tempered, . . .	0·000010792	English flint-glass, ...	0·000008116
Tempered steel reheated to 65°,	0·000012395	Gold, procured by parting, .	0·000014660
Silver obtained by cupellation,	0·000019075	Platina, ...	0·000009918
Silver, Paris standard, . .	0·000019086	Lead, ...	0·000088483
Copper, ...	0·000017173	French glass with lead, . .	0·000008715
Brass, ...	0·000018782	Sheet zinc, ...	0·000029416
Malacca tin, ...	0·000019376	Forged zinc, ...	0·000031083
Falmouth tin, ...	0·000021729		

The coefficient of expansion of a metal is not precisely the same at all temperatures, but it is sensibly constant from 0° to 100° C.

A simpler and probably more accurate method of observing expansions was employed by Ramsden and Roy. It consists in the direct observation of the distances moved by the ends of the bar, by means of two microscopes furnished with micrometers, the microscopes themselves being attached to an apparatus which is kept at a constant temperature by means of ice.

29. Compensated Pendulum.—The rate of a clock is regulated by the motion of its pendulum. Suppose the clock to keep correct time at a certain temperature. Then at higher temperatures the pendulum will be too long and will therefore vibrate too slowly, so that

the clock will lose. At lower temperatures, on the other hand, the clock will gain. To obviate or, at least, diminish this source of irregularity, the following methods of compensation are employed.

1. *Harrison's Gridiron Pendulum.*—This consists of four oblong frames, the uprights of which are alternately of steel F and of brass

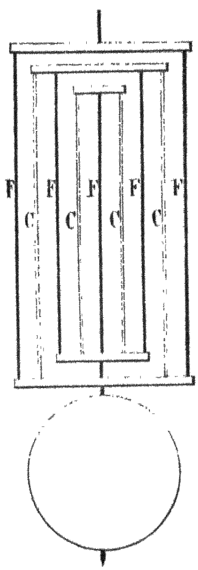


Fig 27
Plan of Gridiron Pendulum.

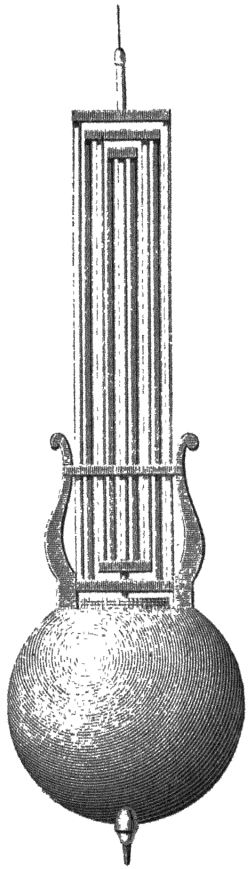


Fig 28
Gridiron Pendulum

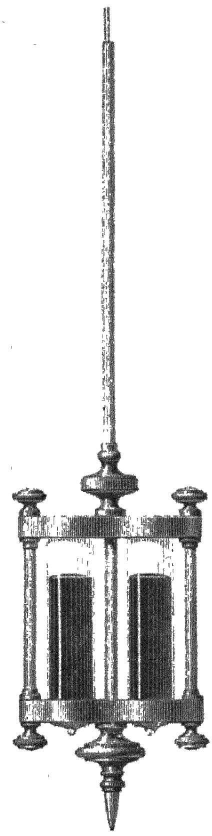


Fig 29
Graham's Mercurial Pendulum

C (Fig. 27), so arranged that the bob will rise or fall through a distance equal to the difference between the total expansion of 3 steel rods and that of 2 brass rods. As the coefficients of expansion of these metals are nearly as 2 to 3, it is possible to make the compensation nearly exact.

2. *Graham's Mercurial Pendulum.*—This consists of an iron rod

carrying at its lower end a frame, in which are fixed one or two glass cylinders containing mercury. When the temperature rises, the lengthening of the rod lowers the centre of gravity and centre of oscillation of the whole; but the expansion of the mercury produces the contrary effect; and if there is exactly the right quantity of mercury the compensation will be nearly perfect.

30. Force of Expansion of Solids.—The *force* of expansion is often very considerable, being equal to the force necessary to compress the body to its original dimensions. Thus, for instance, iron when heated from 0° to 100° increases by ·0012 of its original length. In order to produce a corresponding change of length in a rod an inch square by mechanical means, a force of about 15 tons would be required. This is accordingly the force necessary to prevent such a rod from expanding or contracting when heated or cooled through 100°.

This force has frequently been utilized for bringing in the walls of a building when they have settled outwards. For this purpose the walls are first tied together by iron rods, which pass through the walls, and are furnished at the ends with screws and nuts. All the nuts having been tightened against the wall, alternate bars are heated; and while they are hot, the nuts upon them, which have been thrust away from the wall by the expansion, are screwed home. As these bars cool, they draw the walls in and allow the nuts on the other bars to be tightened. The same operation is then repeated as often as may be necessary.

Iron cannot with safety be used in structures, unless opportunity is given it to expand and contract without doing damage. In laying a railway, small spaces must be left between the ends of the rails to leave room for expansion; and when sheets of lead or zinc are employed for roofing, room must be left for them to overlap.

CHAPTER IV.

EXPANSION OF LIQUIDS.

31. **Method of Equilibrating Columns.**—Most of the methods employed for measuring the expansion of liquids depend upon a previous knowledge of the expansion of glass, the observation itself consisting in a determination of the apparent expansion of the liquid relative to glass. There is, however, one method which is not liable to this objection, and it has been employed by Dulong and Petit, and afterwards by Regnault, for measuring the expansion of mercury—an element of great importance for many physical applications. It depends upon the hydrostatic principle that the heights of two liquid columns which produce equal pressures are inversely as their densities

Fig 30
Principle of Dulong's Method

Let A and B (Fig. 30) be two tubes containing mercury, and communicating with each other by a very narrow horizontal tube CD at the bottom. If the temperature of the liquid be uniform, the mercury will stand at the same height in both branches; but if one column be kept at 0° and the other be heated, their densities will be unequal. Let $d\,d'$ be their densities, and $h\,h'$ their heights. Then since their pressures at the bottom are equal, we must have

$$h\,d = h'\,d'.$$

But if v and v' denote the volumes of one and the same mass of liquid at the two temperatures, we have

$$v\,d = v'\,d'.$$

From these two equations, we have

$$v : v' :: h : h',$$

so that the expansion of volume is directly given by a comparison of the heights. Denoting this expansion by m, we shall have

$$m = \frac{h' - h}{h}.$$

Strictly speaking, the mercury in this experiment is not in equilibrium. There will be two very slow currents through the horizontal tube, the current from hot to cold being above, and the current from cold to hot below. Equilibrium of pressure will exist only at the intermediate level—that of the axis of the tube, and it is from this level that h and h' should be measured.

32. The apparatus employed by Dulong and Petit for carrying out this method is represented in Fig. 31. The two upright tubes

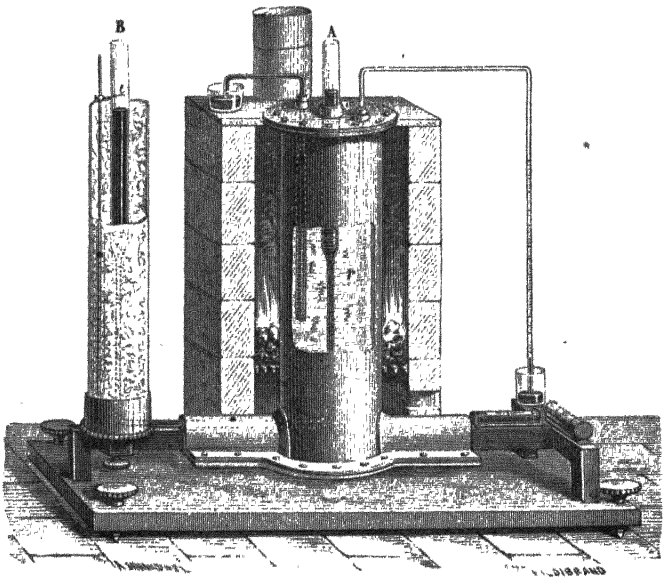


Fig 31.—Apparatus of Dulong and Petit

A, B, and the connecting tube at their base, rest upon a massive support furnished with levelling screws, and with two spirit-levels at right angles to each other, for insuring horizontality. The tube B is surrounded by a cylinder containing melted ice. The other tube A is surrounded by a copper cylinder filled with oil, which is heated by a furnace connected with the apparatus. In making an observation, the first step is to arrange the apparatus so that, when

the oil is heated to the temperature required, the mercury in the tube A may just be seen above the top of the cylinder, so as to be sighted with the telescope of a cathetometer; this may be effected by adding or taking away a small quantity of mercury. The extremity of the column B is next sighted, which gives the difference of the heights h' and h. The absolute height h is determined by means of a fixed reference mark i near the top of the column of mercury in the tube B. This reference mark is carried by an iron rod surrounded by the ice, and its distance from the axis of the horizontal connecting tube has been very accurately measured once for all. The temperature of the oil is given by the weight thermometer t, and by the air thermometer r, which latter we shall explain hereafter.

By means of this method Dulong and Petit ascertained that the expansion of mercury is nearly uniform between 0° and 100° C., as compared with the indications of an air-thermometer, and that though its expansion at higher temperatures is more rapid, the difference is less marked than in the case of other liquids. They found the mean coefficient of expansion from 0° to 100° to be $\frac{1}{5550}$; from 0° to 200°, $\frac{1}{5425}$; and from 0° to 300°, $\frac{1}{5300}$.

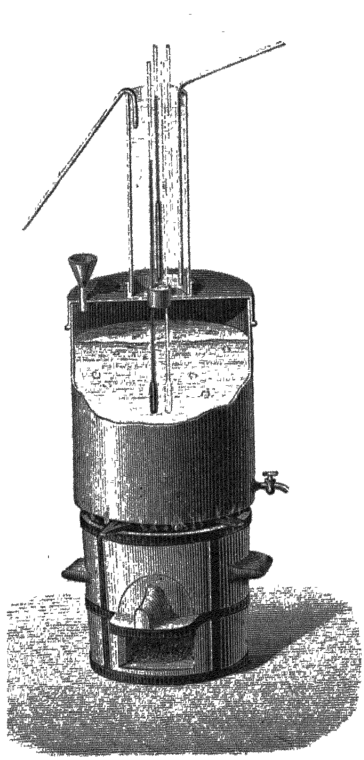
Regnault, without altering the principle of the apparatus of Dulong and Petit, introduced several improvements in detail, and added greatly to the length of the tubes A and B, thereby rendering the apparatus more sensitive. His results are not very different from those of Dulong and Petit. For example, he makes the mean coefficient from 0° to 100° to be $\frac{1}{5509}$; from 0° to 200°, $\frac{1}{5479}$; and from 0° to 300°, $\frac{1}{5360}$. His experiments show that the mean coefficient from 0° to 50° is $\frac{1}{5547}$, a value almost identical with $\frac{1}{5550}$.

33.. Expansion of Glass.—The expansion of mercury being known, we can find the expansion of any kind of glass by observing the apparent expansion of mercury in a weight thermometer (§ 25) constructed of this glass, and subtracting this apparent expansion from the real expansion of the liquid; or more rigorously, by dividing the factor of real expansion of the liquid by the factor of apparent expansion (§ 20), we shall obtain the factor of expansion of the glass.

Dulong and Petit found $\frac{1}{6480}$ as the mean value of the coefficient

of apparent expansion of mercury in glass, and $\frac{1}{5550}$ as the coefficient of real expansion of mercury. The difference of these two fractions is approximately $\frac{1}{38700}$, which may therefore be taken as the coefficient of expansion of glass. It is about one-seventh of the coefficient of expansion of mercury.

34. Expansion of any Liquid.—The expansion of the glass of which a thermometer is made being known, we may use the instrument to measure the expansion of any liquid. For this purpose we must measure the capacity of the bulb and find how many divisions of the stem it is equal to. We can thus determine how many divisions the liquid occupies at two different temperatures, that is, we can determine the apparent expansion of the liquid; and by adding to this the expansion of the glass, we shall obtain the real expansion of the liquid. Or more rigorously, we shall obtain the factor of real expansion of the liquid by multiplying together the factor of apparent expansion and the factor of expansion of the glass.



M. Pierre has performed an extensive series of experiments by this method upon a great number of liquids. The apparatus employed by him is shown in Fig. 32. The thermometer containing the given liquid is fixed beside a mercurial thermometer, which marks the temperature. The reservoir and a small part of the tube are immersed in the bath contained in the cylinder below. The upper parts of the stems are inclosed in a second and smaller cylinder, the water in which is maintained at a sensibly constant temperature indicated by a very delicate thermometer.

From these experiments it appears that the expansions of liquids are in general much greater than those of solids; also that their ex-

pansion does not proceed uniformly, as compared with the indications of a mercurial thermometer, but increases very perceptibly as the temperature rises. This is shown by the following table:—

	Volume at 0°	Volume at 10°.	Volume at 40°
Water...	1	1·000146	1·007492
Alcohol....	1	1·010661	1·044882
Ether	1	1·015408	1 066863
Bisulphide of carbon...	1	1 011554	1 049006
Wood-spirit.............	1	1 012020	1·050509

35. **Other Methods.**—Another method of determining the apparent expansion of a liquid, with a view to deducing its real expansion, consists in weighing a glass bottle full of the liquid at different temperatures. This is virtually employing a weight thermometer.

A third method consists in observing the loss of weight of a piece of glass when weighed in the liquid at different temperatures. Time must be given in each case for the glass to take the temperature of the liquid; and when this condition is fulfilled, the factor of expansion will be equal to the loss of weight at the lower temperature, divided by the loss of weight at the higher.

For if the volume of the glass at the lower temperature be called unity, and its volume at the higher temperature $1+g$, the mass of liquid displaced at the lower temperature will be equal to its density d, and the mass displaced at the higher temperature will be the product of $1+g$ by the density $\frac{d}{1+l}$, where l denotes the expansion of the liquid. The losses of weight, expressed in gravitation measure, are therefore

$$d \text{ and } \frac{(1+g)\,d}{1+l},$$

and the former of these divided by the latter gives $\frac{1+l}{1+g}$, which (§ 20) is the factor of apparent expansion.

36. **Formulæ for the Expansion of Liquids.**—As we have mentioned above, the expansion of liquids does not advance uniformly with the temperature; whence it follows that the mean coefficient of expansion will vary according to the limiting temperatures between which it is taken.

For a great number of liquids, the mean coefficient of expansion may be taken as increasing uniformly with the temperature. If, therefore, Δ be the expansion from 0 to t, we have

$$\frac{\Delta}{t} = a + bt, \text{ whence } \Delta = at + bt^2,$$

a and b being two constants specifying the expansibility of the given liquid.

For some very expansible liquids two constants are not sufficient, and the expansion is represented by the formula

$$\Delta = at + bt^2 + ct^3.$$

We subjoin a few instances of this class taken from the work of M. Pierre:—

Alcohol.....................$\Delta = 0{\cdot}0010486\ t + 0{\cdot}0000017510\ t^2 + 0{\cdot}00000000134518\ t^3$
Ether.......................$\Delta = 0{\cdot}0015132\ t + 0{\cdot}0000023592\ t^2 + 0{\cdot}000000040051\ t^3$
Bisulphide of carbon....$\Delta = 0{\cdot}0011398\ t + 0{\cdot}0000013707\ t^2 + 0{\cdot}00000019123\ t^3$
Bromine.................. $\Delta = 0{\cdot}0010382\ t + 0{\cdot}0000017114\ t^2 + 0{\cdot}0000000054471\ t^3$

37. Maximum Density of Water.—Water, unlike other liquids, contracts as its temperature rises from 0° to 4°, at which point its volume is a minimum, and therefore its density a maximum.

The following experiment, which furnishes a means of determining the temperature of maximum density, is due to Hope.

A glass jar is employed, having two lateral openings, one near the top and the other near the bottom, which admit two thermometers placed horizontally. The jar is filled with water at a temperature higher than 4°, and its middle is surrounded with a freezing-mixture. The following phenomena will then be observed.

The lower thermometer descends steadily to 4°, and there remains stationary. The upper thermometer at first undergoes very little change, but when the lower one has reached the fixed temperature, the upper one begins to fall, reaches the temperature of zero, and, finally, the water at the surface freezes, if the action of the freezing-mixture continues for a sufficiently long time. These facts admit of a very simple explanation.

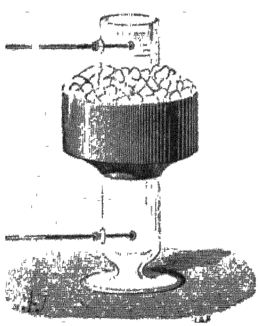


Fig. 33
Hope's Experiment

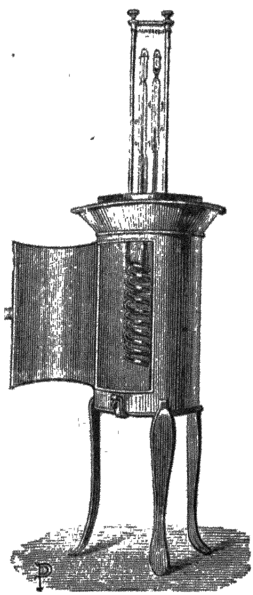
As the water in the middle portion of the jar grows colder, its density increases, and it sinks to the bottom. This process goes on till all the water in the lower part has attained the temperature of 4°. But when all the water from the centre to the bottom has attained this temperature, any further cooling of the water in the centre will produce no circulation in the lower portion, and very little in the upper, until needles of ice are formed. These, being lighter than water, rise to the surface, and thus produce a circulation

which causes the water near the surface to freeze, while that near the bottom remains at the temperature of 4°.

This experiment illustrates what takes place during winter in pools of fresh water. The fall of temperature at the surface does not extend to the bottom of the pool, where the water, whatever be the external temperature, seldom falls below 4°. This is a fact of great interest, as exemplifying the close connection of natural phenomena, and the manner in which they contribute to a common end It is in virtue of this anomaly exhibited by water in its expansion, taken in conjunction with the specific lightness of ice and the low conducting power of liquids generally, that the temperature at the bottom of deep pools remains moderate even during the severest cold and that the lives of aquatic animals are preserved.

38. Saline Solutions.—These remarks are not applicable to sea-water, which contracts as its temperature falls till its freezing-point is attained; this latter being considerably lower than the freezing-point of fresh water.

In the case of saline solutions of different strengths, the temperature of maximum density falls along with the freezing-point, and falls more rapidly than this latter, so that for solutions containing more than a certain proportion of salt the temperature of maximum density is below the freezing-point. In order to show this experimentally, the solution must be placed in such circumstances as to remain liquid at a temperature below its ordinary freezing-point.



Maximum Density of Water

39. Apparent Expansion of Water.—Fig. 34 represents an apparatus for showing the changes of apparent volume of water in a glass vessel. In the centre are two thermometers, one containing alcohol and the other water. The reservoir of the latter is a long spiral, surrounding the reservoir of the alcohol thermometer and having much greater capacity. Both reservoirs are contained in a metal box, which is at first filled with melting ice. The two instruments are so placed that at zero the extremities of the two liquid columns are on the same horizontal line. This being the case, if the

ice be now removed, and the apparatus left to itself, or if the process be accelerated by placing a spirit-lamp below the box, the alcohol will immediately be seen to rise, while the water will descend; and the two liquids will thus continue to move in opposite directions until a temperature of 5° or 6° is attained. From this moment the water ceases to descend, and begins to move in the same direction as the alcohol. The temperature at which the water thermometer becomes stationary is that at which the coefficient of expansion of water is the same as that of glass. The coefficient of expansion of water is zero at 4°, and at temperatures near 4° is approximately

$$\cdot 000016\ (t-4).$$

The average value of the coefficient of expansion of glass is about ·000027, and by equating these two expressions, we have

$$t-4=\frac{27}{16}=1\cdot 7 \text{ nearly;}$$

hence the water thermometer will be stationary at the temperature 5°·7.

40. Density of Water at Various Temperatures.—The volume, at temperatures near 4°, of a quantity of water which would occupy unit volume at 4°, is approximately

$$1+\cdot 000008\ (t-4)^2,$$

and the density of water at these temperatures is therefore

$$1-\cdot 000008\ (t-4)^2,$$

the density at 4° being taken as unity.

The density of water at some other temperatures is given in the following table:—

Temperature.	Density
0°	·999871
4°	1·000000
8°	·999886
12°	·999549
16°	·999002
20°	·998259
50°	·9882
100°	·9586

41. Expansion of Iron and Platinum.—The coefficient of absolute expansion of mercury being known, that of glass is deduced from it in the manner already indicated (§ 33). Dulong and Petit have deduced from it also the coefficients of expan-

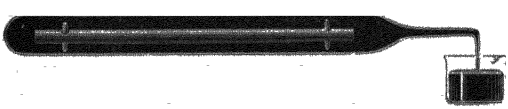


Fig. 35.—Expansion of Iron and Platinum.

sion of iron and platinum, these metals not being attacked by mercury. The method employed is the following.

The metal in question is introduced, in the shape of a cylindrical bar, into the reservoir of a weight thermometer. Let W be the weight of the metal introduced, and D its density at zero. The process is the same as in using the weight thermometer; that is, after having filled the reservoir with mercury at 0° C., we observe the weight w of the metal which issues at a given temperature t. The volume at 0° C. of the mercury which has issued, is $\frac{w}{d}$, d being the density of mercury at zero; the volume at $t°$ is therefore $\frac{w}{d}(1+mt)$, m being the coefficient of expansion of mercury. This volume evidently represents the expansion of the metal, *plus* that of the mercury, *minus* that of the glass. If then M denote the weight of mercury that fills the apparatus at 0° C., and if K be the coefficient of cubical expansion of glass, and x the expansion of unit volume of the given metal, we have the equation

$$\frac{w}{d}(1+mt)=\frac{W}{D}x+\frac{M}{d}mt-\left(\frac{W}{D}+\frac{M}{d}\right)Kt,$$

whence we can find x.

42. Convection of Heat in Liquids.—When different parts of a liquid or gas are heated to different temperatures, corresponding differences of density arise, leading usually to the formation of currents. This phenomenon is called *convection.*

Thus, for instance, if we apply heat to the bottom of a vessel containing water, the parts immediately subjected to the action of the heat expand and rise to the surface; they are replaced by colder portions, which in their turn are heated and ascend; and thus a continual circulation is maintained. The ascending and descending currents can be rendered visible by putting oak sawdust into the water.

43. Heating of Buildings by Hot Water.—This is a simple application of the principle just stated. One of the most common arrangements for this purpose is shown in Fig. 36 The boiler C is heated by a fire below it, and the products of combustion escape through the chimney A B. At the top of the house is a reservoir D, communicating with the boiler by a tube. From this reservoir the liquid flows into another reservoir E in the story immediately below, thence into another reservoir F, and so on. Finally, the last of these

reservoirs communicates with the bottom of the boiler. The boiler, tubes, and reservoirs are all completely filled with water, with the

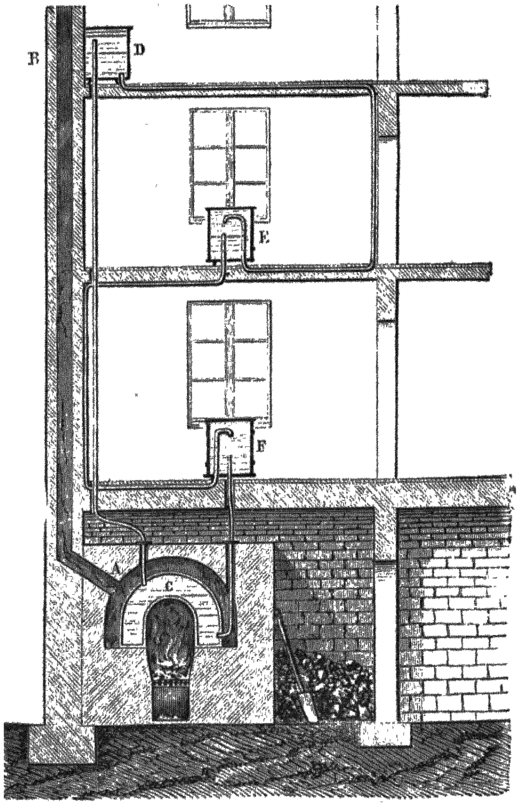


Fig. 36.—Heating by Hot Water

exception of a small space left above in order to give room for the expansion of the liquid. An ascending current flows through the left-hand tube, and the circulation continues with great regularity, so long as the temperature of the water in the boiler remains constant.

CHAPTER V.

EXPANSION OF GASES.

44. Experiments of Gay-Lussac.—Gay-Lussac conducted a series of researches on the expansion of gases, the results of which were long regarded as classical. He employed a thermometer with a large reservoir A, containing the gas to be operated on; an index of mercury mn separated the gas from the external air, while leaving it full liberty to expand. The gas had previously been dried by pass-

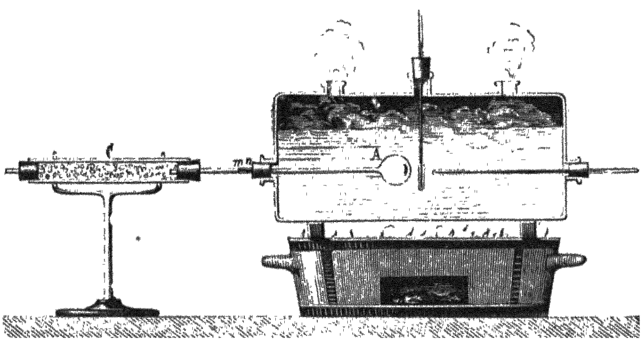


Fig. 37.—Gay-Lussac's Apparatus

ing it through a tube containing chloride of calcium, or some other desiccating substance. The thermometer was first placed in a vessel filled with melting ice, and when the gas had thus been brought to 0° C., the tube was so adjusted that the index coincided with the opening through which the thermometer passed.

The tube and reservoir having been previously gauged, and the former divided into parts of equal capacity, the apparent volume of the gas (expressed in terms of these divisions) is indicated by the position of the index; let the apparent volume observed at 0° C. be called n, and let H denote the external pressure as indicated by a

barometer. The apparatus is then raised to a known temperature t by means of the furnace below the vessel, and the stem of the thermometer is moved until the index reaches the edge of the opening. Let n' be the apparent volume of the gas at this new temperature, and as the external pressure may have varied, let it be denoted by H′. The real volumes of the gas will be as n to $n'(1+gt)$, where g denotes the mean coefficient of expansion of the glass; and the products of volume and pressure will be as $n\,\mathrm{H}$ to $n'(1+gt)\,\mathrm{H}'$. Hence if a denote the mean coefficient of expansion of the air, we have

$$n\,\mathrm{H}\,(1+at)=n'\,(1+gt)\,\mathrm{H}';$$

from which equation a can be determined.

By means of this method Gay-Lussac verified the law previously announced by Sir Humphry Davy for air, that the coefficient of expansion is independent of the pressure. He also arrived at the result that this coefficient is sensibly the same for all gases. He found its value for dry air to be ·00375. This result, which was for a long time the accepted value, is now known to be in excess of the truth. Rudberg, a Swedish philosopher, was the first to point out the necessity for using greater precautions to insure the absence of moisture, which adheres to the glass with great tenacity at the lower temperature, and, by going off into vapour when heated, adds to the volume of the air at the higher temperature. He found that the last traces of vapour could only be removed by repeatedly exhausting the vessel with an air-pump when heated, and refilling it with dried air. Another weak point in the method employed by Gay-Lussac was the shortness of the mercurial index, which, in conjunction with the fact that mercury does not come into close contact with glass (as proved by the fact of its not wetting it), allowed a little leakage in both directions. These imperfections have been remedied in later investigations, of which the most elaborate are those of Regnault. He employed four distinct methods, of which we shall only describe one.

45. Regnault's Apparatus.—The glass vessel BC (Fig. 38) containing the air to be experimented on, is connected with the T-shaped piece EI, the branch I of which communicates, through desiccating tubes, with an air-pump, and is hermetically closed with a blow-pipe after the vessel has received its charge of dry air; while the branch ED communicates with the top of a mercurial manometer. A mark is made at a point b in the capillary portion of the tube, and in every

observation the mercury in the manometer is made to reach exactly to this point, either by pouring in more mercury at the top M′ of the other tube of the manometer, or by allowing some of the liquid to escape through the cock R at the bottom. The air under experiment is thus always observed at the same apparent volume, and the observation gives its pressure. The vessel B is inclosed within a

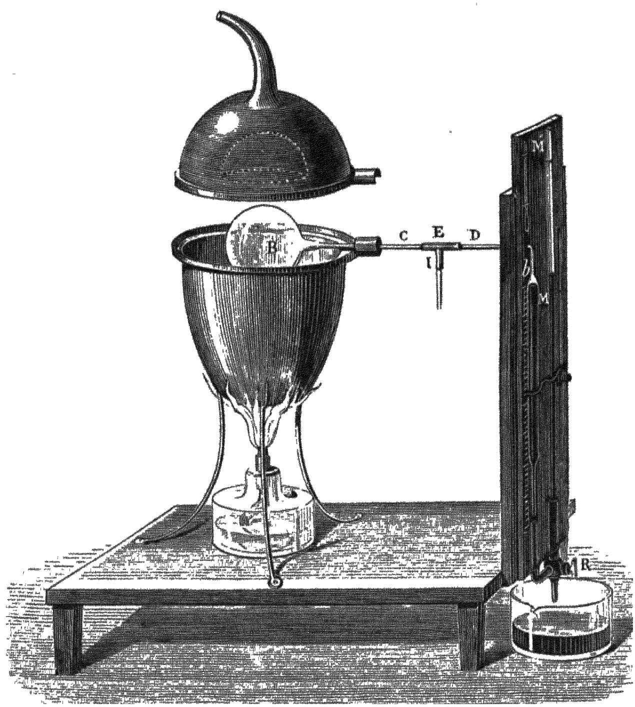


Fig 38 –Regnault's Apparatus.

boiler, which consists of an inner and an outer shell, with a space between them, through which the steam circulates when the water boils.

In reducing the observations, the portion of the glass vessel within the boiler is regarded as having the temperature of the water in the boiler, while the portion of the tube external to the boiler is regarded as having the temperature of the surrounding air.

In this mode of operating, the volume, or at least the apparent volume, is constant, so that the coefficient α which is determined is substantially defined by the equation

$$P_t = P_0 (1 + \alpha t),$$

P_0 and P_t denoting the pressures at constant volume. The coefficient thus defined should be called the *coefficient of increase of pressure.* It is often called the "coefficient of expansion at constant volume," which is a contradiction in terms.

In another mode of operating Regnault observed the expansion at constant pressure, and thus determined the *coefficient of expansion* properly so called. A small but steady difference was found between the two. If Boyle's law were exact they would be identical. As a matter of fact, the coefficient of increase of pressure was found, in the case of air and all gases except hydrogen, to be rather less than the coefficient of expansion. In other words, the product of volume and pressure at one and the same temperature $t°$ was found to be least when the volume was least; a result which accords with Regnault's direct observations on Boyle's law.

46. **Results.**—The following table contains the final results for the various gases which were submitted to experiment:—

	Coefficient of increase of pressure at constant volume	Coefficient of increase of volume at constant pressure
Air	0·003665	0·003670
Nitrogen	0·003668	
Hydrogen	0·003667	0·003661
Carbonic oxide	0·003667	0·003669
Carbonic acid	0·003688	0·003710
Nitrous oxide	0·003676	0·003720
Cyanogen	0·003829	0·003877
Sulphurous acid	0·003845	0·003903

It will be observed that the largest values of the coefficients belong to those gases which are most easily liquefied.

We may add that the coefficients increase very sensibly with the pressure; thus between the pressures of one and of three atmospheres the coefficient of expansion of air increases from ·00367 to ·00369. This increase is still more marked in the case of the more liquefiable gases.

47. **Reduction to the Fahrenheit Scale.**—The coefficient of expansion of any substance per degree Fahrenheit is $\frac{5}{9}$ of the coefficient per degree Centigrade; the volume at 32° F. being made the standard from which expansions are reckoned, so that if V_0 denote the volume at this temperature and V the volume at $t°$ F., the coefficient of expansion α is defined by the equation

$$V = V_0 \left\{ 1 + \alpha \, (t - 32) \right\}.$$

48. Air-thermometer—The close agreement between the expansions of different gases, and between the expansions of the same gas at different pressures, is a strong reason for adopting one of these bodies as the standard substance for the measurement of temperature by expansion, rather than any particular liquid.

Moreover, the expansion of gases being nearly twenty times as great as that of mercury, the expansion of the containing vessel will be less important; the apparent expansion will be nearly the same as the real expansion, and differences of quality in the glass will not sensibly affect the comparability of different thermometers.

Air-thermometers have accordingly been often used in delicate investigations. They consist, like other thermometers, of a reservoir and tube; but the latter, instead of being sealed, is left open. This open end, in one form of the instrument, is pointed downwards, and immersed in a liquid, usually mercury, which rises to a greater or less distance up the tube as the air in the thermometer contracts or expands. As variations of pressure in the surrounding air will also affect the height of this column of liquid, it is necessary to take readings of the barometer, and to make use of them in reducing the indications of the air-thermometer. Even if the barometer continues steady, it is still necessary to apply a correction for changes of pressure, since the difference between the pressure in the air-thermometer and that of the external air is not constant, but is proportional to the height of the column of liquid.

In the form of air-thermometer finally adopted by Regnault, the air in the instrument was kept at constant (apparent) volume, and its variations of pressure were measured, the apparatus employed being precisely that which we have described in § 45.

49. Perfect Gas.—In discussions relating to the molecular constitution of gases, the name *perfect gas* is used to denote a gas which would exactly fulfil Boyle's law; and molecular theories lead to the conclusion that for all such gases the coefficients of expansion would be equal. Actual gases depart further from these conditions as they are more compressed below the volumes which they occupy at atmospheric pressure; and it is probable that when very highly rarefied they approach the state of "perfect gases" very closely indeed.

50. Absolute Temperature by Air-thermometer.—*Absolute temperature by the air-thermometer* is usually defined by the condition that the temperature of a given mass of air at constant pressure is to be regarded as *proportional to its volume.* If the difference of

temperature between the two ordinary fixed points be divided into a hundred degrees, as in the ordinary Centigrade thermometer, the two fixed points themselves will be called respectively 273° and 373°; since air expands by $\frac{1}{273}$ of its volume at the lower fixed point for each degree, and therefore by $\frac{100}{273}$ of this volume for a hundred degrees.

There is some advantage in altering the definition so as to make the temperature of a given mass of air at constant volume *proportional to its pressure.* The two fixed points will then be 273° and 373° as above, and the zero of the scale will be that temperature at which the pressure vanishes.

The advantage of the second form of definition is that it enables us to continue our scale down to this point—called absolute zero—without encountering any physical impossibility, such as the conception of reducing a finite quantity of air to a mathematical point, which would be required according to the first form of definition.

Practically, "absolute temperatures by air-thermometer" are computed by adding 273 to ordinary "temperatures by air-thermometer," these latter being expressed on the Centigrade scale. We shall employ the capital letter T to denote absolute temperature, and the small letter t to denote ordinary temperature. We have

$$T = 273 + t,$$

and the general law connecting the volume, pressure, and temperature of a gas is

$$\frac{VP}{T} = \text{constant};$$

or, introducing the density D instead of the volume V,

$$\frac{P}{DT} = \text{constant}.$$

As above explained, these laws, though closely approximate in ordinary cases, are not absolutely exact.

51. Pyrometers.—The measurement of high temperatures such as those of furnaces is very difficult. Instruments for this purpose are called pyrometers. One of the best is the air-thermometer employed by Deville and Troost, having a bulb of hard porcelain.

52. Density of Gases.—The *absolute density* of a gas—that is, its mass per unit volume—which is denoted by D in the above formula, is proportional, as the formula shows, to $\frac{P}{T}$, and may therefore

undergo enormous variation. In stating the *relative density* of a gas as compared with air, the air and the gas are supposed to be at the same pressure and temperature. For purposes of great accuracy this pressure and temperature must be specified, since, as we have seen, there are slight differences in the changes produced in different gases by the same changes of pressure and temperature. The comparison is generally supposed to be made at the temperature 0° C., and at the pressure of one standard atmosphere.

53. Measurement of the Relative Density of a Gas.—The densities of gases have been the subject of numerous investigations; we shall describe only the method employed by Regnault.

The gas is inclosed in a globe, of about 12 litres capacity (Fig 39), furnished with a stop-cock leading to a three-way tube, one of whose branches is in communication with a manometer, and the other with an air-pump. The globe is exhausted several times, and each time the gas is dried on its way to the globe by passing through a number of tubes containing pieces of pumice-stone moistened with sulphuric acid. When all moisture has been removed, the globe is surrounded with melting ice, and is kept full of gas at the pressure of the atmosphere till sufficient time has been given for its contents to assume the temperature of the melting ice. The stop-cock is then closed, the globe is taken out, carefully dried, and allowed to take the temperature of the atmosphere. It is then weighed with a delicate balance.

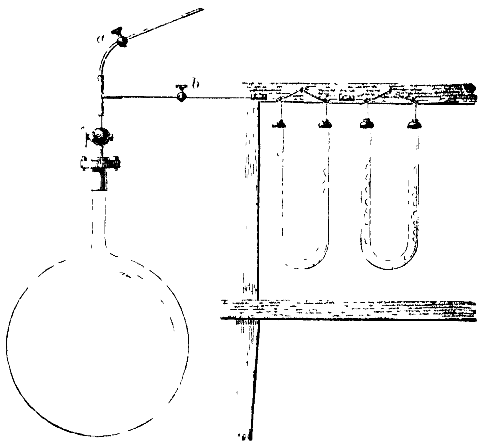


Fig 39 —Measurement of Density of Gases

The experiment is repeated, with no change except that by means of the air-pump the gas in the globe is reduced to as small a pressure as possible. Let this pressure be denoted by h, and the atmospheric pressure in the previous experiment by H. Then the difference of the two weights is the weight of as much gas at temperature 0° and pressure $\mathrm{H}-h$ as would fill the globe. Let w denote this difference, and let w' be the difference between two weighings made in the same

manner with dry air in the globe at pressures H′ and h'. Then the relative density of the gas will be

$$\frac{w}{w'}\,\frac{H'-h'}{H-h}.$$

We must now describe a special precaution which was employed by Regnault (and still earlier by Dr. Prout) to avoid errors in weighing arising from the varying weight of the external air displaced by the globe.

A second globe (Fig. 40) of precisely the same external volume as the first, made of the same glass, and closed air-tight, was used as a counterpoise. The equality of external volumes was ensured in the following way. The globes were filled with water, hung from the two scales of a balance, and equilibrium was brought about by putting a sufficient quantity of some material into one scale. Both globes, thus hanging from the scales in equilibrium, were then immersed in water, and if this operation disturbed the equilibrium it was known that the external volumes were not equal. Let p be the weight which must be put into one scale to restore equilibrium; then this weight of water represents the difference of the two external volumes; and the next operation was to prepare a small piece of glass tube closed at the ends which should lose p when weighed in water. The larger of the two globes was used for containing the gases to be weighed, and the smaller globe along with this piece of tube constituted the counterpoise. Since the volume of the gas globe was exactly the same as that of the counterpoise, the pressure of the external air had no tendency to make either preponderate, and variations in the condition of this air, whether as regards pressure, temperature, or humidity, had no disturbing effect.

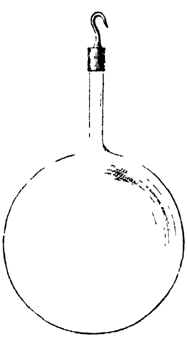


Fig. 40
Compensating Globe.

54. **Absolute Densities.**—In order to convert the preceding relative determinations into absolute determinations, it is only necessary to know the precise internal volume of the globe at the temperature 0° C. In order to determine this with the utmost possible exactness the following operations were performed.

The globe was first weighed in air, with its stop-cock open, the temperature of the air and the height of the barometer being noted.

It was then filled with water, special precautions being taken to expel every particle of air; and was placed for several hours in the midst of melting ice, to insure its being filled with water at 0° C.

The stop-cock was then closed, and the globe was left for two hours in a room which had a very steady temperature of 6°. It was then weighed in this room, the height of the barometer being at the same time observed. The difference between this weight and that of the globe before the introduction of the water, was the weight of the water *minus* the weight of the same volume of air, subject to a small correction for change of density in the external air between the two weighings, which, with the actual heights of the barometer and thermometer, was insensible.

The weight of water at 0° which the globe would hold at 0° was therefore known; and hence the weight of water at 4° (the temperature of maximum density) which the globe would hold at 0° was calculated, from the known expansion of water. This weight, in grammes, is equal to the capacity in cubic centimetres.

The result thus obtained was that the capacity of the globe at 0° was 9881 cubic centimetres; and the weight of the dry air which filled it at 0° and a pressure of 760^{mm} was 12·778 grammes. Hence the weight (or mass) of 1 cubic centimetre of such air is ·0012932 gramme.

This experiment was performed at Paris, where the value of g (the intensity of gravity) is 980·94; and since the density of mercury at 0° is 13·596, the pressure of 76 centimetres of mercury was equivalent to

$$76 \times 13{\cdot}596 \times 980{\cdot}94 = 1{\cdot}0136 \times 10^6$$

dynes per square centimetre.

If we divide the density just found by 1·0136, we obtain the density of air at 0° and a pressure of a million dynes per square centimetre, which is a convenient standard for general reference; we have thus

$$\cdot 0012932 \div 1{\cdot}0136 = \cdot 0012759.$$

A litre or cubic decimetre contains 1000 cubic centims. Hence the weight of a litre of air in the standard condition adopted by Regnault is 1·2932 gramme.

The following table gives the densities of several gases at 0° C. at a pressure of 760 millimetres of mercury at Paris.

Name of Gas.	Relative Density.		Mass of a Litre in Grammes.
Air	1	......	1·2932
Oxygen	1·10563	......	1·4298
Hydrogen	·06926	......	·08957
Nitrogen	·97137	......	1·25615
Chlorine	2·4216	......	3·1328
Carbonic oxide	·9569	......	1·2344
Carbonic acid	1·52901	......	1·9774
Protoxide of nitrogen	1·5269	......	1·9697
Binoxide of nitrogen	1·0388	......	1·3434
Sulphurous acid	2·1930	......	2·7289
Cyanogen	1·8064	......	2·3302
Marsh-gas	·559	......	·727
Olefiant gas	·985	......	1·274
Ammonia	·5967	......	7697

55. **Draught of Chimneys.**—The expansion of air by heat produces the upward current in chimneys, and an approximate expression for the velocity of this current may be obtained by the application of Torricelli's theorem on the efflux of fluids from orifices (see Part I.).

Suppose the chimney to be cylindrical and of height h. Let the air within it be at the uniform temperature t' Centigrade, and the external air at the uniform temperature t. According to Torricelli's theorem, the square of the linear velocity of efflux is equal to the product of $2g$ into the head of fluid, the term *head of fluid* being employed to denote the *pressure* producing efflux, *expressed in terms of depth of the fluid.*

In the present case this head is the difference between h, which is the height of air within the chimney, and the height which a column of the external air of original height h would have if expanded upwards, by raising its temperature from t to t'. This latter height is $h\,\frac{1+at'}{1+at}$; a denoting the coefficient of expansion ·00366; and the head is

$$h\frac{1+at'}{1+at} - h = \frac{ha\,(t'-t)}{1+at}.$$

Hence, denoting by v the velocity of the current up the chimney, we have

$$v^2 = \frac{2gha\,(t'-t)}{1+at}.$$

This investigation, though it gives a result in excess of the truth, from neglecting to take account of friction and eddies, is sufficient to explain the principal circumstances on which the strength of draught

depends. It shows that the draught increases with the height h of the chimney, and also with the difference $t'-t$ between the internal and external temperatures.

The draught is not so good when a fire is first lighted as after it has been burning for some time, because a cold chimney chills the

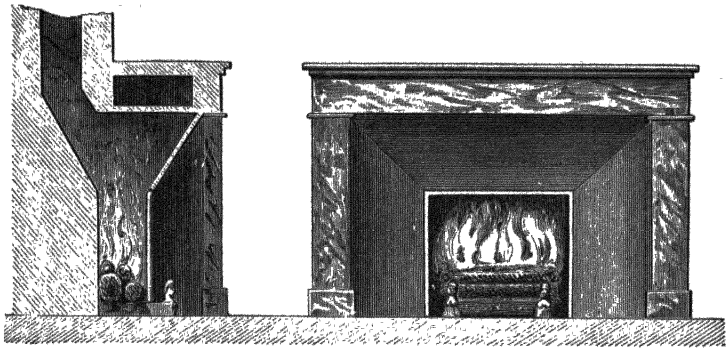


Fig 41.—Rumford's Fireplace.

air within it. On the other hand, if the fire is so regulated as to keep the room at the same temperature in all weathers, the draught will be strongest when the weather is coldest.

The opening at the lower end of the chimney should not be too wide nor too high above the fire, as the air from the room would then enter it in large quantity, without being first warmed by passing through the fire. These defects prevailed to a great extent in old chimneys. Rumford was the first to attempt rational improvements. He reduced the opening of the chimney and the depth of the fireplace, and added polished plates inclined at an angle, which serve both to guide the air to the fire and to reflect heat into the room (Fig. 41).

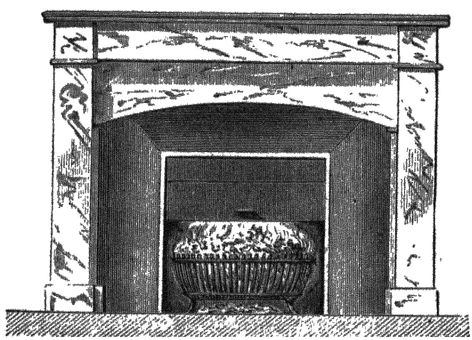


Fig 42 —Fireplace with Blower.

The blower (Fig. 42) produces its well-known effects by compelling all air to pass through the fire before entering the chimney This at once improves the draught of the chimney by raising the

temperature of the air within it, and quickens combustion by increasing the supply of oxygen to the fuel.

56. Stoves.—The heating of rooms by open fireplaces is effected almost entirely by radiation, and much even of the radiant heat is wasted. This mode of heating then, though agreeable and healthful, is far from economical. Stoves have a great advantage in point of economy; for the heat absorbed by their sides is in great measure given out to the room, whereas in an ordinary fireplace the greater part of this heat is lost. Open fireplaces have, however, the advantage as regards ventilation; the large opening at the foot of the chimney, to which the air of the room has free access, causes a large body of air from the room to ascend the chimney, its place being supplied by fresh air entering through the chinks of the doors and windows, or any other openings which may exist.

Stoves are also liable to the objection of making the air of the room too dry, not, of course, by removing water, but by raising the temperature of the air too much above the dew-point (Chap. xi.). The same thing occurs with open fireplaces in frosty weather, at which time the dew-point is unusually low. This evil can be remedied by placing a vessel of water on the stove. The reason why it is more liable to occur with stoves than with open fireplaces, is mainly that the former raise the air in the room to a higher temperature than the latter, the defect of air-temperature being in the latter case compensated by the intensity of the direct radiation from the glowing fuel.

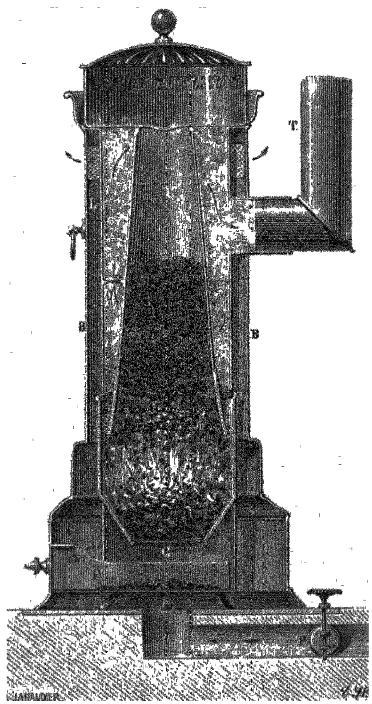


Fig. 43 —Ventilating Stove.

Fire-clay, from its low conducting power, is very serviceable both for the backs of fireplaces and for the lining of stoves. In the former situation it prevents the wasteful escape of heat backwards into the chimney, and keeps the back of the fire nearly as hot as the centre.

As a lining to stoves, it impedes the lateral escape of heat, thus answering the double purpose of preventing the sides of the stove from overheating, and at the same time of keeping up the temperature of the fire, and thereby promoting complete combustion. Its use must, however, be confined to that portion of the stove which serves as the fire-box, as it would otherwise prevent the heat from being given out to the apartment.

The stove represented in Fig. 43 belongs to the class of what are called in France *calorifères*, and in England *ventilating stoves*, being constructed with a view to promoting the circulation and renewal of the air of the apartment. G is the fire-box, over which is the feeder U, containing unburned fuel, and tightly closed at top by a lid, which is removed only when fresh fuel is to be introduced. The ash-pan F has a door pierced with holes for admitting air to support combustion. The flame and smoke issue at the edge of the fire-box, and after circulating round the chamber O which surrounds the feeder, enter the pipe T which leads to the chimney. The chamber O is surrounded by another inclosure L, through which fresh air passes, entering below at A, and escaping into the room through perforations in the upper part of the stove as indicated by the arrows. The amount of fresh air thus admitted can be regulated by the throttle-valve P.

CHAPTER VI.

CALORIMETRY.

57. **Quantity of Heat.**—We have discussed in previous chapters the measurement of temperature, and have seen that it is to a great extent arbitrary, since intervals of temperature which are equal as measured by the expansion of one substance are not equal as measured by the expansion of another.

The measurement of quantities of heat stands upon an entirely different footing. There is nothing arbitrary or conventional in asserting the equality or inequality of two quantities of heat.

58. **Principles Assumed.**—The two following principles may be regarded as axiomatic.

(1) The heat which must be given to a body to raise it through a given range of temperature at constant pressure, is equal to that which the body gives out in falling through the same range of temperature under the same pressure. For instance, the heat which must be given to a gramme of water, to raise its temperature from 5° to 10°, is equal to that which is given out from the same water when it falls from 10° to 5°.

(2) In a homogeneous substance equal portions require equal quantities of heat to raise them from the same initial to the same final temperature; so that, for example, the heat required to raise two grammes of water from 5° to 10° is double of that which is required to raise one gramme of water from 5° to 10°.

59. **Cautions.**—We are not entitled to assume that the quantities of heat required to raise a given body through equal intervals of temperature—for example, from 5° to 10°, and from 95° to 100°—are equal. Indeed we have already seen that the equality of two intervals of temperature is to a considerable extent a matter of mere

convention; temperature being conventionally measured by the expansion (real or apparent) of some selected substance.

It would, however, be quite possible to adopt a scale of temperature based on the elevation of temperature of some particular substance when supplied with heat. We might, for instance, define a degree (at least between the limits 0° and 100°) as being the elevation of temperature produced in water of any temperature by giving it one hundredth part of the heat which would be required to raise it from 0° to 100°.

Experiments which will be described later show that if air or any of the more permanent gases were selected as the standard substance for thus defining equal intervals of temperature, the scale obtained would be sensibly the same as that of the air-thermometer; and the agreement is especially close when the gases are in a highly rarefied condition.

60. Unit of Heat.—We shall adopt as our unit, in stating quantities of heat, the heat required to raise a gramme of cold water through one degree Centigrade. This unit is called, for distinction, the gramme degree. The kilogramme-degree and the pound-degree are sometimes employed, and are in like manner defined with reference to cold water as the standard substance.

There is not at present any very precise convention as to the temperature at which the cold water is to be taken. If we say that it is to be within a few degrees of the freezing-point, the specification is sufficiently accurate for any thermal measurements yet made.

61. Thermal Capacity.—If a quantity Q of heat given to a body raises its temperature from $t_1°$ to $t_2°$, the quotient

$$\frac{Q}{t_2 - t_1}$$

of the quantity of heat given by the rise of temperature which it produces, is called the *mean thermal capacity* of the body between the temperatures $t_1°$ and $t_2°$.

As t_2 is brought nearer to t_1, so as to diminish the denominator, the numerator Q will also diminish, and in general very nearly in the same proportion. The limit to which the fraction approaches as t_2 is brought continually closer to t_1 is called the *thermal capacity* of the body at the temperature $t_1°$. That is, in the language of the differential calculus, the thermal capacity at $t°$ is $\frac{dQ}{dt}$.

From the way in which we have defined our unit of heat, it fol-

lows that the thermal capacity of any quantity of cold water is numerically equal to its mass expressed in grammes; and that the number which expresses the thermal capacity of any body may be regarded as expressing the quantity of water which would receive the same rise of temperature as the body from the addition of the same quantity of heat. This quantity of water is often called the *water-equivalent* of the body.

62. **Specific Thermal Capacities.**—The thermal capacity of unit mass of a substance is called the *specific heat* of the substance; and it is always to be understood that the same unit of mass is employed for the substance as for the water which is mentioned in the definition of the unit of heat. Specific heat is therefore independent of units, and merely expresses the ratio of the two quantities of heat which would raise equal masses of the given substance and of cold water through the same small difference of temperature. Or we may regard it as the ratio of two masses, the first, of cold water, and the second of the substance in question, which have the same thermal capacity.

There is another specific thermal capacity which it is often necessary to consider, namely, the *thermal capacity of unit volume* of a substance. It has not received any brief name. It is equal to the mass of unit volume multiplied by the thermal capacity of unit mass; in other words, it is equal to the *product of the density and the specific heat* of the substance.

It is evident, from what precedes, that the heat required to raise m grammes of a substance through t degrees is mst, where s denotes the mean specific heat between the initial and the final temperature; and the same expression denotes the quantity of heat which the body in question loses in cooling down through t degrees.

63. **Method of Mixtures.**—Let m_1 grammes of a substance of specific heat s_1 and temperature $t_1°$ be mixed with m_2 grammes of a substance of specific heat s_2 and temperature $t_2°$, the mixture being merely mechanical, so that no heat is generated or absorbed by any action between the substances, and all external gain or loss of heat being prevented. Then the warmer substance will give heat to the colder, until they both come to a common temperature, which we will denote by t. The warmer substance, which we will suppose to be the former, will have cooled down through the range $t_1 - t$, and will have lost $m_1 s_1 (t_1 - t)$ units of heat. The colder substance will have risen through the range $t - t_2$, and will have gained $m_2 s_2 (t - t_2)$

units of heat. These two expressions represent the same thing, namely, the heat given by the warmer body to the colder. We may therefore write

$$m_1 s_1 (t_1 - t) = m_2 s_2 (t - t_2), \tag{1}$$

that is,

$$m_1 s_1 t_1 + m_2 s_2 t_2 = (m_1 s_1 + m_2 s_2)\, t, \tag{2}$$

whence

$$t = \frac{m_1 s_1 t_1 + m_2 s_2 t_2}{m_1 s_1 + m_2 s_2}. \tag{3}$$

If there are more than two components in the mixture, similar reasoning will still apply; thus, if there are three components, the resulting temperature will be

$$t = \frac{m_1 s_1 t_1 + m_2 s_2 t_2 + m_3 s_3 t_3}{m_1 s_1 + m_2 s_2 + m_3 s_3}. \tag{4}$$

Strictly speaking, s_1 in these formulæ denotes the *mean* specific heat of the first substance between the temperatures t_1 and t, s_2 the mean specific heat of the second substance between t_2 and t, and so on.

It is not necessary to suppose the two bodies to be literally *mixed.* One of them may be a solid and the other a liquid in which it is plunged. The formulæ apply whenever bodies at different temperatures are reduced to a common temperature by interchange of heat one with another.

64. Practical Application.—The following is an outline of the method most frequently employed for determining the specific heats of solid bodies.

The body to be tested is raised to a known temperature t_1, and then plunged into water of a known temperature t_2 contained in a thin copper vessel called a *calorimeter.* If m_1 be the mass of the body, m_2 that of the water before immersion, and t the final temperature, all of which are directly observed, we have

$$m_1 s_1 (t_1 - t) = m_2 (t - t_2), \tag{5}$$

since s_2, the specific heat of the water, may be taken as unity. Hence we have

$$s_1 = \frac{m_2 (t - t_2)}{m_1 (t_1 - t)}. \tag{6}$$

65. Corrections.—The theoretical conditions which are assumed in the above calculation, cannot be exactly realized in practice.

I. The calculation assumes that the only exchange of heat is between the body and the water, which is not actually the case; for

1. The body is often contained in an envelope which cools along with it, and thus furnishes part of the heat given up.

2. The heat is not given up exclusively to the water, but partly to the calorimeter itself, to the thermometer, and to such other instruments as may be employed in the experiment, as, for instance, a rod to stir the liquid for the purpose of establishing uniformity of temperature throughout it.

In order to take account of these disturbing circumstances, it is only necessary to know the thermal capacity of each of the bodies which takes part in the exchange of heat. We shall then have such an equation as the following:—

$$(m_1 s_1 + c_1)(t_1 - t) = (m_2 + c_2 + c_3 + c_4)(t - t_2),$$

where c_1 denotes the thermal capacity of the envelope, and c_2, c_3, c_4 are the thermal capacities of the calorimeter, thermometer, and stirring rod.

II. The calorimeter gives out heat to the surrounding air, or takes heat from it. This difficulty is often met by contriving that the heat gained by the calorimeter from the air in the first part of the experiment shall be as nearly as possible equal to that which it loses to the air in the latter part.

This condition will be fulfilled if the average temperature of the calorimeter (found by taking the mean of numerous observations at equal small intervals of time) is equal to the temperature of the air. As the immersed body gives out its heat to the water very rapidly at first, and then by degrees more and more slowly, the initial defect of temperature must be considerably greater than the final excess, to make the compensation exact.

Instead of attempting exact compensation, some observers have determined, by a separate experiment, the rate at which interchange of heat takes place between the calorimeter and the air, when there is a given difference of temperature between them. This can be observed by filling the calorimeter with water in which a thermometer is immersed. The rate of interchange is almost exactly proportional to the difference of temperature between the calorimeter and the air, and is independent of the nature of the contents. The law of interchange having thus been determined, the temperature of the calorimeter must be observed at stated times during the progress of the experiment on specific heat; the total heat lost or gained by interchange with the air will thus be known, and this total heat divided by the total thermal capacity of the calorimeter and its contents gives a correction, which is to be added to or subtracted from t the observed final temperature.

66. Regnault's Apparatus.—The subject of specific heat has been investigated with great care by Regnault, who employed for that purpose an apparatus in which the advantages of convenience and precision are combined. The body whose specific heat is required is divided into small fragments, which are placed in a cylindrical basket G (Fig. 44) of very fine brass wire, in the centre of which

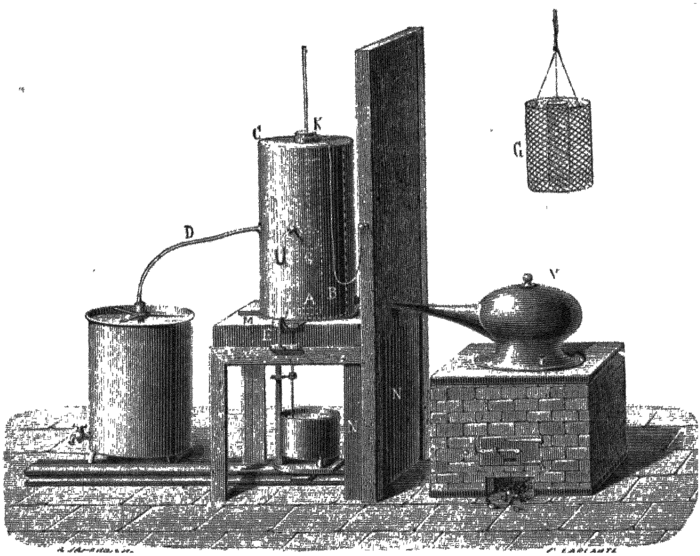


Fig. 44.—Regnault's Apparatus.

is a tube of the same material for the insertion of a thermometer. The basket is shown separately in the figure on a larger scale than the rest of the apparatus. This basket is suspended in the central compartment of the steamer A B C, the suspending thread being fixed by the cork K, through which the stem of the thermometer passes. The steamer consists of three concentric cylinders, the two outer compartments being occupied by steam, which is supplied from the boiler V to the second compartment, and finally escapes from the outermost compartment through the tube D into a condenser. In the bottom of the steamer are a pair of slides E which can be drawn out when required.

The steamer rests, by means of a sheet of cork, upon a hollow metal vessel M N, consisting of a horizontal portion M and a vertical portion N, filled with cold water, and serving as a screen for the calorimeter; the horizontal portion, and the cork above it,

having a hole in the centre large enough for the basket G to pass through.

The calorimeter itself, which is shown beneath the steamer in the figure, is a vessel of very thin polished brass, resting by three points upon a small wooden sled, which runs smoothly along a guiding groove. The thermometer for measuring the temperature of the water in the calorimeter, is carried by a support attached to the sled.

The basket, with its contents, is left in the steamer until the temperature indicated by the thermometer has been for some time stationary. The calorimeter, which, up to this time has been kept as far away as it can slide, is then pushed into the position shown in the figure, the slides E, which close the bottom of the compartment in which the basket is, are drawn out; and the cork at the top having been loosened, the basket is lowered by its supporting thread into the calorimeter, which is immediately slid back to its former place. The basket is then moved about in it until the water attains its maximum temperature, which is read off on the thermometer.

To determine the specific heats of liquids, a thin glass tube is employed instead of the basket. It is nearly filled with the liquid and hermetically sealed.

For solids which are soluble in water, or upon which water has a chemical action, some other liquid—oil of turpentine, for example—is placed in the calorimeter, instead of water; and the experiment is in other respects the same.

The specific heats of several substances are given in the following table:—

Water,	1·00000		
Solids.			
Antimony,	0·05077	Brass,	0·09391
Silver,	0·05601	Nickel,	0·10860
Arsenic,	0·08140	Gold,	0·03244
Bismuth,	0·03084	Phosphorus,	0·18870
Cadmium,	0·05669	Platinum,	0·03243
Charcoal,	0·24150	Lead,	0·03140
Copper,	0·09215	Plumbago,	0·21800
Diamond,	0·14680	Sulphur,	0·20259
Tin,	0·05623	Glass,	0·19768
Iron,	0·11379	Zinc,	0·09555
Iodine,	0·05412	Ice,	0·5040
Liquids.			
Mercury,	0·03332	Benzine,	0·3952
Acetic acid,	0·6589	Ether,	0·5157
Alcohol at 36°,	0·6735	Oil of turpentine,	0·4629

67. Great Specific Heat of Water.—This table illustrates the important fact, that, of all substances, water has the greatest specific heat; that is to say, it absorbs more heat in warming, and gives out more heat in cooling, through a given range of temperature, than an equal weight of any other substance. The quantity of heat which raises a pound of water from 0° to 100° C. would suffice to raise a pound of iron from 0° to about 900° C, that is to a bright red heat, and conversely, a pound of water in cooling from 100° to 0°, gives out as much heat as a pound of iron in cooling from 900° to 0°. This property of water is utilized in the heating of buildings by hot water, and in other familiar instances, such as the bottles of hot water used for warming beds, and railway foot-warmers.

68. Ice Calorimeters.—In the calorimeters above described, the heat which a body loses in cooling is measured by the elevation of temperature which it produces in a mass of water. In ice calorimeters this heat is measured by the quantity of ice (initially at the freezing-point) which it melts. In some ice calorimeters the water produced by melting is collected and weighed; in Bunsen's, the measurement depends upon the diminution of volume which occurs when ice melts.

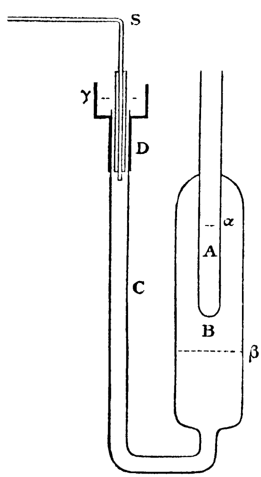


Fig. 45 —Bunsen's Calorimeter

The construction of this instrument is shown in Fig 45. The small thin test-tube A, open to the air at the top, is sealed into the reservoir B, which communicates with the tube C having at its end a movable plug D, through which passes the vertical portion of a fine tube S, whose other portion is horizontal and may be of considerable length. The small body whose specific heat is required is heated, and dropped into ice-cold water which stands at the level α in A. The surrounding space in the reservoir, from the level β to the top, contains ice and ice-cold water, and some of the ice is melted by the heat emitted from the body in A, the whole apparatus being kept immersed in a mixture of ice and water to prevent loss or gain of heat externally. The diminution in the joint volume of water and ice in B produced by the melting is indicated by the movement of the end of a mercurial column in the horizontal part of S, the whole space from B to S being occupied by mercury.

To prevent the formation of air bubbles in the reservoir B, it is filled with boiling water, which is afterwards frozen. The mercury should also be boiled to expel air, and before its introduction the tubes should be thoroughly dried.

The graduation of the tube S on which the indications are read may be arbitrary, and the value of the divisions is determined by observing the effect of a known mass of water at a known temperature when introduced into A. By means of the sliding plug D, the end of the column can be brought to a convenient part of the tube before each experiment.

69. Specific Heats of Gases.—Regnault made very careful determinations of the specific heats of air and other gases, by means of an apparatus in which a measured quantity of gas at a known temperature was passed through a series of spiral tubes surrounded by cold water, and finally escaped at a temperature sensibly the same as that of the water. The elevation produced in the temperature of the water by this process, furnished a measure of the quantity of heat given out by the gas in falling through a known range of temperature. The gas had sensibly the same pressure on entering as on leaving the calorimeter; the specific heat determined by the experiments was therefore the specific heat *at constant pressure.* This element must be carefully distinguished from the specific heat of a gas *at constant volume.* The connection between the two will be discussed in a later chapter.

Regnault's experiments established the following conclusions.

(1) The specific heat of a gas is the same at all pressures; in other words, the thermal capacity per unit volume is directly as the density.

(2) The specific heats of different simple gases are approximately in the inverse ratio of their relative densities.

Let s denote the specific heat and d the absolute density of a gas at a given pressure and temperature; then this law asserts that the product sd is the same (approximately) for all simple gases. But since d is the mass of unit volume, sd is the capacity of unit volume. The law may therefore be thus expressed:—

All simple gases have approximately *the same thermal capacity per unit volume,* when compared at the same pressure and temperature.

(3) The specific heat of a gas is the same at all temperatures, temperature being measured by the air-thermometer, or by the expansion

of the gas itself at constant pressure. This is equivalent to the assertion that *if equal quantities of heat be successively added to a gas at constant pressure, the volume of the gas will increase in arithmetical progression.* We here neglect the slight differences which exist between the expansions of different gases, and also their slight departures from Boyle's law.

The specific heat of dry air (at constant pressure) according to Regnault is ·2375.

The three laws above stated are also true for the specific heat of gases *at constant volume.* The third law may then be stated in the following form:—

If equal quantities of heat be successively added to a gas at constant volume, the pressure will increase in arithmetical progression.

70. Dulong and Petit's Law.—According to the modern molecular theory of gases, all simple gases at the same pressure and temperature have the same number of atoms per unit volume. The mass of an atom of any gas will therefore be proportional to the relative density of the gas, and law (2) of last section will reduce to the following: —The specific heats of different simple gases are inversely as the masses of their atoms.

The second statement of the same law assumes the following still more simple form:—

An atom of one gas has the same thermal capacity as an atom of any other gas.

What is called in chemistry the *atomic weight* of an elementary substance is proportional to the supposed mass of an atom of the substance, and is believed to be proportional to the relative density of the substance when reduced to a state of vapour at high temperature and low pressure.

It was remarked by Dulong and Petit that the specific heats of elementary substances are for the most part in the inverse ratio (approximately) of their atomic weights; or the *product of specific heat and atomic weight is* (approximately) *constant.* The constancy is very rough when the specific heats are taken at ordinary temperatures; but it is probable that at very high temperatures the law would be nearly exact.

71. Method of Cooling.—Attempts have sometimes been made to compare the specific heats of different substances by means of the times which they occupy in cooling through the same range. If

two exactly similar thin metallic vessels are filled with two different substances, and after being heated to a common temperature are allowed to cool in air under the same conditions, the times which they occupy in falling to any other common temperature will be proportional to the quantities of heat which they emit, if we can assume that the contents of the vessels are at sensibly the same temperatures as their surfaces. We have thus a comparison of the thermal capacities of the two substances per unit volume.

In the case of solid substances, their differences in conducting power render the method worthless; but Regnault has found that it gives tolerably correct results in the case of liquids. In fact the extreme mobility of liquids, combined with their expansion when heated, prevents any considerable difference of temperature from existing in the same horizontal layer; so that the centre is sensibly at the same temperature as the circumference.

CHAPTER VII.

FUSION AND SOLIDIFICATION.

72. Fusion.—Many solid bodies, when raised to a sufficiently high temperature, become liquid. This change of state is called *melting* or *fusion*, and the temperature at which it occurs (called the melting-point, or temperature of fusion) is constant for each substance, with the exception of the variations—which in ordinary circumstances are insignificant—due to differences of pressure (§ 86). The melting-points of several substances are given in the following table:—

TABLE OF MELTING-POINTS, IN DEGREES CENTIGRADE

Mercury,	-39	Tin,	230
Ice,	0	Bismuth,	262
Butter,	33	Lead,	326
Lard,	33	Zinc,	412
Spermaceti,	49	Antimony,	432
Stearine,	55	Aluminium	600
Yellow Wax,	62	Bronze,	900
White Wax,	68	Pure Silver,	954
Stearic Acid	70	Gold,	1045
Phosphorus,	44	Copper,	1054
Potassium,	63	Cast Iron,	1050 to 1250
Sodium,	95	Steel,	1300 to 1400
Iodine,	107	Wrought Iron,	1500 to 1600
Sulphur,	110	Platinum,	1775

Some bodies, such as charcoal, have hitherto resisted all attempts to reduce them to the liquid state; but this is to be attributed only to the insufficiency of the means which we are able to employ.

It is probable that, by proper variations of temperature and pressure, all simple substances, and all compound substances which would not be decomposed, could be compelled to assume the three forms, solid, liquid, and gaseous.

The passage from the solid to the liquid state is generally abrupt;

but this is not always the case. Glass, for instance, before reaching a state of perfect liquefaction, passes through a series of intermediate stages in which it is of a viscous consistency, and can be easily drawn out into exceedingly fine threads, or moulded into different shapes.

73. **Definite Temperature.**—When the solid and liquid forms of a substance are present in contact with each other in the same vessel, and time is allowed for uniformity of temperature to be established; the temperature will be that of the melting-point, and will be quite independent of the relative proportions of solid and liquid in the vessel. For example, water and ice, in any proportions, if brought to a uniform temperature, will be at 0° C.

It is sometimes stated that, if heat be applied to a vessel containing ice and water, the temperature of the contents will remain at 0° C. till all the ice is melted; but this statement is not strictly accurate. The portions of the water in contact with the sides and receiving heat from the sides, will be at a somewhat higher temperature than the portions in contact with the ice. If, however, the application of heat be stopped, and uniformity of temperature be established through the whole mass, by stirring or otherwise, the temperature of the whole will then be 0° C.

For each substance that passes, like ice, by a sudden transition, from the solid to the liquid state, without an intermediate pasty condition, there is one definite temperature at which the solid and the liquid forms can exist in contact under atmospheric pressure. This temperature is variously styled the *temperature of fusion*, the *melting-point*, and the *freezing-point.*

74. **Latent Heat of Fusion.**—Although the solid and liquid forms of a substance can exist together at the same temperature, the application of heat is requisite for reducing the solid to the liquid form. If ice at 0° C. be put into a vessel and placed on the fire, it will be gradually melted by the heat which it receives from the fire; but at any time during the operation, if we stop the application of heat, and stir the contents till uniformity of temperature is established, the temperature will be 0° C. as at first. The heat which has been received has left its effect in the shape of the melting of ice, not in the shape of rise of temperature. Heat thus spent is usually called *latent heat,* a name introduced by Black, who was the first to investigate this subject. A similar absorption of heat without rise of temperature occurs when a boiling liquid is converted into vapour. Hence it is necessary to distinguish between the *latent heat of fusion*

and the *latent heat of vaporization.* The former is often called the latent heat *of the liquid,* and the latter *of the vapour.* Thus we speak of the latent heat of water (which becomes latent in the melting of ice), and of the latent heat of steam (which becomes latent in the vaporization of water).

The same amount of heat which is absorbed in the conversion of the solid into the liquid, is given out when the liquid is converted into the solid, and a similar remark applies to the conversion of vapour into liquid.

75. **Measurement of Heat of Fusion.**—The heat required *to convert unit mass* of a substance from the solid to the liquid form is employed as the measure of the latent heat of liquefaction of that substance. Its amount for several substances is given in the last column of the following table:—

Substances.	Melting-point	Specific Heats		Latent Heat of Fusion
		In the Solid State	In the Liquid State	
Ice, ...	0°	5040	1 0000	79·250
Phosphorus, .	44 20	·2000	2000	5 400
Sulphur, . . .	111	2020	·2340	9 368
Bromine, . . .	− 7 32	0840	1670	16 185
Tin, ...	232	0560	0640	14 252
Bismuth, . . .	266	·0308	0363	12 640
Lead, ...	326	0314	·0402	5·369
Mercury, . . .	− 39	·0319	·0333	2 820

The most accurate determinations of latent heat of fusion have been made by a method similar to the "method of mixtures" which is employed in the determination of specific heats.

Let i grammes of ice at 0° be mixed with w grammes of water at $t°$, and when all the ice is melted let the temperature of the whole be $\theta°$. Then if the specific heat of water at all temperatures between 0° and $t°$ can be taken as unity, we have $w(t-\theta)$ units of heat lost by the w grammes of water, and spent partly in melting the ice, and partly in raising the temperature of the water produced by the melting from 0° to $\theta°$. Hence if x denote the latent heat of liquefaction, we have

$$w(t-\theta)=i(x+\theta);$$

whence we find

$$x=\frac{wt}{i}-\frac{w+i}{i}\theta.$$

One gramme of water at between 79° and 80°, or between 79 and 80

grammes of water at 1°, will be just enough to melt one gramme of ice at 0°; and the final temperature of the whole will in each case be 0°.

For any other substance, let T° be the melting-point, s the specific heat of the substance in the liquid form, and x the latent heat of liquefaction. Then if i grammes of the solid at T° be mixed with w grammes of the liquid at t°, and θ° be the temperature of the whole when all the solid is melted, we have

$$s w (t - \theta) = ix + i(\theta - \mathrm{T});$$

whence

$$x = \frac{w}{i} s (t - \theta) - (\theta - \mathrm{T}).$$

In these calculations we have tacitly assumed that no heat is gained or lost externally by the substance under examination. Practically, it is necessary (as in the determination of specific heats) to take account of the thermal capacity of the calorimeter (that is the vessel in which the substance is contained) and of the heat gained or lost by the calorimeter to surrounding bodies. For substances which have a high melting-point, a different method may be employed. The body in the molten state may be inclosed in a small thin metal box and immersed in the water of the calorimeter. Let m be the mass of the body, T′ its initial temperature, T its melting-point, s' its specific heat in the liquid, and s in the solid state, θ the final temperature of the calorimeter, and x the latent heat of the substance, which is required; then the heat lost by the body is

$$m s' (\mathrm{T}' - \mathrm{T}) + m x + m s (\mathrm{T} - \theta),$$

and this quantity, together with the heat lost by the envelope must be equated to the heat gained by the calorimeter and its original contents, subject to a correction for radiation which can be determined by the ordinary methods.

As regards the two specific heats which enter this equation, s the specific heat in the solid state may be regarded as known, and s' the specific heat in the liquid state can be deduced by combining this equation with another of the same kind in which the initial temperature is very different. In the case of bodies which, like mercury and bromine, are liquid at ordinary temperatures, the specific heat in the solid state can be found by a similar but inverse process.

76. Conservatism of Water.—The table in § 75 shows that the

heat of fusion is much greater for ice than for any of the other substances mentioned. It is 14 times as great as for lead, and 28 times as great as for mercury. Ice is, in this sense, the most difficult to melt, and water the most difficult to freeze, of all substances; a fact which is of great importance in the economy of nature, as tending to retard the processes both of freezing and thawing. Even as it is, the effects of a sudden thaw are often disastrous, and yet, for every particle of ice melted, as much heat is required as would raise the water produced through 79° C. or 142° F.

77. **Solution.**—The reduction of a body from the solid to the liquid state may be effected by other means than by the direct action of heat; it may be produced by the action of a liquid. This is what occurs when, for instance, a grain of salt or of sugar is placed in water; the body is said to be *dissolved* in the water. Solution, like fusion, is accompanied by the disappearance of heat consequent on the change from the solid to the liquid state. For example, by rapidly dissolving nitrate of ammonia in water, a fall of from 20° to 25° C. can be obtained.

Unlike fusion, it is attached to no definite temperature, but occurs with more or less freedom over a wide range. Rise of temperature usually favours it; but there are some strongly marked exceptions.

78. **Freezing-mixtures.**—The absorption of heat which accompanies the liquefaction of solids is the basis of the action of freezing-mixtures. In all such mixtures there is at least one solid ingredient which, by the action of the rest, is reduced to the liquid state, thus occasioning a fall of temperature proportional to the latent heat of its liquefaction.

The mixture most commonly employed in the laboratory is one of snow and salt. There is a double absorption of heat caused by the simultaneous melting of the snow and dissolving of the salt. Professor Guthrie found that the proportions of the two ingredients and their initial temperatures may vary between very wide limits without affecting the temperature obtained. This definite temperature is the freezing-point of a definite compound of salt and water. When ordinary sea-water in a vessel is subjected to cold, the ice first formed is fresh; and the brine increases in strength by the freezing out of the water till it has attained the strength of the definite compound above mentioned. Then a change occurs, and the ice formed is no longer fresh, but of the same composition as the brine. From this point onward until all the brine is frozen the temperature of

the liquid is $-22°$ C., which is, accordingly, the temperature obtained by mixing snow and salt.[1]

Fahrenheit intended that the temperature thus obtained should be the zero of his scale, the freezing-point of water being 32° and the boiling-point 212°; but the thermometers with which he worked were extremely rough, and if we define his scale by the two ordinary fixed points, the temperature $-22°$ C. will not be 0° F., but $-7·6°$ F.

79. The following mixtures are also sometimes employed.

	Proportions by Weight	Fall of Temperature Produced
Snow,	3	from 0° to $-48°$.
Crystallized Chloride of Calcium,	4	
Nitrate of Ammonia,	1	from $+10°$ to $-15°$.
Water,	1	
Sal-ammoniac,	5	from $+10°$ to $-15°$.
Nitrate of Potash,	5	
Sulphate of Soda,	8	
Water,	16	
Sulphate of Soda,	8	from $+10°$ to $-17°$.
Hydrochloric Acid,	5	

Fig. 46 represents an apparatus intended for the artificial production of ice. The water to be frozen is inclosed in a mould formed of two concentric vessels—an arrangement which has the advantage

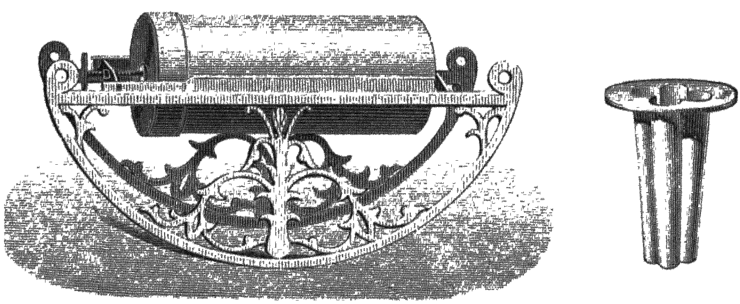


Fig. 46.—Freezing Rocker.

of giving a large surface of contact; and the mould is immersed in the freezing-mixture (hydrochloric acid and sulphate of soda) which is contained within a metal cylinder mounted on a cradle, the rocking of which greatly assists the operation.

80. Solidification or Congelation.—All liquids are probably capable of being solidified; though some of them, for example, alcohol and bisulphide of carbon, have never yet been seen in the solid state.

[1] *Proceedings of Physical Society of London*, January, 1875, p. 78.

The temperature of fusion is the highest temperature at which congelation can occur, and is frequently called the *temperature of congelation* (or the *freezing-point*); but it is possible to preserve substances in the liquid state at lower temperatures. Liquids thus cooled below their so-called freezing-points have, however, if we may so say, a *tendency to freeze, which is only kept in check by the difficulty of making a commencement.* If freezing once begins, or if ever so small a piece of the same substance in the frozen state be allowed to come in contact with the liquid, congelation will quickly extend until there is none of the liquid left at a temperature below that of fusion. The condition of a liquid cooled below its freezing-point has been aptly compared to that of a row of bricks set on end in such a manner that if the first be overturned, it will cause all the rest to fall, each one overturning its successor.

The contact of its own solid infallibly produces congelation in a liquid in this condition, and the same effect may often be produced by the contact of some other solid, especially of a crystal, or by giving a slight jar to the containing vessel.

Despretz has cooled water to -20° C. in fine capillary tubes, without freezing, and Dufour has obtained a similar result by suspending globules of water in a liquid of the same specific gravity with which it would not mix, this liquid being one which had a very low freezing-point.

81. Heat set free in Congelation.—At the moment when congelation takes place, the thermometer immediately rises to the temperature of the melting-point. This may be easily shown by experiment. A small glass vessel is taken, containing water, in which a mercurial thermometer is plunged. By means of a frigorific mixture the temperature is easily lowered to -10° or -12°, without the water freezing; a slight shock is then given to the glass, congelation takes place, and the mercury rises to 0°.

The quantity of ice that will be formed when congelation sets in, in water which has been cooled below the freezing-point, may be computed—very approximately at least—in the following way:—

Suppose we have unit mass of water at the temperature $-t°$, and when congelation sets in suppose that it yields a mass x of ice and a mass $1-x$ of water, both at 0°.

To melt this ice and bring the whole mass to the state of water at 0° would require the addition of 79·25 x units of heat; but to bring the whole mass of water from $-t$ to 0° would require t units of heat.

These two quantities of heat must be the same, subject to a possible correction which will be discussed in the chapter on thermodynamics. Hence we may write

$$79{\cdot}25\,x=t; \qquad x=\frac{t}{79{\cdot}25}.$$

Whatever the original quantity of water may be, this value of x expresses the fraction of it which will be converted into ice.

82. **Crystallization.**—When the passage from the liquid to the solid state is a gradual one, it frequently happens that the molecules group themselves in such a manner as to present regular geometric forms. This process is called crystallization, and the regular bodies thus formed are called crystals. The particular crystalline form assumed depends upon the substance, and often affords a means of recognizing it. The forms, therefore, in which bodies crystallize are among their most important characteristics, and are to some extent analogous to the shapes of animals and plants in the organic world.

In order to make a body crystallize in solidifying, the following method is employed. Suppose the given body to be bismuth; the first step is to melt it, and then leave it to itself for a time. The metal begins to solidify first at the surface and at the sides, where it is most directly exposed to cooling influences from without; accordingly, when the outer layer of the metal is solidified, the interior is still in the liquid state. If the upper crust be now removed, and the liquid bismuth poured off, the sides of the vessel will be seen to be covered with a number of beautiful crystals.

If the metal were allowed to stand too long, the entire mass would become solid, the different crystals would unite, and no regularity of structure would be observable.

83. **Flowers of Ice.**—The tendency of ice to assume a crystalline form is seen in the fern-leaf patterns which appear on the windows in winter, caused by the congealing of moisture on them, and still more distinctly in the symmetrical forms of snow-flakes (see Chap. xi.). In a block of ice, however, this crystalline structure does not show itself, owing to the closeness with which the crystals fit into each other, so that a mass of this substance appears almost completely *amorphous*. Tyndall, however, in a very interesting experiment, succeeded in gradually *decrystallizing* ice, if we may use the expression, and thus exhibiting the crystalline elements of which it is composed. The experiment consists in causing a pencil of solar rays

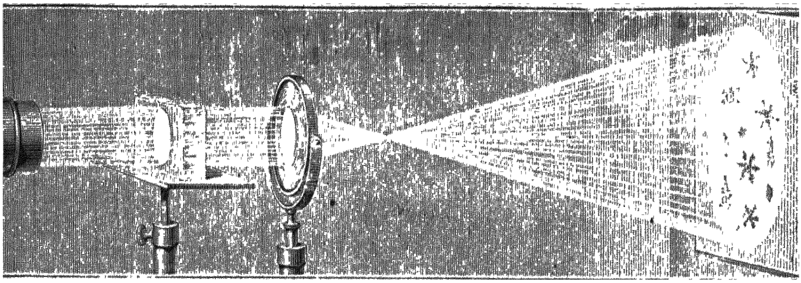


Fig. 47 —Flowers of Ice projected on a Screen

to fall perpendicularly to the surfaces of congelation on a sheet of ice, such as is naturally formed upon the surface of water in winter A lens placed behind the ice (Fig. 47) serves to project upon a screen the image of what is found in the interior of the block. The successive appearances observed upon the screen are shown in Fig. 48. A small luminous circle is first seen, from which branch out rays, resembling the petals of a flower whose pistil is the circle. Frequent changes also occur in the shape of the branches themselves, which are often cut so as to resemble fern-

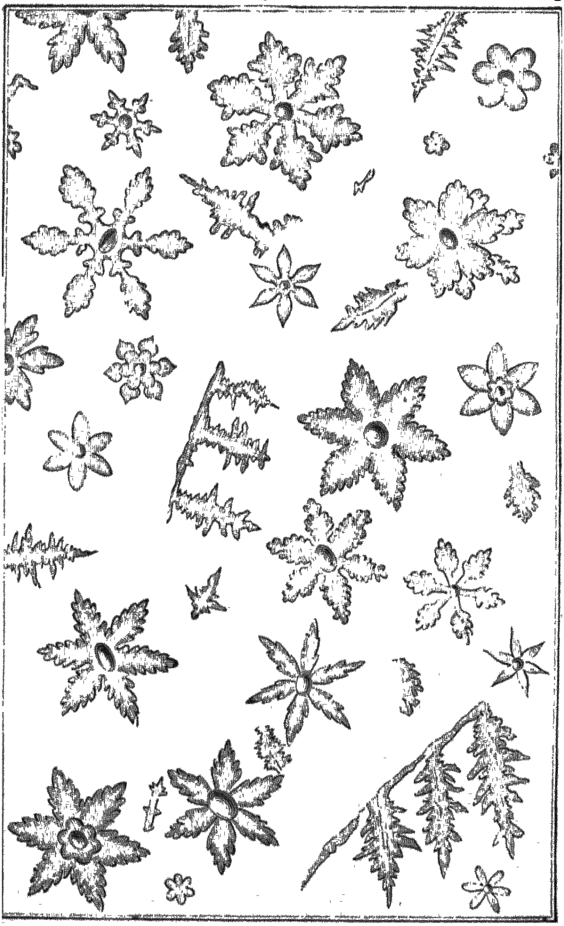


Fig 48 —Flowers of Ice

leaves, like those seen upon the windows during frost. In this experiment, the solar heat, instead of uniformly melting the mass of ice, which it would certainly do if the mass were amorphous, acts successively upon the different crystals of which it is built up, affecting them in the reverse order of their formation. There are thus produced a number of spaces of regular shape, containing water, and producing comparatively dark images upon the screen. In the centre of each there is generally a bright spot, which corresponds to an empty space, depending on the fact that the water occupies a smaller volume than the ice from which it has been produced.

84. **Supersaturation.**—The proportion of solid matter which a liquid can hold in solution varies according to the temperature; and as a general rule, though not by any means in all cases, it increases as the temperature rises. Hence it follows, that if a saturated solution be left to itself, the effect of evaporation or cooling will be gradually to diminish the quantity of matter which can be held in solution. A portion of the dissolved substance will accordingly pass into the solid state, assuming generally a crystalline form. This is an exceedingly common method of obtaining crystals, and is known as the *humid way*.

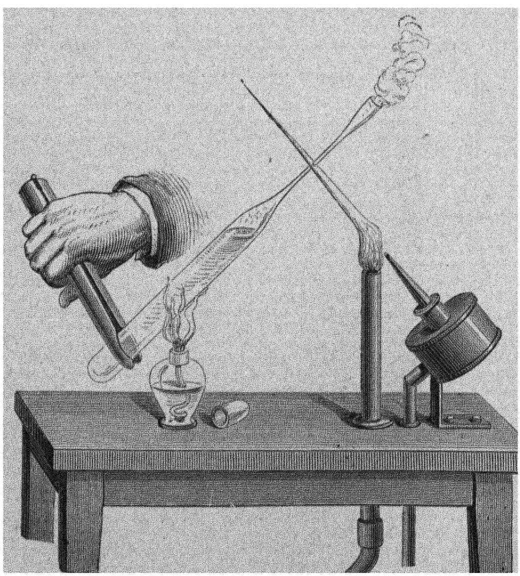


Fig 49.—Preparation of Supersaturated Solution of Sulphate of Soda.

In connection with this process a phenomenon occurs which is precisely analogous to the cooling of a liquid below its freezing-point. It may be exemplified by the following experiment. A tube drawn out at one end (Fig. 49) is filled with a warm concentrated solution of sulphate of soda The solution is boiled, and while ebullition is proceeding freely, the tube is hermetically sealed; by this means the tube is exhausted of air. The solution when

left to itself cools without the solid being precipitated, although the liquid is *supersaturated*. But if the end of the tube be broken off, and the air allowed to enter, crystallization immediately commences at the surface, and is quickly propagated through the whole length of the tube; at the same time, as we should expect, a considerable rise of temperature is observed. If the phenomenon does not at once occur on the admission of the air, it can be produced with certainty by throwing a small piece of the solid sulphate into the solution.

85. Change of Volume at the Moment of Congelation. Expansive Force of Ice.—In passing from the liquid to the solid state, bodies generally undergo a diminution of volume; there are, however, exceptions, such as ice, bismuth, and cast-iron. It is this property which renders this latter substance so well adapted for the purposes of moulding, as it enables the metal to penetrate completely into

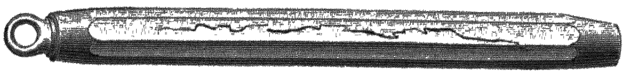


Fig 50.—Bursting of Iron Tube by Expansion of Water in Freezing

every part of the mould. The expansion of ice is considerable, amounting to about $\frac{1}{12}$; its production is attended by enormous

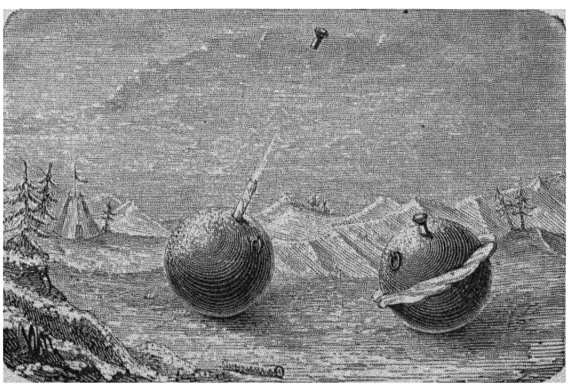


Fig 51 – Experiment of Major Williams

mechanical force, just as in the analogous case of expansion by heat. Its effect in bursting water-pipes is well known. The following experiment illustrates this expansive force. A tube of forged iron (Fig. 50) is filled with water, and tightly closed by a screw-stopper. The tube is then surrounded with a freezing-mixture of snow and

salt. After some time the water congeals, a loud report is often heard, and the tube is found to be rent.

The following experiment, performed by Major Williams at Quebec, is still more striking. He filled a 12-inch shell with water and closed it with a wooden stopper, driven in with a mallet. The shell was then exposed to the air, the temperature being $-28°$ C. ($-18°$ F.). The water froze, and the bung was projected to a distance of more than 100 yards, while a cylinder of ice of about 8 inches in length was protruded from the hole. In another experiment the shell split in halves, and a sheet of ice issued from the rent (Fig. 51).

It is the expansion and consequent lightness of ice which enables it to float upon the surface of water, and thus afford a protection to animal life below.

86. Effect of Pressure on the Melting-point.—Professor James Thomson was led by theoretical considerations to the conclusion that, in the case of a substance which, like water, expands in solidifying, the freezing (or melting) point must of necessity be lowered by pressure, and that a mixture of ice and ice-cold water would fall in temperature on the application of pressure. His reasoning[1] consisted in showing that it would otherwise be possible (theoretically at least) to construct a machine which should be a perpetual source of work without supply; that is, what is commonly called a perpetual motion.

The matter was shortly afterwards put to the test of experiment by his brother, Lord Kelvin, who compressed, in an Œrsted's piezometer, a mixture of ice and water, in which was inserted a very delicate thermometer protected from pressure in the same manner as the instrument represented in Fig. 15 (§ 12). The thermometer showed a regular fall of temperature as pressure was applied, followed by a return to 0° C. on removing the pressure. Pressures of 8·1 and 16·8 atmospheres (in excess of atmospheric pressure) lowered the freezing-point by ·106 and ·232 of a degree Fahr. respectively as indicated by the thermometer, results which agree almost exactly with Prof. J. Thomson's prediction of ·0075 of a degree Cent., or ·0135 of a degree Fahr. per atmosphere.

Mousson has since succeeded in reducing the melting-point several degrees by means of enormous pressure. He employed two forms of apparatus, by the first of which he melted ice at the temperature of $-5°$ C., and kept the water thus produced for a considerable time

[1] *Transactions Royal Society, Edinburgh.* January, 1849.—*Cambridge and Dublin Math. Journal.* November, 1850.

at this temperature. This apparatus had windows (consisting of blocks of glass) in its sides, through which the melting of the ice was seen. His second form of apparatus, which bore a general resemblance to the first, is represented in the annexed figure. It consisted of a steel prism with a cylindrical bore, having one of its extremities closed by a conical stopper strongly screwed in, the rest of the bore being traversed by a screw-piston of steel. The apparatus was inverted, and nearly filled with water recently boiled, into which a piece of copper was dropped, to serve as an index. The apparatus, still remaining in the inverted position, was surrounded by a freezing-mixture, by means of which the water was reduced to ice at the temperature of $-18°$ C. The stopper was then screwed into its place, and the apparatus placed in the erect position. The piston was then screwed down upon the ice with great force, the pressure exerted being estimated in some of the experiments at several thousand atmospheres. The pressure was then relaxed, and, on removing the stopper, the copper index was found to have fallen to the bottom of the bore, showing that the ice had been liquefied.

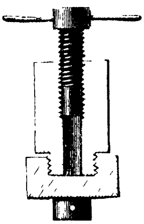


Fig 52
Mousson's Apparatus

Experiments conducted by Bunsen and by Hopkins have shown that wax, spermaceti, sulphur, stearin, and paraffin—substances which, unlike ice, expand in melting—have their melting points *raised* by pressure, a result which had been predicted by Lord Kelvin.

87. **Effect of Stress in general upon Melting and Solution.**—In the experiments above described, the pressure applied was hydrostatical, and was therefore equal in all directions. But a solid may be exposed to pressure in one direction only, or to pull in one or more directions, or it may be subjected to shearing, twisting, or bending forces, all these being included under the general name of *stress*.

Reasoning, based on the general laws of energy, leads to the conclusion that stress of any kind other than hydrostatic, applied to a solid, must lower its melting-point. To quote Professor J. Thomson (*Proc. Roy Soc.* Dec. 1861), "Any stresses whatever, tending to change the form of a piece of ice in ice-cold water, must impart to the ice a tendency to melt away, and to give out its cold, which will tend to generate, from the surrounding water, an equivalent quantity of ice free from the applied stresses," and "stresses tending to change the form of any crystals in the saturated solutions from which they have

been crystallized must give them a tendency to dissolve away, and to generate, in substitution for themselves, other crystals free from the applied stresses or any equivalent stresses."[1] This conclusion he verified by experiments on crystals of common salt. He at the same time suggested, as an important subject for investigation, the effect of hydrostatic pressure on the crystallization of solutions, a subject which was afterwards taken up experimentally by Sorby, who obtained effects analogous to those above indicated as occurring in connection with the melting of ice and wax.

88. **Bottomley's Experiment.**—Mr. J. T. Bottomley has devised an instructive experiment on the effect of applying stress to ice. A block of ice is placed on two supports with a little space between them, and a stout copper wire with heavy weights at its two ends is slung across it. The wire gradually makes its way through the block—occupying, perhaps, an hour or two in its passage—and at last drops upon the floor; but the block is not cut in two; the cut which the wire makes is filled up by the formation of fresh ice as fast as the wire advances. The pressure of the wire lowers the melting-point of the ice in front, and causes it to melt at this lowered melting-point. The wire itself acquires, by contact with the melting ice, a temperature below zero, and the escaping water freezes at the back of the wire.

89. **Regelation of Ice.**—Faraday in 1850 called attention to the fact that pieces of moist ice placed in contact with one another will freeze together even in a warm atmosphere. This phenomenon, to which Tyndall has given the name of *regelation*, admits of ready explanation by the principles just enunciated. Capillary action at the boundaries of the film of water which connects the pieces placed in contact, produces an effect equivalent to attraction between them, just as two plates of clean glass with a film of water between them seem to adhere. Ice being wetted by water, the boundary of the connecting film is concave, and this concavity implies a diminution of pressure in the interior. The film, therefore, exerts upon the ice a pressure less than atmospheric; and as the remote sides of the

[1] Professor Thomson draws these inferences from the following principle, which appears axiomatic:—If any substance or system of substances be in a condition in which it is free to change its state [as ice, for example, in contact with water at 0° C., is free to melt], and if mechanical forces be applied to it in such a way that the occurrence of the change of state will make it lose the potential energy due to these forces without receiving other potential energy as an equivalent; then the substance or system will pass into the changed state.

blocks are exposed to atmospheric pressure, there is a resultant force urging them together and producing stress at the small surface of contact. Melting of the ice therefore occurs at the places of contact, and the cold thus evolved freezes the adjacent portions of the water film, which, being at less than atmospheric pressure, will begin to freeze at a temperature a little above the ordinary freezing-point.

As regards the amount of the force urging the pieces together, if two flat pieces of ice be supported with their faces vertical, and if they be united by a film from whose lower edge water trickles away, the hydrostatic pressure at any point within this film is less than atmospheric by an amount represented, in weight of water, by the height of this point above the part from which water trickles. If, for simplicity, we suppose the film circular, the plates will be pressed together with a force equal to the weight of a cylinder of water whose base is the film and whose height is the radius.

90. Apparent Plasticity of Ice. Motion of Glaciers.—A glacier may be described in general terms as a mass of ice deriving its origin from mountain snows, and extending from the snow-fields along channels in the mountain sides to the valleys beneath.

The first accurate observations on the movements of glaciers were made in 1842, by the late Professor (afterwards Principal) J. D. Forbes, who established the fact that glaciers descend along their beds with a motion resembling that of a pailful of mortar poured into a sloping trough; the surface moving faster than the bottom and the centre faster than the sides. The chief motion occurs in summer. He summed up his view by saying, "A glacier is an imperfect fluid, or a viscous body which is urged down slopes of a certain inclination by the mutual pressure of its parts."

This apparent viscosity is explained by the principles of § 87. According to these principles the ice should melt away at the places where stress is most severe, an equivalent quantity of ice being formed elsewhere. The ice would thus gradually yield to the applied forces, and might be moulded into new forms, without undergoing rupture. Breaches of continuity might be produced in places where the stress consisted mainly of a pull, for the pull would lower the freezing-point, and thus indirectly as well as directly tend to produce ruptures, in the form of fissures transverse to the direction of most intense pull. The effect of compression in any direction would, on the other hand, be, not to crack the ice, but to melt a portion of its interior sufficient to relieve the pressure in the particular part affected,

and to transfer the excess of material to neighbouring parts, which must in their turn give way in the same gradual manner.

In connection with this explanation it is to be observed that the temperature of a glacier is always about 0° C., and that its structure is eminently porous and permeated with ice-cold water. These are conditions eminently favourable (the former, but not the latter, being essential) to the production of changes of form depending on the lowering of the melting-point by stresses.

This explanation is due to Professor J. Thomson[1] (*British Assocition Report*, 1857). Professor Tyndall had previously attempted to account for the phenomena of glacier motion by supposing that the ice is fractured by the forces to which it is subjected, and that the broken pieces, after being pushed into their new positions, are united by regelation. In support of this view he performed several very interesting and novel experiments on the moulding of ice by pres-

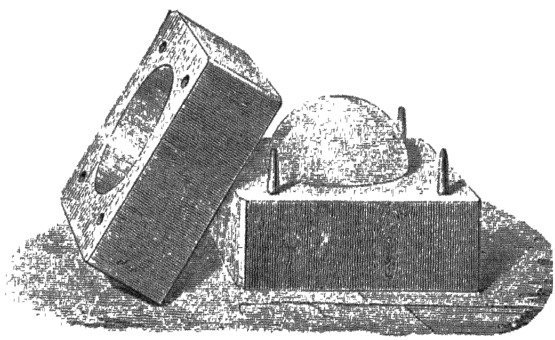


Fig 53 —Ice Moulded by Pressure

sure, such as striking medals of ice with a die, and producing a clear transparent cake of ice by powerfully compressing broken pieces in a boxwood mould (Fig. 53).

Interesting experiments on the plasticity of ice may be performed by filling an iron shell with water and placing it in a freezing-mixture, leaving the aperture open. As the water freezes, a cylinder of ice will be gradually protruded. This experiment is due to Mr.

[1] If it should be objected that the lowering of the melting-point by stress is too insignificant to produce the vast effects here attributed to it, the answer is that, when ice and water are present together, the slightest difference is sufficient to determine which portion of the water shall freeze, or which portion of the ice shall melt. In default of a more powerful cause, those portions of ice which are most stressed will melt first.

Christie. Principal Forbes obtained a similar result by using a very strong glass jar; and by smearing the interior, just below the neck, with colouring matter, he demonstrated that the external layer of ice which was first formed, slid along the glass as the freezing proceeded, until it was at length protruded beyond the mouth.

In the experiments of Major Williams, described in § 85, it is probable that much of the water remained unfrozen until its pressure was relieved by the bursting of the shells.

CHAPTER VIII.

EVAPORATION AND CONDENSATION.

91. **Transformation into the State of Vapour.**—The majority of liquids, when left to themselves in contact with the atmosphere, gradually pass into the state of vapour and disappear. This phenomenon occurs much more rapidly with some liquids than with others, and those which evaporate most readily are said to be the most volatile. Thus, if a drop of ether be let fall upon any substance, it disappears almost instantaneously; alcohol also evaporates very quickly, but water requires a much longer time for a similar transformation. The change is in all cases accelerated by an increase of temperature; in fact, when we *dry* a body before the fire, we are simply availing ourselves of this property of heat to hasten the evaporation of the moisture of the body. Evaporation may also take place from solids. Thus camphor, iodine, and several other substances pass directly from the solid to the gaseous state, and we shall see hereafter that the vapour of ice can be detected at temperatures far below the freezing-point.

Evaporation, unlike fusion, occurs over a very wide range of temperature. There appears, however, to be a temperature for each substance, below which evaporation, if it exist at all, is insensible to ordinary tests. This is the case with mercury at 0° C., and with sulphuric acid at ordinary atmospheric temperatures.

92. **Vapour, Gas.**—The words *gas* and *vapour* have no essential difference of meaning. A vapour is the gas into which a liquid is changed by evaporation. Every gas is probably the vapour of a liquid. The word *vapour* is especially applied to the gaseous condition of bodies which are usually met with in the liquid or solid state, as water, sulphur, &c.; while the word *gas* generally denotes a

body which, under ordinary conditions, is never found in any state but the gaseous.

93. Pressure of Vapours. Maximum Pressure and Density.—The characteristic property of gases is the elastic force[1] with which they tend to expand. This may be exemplified in the case of vapours by the following experiment.

A glass globe A (Fig. 54) is fitted with a metal cap provided with two openings, one of which can be made to communicate with a mercurial manometer, while the other is furnished with a stop-cock R. The globe is first exhausted of air by establishing communication through R with an air-pump. The mercury rises in the left-hand and falls in the right-hand branch of the manometer; the final difference of level in the two branches differing from the height of the barometer only by the very small quantity representing the pressure of the air left behind by the machine. The stop-cock R is then closed, and a second stop-cock R′ surmounted by a funnel is fixed above it. The hole in this second stop-cock, instead of going quite through the metal, extends only half-way, so as merely to form a cavity. This cavity serves to introduce a liquid into the globe, without any communication taking place between the globe and the external air. For this purpose we have only to fill the funnel with a liquid, to open the cock R, and to turn that at R′ backwards and forwards several times. It will be found that after the introduction of a small quantity of liquid into the globe, the mercurial column begins to descend in the left branch of the manometer, thus indicating an increase of elastic force. This elastic force goes on increasing as a greater quantity of liquid is introduced into the globe; and as no liquid is visible in the globe, we must infer that it evaporates as fast as it is introduced, and that the fall of the mercurial column is caused by the elastic force of the vapour thus formed.

This increase of pressure, however, does not go on indefinitely. After a time the difference of level in the two branches of the manometer ceases to increase, and a little of the unevaporated liquid may be seen in the globe, which increases in quantity as more liquid is

[1] The terms "pressure," "tension," and "elastic force" are often used interchangeably to denote the stress existing in a vapour or gas. "Tension" is the ordinary term employed in this sense in French books. The best English authorities upon elasticity, however, employ the two terms "pressure" and "tension" to denote two opposite things; a pressure is a push, and a tension is a pull. Gases and vapours cannot pull, they can only push, and they are constantly pushing in all directions; hence they are never in a state of *tension* but are always in a state of *pressure*.

introduced. From this important experiment we conclude that there is a limit to the quantity of vapour which can be formed at a given temperature in an empty space. When this limit is reached, the space is said to be *saturated*, and the vapour then contained in it is at *maximum pressure*, and at *maximum density*. It evidently fol-

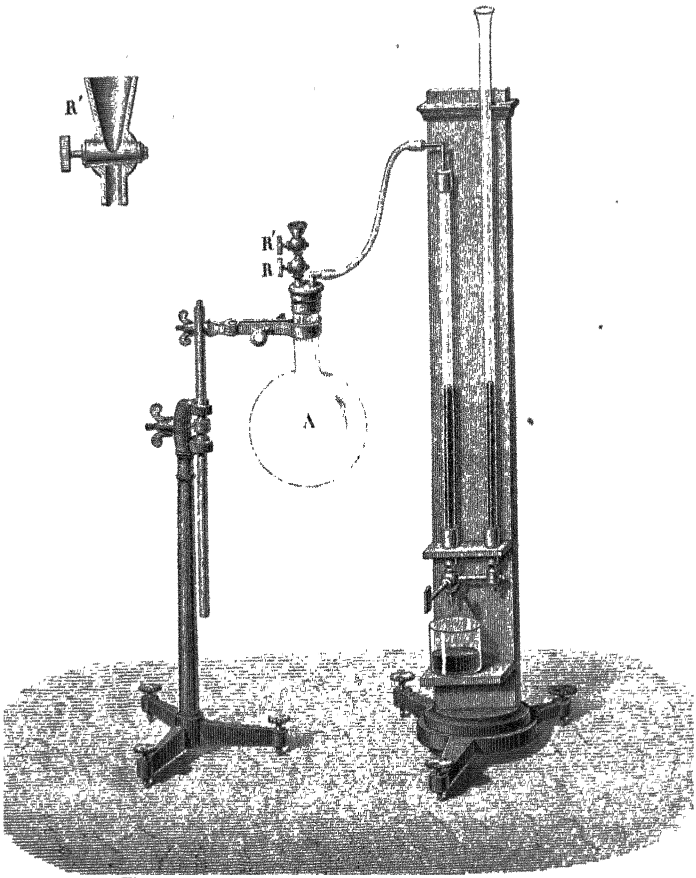


Fig. 54.—Apparatus for studying the Formation of Vapours.

lows from this that if a quantity of vapour at less than its maximum density be inclosed in a given space, and then compressed at constant temperature, its pressure and density will increase at first, but that after a time a point will be reached when further compression, instead of increasing the density and pressure of the vapour, will only cause some of it to pass into the liquid state. This last result may be directly verified by the following experiment. A barometric tube

a b (Fig. 55) is filled with mercury, with the exception of a small space, into which a few drops of ether are introduced, care having first been taken to expel any bubbles of air which may have remained adhering to the mercury. The tube is then inverted in the deep bowl MN, when the ether ascends to the surface of the mercury, is there converted into vapour, and produces a sensible depression of the mercurial column. If the quantity of ether be sufficiently small, and if the tube be kept sufficiently high, no liquid will be perceived in the space above the mercury; this space, in fact, is not saturated. The pressure of the vapour which occupies it is given by the difference between the height of the column in the tube and of a barometer placed beside it. If the tube be gradually lowered, this difference will at first be seen to increase, that is, the pressure of the vapour of ether increases; but if we continue the process, a portion of liquid ether will be observed to collect above the mercury, and after this, if we lower the tube any further, the height of the mercury in it remains invariable. The only effect is to increase the quantity of liquid deposited from the vapour.[1]

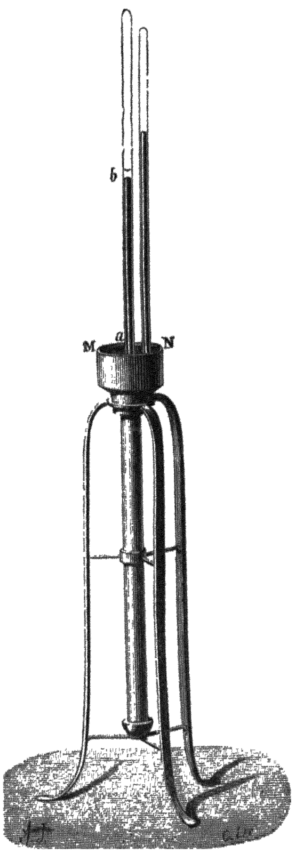


Fig. 55 —Maximum Tension of Vapours.

94. Influence of Temperature on Maximum Density and Pressure.—Returning now to the apparatus represented in Fig. 54, suppose that some of the liquid remains unevaporated in the bottom of the globe, and let the globe be subjected to an increase of temperature. An increase of elastic force will at once be indicated by the manometer, while the quantity of liquid will be diminished. The maximum pressure of a vapour, therefore, and also its maximum density, increase with the temperature; and consequently, in order to saturate

[1] Strictly speaking, there will be a slight additional depression of the mercurial column due to the weight of the liquid thus deposited on its summit; but this effect will generally be very small, owing to the smallness of the quantity of liquid.

a given space, a quantity of vapour is required which increases with the temperature.

Vapour which is at less than the maximum density is called *superheated vapour;* because it can be obtained by giving heat to vapour at maximum density at a lower temperature.

Fig. 56 is a graphical representation of the rate at which the maximum density of aqueous vapour increases with the temperature from $-20°$ to $+35°$ C. Lengths are laid off on the base-line AB, to represent temperatures, and ordinates are erected at every fifth de-

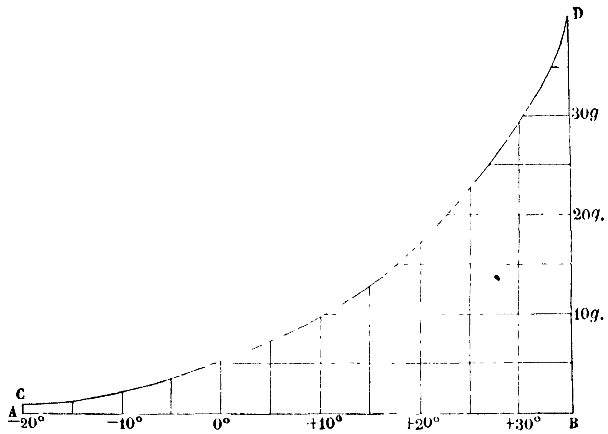


Fig. 56.—Saturation at different Temperatures.

gree, proportional to the masses of vapour required to saturate the same space at different temperatures. The curve CD, drawn through the extremities of these ordinates, is the curve of vapour-density as a function of temperature. The figures on the right hand indicate the number of grammes of vapour required to saturate a cubic metre.

95. Mixture of Gas and Vapour. Dalton's Laws.—The experiments with the apparatus of Fig. 54 may be repeated after filling the globe with dry air, or any other dry gas, and the results finally obtained will be the same as with the exhausted globe. If, as before, we introduce successive small quantities of a liquid, it will be converted into vapour, and the pressure will go on increasing till saturation is attained; the elastic force of vapour will then be found to be exactly the same as in the case of the vacuous globe, and the quantity of liquid evaporated will also be the same.

There is, however, one important difference. In the vacuum the complete evaporation of the liquid is almost instantaneous; in a gas,

on the other hand, the evaporation and consequent increase of pressure proceed with comparative slowness; and the difference between the two cases is more marked in proportion as the pressure of the gas is greater.

We may lay down, then, the two following laws (called, from their discoverer, Dalton's laws) for the mixture of a vapour with a gas:—

1. *The mass of vapour which can be contained in a given space is the same whether this space be empty or filled with gas.*

2. *When a gas is saturated with vapour, the actual pressure of the mixture is the sum of the pressures due to the gas and vapour separately; that is to say, it is equal to the pressure which the gas would exert if it alone occupied the whole space, plus the maximum pressure of vapour for the temperature of the mixture.*

This second law comes under the general rule for determining the pressure of a mixture of gases (see Part I), and the rule still applies to a mixture of gas and vapour when the quantity of the latter *falls short of* saturation. Each element in a mixture of gases and vapours exerts the same pressure on the walls of the containing vessel as it would exert if the other elements were removed.

It is doubtful, however, whether these laws are rigorously true. It would rather appear from some of Regnault's experiments, that the quantity of vapour taken up in a given space is slightly, though almost insensibly, diminished, as the density of the gas which occupies the space is increased.

96. **Liquefaction of Gases.**—When vapour exists in the state of saturation, any diminution in the volume must, if the temperature is preserved constant, involve the liquefaction of as much of the vapour as would occupy the difference of volumes; and the vapour which remains will still be at the original density and tension. A vapour existing by itself may therefore be completely liquefied by subjecting it to a pressure exceeding, by ever so slight an amount, the maximum tension corresponding to the temperature, provided that the containing vessel is prevented from rising in temperature.

Again, if a vapour at saturation be subjected to a fall of temperature, while its volume remains unchanged, a portion of it must be liquefied corresponding to the difference between the density of saturation at the higher and at the lower temperature. This operation will obviously diminish the pressure, since this will now be the maximum pressure corresponding to the lower instead of to the higher temperature.

There are therefore two distinct means of liquefying a vapour—increase of pressure, and lowering of temperature. They are employed sometimes separately, and sometimes in conjunction.

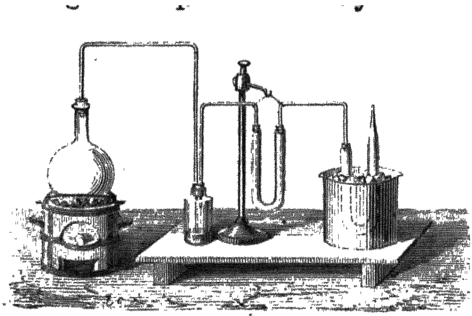
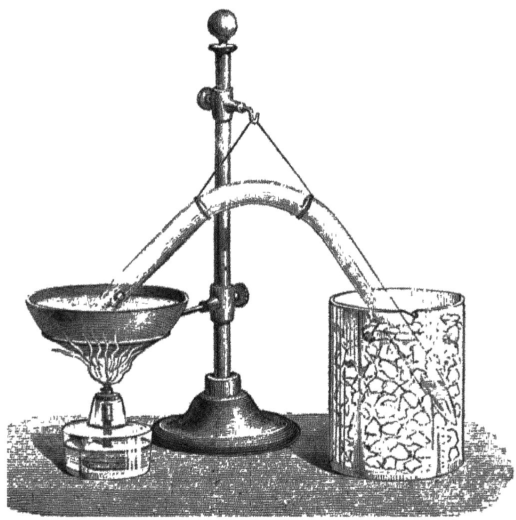


Fig 57.—Liquefaction of Sulphurous Acid.

Fig. 57 represents the apparatus usually employed for obtaining sulphurous acid in the liquid state. The gas, which is generated in a glass globe, passes first into a washing-bottle, then through a drying-tube, and finally into a tube surrounded with a freezing-mixture of snow and salt.

Pouillet's apparatus for showing the unequal compressibilities of different gases (see Part I.) serves to liquefy most gases by means of compression. In order to ascertain the pressures at which liquefaction takes place, or, in other words, the maximum pressures of gases, one of the tubes in that apparatus is replaced by a shorter tube, containing air, and serving as a manometer.



By this means Pouillet found that, at the temperature of 10° C., sulphurous acid is liquefied by a pressure of 2½ atmospheres, nitrous oxide by a pressure of 43, and carbonic acid by a pressure of 45 atmospheres.

97. Faraday's Method.—Faraday, who was the first to conduct methodical experiments on the liquefaction of gases, employed, in the first instance, the simple apparatus represented in Fig. 58. It con-

sists of a very strong bent glass tube, one end of which contains ingredients which evolve the gas on the application of heat, while the other is immersed in a freezing-mixture. The pressure produced by the evolution of the gas in large quantity in a confined space, combines with the cold of the freezing-mixture to produce liquefaction of the gas, and the liquid accordingly collects in the cold end of the tube.

Thilorier, about the year 1834, invented the apparatus represented in Fig. 59, which is based on this method of Faraday, and is intended for liquefying carbonic acid gas. This operation requires the enormous pressure of about fifty atmospheres at ordinary temperatures. If a slight rise of temperature occur from the chemical actions attending the production of the gas, a pressure of 75 or 80 atmospheres may not improbably be required. Hence great care is necessary in testing the strength of the metal employed in the construction of the apparatus. It was formerly made of cast-iron, and strengthened by wrought-iron hoops; but the construction has since been changed on account of a terrible explosion, which cost the life of one of the operators. At present the vessels are formed of three parts; the inner one of lead, the next *e*, which completely envelops this, of copper, and finally, the hoops *ff* of wrought iron (Fig. 59), which bind the whole together. The apparatus consists of two distinct reservoirs. In the generator C is placed bicarbonate of soda, and a vertical tube *a*, open at top, containing sulphuric acid. By imparting an oscillatory movement to the vessel about the two pivots which support it near the middle, the sulphuric acid is gradually discharged, and the carbonic acid is evolved, and becomes liquid in the interior. The generator is then connected with the condenser C′ by the tube *t*, and the stop-cocks R and R′ are opened. As soon as the two vessels are in communication, the liquid carbonic acid passes into the condenser, which is at a lower temperature than the generator, and represents the cold branch of Faraday's apparatus. The generator can then be disconnected and recharged, and thus several pints of liquid carbonic acid may be obtained.

In the foregoing methods, the pressure which produces liquefaction is furnished by the evolution of the gas itself.

In some other forms of apparatus the pressure is obtained by the use of one or more compression-pumps, which force the gas from the vessel in which it is generated into a second vessel, which is kept cool either by ice or a freezing-mixture. The apparatus of this kind

which is most extensively used is that devised by Bianchi. It consists of a compression-pump driven by a crank furnished with a fly-wheel, and turned by hand.

Faraday, in his later experiments, employed two pumps, the first

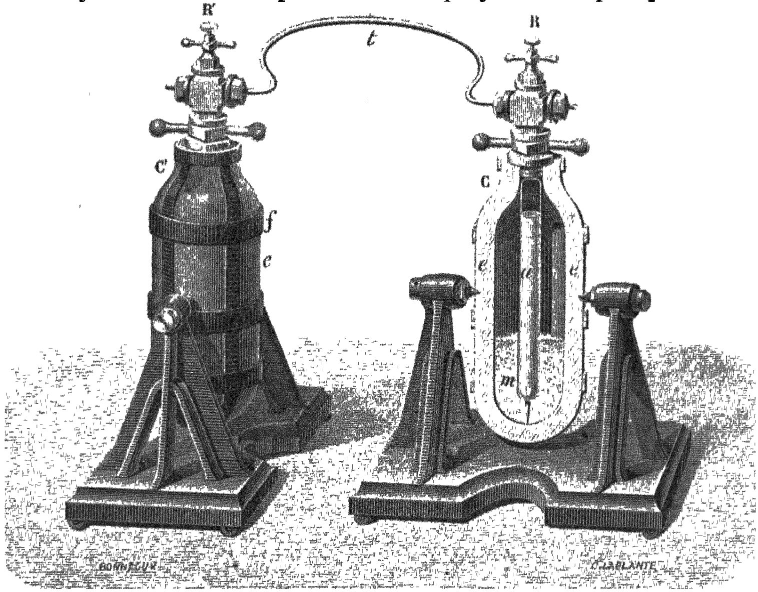


Fig. 59.—Thilorier's Apparatus.

having a piston of an inch, and the second of only half an inch diameter. The first pump in the earlier stage of the operation forced the gas through the second into the receiver. In the later stage the second pump was also worked, so as to force the gas already condensed to 10, 15, or 20 atmospheres into the receiver at a much higher pressure. The receiver was a tube of green bottle-glass, and was immersed in a very intense freezing-mixture, consisting of solid carbonic acid and ether, the cooling effect being sometimes increased by exhausting the air and vapour from the vessel containing the freezing-mixture, so as to promote more rapid evaporation.

98. Latent Heat of Vaporization. Cold produced by Evaporation.—The passage from the liquid to the gaseous state is accompanied by the disappearance of a large quantity of heat. Whenever a liquid evaporates without the application of heat, a depression of temperature occurs. Thus, for instance, if any portion of the skin be kept moist with alcohol or ether, a decided sensation of cold is felt. Water

produces the same effect in a smaller degree, because it evaporates less rapidly.

The heat which thus disappears in virtue of the passage of a liquid into the gaseous condition, is called the *latent heat of vaporization.* Its amount varies according to the temperature at which the change is effected, and it is exactly restored when the vapour returns to the liquid form, provided that both changes have been effected at the same temperature. Its amount for vapour of water at the temperature 100° C. is 536°; that is to say, the quantity of heat which disappears in the evaporation of a pound of water at this temperature, and which reappears in the condensation of a pound of steam at the same temperature, would be sufficient to raise the temperature of 536 pounds of water from 0° to 1°.

The latent heat of vaporization plays an important part in the heating of buildings by steam. A pound of steam at 100°, in becoming reduced to water at 30°, gives out as much heat as about $8\frac{2}{3}$ lbs. of water at 100° in cooling down to the same temperature.

99. Leslie's Experiment.—Water can be easily frozen by the cold resulting from its own evaporation, as was first shown by Leslie in a celebrated experiment. A small capsule (Fig. 60) of copper is taken, containing a little water, and is placed above a vessel containing strong sulphuric acid. The whole is placed under the receiver of an air-pump, which is then exhausted. The water evaporates with great rapidity, the vapour being absorbed by the sulphuric acid as fast as it is formed, and ice soon begins to appear on the surface. The experiment is, however, rather difficult to perform successfully. This arises from various causes.

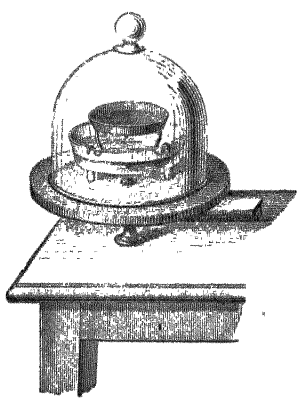


Fig. 60 —Leslie's Experiment

In the first place, the vapour of water which occupies the upper part of the receiver is only imperfectly absorbed; and, in the second place, as the upper layer of the acid becomes diluted by absorbing the vapour, its affinity for water rapidly diminishes.

These obstacles have been removed by an apparatus invented by M. Carré, which enables us to obtain a considerable mass of ice in a few minutes. It consists (Fig. 61) of a leaden reservoir containing

sulphuric acid. At one extremity is a vertical tube, the end of which is bent over and connected with a flask containing water. The other extremity of the reservoir communicates with an air-pump, to the

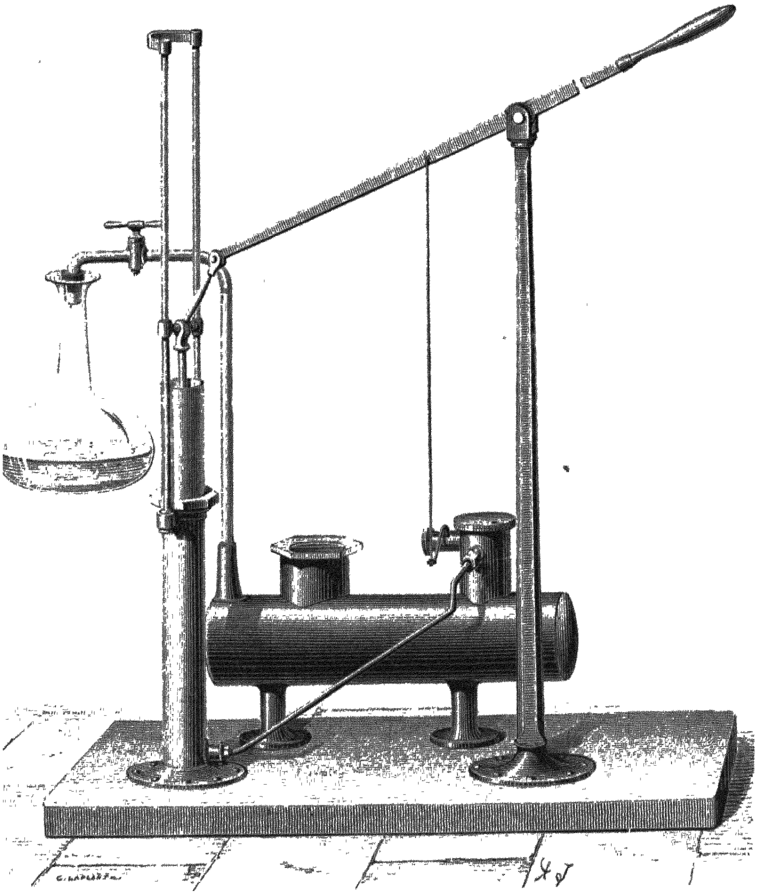


Fig. 61 —Carré's Apparatus for freezing by Sulphuric Acid.

handle of which is fitted a metallic rod, which drives an agitator immersed in the acid. By this means the surface of the acid is continually renewed, absorption takes place with regularity, and the water is rapidly frozen.

100. **Cryophorus.**—Wollaston's cryophorus (Fig. 62) consists of a bent tube with a bulb at each end. It is partly filled with water, and hermetically sealed while the liquid is in ebullition, thus expelling the air.

When an experiment is to be made, all the liquid is passed into the bulb B, and the bulb A is plunged into a freezing-mixture, or into pounded ice. The cold condenses the vapour in A, and thus produces rapid evaporation of the water in B. In a short time needles of ice appear on the surface of the liquid.

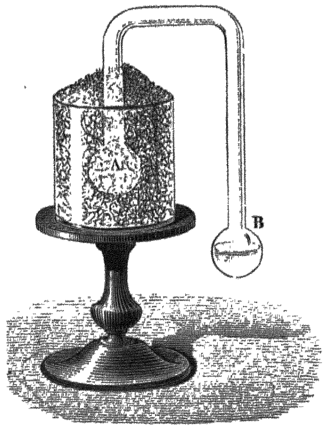


Fig 62.—Cryophorus.

101. Freezing of Water by the Evaporation of Ether.—Water is poured into a glass tube dipped into ether, which is contained in a glass vessel for the purpose (Fig. 63) By means of a pair of bellows a current of air is made to pass through the ether; evaporation is quickly produced, and at the end of a few minutes the water in the tube is frozen.

If, instead of promoting evaporation of the ether by means of a

Fig. 63.—Freezing of Water by Evaporation of Ether

current of air, the vessel were placed under the exhausted receiver of an air-pump, a much greater fall of temperature would be ob-

tained, and even mercury might easily be frozen. This experiment, however, is injurious to the pump, owing to the solvent action of the ether on the oil with which the valves and other moving parts are lubricated.

102. **Freezing of Mercury by means of Sulphurous Acid.**—Mercury may be frozen by means of liquid sulphurous acid, which is much more volatile than ether. In order to escape the suffocating action of the gas, the experiment is performed in the following manner:—

Into a glass vessel (Fig. 64) are poured successively mercury and liquid sulphurous acid. The vessel is closed by an india-rubber stopper, in which two glass tubes are fitted. One of these dips to the bottom of the sulphurous acid, and is connected at its outer end with a bladder full of air. Air is passed through the liquid by compressing the bladder, and escapes, charged with vapour, through the second opening, which is fitted with an india-rubber tube leading to the open air. Evaporation proceeds with great rapidity, and the mercury soon freezes.

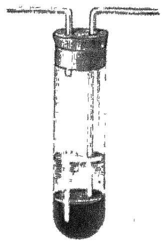
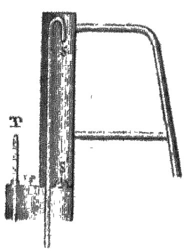


Fig. 64. Freezing of Mercury by Evaporation of Sulphurous Acid.

103. **Carré's Ammoniacal Apparatus.**—The apparatus invented some years ago by M. Carré for making ice is another instance of the ap-



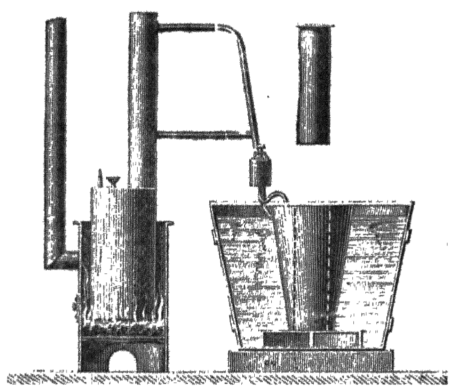


Fig. 65. Carré's Apparatus for Freezing by Ammonia Fig. 66.

plication of cold produced by evaporation. It consists (Figs. 65 and 66) of two parts, a boiler and a cooler. The boiler is of wrought iron, and is so constructed as to give a very large heating surface. It is three-quarters filled with a saturated solution of ammonia.

which contains from six to seven hundred times its volume of gas. The cooler is of an annular form, and in the central space is placed a vessel containing the water to be frozen. In the sides of the cooler are a number of small cells, the object of which is to increase its surface of contact with the water in which it is immersed.

In the first part of the experiment, which is represented in the figure, the boiler is placed upon a fire, and the temperature raised to 130°, while the cooler is surrounded with cold water. Ammoniacal gas is given off, passes into the cooler by the valve s' opening upwards, and is condensed in the numerous cells above mentioned. This first part of the operation, in the small machines for domestic use, occupies about three-quarters of an hour. In the second part of the operation, the cylindrical vessel containing the water to be frozen is placed in the central space; the cooler is surrounded with an envelope of felt, which is a very bad conductor of heat, and the boiler is immersed in cold water. The water in the boiler, as it cools, is able again to receive and dissolve the gas, which enters by the valve s of the bent siphon-shaped tube. The liquid ammonia in the cooler accordingly evaporates with great rapidity, producing a fall of temperature which freezes the water in the inclosed vessel.

104. **Solidification of Carbonic Acid.**—When a small orifice is opened in a vessel containing liquid carbonic acid, evaporation proceeds so rapidly that the cold resulting from it freezes a portion of the vapour, which takes the form of fine snow, and may be collected in considerable quantity.

This carbonic acid snow, which was first obtained by Thilorier, is readily dissolved by ether, and forms with it one of the most intense freezing-mixtures known. By immersing tubes containing liquefied gases in this mixture, Faraday succeeded in reducing several of them, including carbonic acid, cyanogen, and nitrous oxide, to the form of clear transparent ice, the fall of temperature being aided, in some of his experiments, by employing an air-pump to promote more rapid evaporation of carbonic acid from the mixture. By the latter process he was enabled to obtain a temperature of $-166°$ F. ($-110°$ C.) as indicated by an alcohol thermometer, the alcohol itself being reduced to the consistence of oil. Despretz, by means of the cold produced by a mixture of solid carbonic acid, liquid nitrous oxide, and ether, rendered alcohol so viscid that it did not run out when the vessel which contained it was inverted.

105. **Continuity of the Liquid and Gaseous States. Critical Tem-**

perature.—Remarkable results were obtained by Cagniard de la Tour[1] by heating volatile liquids (alcohol, petroleum, and sulphuric ether) in closed tubes of great strength, and of capacity about double the volume of the inclosed liquid. At certain temperatures (36° C. for alcohol, and 42° for ether) the liquid suddenly disappeared, becoming apparently converted into vapour.

Drion,[2] by similar experiments upon hydrochloric ether, hyponitric acid, and sulphurous acid, showed—

1. That the coefficients of apparent expansion of these liquids increase rapidly with the temperature.

2. That they become equal to the coefficient of expansion of air, at temperatures much lower than those at which total conversion into vapour occurs.

3. That they may even become double and more than double the coefficient of expansion of air; for example, at 130° C. the coefficient of expansion of sulphurous acid was ·009571.

Thilorier had previously shown that the expansion of liquid carbonic acid between the temperatures 0° and 30° C. is four times as great as that of air.

Drion further observed, that when the temperature was raised very gradually to the point of total vaporization, the free surface lost its definition, and was replaced by a nebulous zone without definite edges and destitute of reflecting power. This zone increased in size both upwards and downwards, but at the same time became less visible, until the tube appeared completely empty. The same appearances were reproduced in inverse order on gradually cooling the tube.

When the liquid was contained in a capillary tube, or when a capillary tube was partly immersed in it, the curvature of the meniscus and the capillary elevation decreased as the temperature rose, until at length, just before the occurrence of total vaporization, the surface became plane, and the level was the same within as without the tube.

Dr. Andrews, by a series of elaborate experiments on carbonic acid, with the aid of an apparatus which permitted the pressure and temperature to be altered independently of each other, showed that at temperatures above 31° C. this gas cannot be liquefied, but, when subjected to intense pressure, becomes reduced to a condition

[1] *Ann. de Chim.* II. xxi. [2] *Ann. de Chim.* III. lvi.

in which, though homogeneous, it is neither a liquid nor a gas. When in this condition, lowering of temperature under constant pressure will reduce it to a liquid, and diminution of pressure at constant temperature will reduce it to a gas; but in neither case can any breach of continuity be detected in the transition.

On the other hand, at temperatures below 31°, the substance remains completely gaseous until the pressure reaches a certain limit depending on the temperature, and any pressure exceeding this limit causes liquefaction to commence and to continue till the whole of the gas is liquefied, the boundary between the liquefied and unliquefied portions being always sharply defined.

The temperature 31° C., or more exactly 30·92° C. (87·7° F.), may therefore be called the *critical temperature* for carbonic acid; and it is probable that every other substance, whether usually occurring in the gaseous or in the liquid form, has in like manner its own critical temperature. Dr. Andrews found that nitrous oxide, hydrochloric acid, ammonia, sulphuric ether, and sulphuret of carbon, all exhibited critical temperatures, which, in the case of some of these substances, were above 100° C.

It is probable that, in the experiments of Cagniard de la Tour and Drion, the so-called total conversion into vapour was really conversion into the intermediate condition.

The continuous conversion of a gas into a liquid may be effected by first compressing it at a temperature above its critical temperature, until it is reduced to the volume which it will occupy when liquefied, and then cooling it below the critical point.

The continuous conversion of a liquid into a gas may be obtained by first raising it above the critical temperature while kept under pressure sufficient to prevent ebullition, and afterwards allowing it to expand.

When a substance is a little above its critical temperature, and occupies a volume which would, at a lower temperature, be compatible with partial liquefaction, very great changes of volume are produced by very slight changes of pressure.

On the other hand, when a substance is at a temperature a little below its critical point, and is partially liquefied, a slight increase of temperature leads to a gradual obliteration of the surface of demarcation between the liquid and the gas; and when the whole has thus been reduced to a homogeneous fluid, it can be made to exhibit an appearance of moving or flickering striæ throughout its entire mass

by slightly lowering the temperature, or suddenly diminishing the pressure.

The apparatus employed in these remarkable experiments, which are described in the Bakerian Lecture (*Phil. Trans.* 1869), is shown in Fig. 67, where *c c* are two capillary glass tubes of great strength, one of them containing the carbonic acid or other gas to be experimented on, the other containing air to serve as a manometer. These are connected with strong copper tubes *d d*, of larger diameter, containing water, and communicating with each other through *a b*, the water being separated from the gases by a column of mercury occupying the lower portion of each capillary tube. The steel screws *s s* are the instruments for applying pressure. By screwing either of them forward into the water, the contents of both tubes are compressed, and the only use of having two is to give a wider range of compression. A rectangular brass case (not shown in the figure), closed before and behind with plate-glass, surrounds each capillary tube, and allows it to be maintained at any required temperature by the flow of a stream of water.

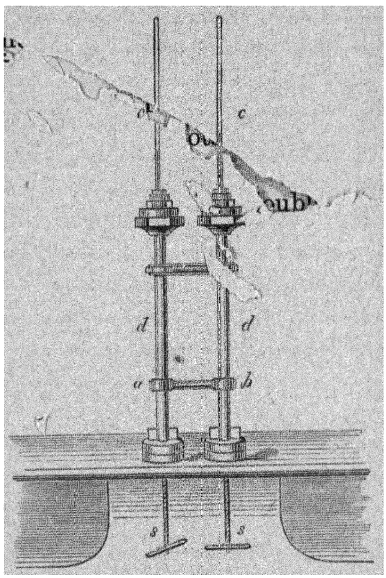


Fig. 67.—Andrews' Apparatus.

106. Liquefaction and Solidification of Oxygen and Hydrogen.—Up to quite recent times, air, oxygen, hydrogen, nitrogen, nitric oxide, and marsh-gas had defied all attempts to liquefy them, and were therefore called "permanent gases." But in the latter part of the year 1877 and the beginning of 1878 they were liquefied by two investigators independently.

M. Cailletet, a French engineer, employed an apparatus similar in principle to that of Dr. Andrews described in the preceding section; the gas being compressed in a strong capillary tube by screwing a plunger into water, which transmitted the pressure to mercury in contact with the gas. When the gas had had time to lose its heat of compression, and to attain the low temperature of the inclosure by

which it was surrounded, it was suddenly allowed to expand by unscrewing a second screw plunger provided for the purpose; and under the influence of the intense cold produced by this expansion, the gas in the tube assumed the form of a cloud, showing that drops of liquid were present in it. For thus liquefying oxygen, he employed a pressure of 300 atmospheres, and, before allowing the gas to expand, cooled it to the temperature −29° C. by means of the evaporation of sulphurous acid. For nitrogen he employed a pressure of 200, and for hydrogen of 280 atmospheres.

M. Raoul Pictet, of Geneva, who has devoted much attention to the artificial production of ice, cooled the gas under pressure, by surrounding it with two tubes one within the other, the outer one containing liquid sulphurous acid, which was rapidly evaporated by pumping away its vapour, while the inner one contained solid carbonic acid, which was also evaporated by means of a pump. The temperature of the outer tube was −65° or −70°, that of the inner about −140°, and this inner tube immediately surrounded the tube containing the gas which it was desired to liquefy. The pressure was produced, as in Faraday's earlier experiments, by the chemical action which evolved the gas. When time had been given for the compressed gas to take the low temperature of its surroundings, a cock was opened which allowed it to escape through a small orifice into the external air, and the issuing jet was seen to be liquid. In the case of oxygen, the pressure before the escape of the jet was 320 atmospheres, in the case of hydrogen it was 650. The jet of liquid hydrogen was of a steel-blue colour, and after a short time it was changed into a hail of solid particles, showing that hydrogen had not only been liquefied but solidified. In a later experiment the jet of oxygen was submitted to optical tests (by polarized light) which showed that it contained solid particles.

106A. Dewar's Experiments.—More recently (1892) Professor Dewar, at the Royal Institution, has liquefied oxygen and some other gases by intense cold at atmospheric pressure. The gas to be liquefied was cooled by the continued evaporation of ethylene and of nitrous oxide, the temperature being sometimes as low as −200° C. Liquid oxygen, in quantities of a pint, was exhibited in an open vessel, its boiling point under atmospheric pressure being −180° C., and its latent heat of evaporation 80° C. After filtration it was a clear transparent liquid with a slight blueish tinge.

It was found to be a non-conductor of electricity, and to be so

strongly magnetic that, when placed in the spheroidal state in a cup of rock-salt beneath the poles of an electro-magnet, it leaped up as soon as the magnetizing current was turned on, and formed a bridge of hour-glass shape between the poles, as shown in Fig. 67A.[1] At

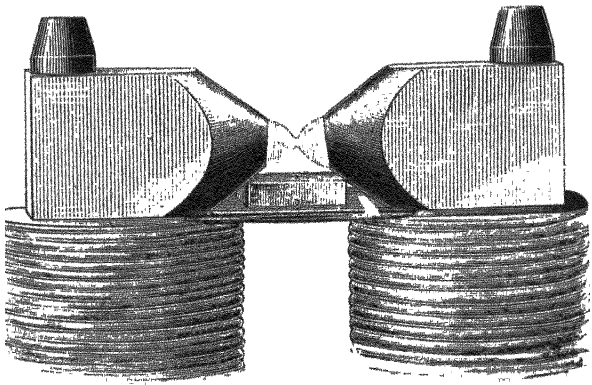


Fig. 67A.—Dewar's Experiment

the temperature $-200°$ C., it was chemically inert, exhibiting no action on phosphorus or potassium dropped into it.

Air was liquefied in a similar manner, its two constituents going down together into one liquid; and when tested by the electro-magnet all the liquid went to the poles. When it was allowed to evaporate, its nitrogen boiled off before its oxygen. When surrounded by a jacket of liquid oxygen, and subjected to rapid and sustained evaporation of its nitrogen by means of a powerful pump, it was reduced to a solid block of transparent ice. The boiling point of liquid nitrogen was found to be $-190°$ C.

[1] For this figure, which is copied from a photograph, we are indebted to Professor Dewar and the publishers of the *Engineer*.

CHAPTER IX.

EBULLITION.

107. Ebullition.—When an open vessel containing a liquid is placed upon a fire or held over the flame of a lamp, evaporation at first goes on quietly and the liquid steadily rises in temperature; but after a time the liquid becomes agitated, gives off vapour much faster, and remains nearly constant in temperature. The liquid is now said to *boil* or to be in a state of *ebullition*.

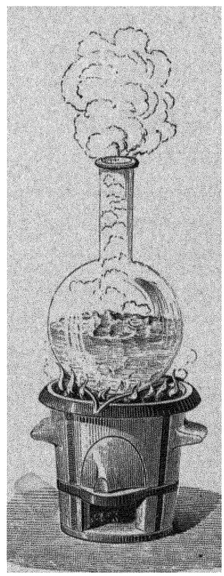


Fig. 68. Ebullition.

If we observe the gradual progress of the phenomena—as we can easily do in a glass vessel containing water, we shall perceive that, after a time, very minute bubbles are given off; these are bubbles of dissolved air. Soon after, at the bottom of the vessel, and at those parts of the sides which are most immediately exposed to the action of the fire, larger bubbles of vapour are formed, which decrease in volume as they ascend, and disappear before reaching the surface. This stage is accompanied by a peculiar sound, indicative of approaching ebullition, and the liquid is said to be *singing*. The sound is probably caused by the collapsing of the bubbles as they are condensed by the colder water through which they pass. Finally, the bubbles increase in number, growing larger as they ascend, until they burst at the surface, which is thus kept in a state of agitation; and the liquid boils.

108. Laws of Ebullition.—The following are the ordinary laws of ebullition.

1. *At the ordinary pressure, ebullition commences at a temperature which is definite for each liquid.*

This law is analogous to that of fusion (§ 72). It follows from this that the boiling-point of any liquid is a *specific* element, serving to determine its nature

The following table gives the boiling-points of several liquids at the pressure of 760 millimetres:—

Sulphurous acid,	−10° C.	Spirits of turpentine,	+130° C.
Hydrochloric ether,	+11°	Phosphorus,	290°
Common ether,	37°	Concentrated sulphuric acid,	325°
Alcohol,	79°	Mercury,	353°
Distilled water,	100°	Sulphur,	440°

2. *The temperature remains constant during ebullition.* If a thermometer be introduced into the glass vessel of Fig. 68, the temperature will be observed to rise gradually during the different stages preceding ebullition; but, when active ebullition has once commenced, no further advance of temperature will be observed.

This phenomenon points to the same conclusion as the cold produced by evaporation. Since, notwithstanding the continuous action of the fire, the temperature remains constant, the conclusion is inevitable, that all the heat produced is employed in doing the work necessary to change the liquid into vapour. The constancy of temperature during ebullition explains the fact that vessels of pewter, tin, or any other easily fusible metal, may be safely exposed to the action of even a very hot fire, provided that they contain water, since the liquid remains at a temperature of about 100°, and its contact prevents the vessel from over-heating. We shall see hereafter that, under certain circumstances, the commencement of ebullition is delayed till the liquid has risen considerably above the permanent temperature which it retains when boiling. The second law also is not absolutely exact. Small fluctuations of temperature occur, and some parts of the liquid are slightly hotter than others. The temperature *of the vapour* is more constant than that of the water, and is accordingly employed in determining the "fixed points" of thermometers.

3. *The pressure of the vapour given off during ebullition is equal to that of the external air.*

Previous to ebullition, the upper part of the vessel in Fig. 68 contains a mixture of air and vapour, the joint pressure being sensibly equal to that of the external air; but when active ebullition

occurs, the air is expelled, and the upper part of the vessel, from the liquid to the mouth, is occupied by vapour alone, which, being in free communication with the external air, must be sensibly equal to it in presssure.

The following experiment furnishes an interesting confirmation of this third law.

We take a bent tube A, open at the longer extremity, and closed at the shorter. The short branch is filled with mercury, all but a small space containing water; in the long branch the mercury stands a little higher than the bend. Water is now boiled in a glass vessel, and, during ebullition, the bent tube is plunged into the steam. The water occupying the upper part of the short branch is partially converted into steam, the mercury falls, and it *assumes the same level in both branches.* Thus the pressure exerted by the atmosphere at the open extremity of the tube is exactly equal to that exerted by the vapour of water at the temperature of ebullition.

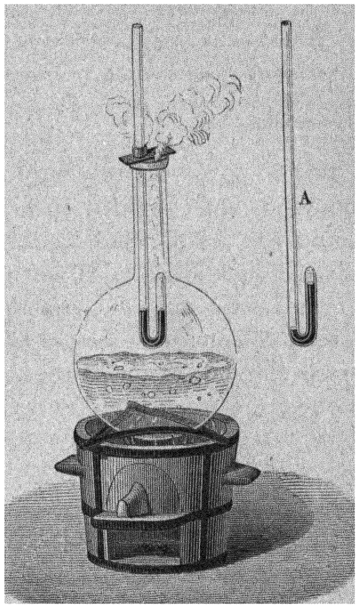


Fig 69 —Tension of Vapour during Ebullition

109. Definition of Ebullition.— This latter circumstance supplies the true physical definition of ebullition. *A liquid is in ebullition when it gives off vapour of the same pressure as the atmosphere above it.*

The necessity of this equality of tension is easily explained. If a bubble of vapour exists in the interior of a liquid (as at *m*, Fig. 70), it is subject to a pressure exceeding atmospheric by the weight of the liquid above it. As the bubble rises, the latter element of pressure becomes less, and the pressure of the vapour composing the bubble accordingly diminishes, until it is reduced to atmospheric pressure on reaching the surface.

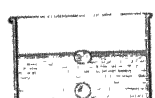


Fig. 70

The boiling-point of a liquid at given pressure is therefore neces-

sarily fixed, since it is the temperature at which the pressure of the vapour at saturation is equal to this pressure. It must be remarked, however, that the boiling-point varies in the different layers of the liquid, increasing with the depth below the surface. Accordingly, in determining the second fixed point of the thermometer, we have stated that the instrument should be plunged into the steam, and not into the water.

110. Effect of Pressure upon the Boiling-point.—It evidently follows from the foregoing considerations that the boiling-point of a liquid must vary with the pressure on the surface; and experiment shows that this is the case. Water, for instance, boils at 100° under the external pressure of 760 millimetres; but if the pressure decreases, ebullition occurs at a lower temperature. Under the receiver of an air-pump, water may be made to boil at any temperature between 0° and 100°. In Carré's apparatus (Fig. 61) the water in the glass bottle is observed to enter into active ebullition a few moments before the appearance of the ice. The reason, therefore, why boiling water has come to be associated in our minds with a fixed temperature is that the variations of atmospheric pressure are comparatively small.

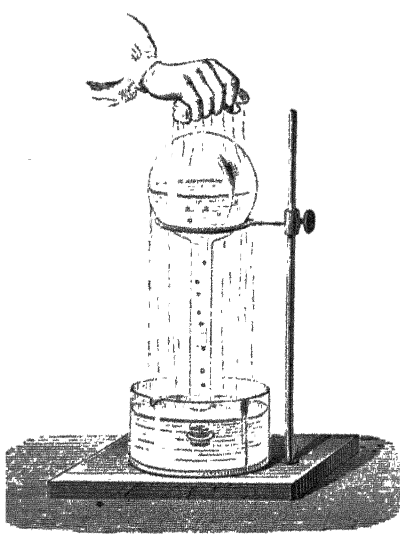


Fig. 71.—Franklin's Experiment.

At Paris, for instance, the external pressure varies between 720 and 790 millimetres (28·3 and 31·1 inches), and the boiling-point, in consequence, varies from 98·5° to 101·1°, the difference being at the rate of about 27^{mm} per degree.

111. Franklin's Experiment.—The boiling of water at a temperature lower than 100° may be shown by the following experiment:—

A little water is boiled in a flask for a sufficient time to expel most of the air contained in it. The flask is then removed from the source of heat, and is at the same time securely corked. To render the exclusion of air still more certain, it may be inverted with the corked end immersed in

water which has been boiled. Ebullition ceases almost immediately; but if cold water be now poured over the vessel, or, better still, if ice be applied to it, the liquid again begins to boil, and continues to do so for a considerable time. This fact may easily be explained: the contact of the cold water or the ice lowers the temperature and pressure of the steam in the flask, and the decrease of pressure causes the renewal of ebullition.

112. Determination of Heights by Boiling-point.—Just as we can determine the boiling-point of water when the external pressure is given, so, if the boiling-point be known, we can determine the external pressure. In either case we have simply to refer to a table of maximum pressures of aqueous vapour at different temperatures.

As the mercurial barometer is essentially unsuitable for portability, Wollaston proposed to substitute the observation of boiling-points as a means of determining pressures. For this purpose he employed a thermometer with a large bulb and with a scale of very long degrees finely subdivided extending only a few degrees above and below 100°. He called this instrument the barometric thermometer.

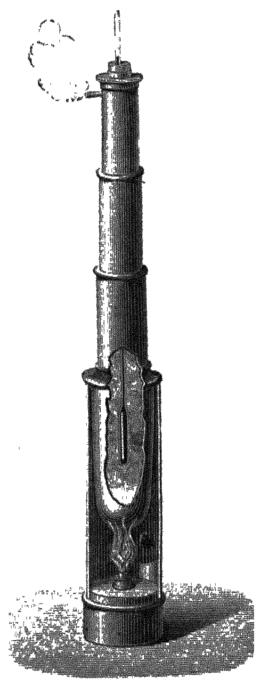


Fig. Hypsometer.

Regnault has constructed a small instrument for the same purpose, which he calls the *hypsometer*. It consists of a little boiler heated by a spirit-lamp, and terminating in a telescope tube with an opening at the side through which the steam escapes. A thermometer dips into the steam, and projects through the top of the tube so as to allow the temperature of ebullition to be read.

This temperature at once gives the atmospheric pressure by reference to a table of vapour-pressures, and the subsequent computations for determining the height are the same as when the barometer is employed (see Part I.).

When only an approximate result is desired, it may be assumed that the height above sea-level is sensibly proportional to the differ-

ence between the observed boiling-point and 100° C., and Soret's formula[1] may be employed, viz.:

$$h = 295\ (100 - t),$$

where h is expressed in metres and t in degrees Centigrade.

Thus, at Quito, where the boiling-point of water is about 90·1°, the height above sea-level would be $9{\cdot}9 \times 295 = 2920$ metres, which agrees nearly with the true height 2808 metres.

At Madrid, at the mean pressure, the boiling-point is 97·8°, which gives $2{\cdot}2 \times 295 = 649$ metres; the actual height being 610 metres.

113. **Papin's Digester.**—While a decrease of pressure lowers the boiling-point, an increase of pressure raises it. Accordingly, by putting the boiler in communication with a reservoir containing air at the pressure of several atmospheres, we can raise the boiling-point to 110°, 115°, or 120°; a result often of great utility in the arts. But in order that the liquid may actually enter into ebullition, the space above the liquid must be sufficiently large and cool to allow of the condensation of the steam. In a confined vessel, water may be raised to a higher temperature than would be possible in the open air, but it will not boil. This is the case in the apparatus invented by the celebrated Papin, and called after him *Papin's digester.* It is a bronze vessel of great strength, covered with a lid secured by a powerful screw. It is employed for raising water to very high temperatures, and thus obtaining effects which would not be possible with water at 100°, such, for example, as dissolving the gelatine contained in bones.

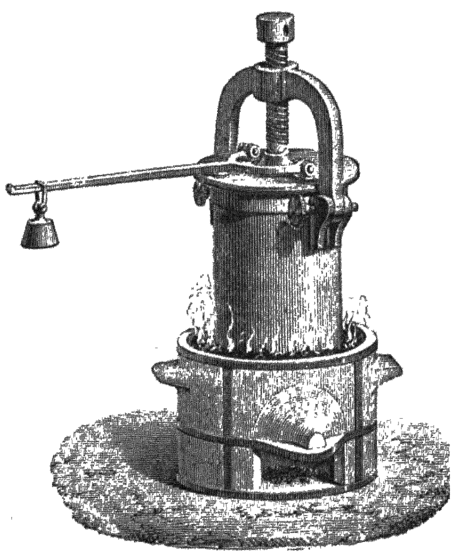


Fig. 78.—Papin's Digester

It is to be observed that the pressure of the steam increases rapidly

[1] If h be expressed in feet, and t in degrees Fahrenheit, the formula becomes

$$h = 538\ (212 - t).$$

with the temperature, and may finally acquire an enormous power. Thus, at 200°, the pressure is that of 16 atmospheres, or about 240 pounds on the square inch. In order to obviate the risk of explosion, Papin introduced a device for preventing the pressure from exceeding a definite limit. This invention has since been applied to the boilers of steam-engines, and is well known as the *safety-valve*. It consists of an opening, closed by a conical valve or stopper, which is pressed down by a lever loaded with a weight. Suppose the area of the lower end of the stopper to be 1 square inch, and that the pressure is not to exceed 10 atmospheres, corresponding to a temperature of 180°. The magnitude and position of the weight are so arranged that the pressure on the hole is 10 times 15 pounds. If the tension of the steam exceed 10 atmospheres, the lever will be raised, the steam will escape, and the pressure will thus be relieved.

It is a remarkable fact that, while the steam from an ordinary kettle scalds the hand, no injury is sustained by holding the hand in the jet of steam which issues from the safety-valve of a high-pressure boiler, although the temperature within the boiler is much higher than that within the kettle. The explanation appears to be that the steam which issues from the safety-valve is dry, and, like any other gas, is slow in raising the temperature of bodies exposed to it, while the steam from the kettle, being at saturation, deposits scalding-hot water on surfaces colder than itself The subject has been much discussed in connection with "the specific heat of saturated steam" (see § 258) The expansion of the steam in escaping through the safety-valve has a powerful cooling effect, which would of itself be sufficient to produce partial condensation; but this effect is counteracted by the heat generated by the violent friction of the steam against the sides of the orifice, and the resultant effect is to leave the steam in a superheated condition.

114 **Boiling-point of Saline Solutions.**—When water holds saline matters in solution, the boiling-point rises as the proportion of saline matter in the water increases. Thus with sea-salt the boiling-point can be raised from 100° to 108°.

When the solution is not saturated, the boiling-point is not fixed, but rises gradually as the mixture becomes concentrated; but at a certain stage the salt begins to be precipitated, and the temperature then remains invariable. This is to be considered the normal boiling-point of the saturated solution. Supersaturation, however, sometimes occurs, the temperature gradually rising above the normal boiling-

point without any deposition of the salt, until all at once precipitation begins, and the thermometer falls several degrees.

The steam emitted by saline solutions consists of pure water, and it is frequently asserted to have the same temperature as the steam of pure water boiling under the same pressure; but the experiments of Magnus and others have shown that this is not the case. Magnus, for example,[1] found that when a solution of chloride of calcium was boiling at 107°, a thermometer in the steam indicated 105¼°, and when by concentration the boiling-point had risen to 116°, the thermometer in the steam indicated 111·2°.

These and other observations seem to indicate that the steam emitted by a saline solution when boiling, is in the condition in which the steam of pure boiling water would be, if heated, under atmospheric pressure, to the temperature of the boiling solution. It can therefore be cooled down to the boiling-point of pure water without undergoing any liquefaction. When cooled to this point, it becomes saturated,[2] and precisely resembles the steam of pure water boiling under the same pressure. When saturated steam loses heat, it does not cool, but undergoes partial liquefaction, and it does not become completely liquefied till it has lost as much heat as would have cooled more than a thousand times its weight of superheated steam one degree Centigrade.

115. **Boiling-point of Liquid Mixtures.**—A mixture of two liquids which have an attraction for each other, and will dissolve each other freely in all proportions—for example, water and alcohol—has a boiling-point intermediate between those of its constituents. But a mechanical mixture of two liquids between which no solvent action takes place—for example, water and sulphide of carbon—has a boiling-point lower than either of its constituents. If steam of water is passed into liquid sulphide of carbon, or if sulphide of carbon vapour is passed into water, a mixture is obtained which boils at 42·6° C., being four degrees lower than the boiling-point of sulphide of carbon alone. This apparent anomaly is a direct consequence of the laws of vapours stated in § 95; for the boiling-point of such a mixture is the temperature at which the sum of the pressures of the two independent vapours is equal to one atmosphere.

116. **Difficulty of Boiling without Air.**—The presence of air in the

[1] Poggendorff's *Annalen*, cxii. p. 415.

[2] *Saturated steam* is the ordinary designation of steam at the maximum density and pressure for its actual temperature. The term *superheated* has been explained in § 94.

midst of the liquid mass is a necessary condition of regularity of ebullition, and of its production at the normal temperature; this is shown by several convincing experiments.

1. *Donny's Experiment.*—We take a glass tube bent twice, and terminated at one of its extremities by a series of bulbs. The first step is to wash it carefully with alcohol and ether, finally leaving in it some diluted sulphuric acid. These operations are for the purpose of removing the solid particles adhering to the sides, which always detain portions of air. Water is then introduced and boiled long enough to expel the air dissolved in it, and while ebullition is proceeding, the end of the apparatus is hermetically sealed. The other

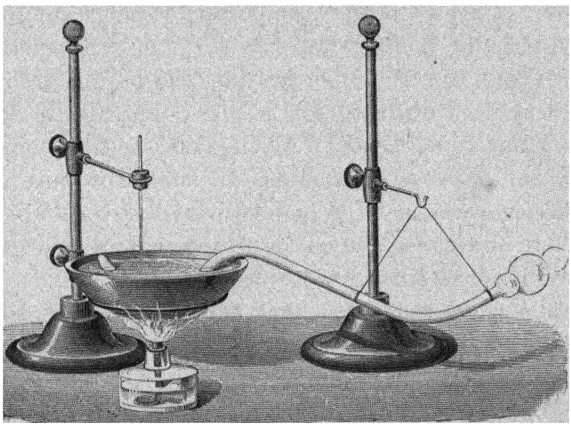


Fig 74 —Donny's Experiment.

extremity is now plunged in a strong solution of chloride of calcium, which has a very high boiling-point, and the tube is so placed that all the water shall lie in this extremity; it will then be found that the temperature may be raised to 135° without producing ebullition. At about this temperature bubbles of steam are seen to be formed, and the entire liquid mass is thrown forward with great violence. The bulbs at the end of the tube are intended to diminish the shock thus produced.

2. *Dufour's Experiment.*—This experiment is still more decisive. A mixture of linseed-oil and oil of cloves, whose respective densities are about ·93 and 1·01, is so prepared that, for temperatures near 100°, the density of the whole is nearly that of water. This mixture is placed in a cubical box of sheet-iron, with two holes opposite each other, which are filled with glass, so as to enable the observer to

perceive what is passing within. The box is placed in a metallic envelope, which permits of its being heated laterally. When the temperature of 120° has been reached, a large drop of water is allowed to fall into the mixture, which, on reaching the bottom of the box, is partially converted into vapour, and breaks up into a number of smaller drops, some of which take up a position between the two windows, so as to be visible to the observer. The temperature may then be raised to 140°, 150°, or even 180°, without producing evaporation of any of these drops. Now the maximum tension of steam at 180° is equal to 10 atmospheres, and yet we have the remarkable phenomenon of a drop of water remaining liquid at this temperature under no other pressure than that of the external air increased by an inch or two of oil. The reason is that the air necessary to evaporation is not supplied. If the drops be touched with a rod of metal, or, better still, of wood, they are immediately converted into vapour with great violence, accompanied by a peculiar noise. This is explained by the fact that the rods used always carry a certain quantity of condensed air upon their surface, and by means of this air the evaporation is produced. The truth of this explanation is proved by the fact, that when the rods have been used a certain number of times, they lose their power of provoking ebullition, owing, no doubt, to the exhaustion of the air which was adhering to their surfaces.

3. *Production of Ebullition by the formation of Bubbles of Gas in the midst of a Liquid.*—A retort is carefully washed with sulphuric acid, and then charged with water slightly acidulated, from which the air has been expelled by repeated boiling. The retort communicates with a manometer and with an air-pump. The air is exhausted until a pressure of only 150 millimetres is attained, corresponding to 60° as boiling-point. Dufour has shown that under these conditions the temperature may be gradually raised to 75° without producing ebullition. But if, while things are in this condition, a current of electricity is sent through the liquid by means of two platinum wires previously immersed in it, the bubbles of oxygen and hydrogen which are evolved at the wires immediately produce violent ebullition, and a portion of the liquid is projected explosively, as in Donny's experiment.

From these experiments we may conclude that liquid, when not in contact with gas, has a difficulty in *making a beginning* of vaporization, and may hence remain in the liquid state even at tempera-

tures at which vaporization would upon the whole involve a fall of potential energy.

That vapour (as well as air) can furnish the means of overcoming this difficulty, is established by the fact noted by Professor G. C. Foster,[1] that when a liquid has been boiling for some time in a retort, it sometimes ceases to exhibit the movements characteristic of ebullition, although the amount of vapour evolved at the surface, as measured by the amount of liquid condensed in the receiver, continues undiminished. In these circumstances, it would appear that the superficial layer of liquid, which is in contact with its own vapour, is the only part that is free to vaporize.

The preceding remarks explain the reluctance of water to boil in glass vessels carefully washed, and the peculiar formation, in these circumstances, of large bubbles of steam, causing what is called *boiling by bumping.* In the case of sulphuric acid, the phenomenon is much more marked; if this liquid be boiled in a glass vessel, enormous bubbles are formed at the sides, which, on account of the viscous nature of the liquid, raise the mass of the liquid above them, and then let it fall back with such violence as sometimes to break the vessel. This inconvenience may be avoided by using an annular brazier (Fig. 75), by means of which the upper part only of the liquid is heated.

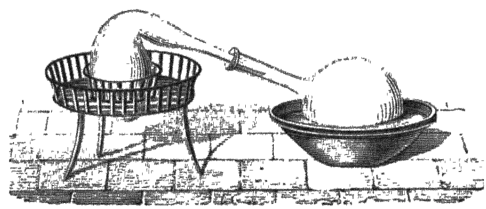


Fig. 75.—Apparatus for Boiling Sulphuric Acid.

The ebullition of ether and alcohol presents some similar features, probably because these liquids dissolve the fatty particles on the surface of the glass, and thus adhere to the sides very strongly.

117. **Spheroidal State.**—This is the name given to a peculiar condition which is assumed by liquids when exposed to the action of very hot metals.

If we take a smooth metal plate, and let fall a drop of water upon it, the drop will evaporate more rapidly as the temperature of the plate is increased up to a certain point. When the temperature of the plate exceeds this limit, which, for water, appears to be about

[1] Watts's *Dictionary of Chemistry*, art. "Heat," p. 88.

150°, the drop assumes a spheroidal form, rolls about like a ball or spins on its axis, and frequently exhibits a beautiful rippling, as represented in the figure. While in this condition, it evaporates much more slowly than when the plate was at a lower temperature. This latter circumstance is important, and is easily verified by experiment. If the plate be allowed to cool, a moment arrives when the globule of water flattens out, and boils rapidly away with a hissing noise.

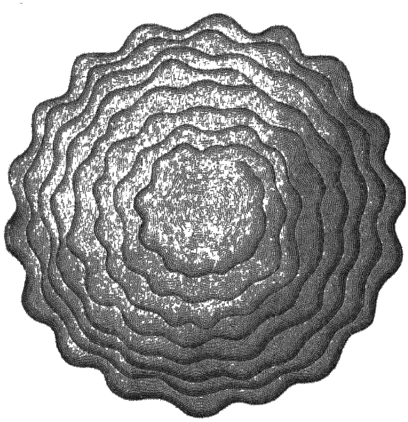


Fig. 76.—Globule in the Spheroidal State

These phenomena have been long known, and were studied by Leidenfrost and Klaproth; but more complete and searching investigations were made by Boutigny. All liquids are probably capable of assuming the spheroidal state. Among those which have been tested are alcohol, ether, liquid sulphurous acid, and liquid nitrous oxide. When in this state they do not boil. Sometimes bubbles of steam are seen to rise and burst at the top of the globule, but these are always owing to some roughness of the surface, which prevents the steam from escaping in any other way; when the surface is smooth, no bubbles are observed.

If the temperature of the liquid be measured by means of a thermometer with a very small bulb, or a thermo-electric junction, it is always found to be below the boiling-point.

118. **Freezing of Water and Mercury in a Red-hot Crucible.**—This latter property enables us to obtain some very striking and paradoxical results. The boiling-point of liquid sulphurous acid is $-10°$ C., and that of liquid nitrous oxide is about $-70°$ C. If a silver or platinum crucible be heated to redness by a powerful lamp, and some liquid sulphurous acid be then poured into it, this latter assumes the spheroidal state; and drops of water let fall upon it are immediately frozen. Mercury can in like manner be frozen in a red-hot crucible by employing liquid nitrous oxide in the spheroidal state.

These experiments are due to Boutigny, who called attention to them as remarkable exceptions to the usual tendency of bodies to

equilibrium of temperature. The exception is of the same kind as that presented by a vessel of water boiling at a constant temperature of 100° over a hot fire, the heat received by the liquid being in both cases expended in producing evaporation.

119. **The Metal not in Contact with the Liquid.**—The basis of the entire theory of liquids in the spheroidal state is the fact that the liquid and the metal plate do not come into contact. This fact can be proved by direct observation.

The plate used must be quite smooth and accurately levelled. When the plate is heated, a little water is poured upon it, and assumes the spheroidal state. By means of a fine platinum wire which passes into the globule, the liquid is kept at the centre of the metal plate. It is then very easy, by placing a light behind the globule, to see distinctly the space between the liquid and the plate. The appearance thus presented may be easily thrown on a screen by means of the electric light.

The interruption of contact can also be proved by connecting (through a galvanometer) one pole of a battery with the hot plate, while a wire from the other pole is dipped in the liquid. The cur-

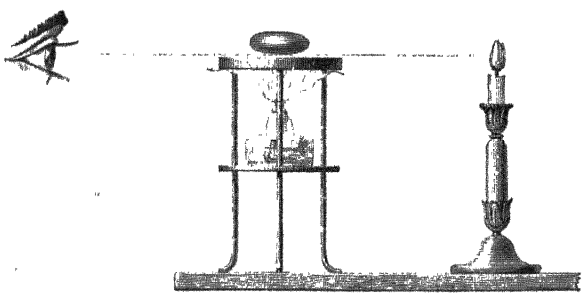


Fig. 77.—Separation between Globule and Plate

rent refuses to circulate if the liquid is in the spheroidal state, but is immediately established when, on cooling the plate, the liquid begins to boil.

Various attempts have been made to account for the absence of contact between the liquid and the metal, but the true explanation is as yet uncertain.

In consequence of the separation, heat can only pass to the globule by radiation, and hence its comparatively low temperature is accounted for.

The absence of contact between a liquid and a metal at a high

temperature may be shown by several experiments. If, for instance, a ball of platinum be heated to bright redness, and plunged (Fig. 78) into water, the liquid is seen to recede on all sides, leaving an envelope of vapour round the ball. This latter remains red for several seconds, and contact does not take place till its temperature has fallen to about 150°. An active ebullition then takes place, and an abundance of steam is evolved.

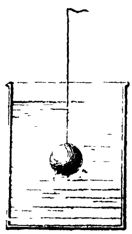


Fig 78.- Red-hot Ball in Water.

Professor Barrett has obtained a more striking effect of the same kind by lowering a red-hot ball of iron into the soapy liquid known as "Plateau's solution."

If drops of melted sugar be let fall on water, they will float for a short time, though their density is greater than that of water, contact being prevented by their high temperature. A similar phenomenon is observed when a fragment of potassium is thrown on water. The water is decomposed; its hydrogen takes fire and burns with a red flame; its oxygen combines with the potassium to form potash; and the globule of potash floats upon the surface without touching it, owing to the high temperature under which it is formed. After a few seconds the globule cools sufficiently to come into contact with the water, and bursts with a slight noise.

120. **Distillation.**—Distillation consists in boiling a liquid and condensing the vapour evolved. It enables us to separate a liquid from the solid matter dissolved in it, and to effect a partial separation of the more volatile constituent of a mixture from the less volatile.

The apparatus employed for this purpose is called a still. One of the simpler forms, suitable for distilling water, is shown in Fig. 79.

It consists of a retort *a*, the neck of which *c* communicates with a spiral tube *d d* called the *worm*, placed in the vessel *e*, which contains cold water. The water in the retort is raised to ebullition, the steam given off is condensed in the worm, and the *distilled water* is collected in the vessel *g*.

As the condensation of the steam proceeds, the water of the cooler becomes heated, and must be renewed; for this purpose a tube descending to the bottom of the cooler is supplied with a continuous stream of cold water from above, while the superfluous water flows out by the tube *i* at the upper part of the cooler. In this way the warm water, which rises to the top, is continually removed. The

boiler is filled about three-quarters full, and the water in it can from time to time be renewed by the opening *f*; but it is advisable not to

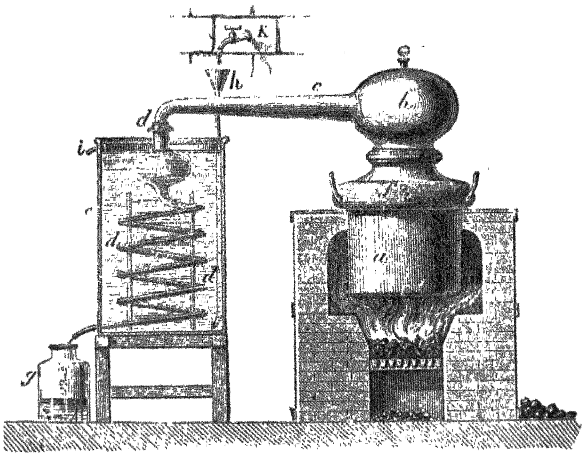


Fig 79 —Still

carry the process of distillation too far, and to throw away the liquid remaining in the boiler when its volume has been reduced to a fourth or a fifth of what it was originally. By exceeding this limit we run the risk of impairing the purity of the water by the carrying over of some of the solid matter contained in the liquid in the boiler.

121. Circumstances which Influence Rapidity of Evaporation.—In the case of a liquid exposed to the air, and at atmospheric temperature, the rapidity of evaporation increases with the extent of free surface, the dryness of the air, and the rapidity of renewal of the air immediately above the surface.

In the case of a liquid evaporated by boiling, the quantity evaporated in a given time is proportional to the heat received. This depends upon the intensity of the source of heat, the facility with which heat passes through the sides of the vessel, and the area of *heating surface*, that is to say, of surface (or more properly lamina) which is in contact with the liquid on one side, and with the source of heat on the other.

CHAPTER X.

QUANTITATIVE MEASUREMENTS RELATING TO VAPOURS.

122. Pressure of Aqueous Vapour.—The knowledge of the maximum pressure of the vapour of water at various temperatures is important, not only from a theoretical, but also from a practical point of view, inasmuch as this pressure is the motive force in the steam-engine. Experiments for determining it have accordingly been undertaken by several experimenters in different countries. The researches conducted by Regnault are especially remarkable for the range of temperature which they embrace, as well as for the number of observations which they include, and the extreme precision of the methods employed. Next to these in importance are the experiments of Magnus in Germany and of Fairbairn and Tate in England.

123. Dalton's Apparatus.—The first investigations in this subject which have any pretensions to accuracy were those of Dalton. The apparatus which he employed is represented in Fig. 80. Two barometric tubes A and B are inverted in the same cistern H; one is an ordinary barometer, the other a vapour-barometer; that is, a barometer in which a few drops of water have been passed up through the mercury. The two tubes, attached to the support C D, are surrounded by a cylindrical glass vessel containing water which can be raised to different temperatures by means of a fire. The first step is to fill the vessel with ice, and then read the difference of level of the mercury in the two tubes. This can be done by separating the fragments of ice. The difference thus observed is the pressure of aqueous vapour at zero Centigrade. The ice is then replaced by water, and the action of the fire is so regulated as to give different temperatures, ranging between 0° and 100° C., each of which is preserved constant for a few minutes, the water being at the same time well stirred by means of the agitator $p\,q$, so as to insure uniformity

of temperature throughout the whole mass. The difference of level in the two barometers is read off in each case; and we have thus the means of constructing, with the aid of graphical or numerical interpolation, a complete table of vapour-pressures from 0° to 100° C. At or about this latter temperature the mercury in the vapour-barometer falls to the level of the cistern; and the method is therefore inapplicable for higher temperatures. Such a table was constructed by Dalton.

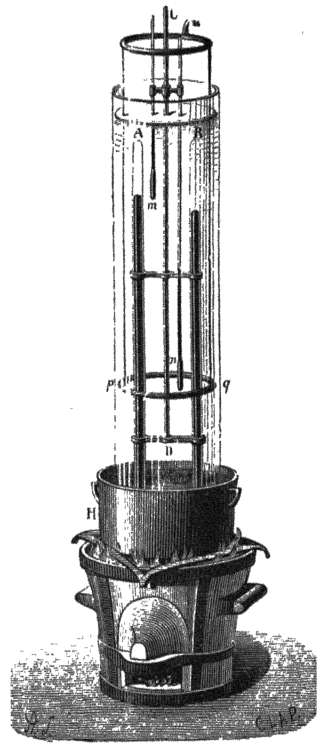


Fig. 80.—Dalton's Apparatus.

124. Regnault's Modifications.—Dalton's method has several defects. In the first place, it is impossible to insure that the temperature shall be everywhere the same in a column as long as that which is formed by the vapour at 70°, 75°, and higher temperatures. In the second place, there is always a good deal of uncertainty in observing the difference of level through the sides of the cylindrical glass vessel. Regnault employed this method only up to the temperature of 50° C. At this temperature the pressure of the vapour is only about 9 centimetres (less than 4 inches) of mercury, and it is thus unnecessary to heat the barometers throughout their entire length. The improved apparatus is represented in Fig. 81. The two barometric tubes, of an interior diameter of 14 millimetres, traverse two holes in the bottom of a metal box. In one of the sides of the box is a large opening closed with plate-glass, through which the necessary observations can be made with great accuracy. On account of the shortness of the liquid column it was very easy, by bringing a spirit-lamp within different distances of the box, to maintain for a sufficient time any temperature between 0° and 50° C.

The difference of level between the two mercurial columns should be reduced to 0° C. by the ordinary correction. We should also take into consideration the short column of water which is above the

mercury in the vapour barometer, and which, by its weight, produces a depression that may evidently be expressed in mercury by dividing the height of the column by 13·59.

To adapt this apparatus to low temperatures, it is modified in the following way. The upper extremity of the vapour barometer tube is drawn out and connected with a small copper tube of three branches, one of which communicates with an air-pump, and another with a glass globe of the capacity of about 500 cubic centimetres. In the interior of this globe is a small bulb of thin glass containing water, from which all the air has been expelled by boiling. The globe is several times exhausted of air, and after each exhaustion is refilled with air which has been passed over desiccating substances. After the last exhaustion, the tube which establishes communication with the air-pump is hermetically sealed, the box is filled with ice, and the pressure at zero of the dry air left behind in the globe by the air-pump is measured; it is of course exceedingly small. Heat is then applied to the globe, the little bulb bursts, and the globe, together with the space above the mercury, is filled with vapour. This form of apparatus can also be employed for temperatures up to 50°, the only difference being that the ice is replaced by water at different temperatures, allowance being made, in each case, for the elastic force of the unexhausted air.

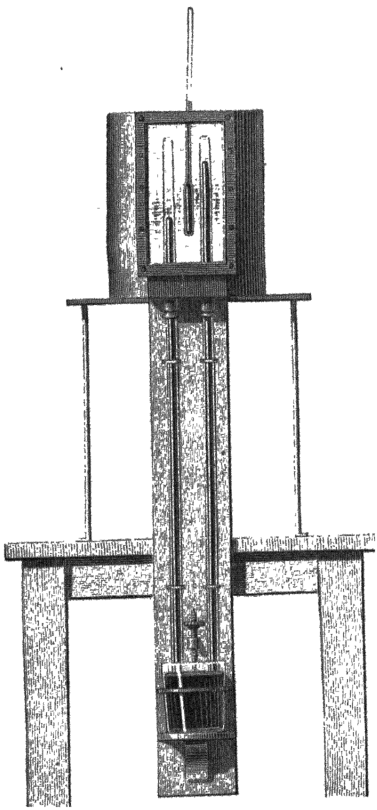


Fig. 81.—Modified Form of Dalton's Apparatus.

In the case of temperatures below zero, the box is no longer required, and the globe alone is placed in a vessel containing a freezing-mixture. The barometric tubes are surrounded by the air of the apartment.

In this case the space occupied by the vapour is at two different temperatures in different parts, but it is evident that equilibrium can exist only when the pressure is the same throughout. But the pressure of the vapour in the globe can never exceed the maximum pressure for the actual temperature; this must therefore be the pressure throughout the entire space, and is consequently that which corresponds to the difference of level observed.

In reality what happens is as follows:—The low temperature of the globe causes some of the vapour to condense; equilibrium is consequently destroyed, a fresh quantity of vapour is produced, enters the globe, and is there condensed, and so on, until the pressure is everywhere the same as the maximum pressure due to the temperature of the globe. This condensation of vapour in the cold part of the space was utilized by Watt in the steam-engine; it is the *principle of the condenser.*

Before Regnault, Gay-Lussac had already turned this principle to account in a similar manner for the measurement of low temperatures.

By using chloride of calcium mixed with successively increasing quantities of snow or ice, the temperature can be brought as low as $-32°$ C. ($-25·6°$ F.), and it can be shown that the pressure of the vapour of water is quite appreciable even at this point.

125. **Measurement of Pressures for Temperatures above 50°.**—In investigating the maximum pressure of the vapour of water at temperatures above 50°, Regnault made use of the fact that the pressure of the steam of boiling water is equal to the external pressure.

His apparatus consists (Fig 82) of a copper boiler containing water which can be raised to different temperatures indicated by very delicate thermometers. The vapour produced passes through a tube inclined upwards, which is kept cool by a constant current of water; in this way the experiment can be continued for any length of time, as the vapour formed by ebullition is condensed in the tube, and flows back into the boiler. The tube leads to the lower part of a large reservoir, in which the air can be either rarefied or compressed at will. This reservoir is in communication with a manometer. The apparatus shown in the figure is that employed for pressures not exceeding 5 atmospheres. Much greater pressures, extending to 28 atmospheres, can be attained by simply altering the dimensions of the apparatus without any change in its principle. The manometer employed in this case was the same as that used in testing Boyle's law, consisting of a long column of mercury.

In using this apparatus, the air in the reservoir is first rarefied until the water boils at about 50° C.; the occurrence of ebullition being recognized by its characteristic sound, and by the temperature

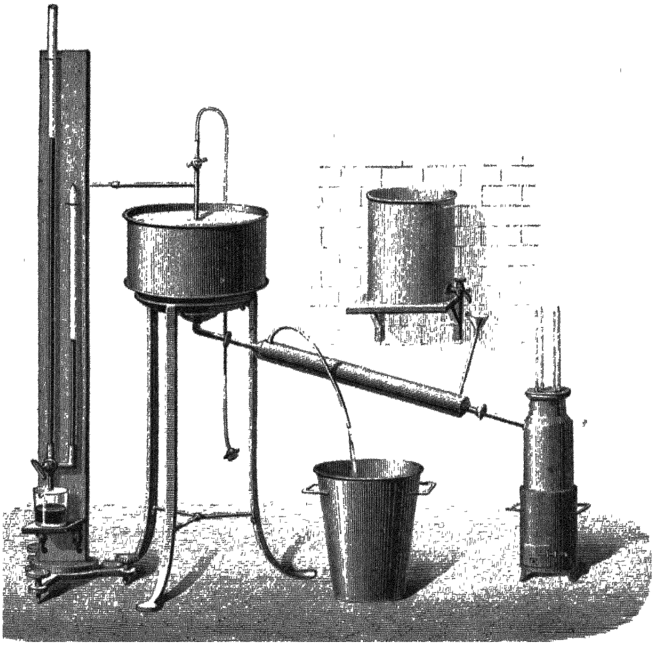


Fig. 82.—Regnault's Apparatus for High Temperatures.

remaining invariable. This steadiness of temperature is of great advantage in making the observations, inasmuch as it enables the thermometers to come into perfect equilibrium of temperature with the water. The pressure indicated by the manometer during ebullition is exactly that of the vapour produced. By admitting air into the reservoir, the boiling-point is raised by successive steps until it reaches 100°. After this, air must be forced into the reservoir by a compression-pump.

The following is an abstract of the results thus obtained:—

Temperatures Centigrade	Pressures in Millimetres of Mercury.	Temperatures Centigrade.	Pressures in Millimetres of Mercury.
−32°	0·32	5°	6·53
−20	0·93	10	9·17
−10	2·09	15	12·70
− 5	3·11	20	17·39
0	4·60	25	23·55

Temperatures Centigrade	Pressures in Millimetres of Mercury.	Temperatures Centigrade.	Pressures in Millimetres of Mercury.
30°	31·55	70°	233·09
35	41·82	75	288·51
40	54·91	80	354·64
45	71·39	85	433·04
50	91·98	90	525·45
55	117·47	95	633 77
60	148·70	100	760·00
65	186·94		

	Pressures in Atmospheres		Pressures in Atmospheres
100°	1	180°	9·929
121	2·025	189	12·125
134	3·008	199	15·062
144	4·000	213	19·997
152	4 971	225	25·125
159	5 966	230	27·534
171	8 036		

126. **Curve of Steam-pressure.**—The comparison of these pressures with their corresponding temperatures affords no clue to any simple relation between them which might be taken as the physical law of the phenomena. It would appear that the law of variation of maximum pressures is incapable of being thrown into any simple expression—judging at least from the failure of all efforts hitherto made An attentive examination of the above table will enable us to assert only that the maximum pressure varies more rapidly than the temperature. Thus between 0° and 100° the variation is only 1 atmosphere, but between 100° and 200° it is about 15, and between 200° and 230° about 13 atmospheres.

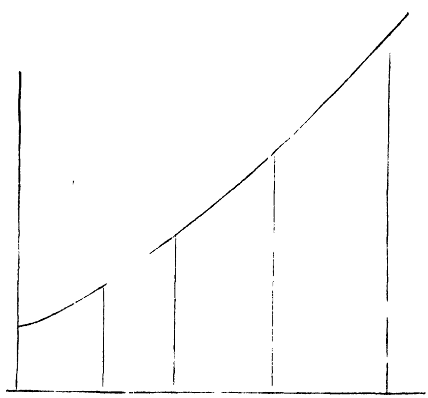


Fig. 83.

The clearest way of representing to the mind the law according to which steam-pressure varies with temperature, is by means of a curve whose ordinates represent steam-pressures, while the abscissæ represent the corresponding temperatures. Such a curve is exhibited in Fig. 83. Lengths proportional to the temperatures, reckoned from 0° C., are laid off on the base-line (called the line of abscissæ), and perpendiculars (called ordi-

nates) are erected at their extremities, the lengths of these perpendiculars being made proportional to the steam-pressures. The scales employed for the two sets of lengths are of course quite independent of one another, their selection being merely a question of convenience. The curve itself is obtained by joining the extremities of the perpendiculars, taking care to avoid sudden changes of direction; and it not only serves to convey to the mind an idea of the amounts of pressure and their rates of variation at different temperatures, but also furnishes the readiest means of determining the pressures at temperatures intermediate between those of observation.

It will be noticed that the curve becomes steeper as the temperature increases, indicating that the pressure increases faster at high than at low temperatures.

127. **Empirical Formulæ.**—Though all attempts at finding a rational formula for steam-pressure in terms of temperature have hitherto failed, it is easy to devise empirical formulæ which yield tolerably accurate results within a limited range of temperature; and by altering the values of the constants in such a formula by successive steps, it may be adapted to represent in succession the different portions of the curve above described.

The simplest of these approximate formulæ[1] is that of Dulong and Arago, which may be written—

$$\left(\frac{40+C}{140}\right)^5 \quad \text{or} \quad \left(\frac{40+F}{252}\right)^5,$$

and gives the maximum pressure *in atmospheres*, corresponding to the temperature C° Centigrade, or F° Fahrenheit. This formula is rigorously correct at 100° C., and gives increasing errors as the temperature departs further from this centre, the errors amounting to about 1½ per cent. at the temperatures 80° C. and 225° C. Hence it appears that between these limits the maximum pressure of aqueous vapour is nearly proportional to the fifth power of the excess of the temperature above −40° C. or −40° F. (for it so happens that this temperature is expressed by the same number on both scales).

128. **Pressures of the Vapours of Different Liquids.**—Dalton held that *the vapours of different liquids have equal pressures at temperatures equally removed from their boiling-points.* Thus the boiling-point of alcohol being 78°, the pressure of alcohol vapour at 70° should be equal to that of the vapour of water at 92°. If this law were correct, it would only be necessary to know the boiling-point

[1] For a general formula, see *Rankine on Steam-engine*, p. 237.

of any liquid in order to estimate the pressure of its vapour at any given temperature; but subsequent experiment has shown that the law is far from being rigorously exact, though it is approximately correct for temperatures differing by only a few degrees from the boiling-points.

Regnault has performed numerous experiments on the vapour-pressures of some of the more volatile liquids, employing for this purpose the same form of apparatus which had served for determining the pressures of aqueous vapour. The following are some of his results:—

VAPOUR OF ALCOHOL.

Temperatures Centigrade	Pressures in Millimetres	Temperatures Centigrade.	Pressures in Millimetres
−20°	3 24	+ 30°	78 52
0	12·70	100	1697 55
+10	24 23	155	6259·19

VAPOUR OF ETHER.

−20°	68 90	+ 30°	634·80
0	184 39	100	4953 30
+10	286 83	120	7719·20

VAPOUR OF SULPHIDE OF CARBON.

−20°	47·30	+ 30°	434·62
0	127 91	100	3325 15
+10°	198·46	150	9095 94

129. **Expression of Vapour-pressure in Absolute Measure.**—The maximum pressure of a given vapour at a given temperature is, from its very nature, independent of geographical position, and should therefore, properly speaking, be denoted by one and the same number at all places. This numerical uniformity will not exist if the pressure be expressed, as in the preceding sections, in terms of the length of a column of mercury which balances it. For example, in order to adapt Regnault's determinations to London, we must multiply them by the fraction $\frac{3456}{3457}$, inasmuch as 3456 millimetres of mercury exert the same pressure at London as 3457 at Paris. In general, to adapt determinations of pressure made at a place A, to another place B, we must multiply them by the fraction

$$\frac{\text{gravity at A}}{\text{gravity at B}}.$$

For if h denote the height (in centimetres) of a column of mercury at 0°, which produces a pressure p (dynes per sq. cm.), and d be the density of mercury at 0°, we have (see *Hydrostatics* in Part I.)

$$p = ghd.$$

Hence, in order that p may be the same at different places, the values of gh must be the same; in other words, h must vary inversely as g.

130. **Laws of Combination by Volume.**—It was discovered by Gay-Lussac, that when two or more gaseous elements at the same temperature and pressure enter into chemical combination with each other, the two following laws apply:—

1. The volumes of the components bear a very simple ratio to each other, such as 2 to 3, 1 to 2, or 1 to 1.

2. The volume of the compound has a simple ratio to the sum of the volumes of the components.

Ammonia, for example, is formed by nitrogen and hydrogen uniting in the proportion of one volume of the former to three of the latter, and the volume of the ammonia, if reduced to the same pressure as each of its constituents, is just half the sum of their volumes. Further investigation has led to the conclusion (which is now generally received, though hampered by some apparent exceptions), that these laws apply to all cases of chemical combination, the volumes compared being those which would be occupied respectively by the combining elements and the compound which they form, *when reduced to the state of vapour*, at such a temperature and pressure as to be very far removed from liquefaction, and consequently to possess the properties of what we are accustomed to call permanent gases.

It is obvious that if all gases and vapours were equally expansible by heat, the volume-ratios referred to in this law would be the same at all temperatures; and that, in like manner, if they were all equally compressible (whether obeying Boyle's law, or departing equally from it at equal pressures), the volume-ratios would be independent of the pressure at which the comparison was made.

In reality great differences exist between different vapours in both respects, and these inequalities are greater as the vapours are nearer to saturation. It is accordingly found that the above laws of volume-ratio often fail to apply to vapours when under atmospheric pressure and within a few degrees of their boiling-points, and that, in such cases, a much nearer fulfilment of the law is obtained by employing very high temperatures, or operating in inclosures at very low pressures.

131. **Relation of Vapour-densities to Chemical Equivalents.**—Chemists have determined with great accuracy the combining proportions by weight of most of the elements. Hence the preceding laws can

be readily tested for bodies which usually exist in the solid or liquid form, if we are able to compare the densities of their vapours. In fact, if two such elements combine in the ratio, by weight, of w_1 to w_2, we have

$$v_1 = \frac{w_1}{d_1}, \qquad v_2 = \frac{w_2}{d_2},$$

v_1 v_2 d_1 d_2 denoting the volumes and densities of the vapours of weights w_1 w_2 of the two substances.

Hence we have the equation—

$$\frac{v_1}{v_2} = \frac{w_1}{w_2} \cdot \frac{d_2}{d_1},$$

which gives the required volume-ratio of the vapours, if the ratio of their densities be known.

The densities themselves will differ enormously according to the pressure and temperature at which they are taken, but their ratio will only vary by comparatively small amounts, and would not differ at all if they were equally expansible by heat, and equally compressible. Hence comparison will be facilitated by tabulating the ratios of the densities to that of some standard gas, namely air, under the same conditions of pressure and temperature, rather than the absolute densities. This is accordingly the course which is generally pursued, so generally indeed, that by the *vapour-density of a substance* is commonly understood the relative density as measured by this ratio.

The process most frequently employed for the determination of this element is that invented by Dumas.

132. Dumas' Method.—The apparatus consists of a glass globe B, containing the substance which is to be converted into vapour.

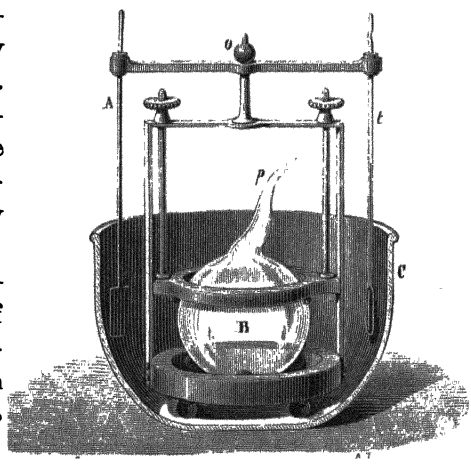


Fig. 84.—Dumas' Apparatus.

The globe is placed in a vessel C, containing some liquid which can be raised to a suitable temperature. If the substance to be operated on is one which can be vaporized at 100° C.,

the bath consists simply of boiling water. When higher temperatures are required, a saline solution, oil, or a fusible alloy is employed. In all cases, the liquid should be agitated, that its temperature may be the same in all parts. This temperature is indicated by the thermometer t.

When the substance in the globe has attained its boiling-point, evaporation proceeds rapidly, and the vapour escapes, carrying out the air along with it. When the vapour ceases to issue, we may assume, if the quantity of matter originally taken has been sufficiently large, that all the air has been expelled, and that the globe is full of vapour at the temperature given by the thermometer, and at the external pressure H. The globe is then hermetically sealed at the extremity p of the neck, which has been previously drawn out into a fine tube.

133. Calculation of the Experiment.—As already remarked, the densities of vapours given in treatises on chemistry express *the ratio of the weight of a given volume of the vapour to that of the same volume of air at the same temperature and pressure.* In order to deduce this ratio from the preceding experiment, we must first find the weight of the vapour. This is done by weighing the globe with its contents, after allowing it to cool. Suppose the weight thus found to be W. Before the experiment the globe had been weighed full of dry air at a known temperature t and pressure h. Suppose this weight to be W′; the difference $W-W'$ evidently represents the excess of the weight of the vapour above that of the air. If, then, we add $W-W'$ to the weight of the air, we shall evidently have the weight of the vapour. Now the weight of the air is easily deduced from the known volume of the globe. If V denote this volume at zero expressed in litres, the weight in grammes of the air contained in the globe at the time of weighing is

$$V\,(1+Kt)\,1{\cdot}293 \times \frac{1}{1+at} \cdot \frac{h}{760},$$

K denoting the coefficient of cubical expansion of glass, and a the coefficient of expansion of air. The weight of the vapour contained in the globe is consequently

$$A = W - W' + V\,(1+Kt) \times 1{\cdot}293 \times \frac{1}{1+at} \cdot \frac{h}{760}.$$

Let H be the pressure and T the temperature at the time of sealing the globe. The volume occupied by the vapour under these circumstances was $V\,(1+KT)$. The density of the vapour will therefore

be obtained by dividing A by the weight of this volume of air at the same temperature and pressure. But this weight is

$$A' = V\,(1+KT)\,.\,1{\cdot}293\,.\,\frac{1}{1+\alpha T}\cdot\frac{H}{760};$$

hence, finally, the required relative density is

$$D = \frac{A}{A'} = \frac{W - W' + V\,(1+Kt)\,.\,1{\cdot}293\,.\,\frac{1}{1+\alpha t}\cdot\frac{h}{760}}{V\,(1+KT)\,.\,1{\cdot}293\,.\,\frac{1}{1+\alpha T}\cdot\frac{H}{760}}.$$

The correctness of this formula depends upon the assumption that no air is left in the globe. In order to make sure that this condition is fulfilled, the point p of the neck of the globe is broken off under mercury; the liquid then rushes in, and, together with the condensed vapour, fills the globe completely, if no air has been left behind.

This last operation also affords a means of calculating the volume V; for we have only to weigh the mercury contained in the globe, or to measure it in a graduated tube, in order to ascertain its volume at the actual temperature, whence the volume at zero can easily be deduced.

134. **Example.**—In order better to illustrate the method, we shall take the following numerical results obtained in an investigation of the vapour-density of sulphide of carbon.—

Excess of weight of vapour above weight of air, $W - W' = {\cdot}3$ gramme; temperature of the vapour $T = 59°$; external pressure $H = 752{\cdot}5$ millimetres; volume of the globe at a temperature of 12°, 190 cubic centimetres; temperature of the dry air which filled the globe at the time of weighing, $t = 15°$; pressure $h = 765$; $K = \frac{1}{38700}$.

The volume V of the globe at zero is

$$\frac{190}{1+\frac{12}{38700}} = 189{\cdot}94 \text{ cubic centimetres} = {\cdot}18994 \text{ litre.}$$

The weight of the air contained in the globe is

$$\cdot 18994 \times 1{\cdot}293\,.\left(1+\frac{15}{38700}\right)\cdot\frac{1}{1+15\times{\cdot}00366}\cdot\frac{765}{760} = {\cdot}23442 \text{ gramme.}$$

Weight of the vapour,

$$\cdot 23442 + W - W' = {\cdot}53442 \text{ gramme.}$$

The weight of the same volume of air at the same temperature and pressure is

$$\cdot 18994 \times 1{\cdot}293\left(1+\frac{59}{38700}\right)\cdot\frac{1}{1+{\cdot}00366\times 59}\cdot\frac{752{\cdot}5}{760} = {\cdot}20019 \text{ gramme.}$$

The density is therefore

$$\frac{\cdot 53442}{\cdot 20019} = 2\cdot 67.$$

Deville and Troost have effected several improvements in the application of Dumas' method to vapours at high temperatures. These temperatures are obtained by boiling various substances, such as chloride of zinc, cadmium, which boils at 860° C., or zinc, which boils at 1040° C. For temperatures above 800°, the glass globe is replaced by a globe of porcelain, which is hermetically sealed with the oxy-hydrogen blowpipe. The globe itself serves as a pyrometer to determine the temperature; and since the weight of air becomes very inconsiderable at high temperatures, some heavier vapour, such as that of iodine, is substituted in its place. If we suppose, as we may fairly do, that at these high temperatures the coefficient of expansion of the vapour of iodine is the same as that of air, the temperature may easily be deduced from the weight of iodine contained in the globe. We subjoin a table of some relative densities of vapours obtained by this method:—

Water, ...	0·622	Phosphorus, ...	4·5
Alcohol, ...	1·6138	Cadmium, ...	3·94
Ether, ...	2·586	Chloride of aluminium, . .	9·347
Spirit of turpentine, . .	5·0130	Bromide of aluminium, . .	18·62
Iodine, ...	8·716	Chloride of zirconium, . .	8·1
Sulphur, ...	2·23	Sesquichloride of iron, . .	11·39

135. Limiting Values of Relative Densities.—In investigating the relative density of acetic acid vapour, Cahours found that it went on decreasing as the temperature increased, up to a certain point, beyond which there was no sensible change. A similar circumstance is observed in the case of all substances, only in different degrees. The vapour of sulphur, for instance, has a relative density of 6·65 at 500° C., while at about 1000° C. it is only 2·23. This indicates that the vapours in question are more expansible by heat than air until the limiting temperatures are attained. It is probable that the nearer a vapour is to its critical point (§ 105) the greater is the change produced in its absolute density by a given change whether of temperature or pressure. The limiting density-ratio is always that which it is most important to determine, and we should consequently take care that the temperature of the vapour is sufficiently high to enable us to obtain it.

136. Gay-Lussac's Method.—Gay-Lussac determined the density of the vapour of water and of some other liquids by a method a little

more complicated than that described above, and which for that reason has not been generally adopted in the laboratory. We proceed to describe it, however, on account both of its historical interest and of the importance of the question which it has assisted in solving.

A graduated tube divided into cubic centimetres, suppose, is filled with mercury, and inverted in a cast-iron vessel containing the same liquid. The inverted tube is surrounded by a glass envelope containing water, as in Dalton's apparatus. A small glass bulb containing a given weight w (expressed in grammes) of distilled water is passed into the tube, and rises to the surface of the mercury. The temperature of the apparatus is then raised by means of a fire below, the bulb bursts, and the water which it contained is converted into vapour. If the quantity of water be not too great, it is all converted into vapour; this is known to be the case when, at the temperature of about 100°, the mercury stands higher in the tube than in the vessel, for if there were any liquid water present, the space would be saturated, and the pressure of the vapour would be equal to the external pressure. This arrangement accordingly gives the weight of a known volume of the vapour of water. This volume, in cubic centimetres, is V (1 + KT), where V denotes the number of divisions of the tube occupied by the vapour, each of which when at the temperature zero represents a cubic centimetre. The temperature T is marked by a thermometer immersed in the water contained in the envelope. The pressure of the vapour is evidently equal to the external pressure *minus* the height of the mercury in the tube.

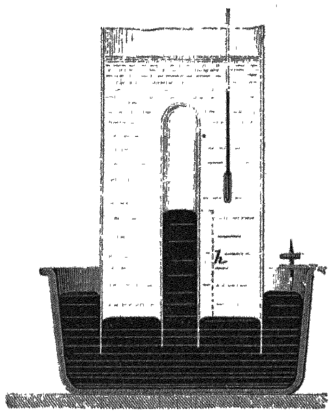


Fig 85 —Gay-Lussac's Apparatus.

In order to find the relative density, we must divide w by the weight of a volume V (1 + KT) of air at the temperature T and pressure H − h, giving

$$\frac{w}{V\,(1+KT)\times \cdot 001293\times \frac{1}{1+aT}\cdot\frac{H-h}{760}}.$$

It may be remarked that the vapour in this experiment is superheated; but superheated vapour of water obeys Boyle's law, and has

therefore the same relative density as saturated vapour at the same temperature.

The relative density of the vapour of water, as thus determined by Gay-Lussac, is about $\frac{5}{8}$, or ·625. Several recent investigations have given as a mean result ·622, which agrees with the theoretical density deduced from the composition of water.[1]

137. **Meyer's Method.**—Victor Meyer has invented a method of determining vapour densities, which is illustrated by Fig. 86. His apparatus consists of a flask B with a long narrow neck, from which a fine tube branches off near the top, and bends down under the surface of mercury. A graduated glass jar D filled with mercury can be inverted over the end of the branch tube.

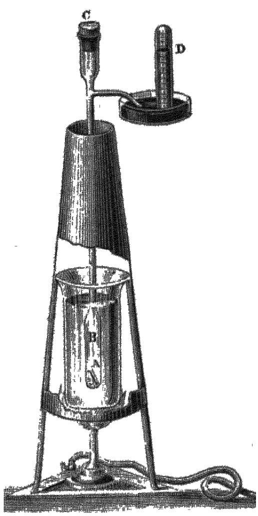


Fig. 86.—Meyer's Apparatus

The first operation is to heat the flask by means of a surrounding bath to the temperature at which it is intended to form the vapour. This operation expands the air and expels a portion in bubbles through the mercury. This portion may be allowed to escape into the atmosphere, and when no more bubbles issue, but equilibrium of pressure has been established, the graduated jar is to be inverted over the end of the tube ready for the second operation, which consists in introducing the substance to be vaporized into the flask, the indian-rubber plug C at the top of the neck being removed for this purpose and quickly replaced. The formation of the vapour expels more air through the mercury, and this air must be collected in the graduated jar.

Comparing the contents of the flask when this operation has been completed with its contents before the plug was drawn, it is obvious that the vapour has taken the place of air at the same temperature and pressure. The relative vapour density will therefore be the quotient of the mass of the vapour by the mass of the air displaced. The mass of the vapour is known, being the same as that of the

[1] Water is composed of 2 volumes of hydrogen, and 1 volume of oxygen, forming 2 volumes of vapour of water. The sum of the density of oxygen and twice the density of hydrogen is 1·244, and the half of this is exactly ·622.—*D.*

substance introduced into the flask; and the mass of the air displaced is known, being the same as that of the air collected in the graduated jar. In the figure, A represents a small tube containing the substance to be vaporized, and asbestos is placed at the bottom of the flask to prevent the latter from being broken when this tube is dropped in.

138 **Volume of Vapour formed by a given Weight of Water.**—When the density of the vapour of water is known, the increase of volume which occurs when a given quantity of water passes into the state of vapour may easily be calculated. Suppose, for instance, that we wish to find the volume which a cubic centimetre of water at 4° will occupy in the state of vapour at 100°. At this temperature the pressure of the vapour is equal to one atmosphere, and its weight is equal to ·622 times the weight of the same volume of air at the same temperature and pressure. If then V be the volume in litres, we have (in grammes)

$$V \times 1\ 293 \times \frac{1}{1+100a} \times {\cdot}622 = 1,$$

whence

$$V = \frac{1+100a}{1{\cdot}293 \times {\cdot}622} = \frac{1{\cdot}366}{{\cdot}804246} = 1{\cdot}698 \text{ lit.} = 1698 \text{ cubic centimetres.}$$

Hence we see that water at 4° gives about 1700 times its volume of vapour at 100° C.

The latent heat of evaporation is doubtless connected with this increase of volume; and it may be remarked that both these elements appear to be greater for water than for any other substance.

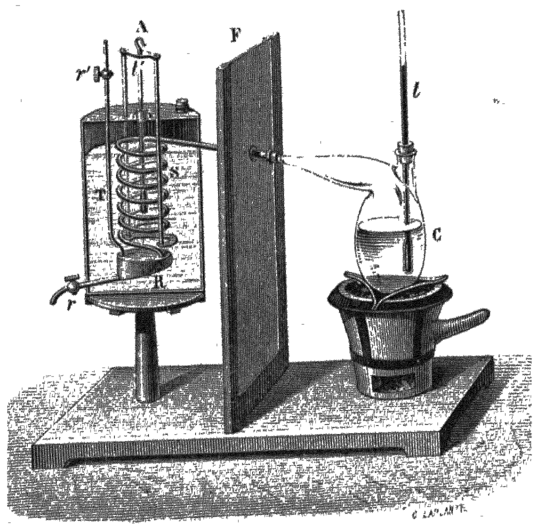
139. **Heat of Evaporation.**—The latent heat of evaporation of water, and of some other liquids, can be determined by means of Despretz's apparatus, which is shown in Fig. 87.

The liquid is boiled in a retort C, which is connected with a worm S surrounded by cold water, and terminating in the reservoir R. The vapour is condensed in the worm, and collects in the reservoir, whence it can be drawn by means of the stop-cock r. The tube T, which is fitted with a stop-cock r', serves to establish communication between the reservoir and the atmosphere, or between the reservoir and a space where a fixed pressure is maintained, so as to produce ebullition at any temperature required, as indicated by the thermometer t. A is an agitator for keeping the water at a uniform temperature, indicated by the thermometer t'.

In using the apparatus, the first step is to boil the liquid in the

retort, and when it is in active ebullition, it is put in communication with the worm. The temperature of the calorimeter has previously been lowered a certain number of degrees below that of the surrounding air, and the experiment ceases when it has risen to the same number of degrees above. The compensation may thus be considered as complete, since the rate of heating is nearly uniform.

If W be the equivalent of the calorimeter in water, t its initial temperature, θ its final temperature; then the quantity of heat gained by it is W $(\theta - t)$. This heat comes partly from the latent heat disengaged at the moment of condensation of the vapour, partly from the loss of temperature of the condensed water, which sinks from T, the boiling-point of the liquid, to the temperature of the calorimeter. If, then, x denote the latent heat of evaporation, w the weight of the



[illegible] Despretz's Apparatus.

liquid collected in the box R, and c its specific heat, we have the equation

$$W(\theta - t) = wx + wc(T - \theta).$$

This experiment is exposed to some serious causes of error. The calorimeter may be heated by radiation from the screen F which protects it from the direct radiation of the furnace. Heat may also be propagated by means of the neck of the retort. Again, the vapour is not *dry* when it passes into the worm, but carries with it small

drops of liquid. Finally, some of the vapour may be condensed at the top of the retort, and so pass into the worm in a liquid state. This last objection is partly removed by sloping the neck of the retort upwards from the fire, but it sometimes happens that this precaution is not sufficient.

140. **Regnault's Experiments.**—The labours of Regnault in connection with the subject of latent heat are of the greatest importance, and have resulted in the elaboration of a method in which all these sources of error are entirely removed. The results obtained by him are the following:—

The quantity of heat required to convert a gramme of water at 100° into vapour, without change of temperature, is 537 gramme-degrees.

If the water were originally at zero, the total amount of heat required to raise it to 100° and then convert it into vapour would evidently be 637 gramme-degrees; and it is this total amount which is most important to know in the application of heat in the arts.

In general, if Q denote the total quantity of heat[1] required to raise water from zero to the temperature T, and then convert it into vapour at this temperature, the value of Q may be deduced with great exactness from the formula

$$Q = 606{\cdot}5 + {\cdot}305T. \qquad (a)$$

From what we have said above, it will be seen that if λ denote the latent heat of evaporation at temperature T, we must have

$$Q = \lambda + T,$$

whence, by substituting for Q in (a), we have

$$\lambda = 606{\cdot}5 - {\cdot}695T. \qquad (b)$$

Hence it appears that latent heat varies in the opposite direction to temperature. This fact had been previously discovered by Watt; but he went too far, and maintained that the increase of the one was *equal* to the diminution of the other, or, in his own words, that "the sum of the sensible and latent heats" (that is $T + \lambda$) "is constant." From (a) we can find the total heat for any given temperature, and from (b) the latent heat of evaporation at any given tem-

[1] Called by Regnault the total heat of saturated vapour at T°, or the total heat of vaporization at T°.

perature. The results for every tenth degree between 0° and 230° are given in the following table:—

Temperatures Centigrade.	Latent Heat	Total Heat.	Temperatures Centigrade	Latent Heat.	Total Heat.
0°	606	606	120°	522	642
10	600	610	130	515	645
20	593	613	140	508	648
30	586	616	150	501	651
40	579	619	160 . . .	494	654
50	572	622	170	486	656
60	565	625	180	479	659
70	558	628	190	472	662
80	551	631	200	464	664
90	544	634	210	457	667
100	537	637	220 . .	449	669
110	529	639	230 . . .	442	672

To reduce latent heat and total heat from the Centigrade to the Fahrenheit scale, we must multiply by $\frac{9}{5}$. Thus the latent and total heat of steam at 212° F. are 966·6 and 1146·6. Total heat is here reckoned from 32° F. If we reckon it from 0° F., 32 must be added.

The following table taken from the researches of Favre and Silbermann, gives the latent heat of evaporation of a number of liquids at the temperature of their boiling-point, referred to the Centigrade scale:—

	Boiling-point.	Latent Heat		Boiling-point	Latent Heat
Wood-spirit, . . .	66·5°	264	Acetic acid, . . .	120°	102
Absolute alcohol,. .	78	208	Butyric acid, . . .	164	115
Valeric alcohol, . .	78	121	Valeric acid, . . .	175	104
Ether, ...	38	91	Acetic ether, . . .	74	100
Ethal, ...	38	58	Oil of turpentine, .	156	69
Valeric ether, . . .	113·5	113·5	Essence of citron, .	165	70
Formic acid, . . .	100	169			

CHAPTER XI.

HYGROMETRY.

141. **Humidity.**—The condition of the air as regards moisture involves two elements:—(1) the amount of vapour present in the air, and (2) the ratio of this to the amount which would saturate the air at the actual temperature. It is upon the second of these elements that our sensations of dryness and moisture chiefly depend, and it is this element which meteorologists have agreed to denote by the term *humidity;* or, as it is sometimes called, *relative humidity.* It is usually expressed as a percentage.

The words *humid* and *moist,* as applied to air in ordinary language, nearly correspond to this technical use of the word *humidity;* and air is usually said to be dry when its *humidity* is considerably below the average. In treatises on physics, "dry air" usually denotes air whose humidity is zero.

The air in a room heated by a hot stove contains as much vapour weight for weight as the open air outside; but it is drier, because its capacity for vapour is greater. In like manner the air is drier at noon than at midnight, though the amount of vapour present is about the same; and it is for the most part drier in summer than in winter, though the amount of vapour present is much greater.

It is to be borne in mind that a cubic foot of air is able to take up the same amount of vapour as a cubic foot of empty space; and "relative humidity" may be defined as *the ratio of the mass of vapour actually present in a given space, to the mass which would saturate the space at the actual temperature.*

Since aqueous vapour fulfils Boyle's law, these masses are proportional to the vapour-pressures which they produce, and relative humidity may accordingly be defined as *the ratio of the actual*

vapour-pressure to the maximum vapour-pressure for the actual temperature.

142. Simultaneous Changes in the Dry and Vaporous Constituents.—When a mixture of air and vapour is subjected to changes of temperature, pressure, or volume which do not condense any of its vapour, the two constituents are similarly affected, since they have both the same coefficient of expansion, and they both obey Boyle's law. If the volume of the whole be reduced from v_1 to v_2 at constant temperature, both the densities will be multiplied by $\frac{v_1}{v_2}$, and hence, by Boyle's law, the pressures will also be multiplied by $\frac{v_1}{v_2}$. If, on the other hand, the temperature be altered from t_1 to t_2 without change of volume, both the pressures will be multiplied by $\frac{1+at_2}{1+at_1}$. The ratio of the vapour-pressure to the dry-air pressure remains unchanged in both cases.

If the changes of volume and temperature are effected simultaneously, each of the pressures will be multiplied by $\frac{v_1}{v_2}\frac{1+at_2}{1+at_1}$, and the total pressure will be multiplied by the same factor. If the total pressure remains unchanged, as is the case when there is free communication between the altered air and the general atmosphere, both the dry-air pressure and the vapour-pressure will therefore remain unchanged.

143. Dew-point.—When a mixture of dry air and vapour is cooled down at constant pressure until the vapour is at saturation, the temperature at which saturation occurs is called the *dew-point* of the original mass; and if the mixture be cooled below the dew-point, some of the vapour will be condensed into liquid water or solid ice.

The reasoning of the preceding section shows that the process of cooling down to the dew-point does not alter the vapour-pressure. The *actual vapour-pressure* in any portion of air *is* therefore *equal to the maximum vapour-pressure at the dew-point.*

When air is confined in a close vessel, and cooled at constant volume, its pressure and density at any given temperature, and the pressures and densities of its dry and vaporous constituents, will be less than if it were in free communication with the atmosphere. Hence its vapour will not be at saturation when cooled down to what is above defined as the dew-point of the original mass, but a lower temperature will be requisite.

144. These conclusions can also be established as follows:—

Let P denote the pressure of the mixture,

p „ the pressure of the vaporous constituent,

V „ the volume,

T „ the temperature reckoned from absolute zero on the air thermometer.

Then for all changes which do not condense any of the vapour

$$\frac{VP}{T} \text{ is constant, and } \frac{Vp}{T} \text{ is constant.}$$

When P is also constant, we have $\frac{V}{T}$ constant, and therefore p constant.

On the other hand, when V is constant, p will vary as T, and will diminish as T diminishes.

145. **Hygroscopes.**—Anything which serves to give rough indications of the state of the air as regards moisture may be called a *hygroscope* (ὑγρος, moist). Many substances, especially those which are composed of organic tissue, have the property of absorbing the moisture of the surrounding air, until they attain a condition of equilibrium such that their affinity for the moisture absorbed is exactly equal to the force with which the latter tends to evaporate. Hence it follows that, according to the dampness or dryness of the air, such a substance will absorb or give up vapour, either of which processes is always attended with a variation in the dimensions of the body. The nature of this variation depends upon the peculiar structure of the substance, thus, for instance, bodies formed of filaments exhibit a greater increase in the direction of their breadth than of their length. Membranous bodies, on the other hand, such as paper or parchment, formed by an interlacing of fibres in all directions, expand or contract almost as if they were homogeneous. Bodies composed of twisted fibres, as ropes and strings, swell under the action of moisture, grow shorter, and are more tightly twisted. The opposite is the case with catgut, which is often employed in popular hygroscopes.

146. **Hygrometers.**—Instruments intended for furnishing precise measurements of the state of the air as regards moisture are called *hygrometers.* They may be divided into four classes:—

1. Hygrometers of absorption, which should rather be called hygroscopes.
2. Hygrometers of condensation, or dew-point instruments.
3. Hygrometers of evaporation, or wet and dry bulb thermometers.

4. Chemical hygrometers, for directly measuring the weight of vapour in a given volume of air.

147. De Saussure's Hygrometer.—The best hygrometer of absorption is that of De Saussure, consisting of a hair deprived of grease, which by its contractions moves a needle (Fig. 88). When the hair relaxes, the needle is caused to move in the opposite direction

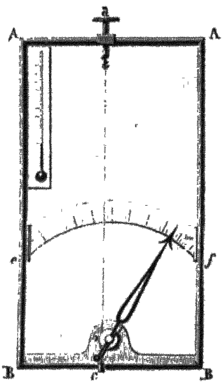


Fig 88.—De Saussure's Hygroscope.

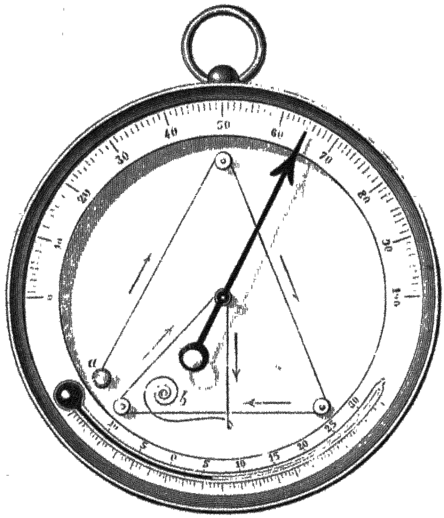


Fig. 89.—Monnier's Hygroscope.

by a weight, which serves to keep the hair always equally tight. The hair contracts as the humidity increases, but not in simple proportion, and Regnault's investigations have shown that, unless the most minute precautions are adopted in the construction and graduation of each individual instrument, this hygrometer will not furnish definite numerical measures.

Fig. 89 represents Monnier's modification of De Saussure's hygrometer, in which the hair, after passing over four pulleys, is attached to a light spring, which serves instead of a weight, and gives the advantage of portability.

These instruments are never employed for scientific purposes in this country.

148. Dew-point Hygrometers.—These are instruments for the direct observation of the dew-point, by causing moisture to be condensed from the air upon the surface of a body artificially cooled to a known temperature.

The dew-point, which is itself an important element, gives directly, as we have seen in § 143, the pressure of vapour; and if the temperature of the air is at the same time observed, the pressure requisite for saturation is known. The, ratio of the former to the latter is the humidity.

149. Dines' Hygrometer.—One of the best dew-point hygrometers is that invented by the late Mr. Dines, shown both in perspective and in section in Figs. 90, 91. Cold water, with ice, if necessary, is put into the reservoir A, and by turning on the tap B this water is allowed to flow through the pipe C into the small double chamber D, the top of which, E, is formed of thin black glass, on which the smallest film of dew is easily perceived. After flowing under the black glass and around the bulb of a thermometer which lies immediately below it, the water escapes through a discharge pipe, and can be received in a vessel, from which it may again be poured into the reservoir A. As soon as any dew is seen on the black glass, the thermometer should be read, and the tap turned off, or partly off, until the dew disappears, when a second reading of the thermometer should be taken.

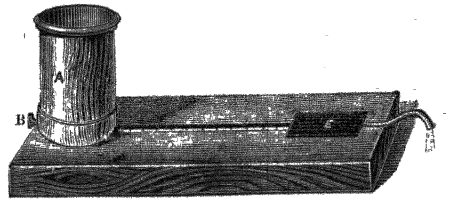


Fig. 90.—Dines' Hygrometer.

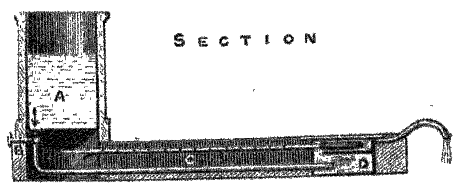


Fig 91 —Dines' Hygrometer.

The mean of the two will be approximately the dew-point; and in order to obtain a good determination, matters should be so managed as to make the temperatures of appearance and disappearance nearly identical.

150. Daniell's Hygrometer.—Daniell's hygrometer has been very extensively used. It consists of a bent tube with a globe at each end, and is partly filled with ether. The rest of the space is occupied with vapour of ether, the air having been expelled. One of the globes A is made of black glass, and contains a thermometer *t*. The method of using the instrument is as follows:—The whole of the liquid is first passed into the globe A, and then the other globe B, which is covered with muslin, is moistened externally with ether.

The evaporation of this ether from the muslin causes a condensation of vapour of ether in the interior of the globe, which produces a fresh evaporation from the surface of the liquid in A, thus lowering the temperature of that part of the instrument. By carefully watching the surface of the globe, the exact moment of the deposition of dew may be ascertained. The temperature is then read on the inclosed thermometer.

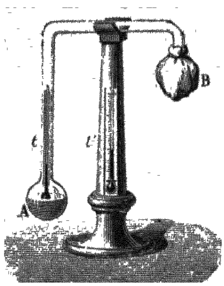


Fig 92
Daniell's Hygrometer.

If the instrument be now left to itself, the exact moment of the disappearance of the dew may be observed; and the usual plan is to take the mean between this temperature and that first observed. The temperature of the surrounding air is given by a thermometer t' attached to the stand.

151. **Regnault's Hygrometer.**—Regnault's hygrometer consists (Fig. 93) of a glass tube closed at the bottom by a very thin silver cap D. The opening at the upper end is closed by a cork, through which

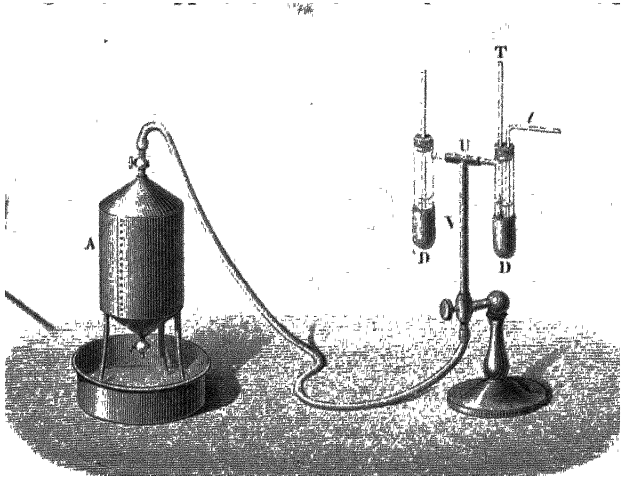


Fig 93.—Regnault's Hygrometer.

passes the stem of a thermometer T, and a glass tube t open at both ends. The lower end of the tube and the bulb of the thermometer dip into ether contained in the silver cap. A side tube establishes communication between this part of the apparatus and a vertical

tube UV, which is itself connected with an aspirator[1] A, placed at a convenient distance. By allowing the water in the aspirator to escape, a current of air is produced through the ether, which has the effect of keeping the liquid in agitation, and thus producing uniformity of temperature throughout the whole. It also tends to hasten evaporation; and the cold thus produced speedily causes a deposition of dew, which is observed from a distance with a telescope, thus obviating the risk of vitiating the observation by the too close proximity of the observer. The observation is facilitated by the contrast offered by the appearance of the second cap, which has no communication with the first, and contains a thermometer for giving the temperature of the external air. By regulating the flow of liquid from the aspirator, the temperature of the ether can be very nicely controlled, and the dew can be made to appear and disappear at temperatures nearly identical. The mean of the two will then very accurately represent the dew-point.

The liquid employed in Regnault's hygrometer need not be ether. Alcohol, a much less volatile liquid, will suffice. This is an important advantage; for, since the boiling-point of ether is 36° C. (97° F.), it is not easy to preserve it in hot climates.

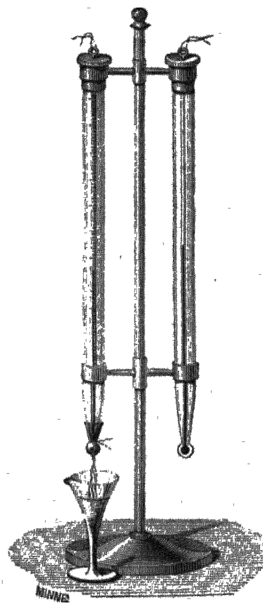


Fig 94
Wet and Dry Thermometers

152. **Wet and Dry Bulb Hygrometer.**—This instrument, which is also called Mason's hygrometer, and is known on the Continent as August's psychrometer, consists (Fig. 94) of two precisely similar thermometers, mounted at a short distance from each other, the bulb of one of them being covered with muslin, which is kept moist by means of a cotton wick leading from a vessel of water. The evaporation which takes place from the moistened bulb produces a depression of temperature, so that this thermometer reads lower than the other by an amount which increases with the dryness of the air. The instrument must be mounted in such a way that the air can circulate

[1] An aspirator is a vessel into which air is sucked at the top to supply the place of water which is allowed to escape at the bottom; or, more generally, it is any apparatus for sucking in air or gas.

very freely around the wet bulb; and the vessel containing the water should be small, and should be placed some inches to the side. The level of this vessel must be high enough to furnish a supply of water which keeps the muslin thoroughly moist, but not high enough to cause drops to fall from the bottom of the bulb. Unless these precautions are observed, the depression of temperature will not be sufficiently great, especially in calm weather.

In frosty weather the wick ceases to act, and the bulb must be dipped in water some time before taking an observation, so that all the water on the bulb may be frozen, and a little time allowed for evaporation from the ice, before the reading is taken.

The great facility of observation afforded by this instrument has brought it into general use, to the practical exclusion of other forms of hygrometer. As the theoretical relation between the indications of its two thermometers and the humidity as well as the dew-point of the air is rather complex, and can scarcely be said to be known with certainty, it is usual, at least in this country, to effect the reduction by means of tables which have been empirically constructed by comparison with the indications of a dew-point instrument. The tables universally employed by British observers were constructed by Mr. Glaisher, and are based upon a comparison of the simultaneous readings of the wet and dry bulb thermometers and of Daniell's hygrometer taken for a long series of years at Greenwich observatory, combined with some similar observations taken in India and at Toronto.[1]

According to these tables, the difference between the dew-point and the wet-bulb reading bears a constant ratio to the difference between the two thermometers, when the temperature of the dry-bulb thermometer is given. When this temperature is 53° F., the dew-point is as much below the wet-bulb as the wet-bulb is below the temperature of the air. At higher temperatures the wet-bulb reading is nearer to the dew-point than to the air-temperature, and the reverse is the case at temperatures below 53°.

153. In order to obtain a clue to the construction of a rational formula for deducing the dew-point from the indications of this instrument, we shall assume that the wet-bulb is so placed that its temperature is not sensibly affected by radiation from surrounding objects, and hence that the heat which becomes latent by the

[1] The first edition of these Tables differs considerably from the rest, and is never used; but there has been no material alteration since the second edition (1856).

evaporation from its surface is all supplied by the surrounding air. When the temperature of the wet-bulb is falling, heat is being consumed by evaporation faster than it is supplied by the air; and the reverse is the case when it is rising. It will suffice to consider the case when it is stationary, and when, consequently, the heat consumed by evaporation in a given time is exactly equal to that supplied by the air.

Let t denote the temperature of the air, which is indicated by the dry-bulb thermometer; t' the temperature of the wet-bulb; T the temperature of the dew-point, and let f, f', F be the vapour-pressures corresponding to saturation at these three temperatures. Then, as shown in § 143, the tension of the vapour present in the air at its actual temperature t is also equal to F.

We shall suppose that wind is blowing, so that continually fresh portions of air come within the sphere of action of the wet-bulb. Then each particle of this air experiences a depression of temperature and an increase of vapour-pressure as it comes near the wet-bulb, from both of which it afterwards recovers as it moves away and mixes with the general atmosphere.

If now it is legitimate to assume[1] that this depression of temperature and exaltation of vapour-pressure are always proportional to one another, not only in comparing one particle with itself at different times, but also in comparing one particle with another, we have the means of solving our problem; at all events, if we may make the additional assumptions that a portion of the air close to the wet-bulb is at the temperature of the wet-bulb, and is saturated.

On these assumptions the greatest reduction of temperature of the air is $t-t'$, and the greatest increase of vapour-pressure is $f'-\mathrm{F}$, and the corresponding changes in the whole mass are proportional to these. The three temperatures t, t', T must therefore be so related, that the heat lost by a mass of air in cooling through the range $t-t'$, is just equal to the heat which becomes latent in the formation of as much vapour as would raise the vapour-pressure of the mass by the amount $f'-\mathrm{F}$.

[1] The assumption which Dr. Apjohn actually makes is as follows:—"When in the moist-bulb hygrometer the stationary temperature is attained, the caloric which vaporizes the water is necessarily exactly equal to that which the air imparts in descending from the temperature of the atmosphere to that of the moistened bulb; and the air which has undergone this reduction becomes saturated with moisture" (*Trans. R.I.A.* Nov. 1834).

This implies that all the air which is affected at all is affected to the maximum extent—a very harsh supposition; but August independently makes the same assumption.

Let h denote the height of the barometer, s the specific heat of air, D the relative density of vapour (§ 131), L the latent heat of steam, and let the vapour-pressures be expressed by columns of mercury.

Then the mass of the air is to that of the vapour required to produce the additional tension, as h to D $(f'-\text{F})$, and we are to have

$$\text{LD}\,(f'-\text{F}) = s\,(t-t')\,h,$$

or

$$f'-\text{F} = (t-t')\,h\,.\,\frac{s}{\text{LD}}, \qquad (1)$$

which is the required formula, enabling us, with the aid of a table of vapour-pressures, to determine F., and therefore the dew-point T, when the temperatures t, t' of the dry and wet bulb, and the height h of the barometer, have been observed. The expression for the relative humidity will be $\frac{\text{F}}{f}\cdot$

Properly speaking, s denotes the specific heat not of dry air but of air containing the actual amount of vapour, and therefore depends to some extent upon the very element which is to be determined; but its variation is inconsiderable. L also varies with the known quantity t', but its variations are also small within the limits which occur in practice. The factor $\frac{s}{\text{LD}}$ may therefore be regarded as constant, and its value, as adopted by Dr. Apjohn[1] for the Fahrenheit scale, is $\frac{1}{2610}$ or $\frac{1}{30} \times \frac{1}{87}\cdot$ We thus obtain what is known as *Apjohn's formula*,

$$\text{F} = f' - \frac{t-t'}{87}\cdot\frac{h}{30}. \qquad (2)$$

When the wet-bulb is frozen, L denotes the sum of the latent heats of liquefaction and vaporization, and the formula becomes

$$\text{F} = f' - \frac{t-t'}{96}\cdot\frac{h}{30}. \qquad (3)$$

[1] This value was founded on the best determinations which had been made at the time, the specific heat of air being taken as ·267, the value obtained by Delaroche and Berard. The same value was employed by Regnault in his hygrometrical investigations. At a still later date Regnault himself investigated the specific heat of air and found it to be ·237. When this correct value is introduced into Regnault's theoretical formula (which is substantially the same as Apjohn's), the discrepancies which he found to exist between calculation and observation are increased, and amount, on an average, to about 25 per cent of the difference between wet-bulb temperature and dew-point. The inference is that the assumptions on which the theoretical formulæ are based are not accurate; and the discrepancy is in such a direction as to indicate that diffusion of heat is more rapid than diffusion of vapour.

In calm weather, and also in very dry weather, the humidity, as deduced from observations of wet and dry thermometers, is generally too great, probably owing mainly to the radiation from surrounding objects on the wet-bulb, which makes its temperature too high.

154. Chemical Hygrometer.—The determination of the quantity of aqueous vapour in the atmosphere may be effected by ordinary chemical analysis in the following manner:—

An aspirator A, of the capacity of about 50 litres, communicates at its upper end with a system of U-tubes 1, 2, 3, 4, 5, 6, filled with

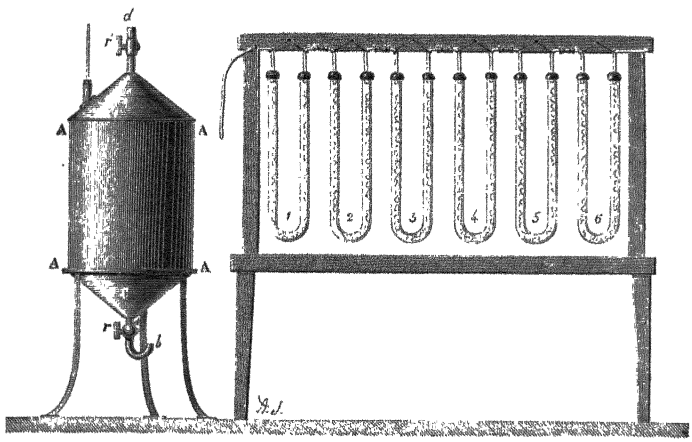


Fig. 95 —Chemical Hygrometer

pieces of pumice soaked in sulphuric acid. The aspirator being full of water, the stop-cock at the bottom is opened, and the air which enters the aspirator to take the place of the water is obliged to pass through the tubes, where it leaves all its moisture behind. This moisture is deposited in the first tubes only. The last tube is intended to absorb any moisture that may come from the aspirator. Suppose w to be the increase of weight of the first tubes 4, 5, 6; this is evidently the weight of the aqueous vapour contained in the air which has passed through the apparatus. The volume V of this air, which we will suppose to be expressed in litres, may easily be found by measuring the amount of water which has escaped. This air has been again saturated by contact with the water of the aspirator, and the aqueous vapour contained in it is consequently at the maximum pressure corresponding to the temperature indicated by a thermometer attached to the apparatus. Let this pressure be denoted

by f. The volume occupied by this air when in the atmosphere, where the temperature is T, is known by the regular formulæ to have been

$$V \cdot \frac{H-f}{H-x} \cdot \frac{1+\alpha T}{1+\alpha t},$$

x denoting the pressure of the aqueous vapour in the atmosphere, and H the total atmospheric pressure as indicated by the barometer; and, since the relative density of steam is ·622, and the weight of a litre of air at temperature 0° C. and pressure 760 mm. is 1·293 gramme, the weight of vapour which this air contained must have been

$$V \cdot \frac{H-f}{H-x} \cdot \frac{1+\alpha T}{1+\alpha t} \times 1{\cdot}293 \times {\cdot}622 \cdot \frac{x}{760} \cdot \frac{1}{1+\alpha T},$$

which must be equal to the known weight w, and thus we have an equation from which we find

$$x = \frac{w(1+\alpha t)\,760\,H}{V(H-f) \times {\cdot}622 \times 1{\cdot}293 + w(1+\alpha t)\,760}.$$

A good approximation will be obtained by writing

$$w = V \times 1{\cdot}293 \times {\cdot}622 \,\frac{x}{760}.$$

whence

$$x = 945\,\frac{w}{V}.$$

This method has all the exactness of a regular chemical analysis, but it involves great labour, and is, besides, incapable of showing the sudden variations which often occur in the humidity of the atmosphere. It can only give the mean quantity of moisture in a given volume of air during the time occupied by the experiment. Its accuracy, however, renders it peculiarly suitable for checking the results obtained by other methods, and it was so employed by Regnault in the investigations to which we have referred in the foot-note to the preceding section.

155. Weight of a given Volume of Moist Air.—The laws of vapours and the known formulæ of expansion enable us to solve a problem of very frequent occurrence, namely, the determination of the weight of a given volume of moist air. Let V denote the volume of this air, H its pressure, f the pressure of the vapour of water in it, and t its temperature. The entire gaseous mass may be divided into two parts, a volume V of dry air at the temperature t and the pressure $H-f$, whose weight is, by known formulæ,

$$V \times 1{\cdot}293 \times \frac{1}{1+\alpha t} \cdot \frac{H-f}{760},$$

and a volume V of aqueous vapour at the temperature t and the pressure f; the weight of this latter is

$$\frac{5}{8}V \times 1{\cdot}293 \times \frac{1}{1+at} \cdot \frac{f}{760}.$$

The sum of these two weights is the weight required, viz.

$$V \times 1{\cdot}293 \times \frac{1}{1+at} \cdot \frac{H - \frac{3}{8}f}{760}.$$

156. **Ratio of the Volumes occupied by the same Air when saturated at Different Temperatures and Pressures.**—Suppose a mass of air to be in presence of a quantity of water which keeps it always saturated; let H be the total pressure of the saturated air, t its temperature, and V its volume.

At a different temperature and pressure t' and H′, the volume occupied V′ will in general be different. The two quantities V and V′ may be considered as the volumes occupied by a mass of dry air at temperatures t and t' and pressures $H-f$ and $H'-f'$; we have then the relation

$$\frac{V}{V'} = \frac{H'-f'}{H-f} \cdot \frac{1+at}{1+at'} \qquad (1)$$

In passing from one condition of temperature and pressure to another, it may be necessary, for the maintenance of saturation, that a new quantity of vapour should be formed, or that a portion of the vapour should be condensed, or again, neither the one nor the other change may take place. To investigate the conditions on which these alternatives depend, let D and D′ be the maximum densities of vapour at the temperatures t and t' respectively. Suppose we have $t' > t$, and that, without altering the pressure f, the temperature of the vapour is raised to t', all contact with the generating liquid being prevented. The vapour will no longer remain saturated; but, on increasing the pressure to f', keeping the temperature unchanged, saturation will again be produced. This latter change does not alter the actual quantity of vapour, and if we suppose its coefficient of expansion to be the same as that of air, we shall have

$$\frac{D}{D'} = \frac{f}{f'} \cdot \frac{1+at'}{1+at}, \qquad (2)$$

and, by multiplying together equations (1) and (2), we have

$$\frac{VD}{V'D'} = \frac{H'f - ff'}{Hf' - ff'}. \qquad (3)$$

From this result the following particular conclusions may be deduced:—

1. If $H'f = Hf'$, $VD = V'D'$, that is, the mass of vapour is the same in both cases; consequently, neither condensation nor evaporation takes place.

2. If $H'f > Hf'$, $VD > V'D'$, that is, partial condensation occurs.

3. If $H'f < Hf'$, $VD < V'D'$, that is, a fresh quantity of vapour is required to maintain saturation. In this case the formula (1) can only be applied when we are sure that there is a sufficient excess of liquid to produce the fresh quantity of vapour which is required.

The general formulæ (1), (2), (3) furnish the solution of many particular problems which may be proposed by selecting some one of the variables for the unknown quantity.

157. **Aqueous Meteors.**—The name *meteor*, from the Greek μετεωρος, *aloft*, though more especially applied to the bright objects otherwise called shooting-stars and their like, likewise includes all the various phenomena which have their seat in the atmosphere; for example, clouds, rain, and lightning. This use of the word *meteor* is indeed somewhat rare; but the correlative term *meteorology* is invariably employed to denote the science which treats of these phenomena, in fact, the *science of matters pertaining to weather*.

By *aqueous meteors* are to be understood the phenomena which result from the condensation of aqueous vapour contained in the air, such as rain, dew, and fog. This condensation may occur in either of two ways. Sometimes it is caused by the presence of a cold body, which reduces the film of air in contact with it to a temperature below the dew-point, and thus produces the liquefaction or solidification of a portion of its vapour in the form of dew or hoar-frost.

When, on the contrary, the condensation of vapour takes place in the interior of a large mass of air, the resulting liquid or solid *falls* in obedience to gravity. This is the origin of rain and snow.

158. **Cloud and Mist.**—When vapour is condensed in the midst of the air, the first product is usually *mist* or *cloud*, a cloud being merely a mist at a great elevation in the air.

Natural clouds are similar in constitution to the cloudy substance which passes off from the surface of hot water, or which escapes in puffs from the chimney of a locomotive. In common language this substance is often called steam or vapour, but improperly, for steam

is, like air, transparent and invisible, and the appearance in question is produced by the presence of particles of liquid water, which have been formed from vapour by cooling it below its dew-point.

Different opinions have been put forward as to the nature of these particles, the difference having arisen in the attempt to explain their suspension in the atmosphere. Some have endeavoured to account for it by maintaining that they are hollow;[1] but even if we could conceive of any causes likely to lead to the formation of such bubbles, it would furnish no solution of the difficulty, for the air inclosed in a bubble is no rarer, but in fact denser, than the external air (see *Capillarity* in Part I), the bubble and its contents are therefore heavier than the air which it displaces.

It is more probable that the particles are solid spheres differing only in size from rain-drops. It has been urged against this view, that such drops ought to exhibit rainbows, and the objection must be allowed to have some weight. The answer to it is probably to be found in the excessive smallness of the globules. Indeed, the non-occurrence of bows may fairly be alleged as proving that the diameters of the drops are comparable with the lengths of waves of light.

This smallness of the particles is amply sufficient to explain all the observed facts of cloud suspension, without resorting to any special theory. It probably depends on the same principle as the suspension of the motes which are rendered visible when a beam of sunlight traverses a darkened room. It is true that these motes, which are small particles of matter of the most various kinds, are never seen resting stationary in the air; but neither are the particles which compose clouds. All who have ever found themselves in mountain mists must have observed the excessive mobility of their constituent parts, which yield to the least breath of wind, and are carried about by it like the finest dust. Sometimes, indeed, clouds have the *appearance* of being fixed in shape and position; but this is an illusion due to distance which renders small movements invisible. In many cases, the fixity is one of form and not of material; for example, the permanent cloud on a mountain-top often consists of successive portions of air, which become cloudy by condensation as they pass through the cold region at the top of the mountain, and recover their transparency as they pass away.

[1] Those who adopt this view call them *vesicles* (*vesica*, a bladder), and call mist or cloud *vapour in the vesicular state.*

159. Varieties of Cloud.—The cloud nomenclature generally adopted by meteorologists was devised by Howard, and is contained in his work on the climate of London. The fundamental forms, according to him, are three—*cirrus*, *cumulus*, and *stratus*.

1. *Cirrus* consists of fibrous, wispy, or feathery clouds, occupying the highest region of the atmosphere. The name *mare's-tails*, which is given them by sailors, describes their aspect well. They are higher than the greatest elevations attained by balloons, and are probably composed of particles of ice. It is in this species of cloud, and its derivatives, that haloes are usually seen; and their observed forms and dimensions seem to agree with the supposition that they are formed by refractions and reflections from ice-crystals.

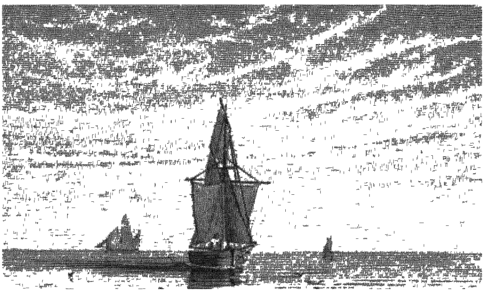


Fig. 96.—Cirrus.

2. *Cumulus* consists of rounded masses, convex above and comparatively flat below. Their form bears a strong resemblance to heaps of cotton wool, hence the name *balls of cotton* and *wool-packs* applied to these clouds by sailors. They are especially prevalent in summer, and are probably formed by columns of ascending vapour which become condensed at their upper extremities.

Fig. 97.—Cumulus.

3. *Stratus* consists of horizontal sheets. Its situation is low in the atmosphere, and its formation is probably due to the cooling of the earth and the lower portion of the air by radiation. It is very frequently formed at sunset, and disappears at sunrise.

Of the intermediate forms it may suffice to mention *cirro-cumulus*, which floats at a higher level than cumulus, and consists usually of

small roundish masses disposed with some degree of regularity. This is the cloud which forms what is known as a *mackerel sky*.

As a distinct form not included in Howard's classification, may be mentioned *scud*, the characteristic of which is that, from its low elevation, it *appears* to move with excessive rapidity.

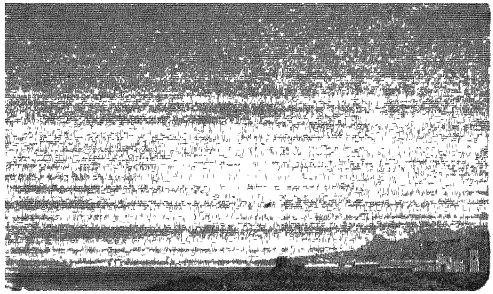


Fig 98 —Stratus

Howard gives the name of *nimbus* to any cloud which is discharging rain; and, for no very obvious reason, he regards this rain-cloud as compounded of (or intermediate between) the three elementary types above defined.

The classification of clouds is a subject which scarcely admits of precise treatment; the varieties are so endless, and they shade so gradually into one another.

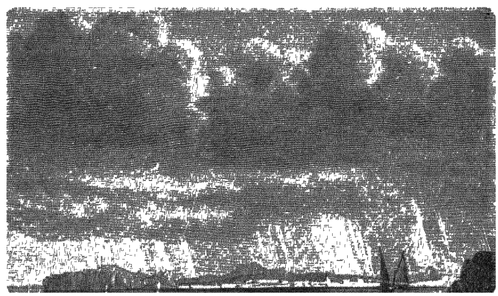


Fig. 99 —Nimbus

160. **Causes of the Formation of Cloud and Mist.**—Since clouds are merely condensed vapour, their formation is regulated by the causes which tend to convert vapour into liquid. Such liquefaction implies the presence of a quantity of vapour greater than that which, at the actual temperature, would be sufficient for saturation, a condition of things which may be brought about by the cooling of a mass of moist air in any of the following ways:—

(1.) By radiation from the mass of air to the cold sky.

(2.) By the neighbourhood of cold ground, for example, mountain-tops.

(3.) By the cooling effect of expansion, when the mass of air ascends into regions of diminished pressure. This cooling of the ascending mass is accompanied by a corresponding warming of the air which

descends—it may be in some distant locality—to supply its place.

Causes (2) and (3) combine to produce the excessive rainfall which generally characterizes mountainous districts.[1]

It is believed that waterspouts are produced by the rapid ascent of a stream of air up the axis of an aerial vortex.

(4) By the contact and mixture of cooler air.[2] It is obvious, however, that this cooler air must itself be warmed by the process; and as both the temperature and vapour-density of the mixture will be intermediate between those of the two components, it does not obviously follow (as is too often hastily assumed) that such contact tends to produce precipitation. Such is however the fact, and it depends upon the principle that the density of saturation increases faster as the temperature is higher; or, what is the same thing, that the curve in which temperature is the abscissa and maximum vapour-density the ordinate, is everywhere concave upwards.

It will be sufficient to consider the case of the mixing of two equal volumes of saturated air at different temperatures, which we will denote by t_1 and t_2. Let the ordinates AA′, BB′ represent the densities of vapour for saturation at these temperatures, A′mB′ being the intermediate portion of the curve, and Cm the ordinate at the middle point of AB, representing therefore the density of saturation for the temperature $\frac{1}{2}(t_1+t_2)$. When the equal volumes are mixed, since the colder *mass* is slightly the greater, the temperature of the mixture will be something less than $\frac{1}{2}(t_1+t_2)$, and, if there were no condensation of vapour, the density of vapour in the mixture would be $\frac{1}{2}(\text{AA}'+\text{BB}')=\text{C}n$. But the density for saturation is something less than Cm. The excess of vapour is therefore represented by something more than mn. The amount actually precipitated will, however, be less than this, since the portion which is condensed

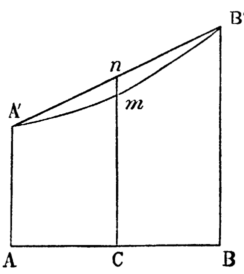


Fig. 100.

[1] The rainiest place at present known in Great Britain is about a mile south of Seathwaite in Cumberland, where the annual rainfall is about 165 inches. The rainiest place in the world is believed to be Cherra Ponjee, in the Khasyah Mountains, about 300 miles N.E. of Calcutta, where the annual fall is about 610 inches.

[2] *Contact* with cooler air may be regarded as equivalent to mixing; for vapour diffuses readily.

gives out its latent heat, and thus contributes to keep up the temperature of the whole.

The cause here indicated combines with (3) to produce condensation when masses of air ascend.

On the surface of the earth mists are especially frequent in the morning and evening; in the latter case extending over all the surface; in the former principally over rivers and lakes. The mists of evening are due simply to the rapid cooling of the air after the heat of the sun has been withdrawn. In the morning another cause is at work. The great specific heat of water causes it to cool much more slowly than the air, so that the vapour rising from a body of water enters into a colder medium, and is there partly condensed, forming a mist, which, however, confines itself to the vicinity of the water, and is soon dissipated by the heat of the rising sun.

161. **Rain.**—In what we have stated regarding the constitution of clouds, it is implied that clouds are always raining, since the drops of which they are composed always tend to obey the action of gravity. But, inasmuch as there is usually a non-saturated region intervening between the clouds and the surface of the earth, these drops, when very small, are usually evaporated before they have time to reach the ground. Ordinary rain-drops are formed by the coalescing of a number of these smaller particles.

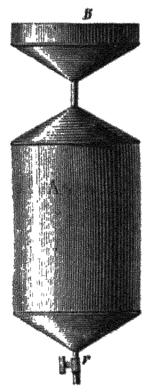


Fig 101
Rain-gauge.

By the amount of annual rainfall at a given place is meant the depth of water that would be obtained if all the rain which falls there in a year were collected into one horizonal sheet; and the depth of rain that falls in any given shower is similarly reckoned. It is the depth of the pool which would be formed if the ground were perfectly horizontal, and none of the water could get away. The instrument employed for determining it is called a *rain-gauge*. It has various forms, one of which is represented in the adjoining figure. B is a funnel into which the rain falls, and from which it trickles into the reservoir A. It is drawn off by means of the stopcock *r*, and measured in a graduated glass.[1]

[1] The best work on the subject of rain and its measurement is Mr. Symons' little treatise [out of print] entitled *Rain*. Mr. Symons, who is at the head of an immense corps of volunteer observers of rain in all parts of the United Kingdom, also publishes an annual volume entitled *British Rainfall*.

The form recommended for use in ordinary localities by Mr. G. J. Symons the best authority on the subject, is called the Snowdon gauge, and is represented in Fig. 102. Its top is a cylinder with a sharp edge. A funnel is soldered to the inside of this cylinder at the distance of about one diameter from the top, and the neck of the funnel descends nearly to the bottom of a bottle which serves as reservoir. A second cylinder, closed below and just large enough for the first to be slipped over it, contains the bottle, and is held in its place by four stakes driven into the ground. The upper cylinder with its attached funnel is slipped over the lower one, and pushed down till its further descent is stopped by the rim of the funnel meeting the edge of the lower cylinder.

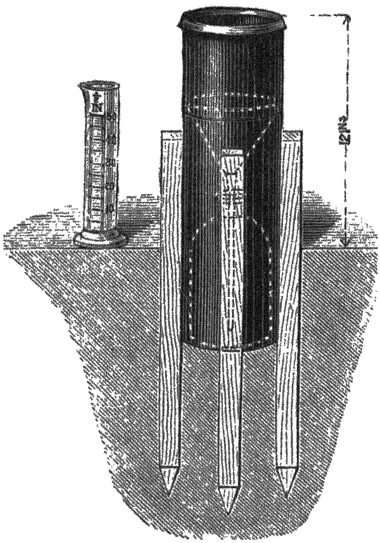


Fig. 102.—Snowdon Rain-gauge.

The height of the receiving surface above the ground is 1 foot, and its diameter 5 inches. The graduated jar reads to hundredths of an inch, and measures up to half an inch. The bottle holds about 3 inches of rain, and in the rare case of a fall exceeding that, the excess is saved by the lower cylinder.

Snow can be measured in either of the following ways:—

(1.) Melt what is caught in the gauge by adding to the snow a previously ascertained quantity of warm water, and then, after deducting this quantity from the total measurement, enter the residue as rain.

(2.) Select a place where the snow has not drifted, invert the upper cylinder with its attached funnel, and, turning it round, lift and melt what is inclosed.

It is essential that the receiving surface should be truly horizontal, otherwise the gauge will catch too much or too little according to the direction of the wind.

The best place for a rain-gauge is the centre of a level and open plot; and the height of its receiving surface should be not less than

6 inches, to avoid in-splashing. The roof of a house is a bad place on account of the eddies which abound there.

A circumstance which has not yet been fully explained is that the higher a gauge is above the ground the less rain it catches. In the case of gauges on the top of poles in an open situation, the amount collected is diminished by $\frac{1}{10}$th part of itself by doubling the height of the receiving surface, as shown by comparing gauges in the same plot of ground at heights ranging from 6 inches to 20 feet.[1]

By means of tipping-buckets and other arrangements, automatic records of rainfall are obtained at the principal observatories.

The mean annual rainfall, according to Mr. Symons, is 20 inches at Lincoln and Stamford; 21 at Aylesbury, Bedford, and Witham; 24 at London and Edinburgh; 30 at Dublin, Perth, and Salisbury; 33 at Exeter and Clifton; 35 to 36 at Liverpool and Manchester; 40 at Glasgow and Cork; 50 at Galway; 64 at Greenock and Inverary; 86 at Dartmoor; 91 on Benlomond; and upwards of 150 inches in some parts of the English lake district.

162. **Snow and Hail.**—Snow is probably formed by the direct passage of vapour into the solid state. Snow-flakes, when examined under the microscope, are always found to be made up of elements possessing hexagonal symmetry. In Fig. 103 are depicted various forms observed by Captain Scoresby during a long sojourn in the Arctic regions.

In these cold countries the air is often filled with small crystals of ice which give rise to the phenomena of haloes and parhelia.

Hail is probably due to the freezing of rain-drops in their passage through strata of air colder than those in which they were formed. Even in fine summer weather, a freezing temperature exists at the height of from 10,000 to 20,000 feet, and it is no unusual thing for a colder stratum to underlie a warmer, although, as a general rule, the temperature diminishes in ascending.

163. **Dew.**—By this name we denote those drops of water which are seen in the morning on the leaves of plants, and are especially noticeable in spring and autumn. We have already seen (§ 157) that dew does not *fall*, as it is not formed in the atmosphere, but in contact with the bodies on which it appears, being in fact due to their cooling after the sun has sunk below the horizon, when they lose heat by radiation to the sky. The lowering of temperature which thus occurs is much more marked for grass, stones, or bare earth than

[1] This appears from the table in Symons on *Rain*, p. 19.

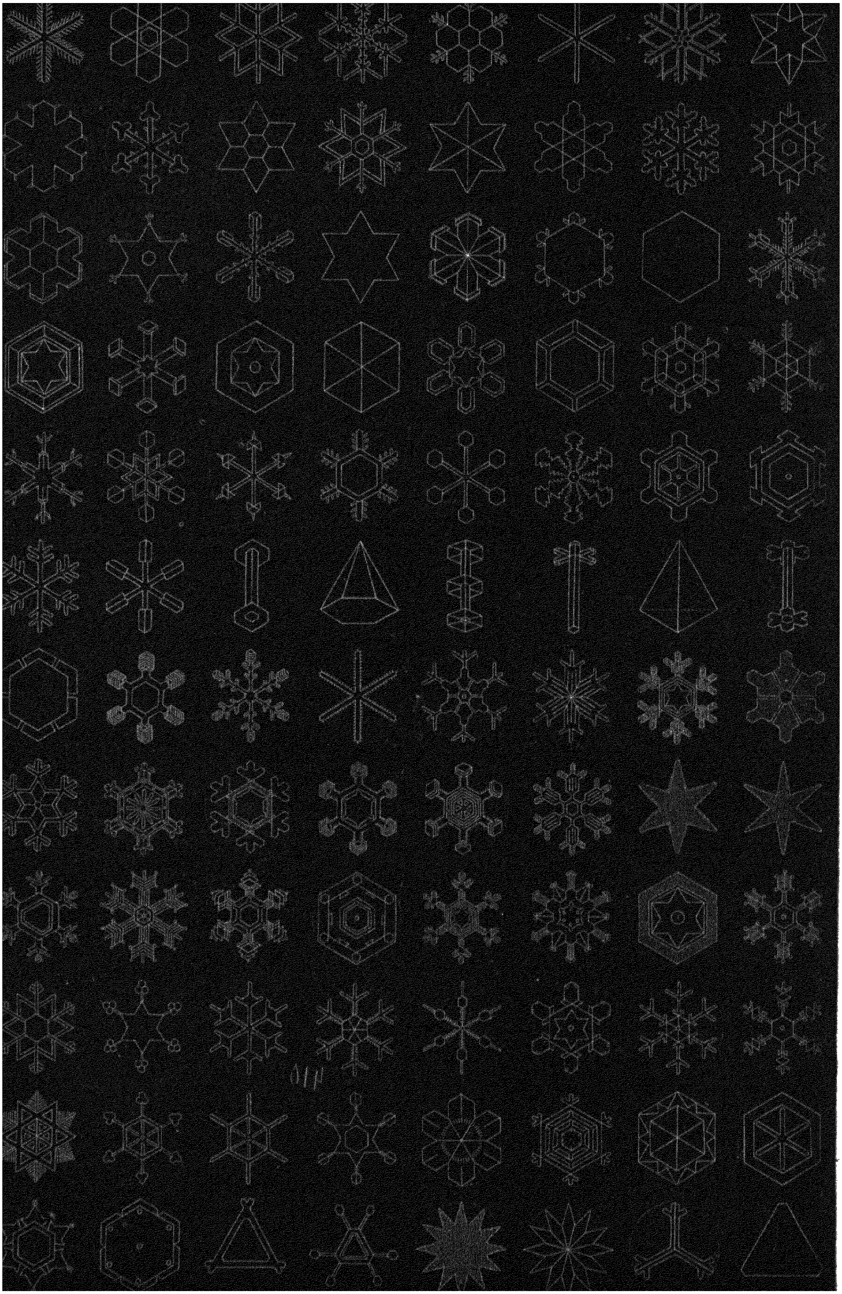


Fig. 103.—Snow-crystals.

for the air, whose radiating power is considerably less. The consequence is a considerable difference of temperature between the surface of the ground and the air at the height of a few feet, a difference which is found by observation to amount sometimes to 8° or 10° C., and it is this which causes the deposition of dew. The surface of the earth, as it gradually cools, lowers the temperature of the adjacent air, which thus becomes saturated, and, on further cooling, yields up a portion of its vapour in the liquid form. If the temperature of the surface falls below 0° C., the dew is frozen, and takes the form of *hoar-frost.*

According to this theory, it would appear that the quantity of dew deposited upon a body should increase with the radiating power of its surface, and with its insulation from the earth or other bodies from which it might receive heat by conduction, both which conclusions are verified by observation.

The amount of deposition depends also in a great measure on the degree of exposure to the sky. If the body is partially screened, its radiation and consequent cooling are checked. This explains the practice which is common with gardeners of employing light coverings to protect plants from frost—coverings which would be utterly powerless as a protection against the cold of the surrounding air. The lightness of the dew on cloudy nights is owing to a similar cause; clouds, especially when overhead, acting as screens.

The deposition of dew is favoured by a slight motion of the atmosphere, which causes the lower strata of air to cool down more rapidly; but if the wind is very high, the different strata are so intermingled that very little of the air is cooled down to its dew-point, and the deposit is accordingly light. When these two obstacles are combined, namely a high wind and a cloudy sky, there is no dew at all.

CHAPTER XII.

CONDUCTION OF HEAT

164. Conduction.—When heat is applied to one end of a bar of metal it is propagated through the substance of the bar, producing a rise of temperature which is first perceptible at near and afterwards at remote portions. This transmission of heat is called *conduction.* The best conductors are metals, but all bodies conduct heat more or less.

165. Variable and Permanent Stages.—Whenever heat is applied steadily to one end of a bar for a sufficient length of time, we may distinguish two stages in the experiment: 1st, the variable stage, during which all portions of the bar are rising in temperature; and, 2nd, the permanent state, which may subsist for any length of time without alteration. In the former stage the bar is gaining heat; that is, it is receiving more heat from the source than it gives out to surrounding bodies. In the latter stage the receipts and expenditure of heat are equal, and are equal not only for the bar as a whole, but for every small portion of which it is composed.

In this permanent state no further accumulation of heat takes place. All the heat which reaches an internal particle is transmitted by conduction, and the heat which reaches a superficial particle is given off partly by radiation and air-contact, and partly by conduction to colder neighbouring particles. In the earlier stage, on the contrary, only a portion of the heat received by a particle is thus disposed of, the remainder being accumulated in the particle, and serving to raise its temperature. Hence in this earlier stage the transmission of heat from the hot to the cold portions of the bar is checked by the absorption which goes on in the intervening parts. The amount of this absorption which occurs before the final condi-

tion is attained will depend upon the capacity of the substance for heat.

166. Conductivity and Diffusivity.—We may thus distinguish between two modes of estimating conducting power. What is especially understood as "conductivity" is independent of absorption, and therefore of thermal capacity. In order to obtain direct measures of it we must observe the flow of heat when the temperatures have become permanent. On the other hand "diffusivity" (to use the name introduced by Lord Kelvin) measures the *tendency to equalization of temperature*, which varies directly as conductivity, and inversely as the thermal capacity of unit volume of the body.

If we compare the times occupied by two equal and similar bodies in passing from the same initial distribution of temperature to the same final distribution, these times will be in the inverse ratio of the diffusivities. If the diffusivities are equal, the times will be the same, and in this case the quantities of heat gained or lost by corresponding portions of the two bodies are directly as the thermal capacities of equal volumes.[1]

167. Definition of Conductivity.—In order to give an accurate definition of conductivity, we must suppose a plate having one face at a uniform temperature v_1, and the other at a higher uniform temperature v_2, and we must suppose all parts of the plate to have attained their permanent temperatures. Then if x denote the thickness of the plate, and k the conductivity of the substance of which it is composed, the quantity, Q, of heat that flows through an area, A, of the plate in the time t will be

$$Q = kA\frac{v_2 - v_1}{x}t; \qquad (1)$$

whence we have

$$k = \frac{Qx}{A(v_2 - v_1)t}; \qquad (2)$$

and the conductivity may be defined as the quantity of heat that flows in unit time through unit area of a plate of unit thickness, with 1° of difference between the temperatures of its faces.

[1] The name *diffusivity* is employed by Lord Kelvin in the article "Heat" in the new edition of the *Encyclopædia Britannica*. The name *thermometric conductivity* had previously been used in the same sense by Professor Clerk Maxwell, ordinary conductivity being called *thermal conductivity* for distinction. There is a close analogy between the conduction of heat and the diffusion of liquids; and the coefficient which expresses the facility with which one liquid diffuses into another is precisely analogous to "thermometric conductivity." Hence the name "diffusivity."

When the unit of heat employed in the reckoning is that which raises the temperature of unit volume of water by 1° (a unit which is practically the same as the gramme-degree), the conductivity k may be defined as the *thickness of a stratum of water* which would be raised 1° in temperature by the heat conducted in unit time through a plate of the substance of unit thickness having 1° of difference between its faces.

If for the words *thickness of a stratum of water* we substitute *thickness of a stratum of the substance,* we have the definition of *diffusivity.*

The thicknesses of the two strata will evidently be inversely as the thermal capacities of equal volumes. But the thermal capacity of unit volume of water is unity. Hence the "diffusivity" is equal to the "conductivity" divided by the thermal capacity of unit volume of the substance. If this thermal capacity be denoted by c, we have $c=sd$, where s denotes the specific heat (or thermal capacity of unit mass) and d the density (or mass of unit volume), and the diffusivity κ is

$$\kappa=\frac{k}{c}=\frac{k}{sd} \tag{3}$$

Strictly speaking, k in equations (1), (2) is the *mean conductivity* between the two temperatures v_1, v_2, and the conductivity *at* any temperature v will be what k becomes when v_1 and v_2 are very nearly equal to each other and to v. The fact that conductivity varies with temperature was discovered by Forbes. He found that a specimen of iron which had a conductivity ·207 at 0° C. had only a conductivity ·124 at 275° C.

168. **Effect of Change of Units.**—In the C.G.S. (Centimetre-Gramme-Second) system, which we have explained in Part I., A is expressed in square centimetres, x in centimetres, and Q in gramme-degrees. It is immaterial whether the degree be Centigrade or Fahrenheit; for a change in the length of the degree will affect the numerical values of Q and of v_2-v_1 alike, and will leave the numerical value of $\frac{Q}{v_2-v_1}$, and hence of $\frac{Qx}{A(v_2-v_1)t}$, or k unaltered.

To find the effect of changes in the units of length and time, we must note that if the unit of length be x centimetres, the unit of area will be x^2 square centimetres, and the unit of mass, being the mass of unit volume of cold water, will be x^3 grammes. The new unit of heat will therefore be x^3 gramme-degrees.

The new unit of conductivity will be the conductivity of a substance such that x^3 gramme-degrees of heat flow in the new unit of time—which we will call t seconds—through x^2 sq. cm. of a plate x cm. thick, with a difference of 1° between its faces. The conductivity of such a plate, when expressed in C.G.S. units, would be found by putting

$$Q = x^3, \ A = x^2, \ v_2 - v_1 = 1$$

in the formula

$$\frac{Q\,x}{A\,(v_2 - v_1)\,t},$$

and would be $\frac{x^4}{x^2 t}$ or $\frac{x^2}{t}$.

Hence to reduce conductivities from the new scale to the C.G.S. scale we must multiply them by $\frac{x^2}{t}$; and the same rule will apply to diffusivities, since the quantity c in equation (3) being the ratio of the thermal capacity of the substance to that of water, bulk for bulk, is independent of units.

169. Illustrations of Conduction.—The following experiments are often adduced in illustration of the different conducting and diffusing powers of different metals.

Two bars of the same size, but of different metals (Fig. 104), are placed end to end, and small wooden balls are attached by wax to

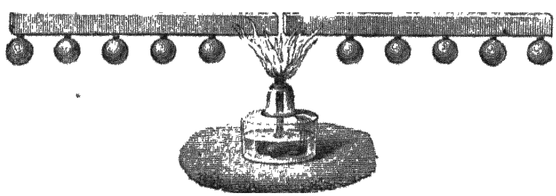


Fig 104 —Balls Melted off

their under surfaces at equal distances. The bars are then heated at their contiguous ends, and, as the heat extends along them, the balls successively drop off. If the conditions are in other respects equal, the balls will begin to drop off first from that which has the greater diffusivity, and the greatest number of balls will ultimately drop off from that which has the greater conductivity.

The well-known experiment of Ingenhousz (Fig. 105) is of the same kind. The apparatus consists of a box, with a row of holes in one of its sides, in which rods of different metals can be fixed. The rods having previously been coated with wax, the box is filled with

boiling water or bo''ng oil, which comes into contact with the inner ends of the rods. The wax gradually melt as the heat travels along the rods. The order in which the melting begins is the order of the diffusivities of the metals employed, and when it has reached its limit (if the temperature of the liquid be maintained constant) 'ie order of the lengths melted is the order of their conductivities.

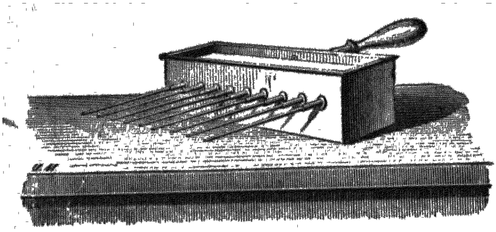


Fig. 105.—Ingenhousz's Apparatus.

170. Metals the Best Conductors.—Metals, though differing considerably one from another, are as a class greatly superior both in conductivity and diffusivity to other substances, such as wood, marble, brick. This explains several familiar phenomena. If the hand be placed upon a metal plate at the temperature of 10° C., or plunged into mercury at this temperature, a very marked sensation of cold is experienced. This sensation is less intense with a plate of marble at the same temperature, and still less with a piece of wood. The reason is that the hand, which is at a higher temperature than the substance to which it is applied, gives up a portion of its heat, which is conducted away by the substance, and consequently a larger portion of heat is parted with, and a more marked sensation of cold experienced, in the case of the body of greater conducting power.

171. Davy Lamp.—The conducting power of metals explains the curious property possessed by wire-gauze of cutting off a flame. If, for example, a piece of wire-gauze be placed above a jet of gas, the flame is prevented from rising above the gauze. If the gas be first allowed to pass through the gauze, and then lighted above, the flame is cut off from the burner, and is unable to extend itself to the under surface of the gauze. These facts depend upon the conducting power of

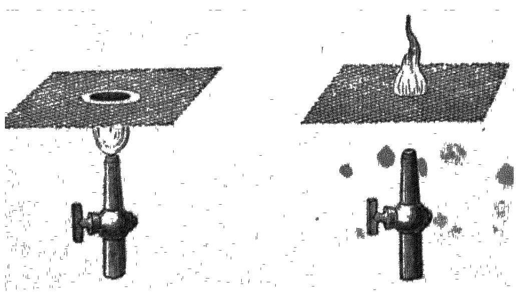
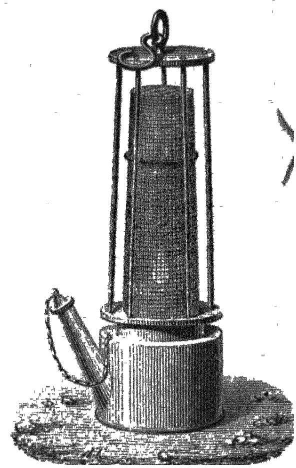


Fig. 106.—Action of Wire-gauze on Flame.

metallic gauze, in virtue of which the heat of the flame is rapidly dissipated at the points of contact, the result being a diminution of temperature sufficient to prevent ignition.



This property of metallic gauze has been turned to account for various purposes, but its most useful application is in the safety-lamp of Sir Humphry Davy.

It is well known that a gas called *fire-damp* is often given off in coal-mines. It is a compound of carbon and hydrogen, and is a large ingredient in ordinary coal-gas.

This fire-damp, when mixed with eight or ten times its volume of air, explodes with great violence on coming in contact with a lighted body. To obviate this danger, Davy invented the safety-lamp, which is an ordinary lamp with the flame inclosed by wire-gauze. The explosive gases pass through the gauze, and burn inside the lamp, in such a manner as to warn the miner of their presence; but the flame is unable to pass through the gauze.

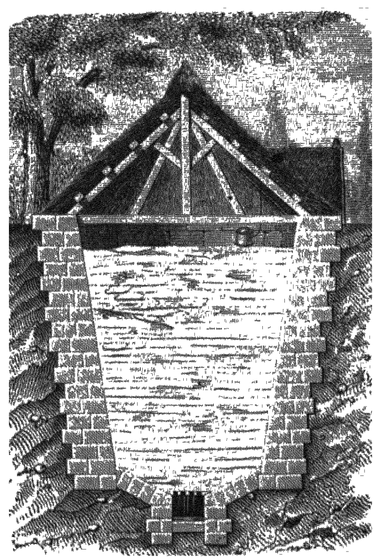


Fig. 108.—Ice-house.

172. Walls of Houses.—The knowledge of the relative conducting powers of different bodies has several important practical applications.

In cold countries, where the heat produced in the interior of a house should be as far as possible prevented from escaping, the walls should be of brick or wood, which have feeble conducting powers. If they are of stone, which is a better conductor, a greater thickness is required. Thick walls are also useful in hot countries in resisting the power of the solar rays during the heat of the day.

We have already alluded (§ 56) to the advantage of employing fire-brick, which is a bad conductor, as a lining for stoves.

The feeble conducting power of brick has led to its employment in the construction of ice-houses. These are round pits (Fig. 108), generally from 6 to 8 yards in diameter at top, and somewhat narrower at the bottom, where there is a grating to allow the escape of water. The inside is lined with brick, and the top is covered with straw, which, as we shall shortly see, is a bad conductor. In order to diminish as much as possible the extent of surface exposed to the

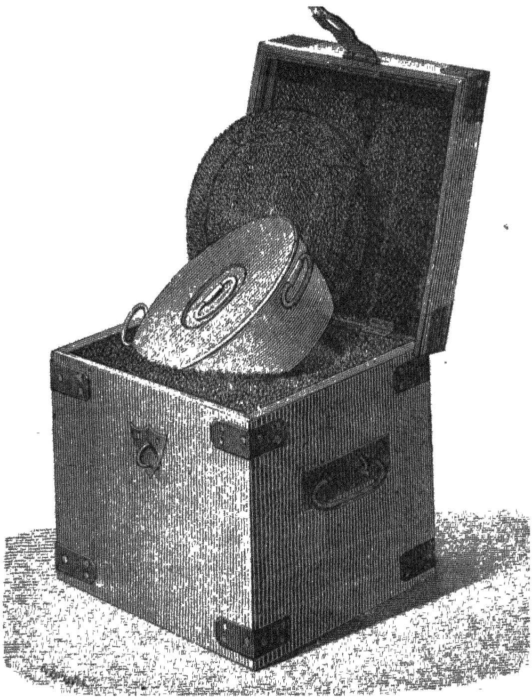


Fig. 109.—Norwegian Cooking-box.

action of the air, the separate pieces are dipped in water before depositing them in the ice-house, and, by their subsequent freezing together, a solid mass is produced, capable of remaining unmelted for a very long time.

173. Norwegian Cooking-box.—A curious application of the bad conducting power of felt is occasionally to be seen in the north of Europe in a kind of self-acting cooking-box. This is a box lined

inside with a thick layer of felt, into which fits a metallic dish with a cover. The dish is then covered with a cushion of felt, so as to be completely surrounded by a substance of very feeble conducting power. The method of employing the apparatus is as follows:—The meat which it is desired to cook is placed along with some water in the dish, the whole is boiled for a short time, and then transferred from the fire to the box, where the cooking is completed *without any further application of heat.* The resistance of the stuffing of the box to the escape of heat is exceedingly great; in fact, it may be shown that at the end of three hours the temperature of the water has fallen by only about 10° or 15° C. It has accordingly remained during all that time sufficiently high to conduct the operation of cooking.

174. Experimental Determination of Conductivity.—Several experimenters have investigated the conductivity of metals, by keeping one end of a metallic bar at a high temperature, and, after a sufficient lapse of time, observing the permanent temperatures assumed by different points in its length.

If the bar is so long that its further end is not sensibly warmer than the surrounding air, and if, moreover, Newton's law of cooling (§ 186) be assumed true for all parts of the surface, and all parts of a cross section be assumed to have the same temperature, the conductivity being also assumed to be independent of the temperature, it is easily shown that the temperatures of the bar at equidistant points in its length, beginning from the heated end, must exceed the atmospheric temperature by amounts forming a decreasing geometric series. Wiedemann and Franz, by the aid of the formula to which these assumptions lead,[1] computed the relative conducting powers of several of the metals, from experiments on thin bars, which were steadily heated at one end, the temperatures at various points in the length being determined by means of a thermo-electric junction clamped to the bar. The following were the results thus obtained:—

RELATIVE CONDUCTING POWERS.

Silver,	100	Steel,	12
Copper,	77·6	Iron,	11·9
Gold,	53·2	Lead,	8 5
Brass,	33	Platinum,	8·2
Zinc,	19·9	Palladium,	6·3
Tin,	14·5	Bismuth,	1·9

[1] See note B at the end of this chapter.

The *absolute* conductivity of wrought iron was investigated with great care by Principal Forbes, by a method which avoided some of the questionable assumptions above enumerated. The end of the bar was heated by a bath of melted lead kept at a uniform temperature, screens being interposed to protect the rest of the bar from the heat radiated by the bath. The temperatures at other points were observed by means of thermometers inserted in small holes drilled in the bar, and kept in metallic contact by fluid metal. In order to determine the loss of heat by radiation at different temperatures, a precisely similar bar, with a thermometer inserted in it, was raised to about the temperature of the bath, and the times of cooling down through different ranges were noted.

The conductivity of one of the two bars experimented on, varied from ·01337 at 0° C. to ·00801 at 275° C., and the corresponding numbers for the other bar were ·00992 and ·00724, the units being the foot, the minute, the degree (of any scale), and the foot-degree[1] (of the same scale). In both instances, the conductivity decreased regularly with increase of temperature.

To reduce these results to the C.G.S. scale, we must (as directed in § 168) multiply them by $\frac{x^2}{t}$, where x denotes the number of centimetres in a foot, or 30·48, and t the number of seconds in a minute; $\frac{x^2}{t}$ will therefore be

$$\frac{(30{\cdot}48)^2}{60}, \text{ or } 15{\cdot}48.$$

The reduced values will therefore be as follows:—

	At 0°.	At 275°
1st bar,...........................	·207	·1240
2d bar,.........................	·1536	·1121

175. **Experimental Determination of Diffusivity.**—Absolute determinations of the diffusivity κ or $\frac{k}{c}$ for the soil or rock at three localities in or near Edinburgh were made by Principal Forbes and Lord Kelvin. They were derived from observations on the temperature of the soil as indicated by thermometers having their bulbs buried at depths of 3, 6, 12 and 24 French feet. The annual range of temperature diminished rapidly as the depth increased, and this diminution of range was accompanied by a retardation of the times of maximum and minimum. The greater the diffusivity the more slowly will the range diminish and the less will be the retardation

[1] See § 60.

of phase. By a process described in note C at the end of this chapter the value of κ was deduced; and by combining this with the value of c (the product of specific heat and density), which was determined by Regnault, from laboratory experiments, the value of k or $c\kappa$ was found. The following are the results, expressed in the C.G.S. scale:—

	$\frac{k}{c}$ or Diffusivity	k or Conductivity
Trap rock of Calton Hill, ...	·00786	·00415
Sand of Experimental Garden,	·00872	·00262
Sandstone of Craigleith Quarry,	·02311	·01068

Similar observations made at Greenwich Observatory, and reduced by the editor of the present work, gave ·01249 as the diffusivity of the gravel of Greenwich Observatory Hill.

A method based upon similar principles has since been employed by Ångstrom and also by Neumann for laboratory experiments; a bar of the substance under examination being subjected to regular periodical variations of temperature at one end, and the resulting periodic variations at other points in its length being observed. These gave the means of calculating the diffusivity, and then observations of the specific heat and density gave the conductivity. The following conductivities were thus obtained by Neumann:—

	Conductivity in C G S units
Copper,	1·108
Brass,	·302
Zinc,	·307
Iron,	·164
German silver,	·109

176. Conductivity of Rocks.—The following values of thermal and thermometric conductivity in C.G.S. units are averages based on the experiments of Professor Alexander Herschel.

	k	$\frac{k}{c}$
Granite,	·0053	·015
Limestone,	·005	·009
Sandstone, dry,	·0056	·012
Sandstone, thoroughly wet,	·0060	·010
Slate, along cleavage,	·0060	·010
Slate, across cleavage,	·0034	·006
Clay, sun-dried,	·0022	·0048
Red brick,	·0015	·0044
Plate-glass,	·0023	·0040

177. Conducting Powers of Liquids.—With the exception of mercury and other melted metals, liquids are exceedingly bad conduc-

tors of heat. This can be shown by heating the upper part of a column of liquid, and observing the variations of temperature below. These will be found to be scarcely perceptible, and to be very slowly produced. If the heat were applied below (Fig. 110), we should have the process called *convection of heat;* the lower layers of liquid would rise to the surface, and be replaced by others which would rise in their turn, thus producing a circulation and a general heating of the liquid. On the other hand, when heat is applied above, the expanded

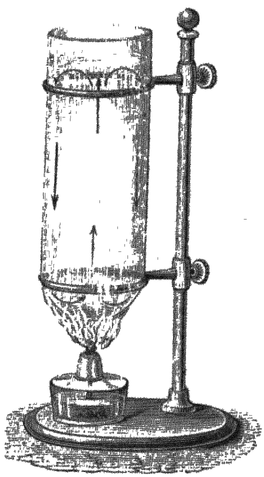


Fig. 110 —Liquid heated from below.

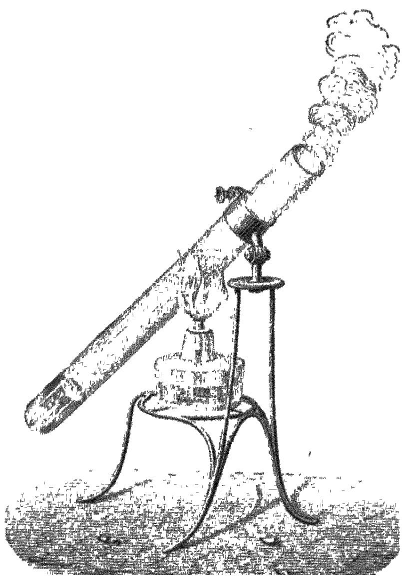


Fig. 111.—Boiling of Water over Ice

layers remain in their place, and the rest of the liquid can he heated by conduction and radiation only.

The following experiment is one instance of the very feeble conducting power of water. A piece of ice is placed at the bottom of a glass tube (Fig 111), which is then partly filled with water; heat is applied to the middle of the tube, and the upper portion of the water is readily raised to ebullition, without melting the ice below.

178. Conducting Power of Water.—The power of conducting heat possessed by water, though very small, is yet quite appreciable. This was established by Despretz by the following experiment. He took a cylinder of wood (Fig. 112) about a yard in height and eight inches in diameter, which was filled with water. In the side of this

cylinder were arranged twelve thermometers one above another, their bulbs being all in the same vertical through the middle of the liquid column. On the top of the liquid rested a metal box, which was filled with water at 100°, frequently renewed during the course of the experiment. Under these circumstances Despretz observed that the temperature of the thermometers rose gradually, and that a long time—about 30 hours—was required before the permanent state was

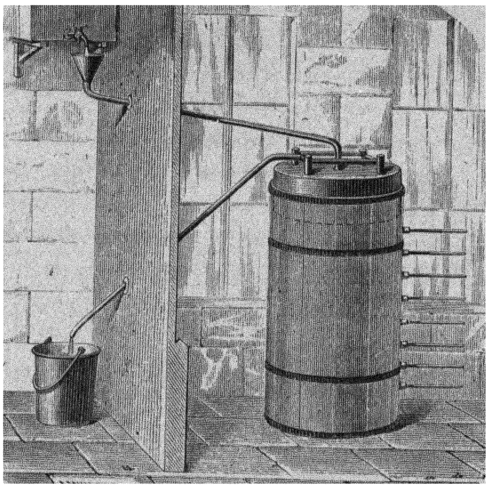


Fig. 112.—Despretz's Experiment.

assumed. Their permanent differences, which formed a decreasing geometric series, were very small, and were inappreciable after the sixth thermometer.

The increase of temperature indicated by the thermometers might be attributed to the heat received from the sides of the cylinder, though the feeble conducting power of wood renders this idea somewhat improbable. But Despretz observed that the temperature was higher in the axis of the cylinder than near the sides, which proves that the elevation of temperature was due to the passage of heat downwards through the liquid.

From experiments by Professor Guthrie,[1] it appears that water conducts better than any other liquid except mercury.

179. **Absolute Measurement of Conductivity of Water.**—The abso-

[1] *B. A. Report*, 1868, and *Trans. R. S.* 1869.

lute value of k for water has been determined by Mr. J. T. Bottomley. Hot water was gently placed on the top of a mass of water nearly filling a cylindrical wooden vessel. Readings were taken from time to time of two horizontal thermometers, one of them a little lower than the other, which gave the difference of temperature between the two sides of the intervening stratum. The quantity of heat conducted in a given time through this stratum was known from the rise of temperature of the whole mass of water below, as indicated by an upright thermometer with an exceedingly long cylindrical bulb extending downwards from the centre of the stratum in question nearly to the bottom of the vessel. A fourth thermometer, at the level of the bottom of the long bulb, showed when the increase of temperature had extended to this depth, and as soon as this occurred (which was not till an hour had elapsed) the experiment was stopped.

The result of these experiments is that the value of k for water is from ·0020 to ·0023, which is nearly identical with its value for ice, this latter element, as determined by Professor George Forbes, being ·00223.

The conductivity of water seems to be much greater than that of wood.

180. Conducting Power of Gases.—Of the conducting powers of gases it is almost impossible to obtain any direct proofs, since it is exceedingly difficult to prevent the interference of convection and direct radiation. However, we know at least that they are exceedingly bad conductors. In fact, in all cases where gases are inclosed in small cavities where their movement is difficult, the system thus formed is a very bad conductor of heat. This is the cause of the feeble conducting powers of many kinds of cloth, of fur, eider-down, felt, straw, saw-dust, &c. Materials of this kind, when used as articles of clothing, are commonly said to be *warm*, because they hinder the heat of the body from escaping. If a garment of eider-down or fur were compressed so as to expel the greater part of the air, and to reduce the substance to a thin sheet, it would be found to be a much less warm covering than before, having become a better conductor. We thus see that it is the presence of air which gives these substances their feeble conducting power, and we are accordingly justified in assuming that air is a bad conductor of heat.

181. Conductivity of Hydrogen.—The conducting power of hydrogen is much superior to that of the other gases—a fact which agrees

with the view entertained by chemists, that this gas is the vapour of a metal. The good conductivity of hydrogen is shown by the following experiments:—

1. Within a glass tube (Fig. 113) is stretched a thin platinum wire, which is raised to incandescence by the passage of an electric current. When air, or any gas other than hydrogen, is passed through the tube, the incandescence continues, though with less vividness than in vacuo; but it disappears as soon as hydrogen is employed.

2. A thermometer is placed at the bottom of a vertical tube, and heated by a vessel containing boiling water which is placed at the top of the tube. The tube is exhausted of air, and different gases are successively admitted. In each case the indication of the thermometer is found to be lower than for vacuum, except when the gas is hydrogen. With this gas, the difference is in the opposite direction, showing that the diminution of radiation has been more than compensated by the conducting power of the hydrogen.

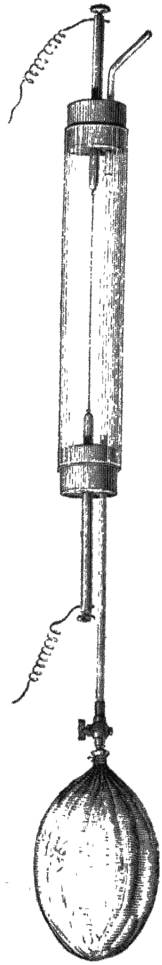


Fig 113 —Cooling by Contact of Hydrogen.

NOTE A. DIFFERENTIAL EQUATION FOR LINEAR FLOW OF HEAT.—The mode of obtaining differential equations for the variation of temperature at each point of a body during the variable stage, may be illustrated by considering the simplest case, that in which the isothermal surfaces (surfaces of equal temperature) are parallel planes, and therefore the lines of flow (which must always be normal to the isothermal surfaces) parallel straight lines.

Let x denote distance measured in the direction in which heat is flowing, v the temperature at the time t at a point specified by x, k the conductivity, and c the thermal capacity per unit volume (both at the temperature v). Then the flow of heat per unit time past a cross section of area A is $-k\mathrm{A}\frac{dv}{dx}$, and the flow past an equal and parallel section further on by the small distance δx is greater by the amount

$$\mathrm{A}\frac{d}{dx}\left(-k\frac{dv}{dx}\right)\delta x.$$

This latter expression therefore represents the loss of heat from the intervening prism $A\delta x$, and the resulting fall of temperature is the quotient of the loss by the thermal capacity $cA\delta x$, which quotient is

$$\frac{1}{c}\frac{d}{dx}\left(-k\frac{dv}{dx}\right).$$

This, then, is the fall of temperature per unit time, or is $-\frac{dv}{dt}$. If the variation of k is insensible, so that $\frac{dk}{dx}$ can be neglected, the equation becomes

$$\frac{dv}{dt}=\frac{k}{c}\frac{d^2v}{dx^2},$$

which applies approximately to the variations of temperature in the soil near the surface of the earth, x being in this case measured vertically. For the integral of this equation, see Note C.

NOTE B. FLOW OF HEAT IN A BAR (§ 174).—If p and s denote the perimeter and section of the bar, k the conductivity, and h the coefficient of emission of the surface at the temperature v, the heat emitted in unit time from the length δx is $hvp\delta x$, if we assume as our zero of temperature the temperature of the surrounding air. But the heat which passes a section is $-sk\frac{dv}{dx}$, and that which passes a section further on by the amount δx is less by the amount $sk\frac{d^2v}{dx^2}\delta x$; and this difference must equal the amount emitted from the intervening portion of the surface. Hence we have the equation $\frac{d^2v}{dx^2}=\frac{hp}{ks}v$, the integral of which for the case supposed is

$$v=Ve^{-x\sqrt{\frac{hp}{ks}}}$$

V denoting the temperature at the heated end.

NOTE C. DEDUCTION OF DIFFUSIVITY FROM OBSERVATIONS OF UNDERGROUND TEMPERATURE (§ 175).—Denoting the diffusivity $\frac{k}{c}$ by κ, the equation of Note A is

$$\frac{dv}{dt}=\kappa\frac{d^2v}{dx^2}. \qquad (4)$$

This equation is satisfied by

$$v=e^{-\alpha x}\sin(\beta t-\alpha x), \qquad (5)$$

where α and β are any two constants connected by the relation

$$\frac{\beta}{2\alpha^2}=\kappa; \qquad (6)$$

for we find, by actual differentiation,

$$\frac{dv}{dx}=e^{-\alpha x}\{-\alpha\sin(\beta t-\alpha x)-\alpha\cos(\beta t-\alpha x)\};$$

$$\frac{d^2v}{dx^2}=e^{-\alpha x}\{\alpha^2\sin(\beta t-\alpha x)+\alpha^2\cos(\beta t-\alpha x)+\alpha^2\cos(\beta t-\alpha x)-\alpha^2\sin(\beta t-\alpha x)\}$$

$$=e^{-\alpha x}2\alpha^2\cos(\beta t-\alpha x);$$

$$\frac{dv}{dt}=e^{-\alpha x}\beta\cos(\beta t-\alpha x)=\frac{\beta}{2\alpha^2}\frac{d^2v}{dx^2}.$$

More generally, equation (4) will be satisfied by making v equal to the sum of any

number of terms similar to the right-hand member of (5), each multiplied by any constant, and a constant term may be added. In fact we may have

$$v = A_0 + A_1 e^{-\alpha_1 x} \sin(\beta_1 t - \alpha_1 x + E_1) + A_2 e^{-\alpha_2 x} \sin(\beta_2 t - \alpha_2 x + E_2)$$
$$+ A_3 e^{-\alpha_3 x} \sin(\beta_3 t - \alpha_3 x + E_3) + \text{\&c.,} \qquad (7)$$

where A_0, A_1, E_1, &c., are any constants.

Let x be measured vertically downwards from the surface of the ground (supposed horizontal); then at the surface the above expression becomes

$$v = A_0 + A_1 \sin(\beta_1 t + E_1) + A_2 \sin(\beta_2 t + E_2) + A_3 \sin(\beta_3 t + E_3) + \text{\&c.} \qquad (8)$$

Now, if T denote a year, it is known that the average temperature of the surface at any time of year can be expressed, in terms of t the time reckoned from 1st of January or any stated day, by the following series —

$$v = A_0 + A_1 \sin\left(\frac{2\pi t}{T} + E_1\right) + A_2 \sin\left(\frac{4\pi t}{T} + E_2\right) + A_3 \sin\left(\frac{6\pi t}{T} + E_2\right) + \text{\&c.,} \qquad (9)$$

where A_0 is the mean temperature of the whole year, and A_1, A_2, A_3, &c., which are called the *amplitudes* of the successive terms, diminish rapidly. The term which contains A_1 and E_1 (called the annual term), completes its cycle of values in a year, the next term in half a year, the next in a third of a year, and so on. The annual term is much larger, and more regular in its values from year to year than any of those which follow it. Each term affords two separate determinations of the diffusivity. Thus, for the annual term, we have, by comparing (8) and (9)—

$$\beta_1 = \frac{2\pi}{T}, \text{ whence, by (6),}$$

$$\alpha_1 = \sqrt{\frac{\beta_1}{2\kappa}} = \sqrt{\frac{\pi}{T\kappa}}.$$

At the depth x, the amplitude of this term will be

$$A_1 e^{-\alpha_1 x},$$

the logarithm of which is

$$\log A_1 - \alpha_1 x.$$

Hence α_1 can be deduced from a comparison of the annual term at two different depths, by dividing the difference of the Napierian logarithms of the amplitudes by the difference of depth.

But α_1 can also be determined by comparing the values of $\beta_1 t - \alpha_1 x + E_1$ at two depths for the same value of t, and taking their difference (which is called the *retardation of phase*, since it expresses how much later the maximum, minimum, and other phases, occur at the lower depth than at the upper). This difference, divided by the difference of depth, will be equal to α_1.

These two determinations of α_1 ought to agree closely, and κ will then be found by the equation

$$\alpha_1 = \sqrt{\frac{\pi}{T\kappa}}.$$

CHAPTER XIII.

RADIATION.

182. **Radiation distinct from Conduction.**—When two bodies at different temperatures are placed opposite to each other, with nothing between them but air or some other transparent medium, the hotter body gives heat to the colder by *radiation*. It is by radiation that the earth receives heat from the sun and gives out heat to the sky; and it is by radiation that a fire gives heat to a person sitting in front of it.

Radiation is broadly distinguished from conduction. In conduction, the transmission of heat is effected by the warming of the intervening medium, each portion of which tends to raise the succeeding portion to its own temperature.

On the other hand heat transmitted from one body to another by radiation does not affect the temperature of the intervening medium. The heat which we receive from the sun has traversed the cold upper regions of the air; and paper can be ignited in the focus of a lens of ice, though the temperature of ice cannot exceed the freezing-point.

Conduction is a gradual, radiation an instantaneous process. A screen interposed between two bodies instantly cuts off radiation between them; and on the removal of such a screen radiation instantly attains its full effect. Radiant heat, in fact, travels with the velocity of light, and it is subject to laws similar to the laws of light; for example, it is usually propagated only in straight lines.

Strictly speaking, radiant heat, like latent heat, is not heat at all, but is a form of energy which is readily converted into heat. Its nature is precisely the same as that of light, the difference between them being only a difference of degree, as will be more fully explained in treating of the analysis of light by the prism and spectro-

scope. The present chapter will contain numerous instances of the analogy between the properties of non-luminous radiant heat and well-known characteristics of light.

183. A Ponderable Medium not Essential.—The transmission of the sun's heat to the earth shows that radiation is independent of any ponderable medium. But since the solar heat is accompanied by light, it might still be questioned whether dark heat could be propagated through a vacuum.

This was tested by Rumford in the following way:—He constructed a barometer (Fig. 114), the upper part of which was expanded into a globe, and contained a thermometer hermetically sealed into a hole at the top of the globe, so that the bulb of the thermometer was at the centre of the globe. The globe was thus a Torricellian vacuum-chamber. By melting the tube with a blow-pipe, the globe was separated, and was then immersed in a vessel containing hot water, when the thermometer was immediately observed to rise to a temperature evidently higher than could be due to the conduction of heat through the stem. The heat had therefore been communicated by direct radiation through the vacuum between the sides of the globe and the bulb *a* of the thermometer.

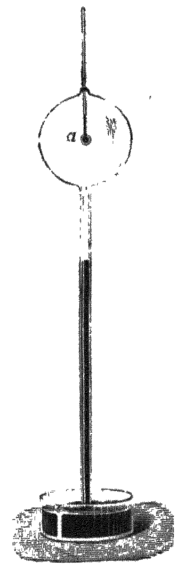


Fig. 114.—Rumford's Experiment.

184. Radiant Heat travels in Straight Lines.—In a uniform medium the radiation of heat takes place in straight lines. If, for instance, between a thermometer and a source of heat, there be placed a number of screens, each pierced with a hole, and if the screens be so arranged that a straight line can be drawn without interruption from the source to the thermometer, the temperature of the latter immediately rises; if a different arrangement be adopted, the heat is stopped by the screens, and the thermometer indicates no effect.

Hence we can speak of *rays* of heat just as we speak of rays of light. Thus we say that rays of heat issue from all points of the surface of a heated body, or that such a body emits rays of heat. The word *ray* when thus used scarcely admits of precise definition. It is a popular rather than a scientific term; for no finite quantity of heat or light can travel along a mathematical line. In a mere

geometrical sense the rays are the lines which indicate the direction of propagation.

It is now generally admitted that both heat and light are due to a vibratory motion which is transmitted through space by means of a fluid called ether. According to this theory the rays of light and heat are lines drawn in all directions from the origin of motion, and along which the vibratory movement advances.

185. **Surface Conduction.**—The cooling of a hot body exposed to the air is effected partly by radiation, and partly by the conduction of heat from the surface of the body to the air in contact with it. The activity of the surface-conduction is greatly quickened by wind, which brings continually fresh portions of cold air into contact with the surface, in the place of those which have been heated.

The cooling of a body *in vacuo* is effected purely by radiation, except in so far as there may be conduction through its supports.

186. **Newton's Law of Cooling.**—In both cases, if the body be exposed in a chamber of uniform temperature, the rate at which it loses heat is approximately proportional to the excess of the temperature of its surface above that of the chamber, and the proportionality is sensibly exact when the excess does not exceed a few degrees. If the body be of sensibly uniform temperature throughout its whole mass, as in the case of a thin copper vessel full of water which is kept stirred, its fall of temperature is proportional to its loss of heat, and hence the rate at which its temperature falls is proportional to the excess of its temperature above that of the chamber. Practically if the body be a good conductor and of small dimensions—say a copper ball an inch in diameter, or an ordinary mercurial thermometer—the fall of its temperature is nearly in accordance with this law, which is called *Newton's law of cooling.* The observed fact is that when the readings of the thermometer are taken at equal intervals of time, their excesses above the temperature of the inclosure (which is kept constant) form a diminishing geometrical progression.

To show that this is equivalent to Newton's law, let θ denote the excess of temperature at time t; then, in the notation of the differential calculus, $-\frac{d\theta}{dt}$ is the rate of cooling; and Newton's law asserts that this is proportional to θ, or that

$$-\frac{d\theta}{dt}=A\theta, \qquad (1)$$

where A is a constant multiplier. This is equivalent to

$$-\frac{d\theta}{\theta} = A\,dt, \qquad (2)$$

which asserts that for equal small intervals of time the differences between the temperatures are proportional to the temperatures. But if the differences between the successive terms of a series are proportional to the terms themselves, the series is geometrical; for if we have

$$\frac{\theta_1 - \theta_2}{\theta_1} = \frac{\theta_2 - \theta_3}{\theta_2} = \frac{\theta_3 - \theta_4}{\theta_3},$$

we obtain, by subtracting unity from each member,

$$\frac{\theta_2}{\theta_1} = \frac{\theta_3}{\theta_2} = \frac{\theta_4}{\theta_3};$$

that is, θ_1, θ_2, θ_3, θ_4 are in geometrical progression.

The expression $-\frac{d\theta}{\theta}$ in equation (2) is, by the rules of the differential calculus, equal to $-d \log \theta$; hence equation (2) shows that $\log \theta$ diminishes by equal amounts in equal times. Log θ here denotes the Napierian logarithm of θ, and since common logarithms are equal to Napierian logarithms multiplied by a constant factor, the common logarithm of θ will also diminish by equal amounts in equal times. The constant A in equation (1) or (2) will be determined from the experimental results by dividing the decrement of $\log \theta$ by the interval of time.

We have been assuming that the body is hotter than the chamber or inclosure; but a precisely similar law holds for the warming of a body which is colder than the inclosure in which it is placed.

187. Dulong and Petit's Law of Cooling.—Newton's law is sensibly accurate for *small* differences of temperature between the body and the inclosure. Dulong and Petit conducted experiments on the cooling of a thermometer by radiation in vacuo with excesses of temperature varying from 20° to 240° C., from which they deduced the formula

$$-\frac{d\theta}{dt} = c a^v (a^\theta - 1);$$

or, as it may be otherwise written,

$$-\frac{d\theta}{dt} = c(a^{v+\theta} - a^v),$$

where v denotes the temperature of the walls of the inclosure, which was preserved constant during each experiment, $v+\theta$ the temperature of the thermometer, and $-\frac{d\theta}{dt}$ the rate of cooling. The other letters, c and a, denote constants. When the temperatures are Centi-

grade, the constant a is 1·0077; when they are Fahrenheit it is 1·0043, the form of the expression for the rate of cooling being unaffected by a change of the zero from which temperatures are reckoned. The value of c depends upon the size of the bulb and some other circumstances, and is changed by a change of zero.

188. **Consequences of this Law.**—The formula in its first form shows that, for the same excess θ, the cooling is more rapid at high than at low temperatures.

Employing the Centigrade scale, we have $a = 1{\cdot}0077$, whence $\log a = {\cdot}0077$ nearly, and since

$$a^\theta = 1 + \theta \log a + \tfrac{1}{2}(\theta \log a)^2 + \tfrac{1}{6}(\theta \log a)^3 + \text{\&c.},$$

Dulong and Petit's formula, in its first form, gives

$$-\frac{d\theta}{dt} = c\,(1{\cdot}0077)^v \{{\cdot}0077\,\theta + \tfrac{1}{2}({\cdot}0077\,\theta)^2 + \text{\&c.}\};$$

which shows that, for a given temperature of the inclosure, the rate of cooling is not strictly proportional to θ, but is equal to θ multiplied by a factor which increases with θ, this factor being proportional to $1 + \frac{1}{2}({\cdot}0077\,\theta) + \frac{1}{6}({\cdot}0077\,\theta)^2 +$ &c.

When θ is small enough for ·0077 θ to be neglected in comparison with unity, the factor will be sensibly constant, in accordance with Newton's law.

189. **Theory of Exchanges.**—The second form of Dulong and Petit's formula, namely

$$-\frac{d\theta}{dt} = c(a^{v+\theta} - a^v),$$

suggests that an unequal *exchange* of heat takes place between the thermometer and the walls, the thermometer giving to the walls a quantity of heat $ca^{v+\theta}$ (where $v+\theta$ denotes the temperature of the thermometer), and the walls giving to the thermometer the smaller quantity ca^v.

This is the view now commonly adopted with respect to radiation in general. It has been fully developed by Professor Balfour Stewart under the name of the *theory of exchanges.* Its original promulgator, Prévost of Geneva, called it the theory of *mobile equilibrium of temperature.*

The theory asserts that all bodies are constantly giving out radiant heat, at a rate depending upon their substance and temperature, but independent of the substance or temperature of the bodies which surround them; and that when a body is kept at a uniform temperature, it receives back just as much heat as it gives out.

According to this view, two bodies at the same temperature, exposed to mutual radiation, exchange equal amounts of heat; but if two bodies have unequal temperatures, that which is at the higher temperature gives to the other more than it receives in exchange.[1]

190. Law of Inverse Squares.—If we take a delicate thermometer and place it at successively increasing distances from a source of heat, the temperature indicated by the instrument will exceed that of the atmosphere by decreasing amounts, showing that the intensity of radiant heat diminishes as the distance increases. The law of variation may be discovered by experiment. In fact, when the excess of temperature of the thermometer becomes fixed, we know that the heat received is equal to that lost by radiation; but this latter is, by Newton's law, proportional to the excess of temperature above that of the surrounding air; we may accordingly consider this excess as the measure of the heat received. It has been found, by experiments at different distances,[2] that the excess is inversely proportional to the square of the distance; we may therefore conclude that *the intensity of the heat received from any source of heat varies inversely as the square of the distance.*

The following experiment, devised by Tyndall, supplies another simple proof of this fundamental law:—

The thermometer employed is a Melloni's pile, the nature of which we shall explain in § 197. This is placed at the small end of a hollow cone, blackened inside, so as to prevent any reflection of heat from its inner surface. The pile is placed at S and S′ in front of a vessel filled with boiling water, and coated with lamp-black on the side next the pile. It will now be observed that the temperature indicated by the pile remains constant for all distances. This result proves the law of inverse squares. For the arrangement adopted prevents the pile from receiving more heat than that due to the area of A B in the first case, and to the area A′ B′ in the second. These are the areas of two circles, whose radii are respectively proportional to S O and S′ O; and the areas are consequently proportional to the squares of S O and S′C. Since, therefore, these two areas communi-

[1] For a full account of this subject see "Report on the Theory of Exchanges," by Balfour Stewart, in *British Association Report*, 1861, p. 97; and *Stewart on Heat*, book ii. chap. iii.

[2] The dimensions of the source of heat must be small in comparison with the distance of the thermometer, as otherwise the distances of different parts of the source of heat from the thermometer are sensibly different. In this case, the amount of heat received varies directly as the solid angle subtended by the source of heat.

cate the same quantity of heat to the pile, the intensity of radiation must vary inversely as the squares of the distances S O and S′O.

The law of inverse squares may also be established *a priori* in the following manner:—

Suppose a sphere of given radius to be described about a radiating particle as centre. The total heat emitted by the particle will be received by the sphere, and all points on the sphere will experience

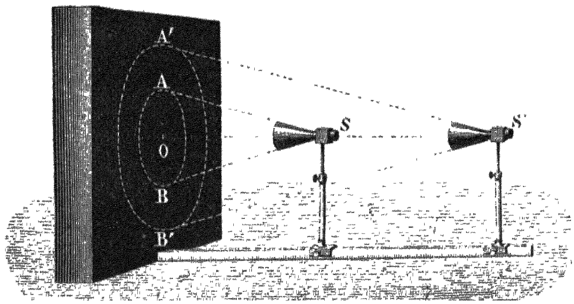


Fig. 115.—Law of Inverse Squares.

the same calorific effect. If now the radius of the sphere be doubled, the surface will be quadrupled, but the total amount of heat remains the same as before, namely, that emitted by the radiating particle. Hence we conclude that the quantity of heat absorbed by a given area on the surface of the large sphere is one-fourth of that absorbed by an equal area on the small sphere; which agrees with the law stated above.

This demonstration is valid, whether we suppose the radiation of heat to consist in the emission of matter or in the emission of energy; for energy as well as matter is indestructible, and remains unaltered in amount during its propagation through space.

191. Law of the Reflection of Heat.— When a ray of heat strikes a polished surface, it is reflected according to the same law as a ray of light.

192. Burning Mirrors.—All rays, either of heat or light, falling on a parabolic mirror in directions parallel to its axis (AC, Fig. 116) are reflected accurately to its focus F, and all rays from F falling on the mirror are reflected parallel to the axis. A spherical concave mirror is a small portion of a sphere, and rays parallel to its axis are reflected so as approximately to pass through its "principal focus" F (same figure), which is midway between A, the central point of the mirror, and C, the centre of the sphere.

When the axis of a concave mirror, of either form, is directed towards the sun, intense heat is produced at the focus, especially if

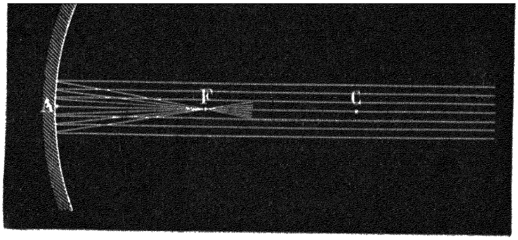


Fig. 116.—Focus of Concave Mirror.

the mirror be large. Fig. 117 represents such a mirror suitably mounted for producing ignition of combustible substances. Tschirnhausen's mirror, which was constructed in 1687, and was about 6½ feet in diameter, was able to melt copper or silver, and to vitrify brick. Instead of curved mirrors, Buffon employed a number of movable plane mirrors, which were arranged so that the different pencils of heat-rays reflected by them converged to nearly the same point. In this way he obtained an extremely powerful effect, and was able, for instance, to set wood on fire at a distance of between 80 and 90 yards. This is the method which Archimedes is said to have employed for the destruction of the Roman fleet in the siege of Syracuse; and though the truth of the story is considered doubtful, it is not altogether absurd.

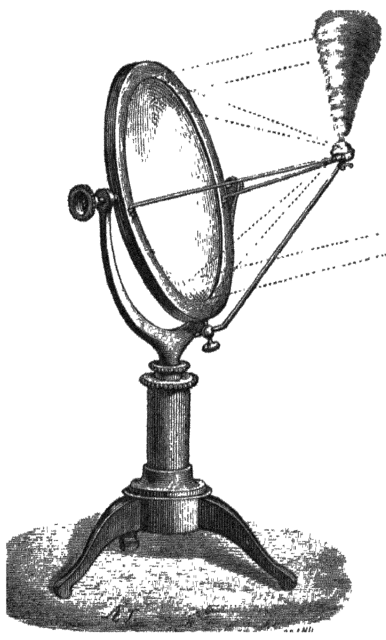


Fig 117.—Burning Mirror.

193. Conjugate Mirrors.—Fig. 118 represents an experiment which is said to have been first performed by Pictet of Geneva.

Two large parabolic mirrors are placed facing each other, at any convenient distance. with their axes in the same straight line. In

the focus of one of them is placed a small furnace, or a red-hot cannon-ball, and in the focus of the other some highly inflammable material, such as phosphorus or gun-cotton. On exciting the furnace with bellows, the substance in the other focus immediately takes fire. With two mirrors of 14 inches diameter, gun-cotton may thus be set on fire at a distance of more than 30 feet. The explanation is very easy. The rays of heat coming from the focus of the first

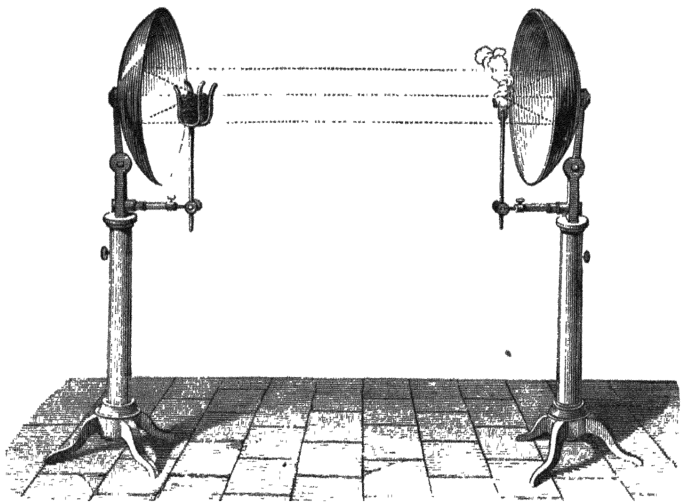


Fig. 118.—Conjugate Mirrors.

mirror are reflected in parallel lines, and, on impinging upon the surface of the second mirror, converge again to its focus, and are thus concentrated upon the inflammable material placed there.

Careful adjustment is necessary to the success of the experiment, and the adjustment is most easily made by first placing a source of light (such as the flame of a candle) in one focus, and forming a luminous image of it in the other. We have thus a convincing proof that heat and light obey the same law as regards direction of reflection.

194. Reflection, Diffusion, Absorption, and Transmission.—When radiant heat is incident upon the surface of a body it is divided into several distinct parts. A portion is *regularly reflected* according to the law given above. A portion is irregularly or *diffusely reflected* and is scattered through space in all directions. A portion penetrates into the body so as to be *absorbed* by it, and to contribute to

raise its temperature; and in some cases a fourth portion passes through the body without contributing to raise its temperature. This portion is said to be *transmitted.*

195. Coefficient of Absorption and Coefficient of Emission.—Applying Newton's law (§ 186), let θ be the small difference of temperature between the surface of the body and the inclosure, and S the area of this surface, which we suppose to have no concavities, then the quantity of heat gained or lost by the body per unit of time is expressed by the formula

$$AS\theta,$$

where A is a constant depending on the nature of the body and more especially on the nature of its surface. This constant A may be called indifferently the *coefficient of emission* or the *coefficient of absorption*, inasmuch as it has the same value (the temperature of the body being given) whether the inclosure be colder or warmer than the body. Experiments conducted by Mr M'Farlane under the direction of Lord Kelvin, showed that when the surface of the body (a copper ball) and the walls of the inclosure were both covered with lamp-black, the inclosure being full of air at atmospheric pressure, the value of the coefficient A in C.G.S. units is about $\frac{1}{4000}$, that is to say $\frac{1}{4000}$ of a gramme-degree of heat is gained or lost per second for each square centimetre of surface of the body, when there is 1° of difference between its temperature and that of the walls of the inclosure. When the surface of the body (the copper ball) was polished, the walls of the inclosure being blackened as before, the coefficient had only $\frac{7}{10}$ of its former value It was estimated that of the value $\frac{1}{4000}$ for blackened surfaces, one-half is due to atmospheric contact and the other half to radiation. As the excess of temperature of the body above that of the walls increased from 5° to 60°, the quantity of heat emitted, instead of being increased only twelve-fold, was increased about sixteen-fold for the blackened and fifteen-fold for the polished ball.

When air is excluded, and the gain or loss of heat is due to pure radiation between the body and the walls, the coefficient A represents, according to the theory of exchanges, the difference between the absolute emission at the temperature of the body and at a temperature 1° higher or lower.

196. Limit to Radiating Power.—It is obviously impossible for a body to absorb more radiant heat than falls upon it. There must,

therefore, be a limiting value of A applicable to a body which would absorb all the heat that falls upon it and not absorb or transmit any. Such a body would possess perfect emissive power for radiant heat. Hence it appears that good radiation depends rather upon defect of resistance than upon any positive power. A perfect radiator would be a substance whose surface offered no resistance to the passage of radiant heat in either direction; while an imperfect radiator is one whose surface allows a portion to be communicated through it, and reflects another portion regularly or irregularly.

The reflecting and diffusive powers of lamp-black are so insignificant, at temperatures below 100°, that this substance is commonly adopted as the type of a perfect radiator, and the emissive and absorptive powers of other substances are usually expressed by comparison with it.

CHAPTER XIV.

RADIATION (CONTINUED).

197. Thermoscopic Apparatus employed in researches connected with Radiant Heat.—An indispensable requisite for the successful study of radiant heat is an exceedingly delicate thermometer. For this purpose Leslie, about the beginning of the present century, invented the differential thermometer, with which he conducted some very important investigations, the main results of which are still acknowledged to be correct. Modern investigators, as Melloni, Laprovostaye, &c., have exclusively employed Nobili's thermo-multiplier, which is an instrument of much greater delicacy than the differential thermometer.

The thermo-pile, invented by Nobili, and improved by Melloni, consists essentially of a chain (Fig. 119) formed of alternate elements of bismuth and antimony. If the ends of the chain be connected by a wire, and the alternate joints slightly heated, a thermo-electric current will be produced, as will be explained hereafter. The amount of current increases with the number of elements, and with the difference of temperatures of the opposite junctions.

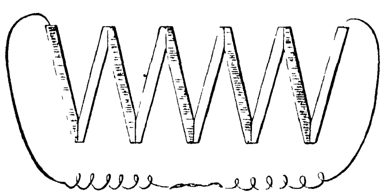


Fig 119 —Nobili's Thermo-electric Series

In the pile as improved by Melloni, the elements are arranged side by side so as to form a square bundle (Fig. 120), whose opposite ends consist of the alternate junctions. The whole is contained in a copper case, with covers at the two ends, which can be removed when it is desired to expose the faces of the pile to the action of heat. Two metallic rods connect the terminals of the thermo-electric series

with wires leading to a galvanometer,[1] so that the existence of any current will immediately be indicated by the deflection of the needle. The amounts of current which correspond to different deflections are known from a table compiled by a method which we shall explain hereafter. Consequently, when a beam of radiant heat strikes the pile, an electric current is produced, and the amount of this current

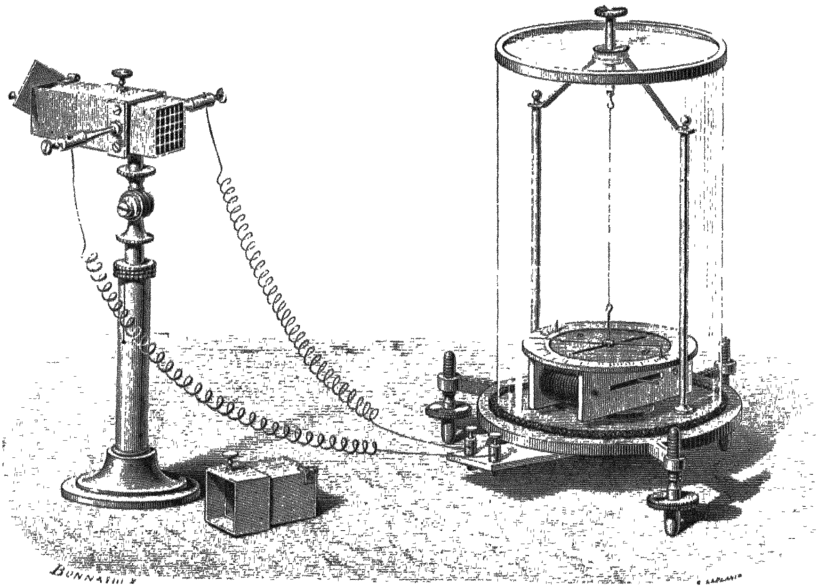


Fig. 120. Melloni's Thermo-multiplier.

is given by the galvanometer. We shall see hereafter, when we come to treat of thermo-electric currents, that within certain limits, which are never exceeded in investigations upon radiant heat, the current is proportional to the difference of temperature between the two ends of the pile. As soon as all parts of the pile have acquired their permanent temperatures, the quantity of heat received during any interval of time from the source of heat will be equal to that lost to the air and surrounding objects. But this latter is, by Newton's law, proportional to the excess of temperature above the surrounding air, and therefore to the difference of temperature between the two ends of the pile. The current is therefore proportional to the quantity of heat received by the instrument. We have thus in Nobili's pile a thermometer of great delicacy, and admirably adapted

[1] The pile and galvanometer together constitute the thermo-multiplier.

to the study of radiant heat; in fact, the immense progress which has been made in this department of physics is mainly owing to this invention of Nobili.

198. Measurement of Emissive Powers.—The following arrangement was adopted by Melloni for the comparison of emissive powers. A graduated horizontal bar (Fig. 121) carries a cube, the different sides of which are covered with different substances. This is filled with water, which is maintained at the boiling-point by means of a spirit-lamp placed beneath. The pile is placed at a convenient distance,

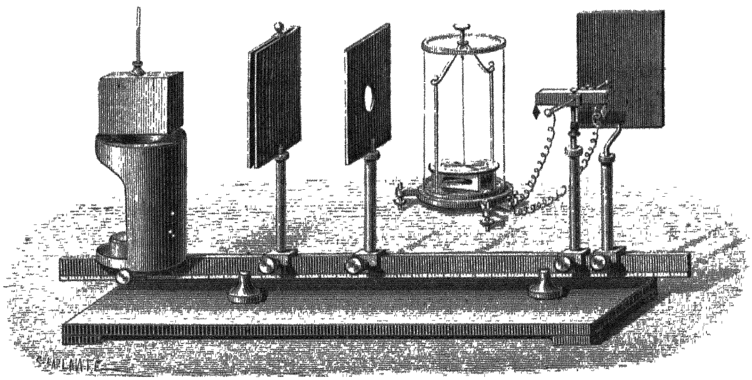


Fig. 121.—Measurement of Emissive Powers.

and the radiation can be intercepted at pleasure by screens arranged for the purpose. The whole forms what is called Melloni's apparatus.

If we now subject the pile to the heat radiated from each of the faces in turn, we shall obtain currents proportional to the emissive powers of the substances with which the different faces are coated.

From a number of experiments of this kind it has been found that lamp-black has the greatest radiating power of all known substances, while the metals are the worst radiators. Some of the most important results are given in the following table, in which the emissive powers of the several substances are compared with that of lamp-black, which is denoted by 100:—

RELATIVE EMISSIVE POWERS AT 100° C.

Lamp-black,	100	Steel,	17
White-lead,	100	Platinum,	17
Paper,	98	Polished brass,	7
Glass,	90	Copper,	7
Indian ink,	85	Polished gold,	3
Shellac,	72	Polished silver,	3

By modified arrangements of the same apparatus, he measured the absorbing, reflecting, diffusing, and transmitting powers of different substances, and established the fact that all those powers vary according to the source from which the incident radiation is derived.

For example when the source was a hot-water cube, lamp-black and white-lead showed equal absorptions; but when it was the flame of an oil lamp, the absorbing power of white-lead was only 53 per cent of that of lamp-black.

The upshot of the matter is that radiant heat exhibits differences precisely analogous to the different colours of light—a fact which Melloni expressed by the name *thermochrose* or heat-colour. A hot-water cube or other non-luminous source emits only long waves; and as the temperature of the source rises, shorter waves are added, till some of the waves are so short as to come within the limits of visibility, and the body is then said to be *incandescent*. Waves of radiant heat, when short enough to produce the sensation of vision, are called waves of light, and there is no difference between radiant heat and light except a difference of wave-length.

199. Diathermancy. — The fact that radiant heat from non-luminous sources could be transmitted through certain transparent substances was established by Pictet of Geneva; and Prévost confirmed the fact by showing that such transmission could occur even through a sheet of ice. Substances which transmit radiant heat are called *diathermanous*. Diathermancy is merely transparency to rays of long wave-length.

Rock-salt is noted for its diathermancy. Even when dirty-looking, it is much more transparent to dark rays than the clearest glass. Alum, on the other hand, is noted for its opacity to dark rays, and quartz (rock crystal) for its transparency to ultra-violet rays.

Tyndall has shown that a solution of iodine in bisulphide of carbon, though excessively opaque to light, allows heat to pass in large quantity. He raised platinum foil to incandescence by placing it in the focus of the mirror of an electric lamp whose light was stopped by interposing a rock-salt trough containing this solution. To this transformation of dark radiant heat into light he gave the name of *calorescence*.

200. Selective Emission and Absorption.—In order to connect together the various phenomena which may be classed under the general title of selective radiation and absorption, it is necessary to

form some such hypothesis as the following. The atoms or molecules of which any particular substance is composed, must be supposed to be capable of vibrating freely in certain periods, which, in the case of gases, are sharply defined, so that a gas is like a musical string, which will vibrate in unison with certain definite notes and with no intermediate ones. The particles of a solid or liquid, on the other hand, are capable of executing vibrations of any period lying between certain limits, so that they may perhaps be compared to the body of a violin, or to the sounding-board of a piano, and these limits (or at all events the upper limit) alter with the temperature, so as to include shorter periods of vibration as the temperature rises.

These vibrations of the particles of a body are capable of being excited by vibrations of like period in the external ether, in which case the body absorbs radiant heat. But they may also be excited by the internal heat of the body, for whenever the molecules experience violent shocks, which excite tremors in them, these are the vibrations which they tend to assume In this case the particles of the body excite vibrations of like period in the surrounding ether, and the body is said to emit radiant heat.

One consequence of these principles is that a diathermanous body is particularly opaque to its own radiation. Rock-salt transmits 92 per cent of the radiation from most sources of heat, but if the source of heat be another piece of rock-salt, especially if it be a thin plate, the amount transmitted is much less, a considerable proportion being absorbed. The heat emitted and absorbed by rock-salt is of exceedingly low refrangibility.

Glass largely absorbs heat of long period, such as is emitted by bodies whose temperatures are not sufficiently high to render them luminous, but allows rays of shorter period, such as compose the luminous portion of the radiation from a lamp-flame, to pass almost entire. Accordingly glass when heated emits a copious radiation of non-luminous heat, but comparatively little light.

Experiment shows that if various bodies, whether opaque or transparent, colourless or coloured, are heated to incandescence in the interior of a furnace, or of an ordinary coal-fire, they will all, while in the furnace, exhibit the same tint, namely the tint of the glowing coals. In the case of coloured transparent bodies, this implies that the rays which their colour prevents them from transmitting from the coals behind them are radiated by the bodies themselves most

copiously; for example, a glass coloured red by oxide of copper permits only red rays to pass through it, absorbing all the rest, but it does not show its colour in the furnace, because its own heat causes it to radiate just those rays which it has the power of absorbing, so that the total radiation which it sends to the eye of a spectator, consisting partly of the radiation due to its own heat, and partly of rays which it transmits from the glowing fuel behind it, is exactly the same in kind and amount as that which comes direct from the other parts of the fire. This explanation is verified by the fact that such glass, if heated to a high temperature in a dark room, glows with a green light.

A plate of tourmaline cut parallel to the axis has the property of breaking up the rays of heat and light which fall upon it into two equal parts, which exhibit opposite properties as regards polarization. One of these portions is very largely absorbed, while the other is transmitted almost entire. When such a plate is heated to incandescence, it is found to radiate just that description of heat and light which it previously absorbed; and if it is heated in a furnace, no traces of polarization can be detected in the light which comes from it, because the transmitted and emitted light exactly complement each other, and thus compose ordinary or unpolarized light.

Spectrum analysis as applied to gases furnishes perhaps still more striking illustrations of the equality of selective radiation and absorption. The radiation from a flame coloured by vapour of sodium—for example, the flame of a spirit-lamp with common salt sprinkled on the wick—consists mainly of vibrations of a definite period, corresponding to a particular shade of yellow. When vapour of sodium is interposed between the eye and a bright light yielding a continuous spectrum, it stops that portion of the light which corresponds to this particular period, and thus produces a dark line in the yellow portion of the spectrum.

An immense number of dark lines exist in the spectrum of the sun's light, and no doubt is now entertained that they indicate the presence, in the outer and less luminous portion of the sun's atmosphere, of gaseous substances which vibrate in periods corresponding to the position of these lines in the spectrum.

201. Our knowledge of solar radiation has been greatly extended in recent years by the researches of Professor Langley of the Smithsonian Institution, conducted by means of an instrument of his own invention called the *bolometer*, which is more sensitive than a

thermopile. The instrument contains an exceedingly fine platinum wire, which is placed successively in different portions of the spectrum; and any change in the temperature of this wire, however slight, is immediately revealed by the deflection of a galvanometer, the wire being, in fact, one of the two arms of a "Wheatstone's Bridge" (§ 209, Part III.).

In order to avoid the absorption of some of the sun's rays which necessarily occurs in transmission through lenses and prisms, he availed himself of the concave "gratings" recently invented by Rowland (§ 271, Part IV.), which produce a spectrum without the aid of a lens.

He has thus been able to trace the ultra-red portion of the solar spectrum much further than it was ever traced before. The wave-length of the extreme violet, in terms of the unit generally employed, being about 3900, and that of the extreme red about 7600, he has traced the ultra-red as far as wave-length 28,000.

By comparing observations made at different heights, some of them being at an elevation of 13,000 feet, on Mount Whitney in Southern California, he has shown that the atmosphere is more transparent to these ultra-red rays than to any others, and that all through the spectrum the absorption is in some inverse ratio to the wave-length. The notion, which has been entertained by many competent authorities, that the atmosphere acts like the glass of a green-house and keeps the earth warm by its opacity to long waves, must, therefore, be discarded, at all events for such climates as those of Pennsylvania and Southern California.

CHAPTER XV.

THERMO-DYNAMICS.

o **202. Connection between Heat and Work.**—That heat can be made to produce work is evident when we consider that the work done by steam-engines and other heat-engines is due to this source.

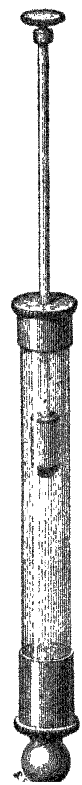


Fig 122.
Fire-syringe.

Conversely, by means of work we can produce heat. Fig. 122 represents an apparatus called the fire-syringe or pneumatic tinder-box, consisting of a piston working tightly in a glass barrel. If a piece of cotton wool moistened with bisulphide of carbon be fixed in the cavity of the piston, and the air be then suddenly compressed, so much heat will be developed as to produce a visible flash of light.

A singular explanation of this effect was at one time put forward. It was maintained that heat or *caloric* was a kind of imponderable fluid, which, when introduced into a body, produced at once an increase of volume and an elevation of temperature. If, then, the body was compressed, the caloric which had served to dilate it was, so to speak, *squeezed out*,[1] and hence the development of heat. An immediate consequence of this theory is that heat cannot be increased or diminished in quantity, but that any addition to the quantity of heat in one part of a system must be compensated by a corresponding loss in another part. But we know that there are cases in which heat is produced by two bodies in contact, without our being able to observe any traces of this compensating process. An instance of this is the production of heat by friction.

[1] In other words, the thermal capacity of the body was supposed to be diminished, so that the amount of heat contained in it, without undergoing any increase, was able to raise it to a higher temperature.

203. Heat produced by Friction.—Friction is a well-known source of heat. Savages are said to obtain fire by rubbing two pieces of dry wood together. The friction between the wheel and axle in railway-carriages frequently produces the same effect, when they have been insufficiently greased; and the stoppage of a train by applying a brake to the wheels usually produces a shower of sparks.

The production of heat by friction may be readily exemplified by the following experiment, due to Tyndall. A glass tube containing water (Fig. 123) and closed by a cork, can be rotated rapidly about

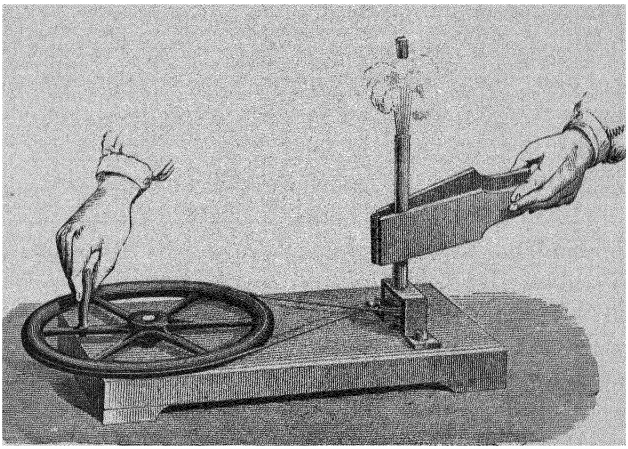


Fig. 123.—Heat produced by Friction.

its axis. While thus rotating, it is pressed by two pieces of wood, covered with leather. The water is gradually warmed, and finally enters into ebullition, when the cork is driven out, followed by a jet of steam. Friction, then, may produce an intense heating of the bodies rubbed together, without any corresponding loss of heat elsewhere.

At the close of last century, Count Rumford (an American in the service of the Bavarian government) called attention to the enormous amount of heat generated in the boring of cannon, and found, in a special experiment, that a cylinder of gun-metal was raised from the temperature of 60° F. to that of 130° F. by the friction of a blunt steel borer, during the abrasion of a weight of metal equal to about $\frac{1}{950}$ of the whole mass of the cylinder. In another experiment, he surrounded the gun by water (which was prevented from entering the

bore), and, by continuing the operation of boring for $2\frac{1}{2}$ hours, he made this water boil. In reasoning from these experiments, he strenuously maintained that heat cannot be a material substance, but must consist in motion.

The advocates of the caloric theory endeavoured to account for these effects by asserting that caloric, which was latent in the metal when united in one solid mass, had been forced out and rendered sensible by the process of disintegration under heavy pressure. This supposition was entirely gratuitous, no difference having ever been detected between the thermal properties of entire and of comminuted metal; and, to account for the observed effect, the latent heat thus supposed to be rendered sensible in the abrasion of a given weight of metal, must be sufficient to raise 950×70, that is 66,500 times its own weight of metal through 1°.

Yet, strange to say, the caloric theory survived this exposure of its weakness, and the, if possible, still more conclusive experiment of Sir Humphry Davy, who showed that two pieces of ice, when rubbed together, were converted into water, a change which involves not the evolution but the absorption of latent heat, and which cannot be explained by diminution of thermal capacity, since the specific heat of water is much greater than that of ice.

Davy, like Rumford, maintained that heat consisted in motion, and the same view was maintained by Dr. Thos. Young; but the doctrine of caloric nevertheless continued to be generally adopted until about the year 1840, since which time, the experiments of Joule, the eloquent advocacy of Mayer, and the mathematical deductions of Thomson, Rankine, and Clausius, have completely established the mechanical theory of heat, and built up an accurate science of thermodynamics.

204. Foucault's Experiment.—The relations existing between electrical and thermal phenomena had considerable influence in leading to correct views regarding the nature of heat. An experiment devised by Foucault illustrates these relations, and at the same time furnishes a fresh example of the production of heat by the performance of mechanical work.

The apparatus consists (Fig. 124) of a copper disc which can be made to rotate with great rapidity by means of a system of toothed wheels. The motion is so free that a very slight force is sufficient to maintain it. The disc rotates between two pieces of iron, constituting the armatures of one of those temporary magnets which are obtained

by the passage of an electric current (called electro-magnets). If, while the disc is turning, the current is made to pass, the armatures become strongly magnetized, and a peculiar action takes place between them and the disc, consisting in the formation of induced currents in the latter, accompanied by a resistance to motion. As

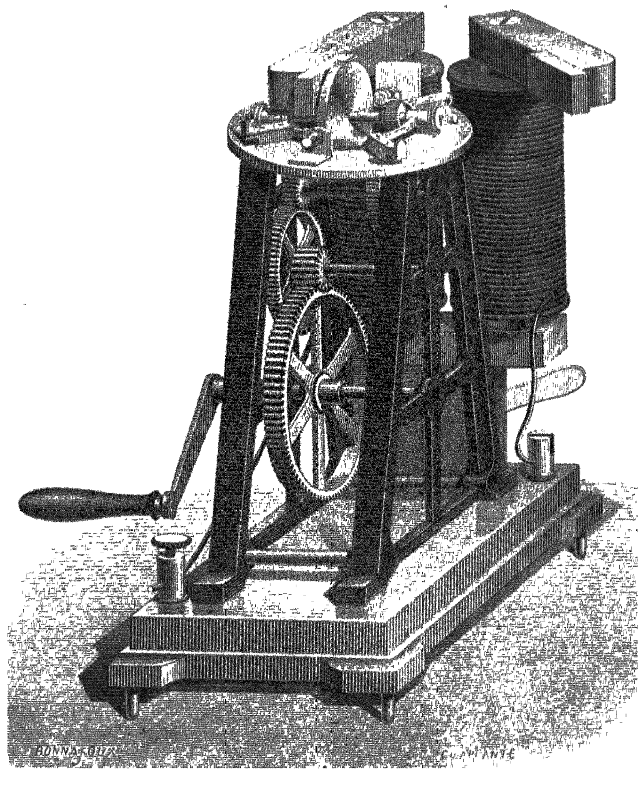


Fig 124—Foucault's Apparatus

long as the magnetization is continued, a considerable effort is necessary to maintain the rotation of the disc; and if the rotation be continued for two or three minutes, the disc will be found to have risen some 50° or 60° C. in temperature, the heat thus acquired by the disc being the equivalent of the work done in maintaining the motion. It is to be understood that, in this experiment, the rotating disc does not touch the armatures; the resistance which it experiences is due entirely to invisible agencies.

The experiment may be varied by setting the disc in very rapid rotation, while no current is passing, then leaving it to itself, and immediately afterwards causing the current to pass. The result will be, that the disc will be brought to rest almost instantaneously, and will undergo a very slight elevation of temperature, the heat gained being the equivalent of the motion which is destroyed.

205. Mechanical Equivalent of Heat.—The first precise determination of the numerical relation subsisting between heat and mechanical work was obtained by the following experiment of Joule. He constructed an agitator which is somewhat imperfectly represented in Fig. 125, consisting of a vertical shaft carrying several sets of paddles revolving between stationary vanes, these latter serving to

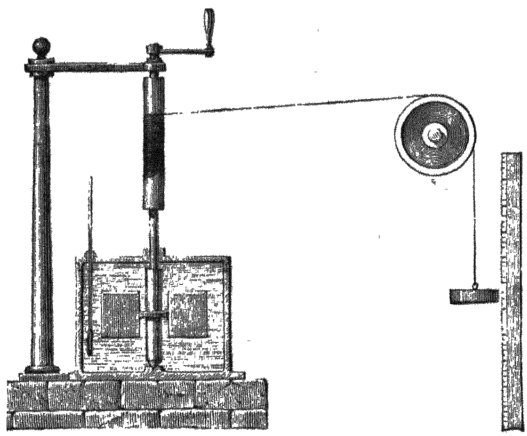


Fig. 125.—Determination of the Mechanical Equivalent of Heat.

prevent the liquid in the vessel from being bodily whirled in the direction of rotation. The vessel was filled with water, and the agitator was made to revolve by means of a cord, wound round the upper part of the shaft, carried over a pulley, and attached to a weight, which by its descent drove the agitator, and furnished a measure of the work done. The pulley was mounted on friction-wheels, and the weight could be wound up without moving the paddles. When all corrections had been applied, it was found that the heat communicated to the water by the agitation amounted to one pound-degree Fahrenheit for every 772 foot-pounds of work spent in producing it. This result was verified by various other forms of experiment, and is certainly very near the truth.

For an elevation of 1° Centigrade, the corresponding number will be $772 \times 9/5$, that is, 1390.

For an elevation of 1° C. in a kilogramme of water, the number of kilogrammetres of work required will be the number of metres in 1390 feet, that is, 424. Any one of these numbers is called a value of *Joule's equivalent*, and is usually denoted by the symbol J. It is sometimes called the *mechanical equivalent of heat*, or more fully, the *mechanical equivalent of the unit of heat.*

206. Rowland's Determination.—Among subsequent verifications of Joule's result, the experiments of Professor Rowland[1] at Baltimore are specially important. He drove, by means of a steam-engine, a revolving stirrer, having numerous blades pierced with holes and passing between similar blades fixed to the water-vessel. The revolutions were counted by means of an endless screw on the shaft of the stirrer; and the mutual couple between the stirrer and the vessel was determined by measuring the counterbalancing couple which prevented the vessel from turning; this counterbalancing couple consisting of (1) a pair of equal weights acting (by means of pulleys) on the circumference of a horizontal wheel and composing a couple of any constant magnitude, (2) the torsion of a suspending wire, which, by increasing or diminishing with the couple to be equilibrated, made the arrangement stable. The average elevation of temperature in each trial was about 20°, whereas in Joule's classical experiment it was only a fraction of a degree.

Rowland's result, for water at 10° C., was 428·5, when expressed in kilogrammetres at Baltimore, which is equivalent to 428·0 kilogrammetres at Manchester, where Joule's determination was made. Eliminating the local element of gravitation by reducing to absolute measure, his result for water at 10° C. was *42 millions of ergs for 1° C. of change of temperature in a gramme of water*, temperature being reckoned on the absolute thermo-dynamic scale which will be explained further on in this chapter. At 5° C. the number (instead of 42) was 42·12, at 15° C., 41·89; at 20° C., 41·79; at 30° C., 41·71; and at 35° C., 41·73, showing that the specific heat of water passes through a minimum at about 30° C.

42 million ergs may conveniently be adopted as the value of Joule's equivalent in the C.G.S. system.

207. First Law of Thermo-dynamics.—Whenever work is per-

[1] Rowland on *the Mechanical Equivalent of Heat.* Cambridge (U.S.) University Press, 1880.

formed by the agency of heat, an amount of heat disappears equivalent to the work performed; and whenever mechanical work is spent in generating heat, the heat generated is equivalent to the work thus spent; that is to say, we have in both cases

$$W = JH;$$

W denoting the work, H the heat, and J Joule's equivalent. This is called the *first law of thermo-dynamics*, and it is a particular case of the great natural law which asserts that energy may be transmuted, but is never created or destroyed.

It may be well to remark here that work is not energy, but is rather the process by which energy is transmuted, amount of work being measured by the amount of energy transmuted. Whenever work is done, it leaves an effect behind it in the shape of energy of some kind or other, equal in amount to the energy consumed in performing the work, or, in other words, equal to the work itself.

As regards the nature of heat, there can be little doubt that heat properly so called, that is, sensible as distinguished from latent heat, consists in some kind of motion, and that quantity of heat is quantity of energy of motion, or kinetic energy, whereas latent heat consists in energy of position or potential energy.

We have already had in the experiments of Rumford, Davy, Foucault, and Joule, some examples of transmutation of energy, but it will be instructive to consider some additional instances.

When a steam-engine is employed in hauling up coals from a pit, an amount of heat is destroyed in the engine equivalent to the energy of position which is gained by the coal.

In the propulsion of a steam-boat with uniform velocity, or in the drawing of a railway train with uniform velocity on a level, there is no gain of potential energy, neither is there, as far as the vessel or train is concerned, any gain of kinetic energy. In the case of the steamer, the immediate effect consists chiefly in the agitation of the water, which involves the generation of kinetic energy; and the ultimate effect of this is a warming of the water, as in Joule's experiment. In the case of the train, the work done in maintaining the motion is spent in friction and concussions, both of which operations give heat as the ultimate effect. Here, then, we have two instances in which heat, after going through various transformations, reappears as heat at a lower temperature.

In starting a train on a level the heat destroyed in the engine

finds its equivalent mainly in the energy of motion gained by the train; and this energy can again be transformed into heat by turning off the steam and applying brakes to the wheels.

When a cannon-ball is fired against an armour plate, it is heated red-hot if it fails to penetrate the plate, the energy of the moving ball being in this case obviously converted into heat. If the plate is penetrated, and the ball lodges in the wooden backing, or in a bank of earth, the ball will not be so much heated, although the total amount of heat generated must still be equivalent to the energy of motion destroyed. The ruptured materials, in fact, receive a large portion of the heat. The heat produced in the rupture of iron is well illustrated by punching and planing machines, the pieces of iron punched out of a plate, or the shavings planed off it, being so hot that they can scarcely be touched, although the movements of the punch and plane are exceedingly slow. The heat gained by the iron is, in fact, the equivalent of the work performed, and this work is considerable on account of the great force required.

208. Heat of Compression and Cold of Expansion.—The heating of a gas by compression or its cooling by expansion is nearly the same in amount as if a quantity of heat equivalent to the work of compression or expansion were given to or taken from the gas at constant volume. This approximate equality was established by an experiment of Joule's. He immersed two equal vessels in water, one of them containing highly-compressed air, and the other being vacuous; and when they were both at the temperature of the water he opened a stop-cock which placed the vessels in communication. The compressed air thus expanded to double its volume, but no change could be detected in the temperature of the surrounding water. The work of expansion produced its equivalent, first in kinetic energy *plus* friction, and finally in heat, and this heat sensibly compensated the cooling effect of the expansion.

Subsequent experiments by Thomson and Joule showed that the cooling effect slightly predominates; hence, conversely, the heating effect of compression slightly exceeds the equivalent of the work done in compressing the gas. The excess of the cooling effect amounted to ·26 of a degree Centigrade in the case of air, when the difference between the initial and final pressures was 1 atmosphere, and to ·26n when the difference was n atmospheres.

The mode of experimenting consisted in steadily forcing air through a plug of cotton wool, and comparing the temperatures of

the entering and the issuing air. The friction of the air in passing through the plug generates heat, which in the long run is imparted to the air as it flows through; and this warming effect is combined with the cooling effect of expansion. The cooling effect preponderated, not only in the case of air but in the case of every gas that was tried except hydrogen, which showed a slight rise of temperature. For carbonic acid at about 10° C. the fall of temperature was about $4\frac{1}{2}$ times as great as for air.

209. Work in Expansion.—The work done by a gas in expanding against uniform hydrostatic or pneumatic pressure may be computed by *multiplying the increase of volume by the pressure per unit area.* For, if we suppose the expanding body to be immersed in an incompressible fluid without weight, confined in a cylinder by means of a movable piston under constant pressure, the work done by the expanding body will be spent in driving back the piston. Let A be the area of the piston, x the distance it is pushed back, and p the pressure per unit area. Then the increment of volume is Ax, and the work done is the product of the force pA by the distance x, which is the same as the product of p by Ax.

210. Difference of the two Specific Heats.—Let a gramme of air, occupying a volume V cub. cm. at the absolute temperature T°, be raised at the constant pressure of P grammes per sq. cm. to the temperature T + 1°. It will expand by the amount $\frac{V}{T}$, and will do work to the amount $\frac{VP}{T}$ in pushing back the surrounding resistances. Now the value of $\frac{VP}{T}$ is (§ 50) the same for all pressures and temperatures. But at 0° C. and 760 mm. we have $T=273$, $P=1033$, and since the volume of 1·293 grammes is 1 litre or 1000 cub. cm., we have

$$V=\frac{1000}{1\cdot 293},$$

and

$$\frac{VP}{T}=\frac{1000}{1\cdot 293}\times\frac{1033}{273}=2926 \text{ gramme-centimetres.}$$

This is the work done in the expansion of 1 gramme of air at any constant pressure when raised 1° C. in temperature, and its thermal equivalent

$$\frac{2926}{42400}=\cdot 0690$$

is the excess of the specific heat at constant pressure above the specific heat at constant volume.

Since the difference of the two thermal capacities of volume V is VP/T, the *difference of the two thermal capacities of unit volume* is P/T and *is the same for all gases at the same pressure and temperature.* We neglect here the small departures of actual gases from the simple theoretical laws.

Assuming Regnault's value of the specific heat of air at constant pressure, ·2375, the specific heat at constant volume will be

$$\cdot 2375 - \cdot 0690 = \cdot 1685.$$

The heat required to produce a given change of temperature in a gas, when its volume changes in any specified way, may be computed to a very close approximation by calculating the work done by the gas against external resistances during its change of volume, and adding the heat-equivalent of this work to the heat which would have produced the same change of temperature at constant volume.

The above calculation of the difference of the two specific heats rests upon the previously known value of Joule's equivalent. Conversely, from the work done in the expansion of air at constant pressure, combined with the ratio of the two specific heats and the observed value of one of them, the value of Joule's equivalent can be computed. A calculation of this kind, but with an erroneous value of the specific heat of air, was made by Mayer, before Joule's equivalent had been determined.

211. Thermic Engines.—In every form of thermic engine, work is obtained by means of expansion produced by heat, the force of expansion being usually applied by admitting a hot elastic fluid to press alternately on opposite sides of a piston travelling in a cylinder. Of the heat received by the elastic fluid from the furnace, a part leaks out by conduction through the sides of the containing vessels, another part is carried out by the fluid when it escapes into the air or into the condenser, the fluid thus escaping being always at a temperature lower than that at which it entered the cylinder, but higher than that of the air or condenser into which it escapes; but a third part has disappeared altogether, and ceased to exist as heat, having been spent in the performance of work. This third part is the exact equivalent of the work performed by the elastic fluid in driving the piston,[1] and may therefore be called the *heat utilized,* or the *heat converted.*

[1] If negative work is done by the fluid in any part of the stroke (that is, if the piston presses back the fluid), the algebraic sum of work is to be taken.

The *efficiency of an engine* may be measured by *the ratio of the heat thus converted to the whole amount of heat which enters the engine;* and we shall use the word *efficiency* in this sense.

212. **Carnot's Investigations.**—The first approach to an exact science of thermo-dynamics was made by Carnot in 1824. By reasoning based on the theory which regards heat as a substance, but which can be modified so as to remain conclusive when heat is regarded as a form of energy, he established the following principles:—

I. *The thermal agency by which mechanical effect may be obtained is the transference of heat from one body to another at a lower temperature.* These two bodies he calls the *source* and the *refrigerator.* Adopting the view generally received at that time regarding the nature of heat, he supposed that all the heat received by an engine was given out by it again as heat; so that, if all lateral escape was prevented, all the heat drawn by the engine from the source was given by the engine to the refrigerator, just as the water which by its descent turns a mill-wheel, runs off in undiminished quantity at a lower level. We now know that, when heat is let down through an engine from a higher to a lower temperature, it is diminished in amount by the equivalent of the work done by the engine against external resistances.

He further shows that the amount of work which can be obtained by letting down a given quantity of heat—or, as we should say with our present knowledge, by partly letting it down and partly consuming it in work, is increased by raising the temperature of the source, or by lowering the temperature of the refrigerator; and establishes the following important principle:—

II. *A perfect thermo-dynamic engine is such that, whatever amount of mechanical effect it can derive from a certain thermal agency; if an equal amount be spent in working it backwards, an equal reverse thermal effect will be produced.* This is often expressed by saying that a *completely reversible engine* is a *perfect engine.*

By a *perfect engine* is here meant an engine which possesses the maximum of efficiency compatible with the given temperatures of its source and refrigerator; and Carnot here asserts that all completely reversible engines attain this maximum of efficiency. The proof of this important principle, when adapted to the present state of our knowledge, is as follows:—

Let there be two thermo-dynamic engines, A and B, working

between the same source and refrigerator; and let A be completely reversible.—Let the efficiency of A be m, so that, of the quantity Q of heat which it draws from the source, it converts $m\,Q$ into mechanical effect, and gives $Q-m\,Q$ to the refrigerator, when worked forwards. Accordingly, when worked backwards, with the help of work $m\,Q$ applied to it from without, it takes $Q-m\,Q$ from the refrigerator, and gives Q to the source.

In like manner, let the efficiency of B be m', so that, of heat Q' which it draws from the source, it converts $m'Q'$ into mechanical effect, and gives $Q'-m'Q'$ to the refrigerator.

Let this engine be worked forwards, and A backwards. Then, upon the whole, heat to the amount $Q'-Q$ is drawn from the source, heat $m'Q'-m\,Q$ is converted into mechanical effect, and heat $Q'-Q-(m'Q'-m\,Q)$ is given to the refrigerator

If we make $m'Q'=m\,Q$, that is, if we suppose the external effect to be nothing, heat to the amount $Q'-Q$ or $\left(\frac{m}{m'}-1\right)Q$ is carried from the source to the refrigerator, if m be greater than m', that is, if the reversed engine be the more efficient of the two. If the other engine be the more efficient, heat to the amount $\left(1-\frac{m}{m'}\right)Q$ is transferred from the refrigerator to the source, or heat pumps itself up from a colder to a warmer body, and *that* by means of a machine which is self-acting, for B does work which is just sufficient to drive A. Such a result we are entitled to assume impossible, therefore B cannot be more efficient than A.

Another proof is obtained by making $Q'=Q$. The source then neither gains nor loses heat, and the refrigerator gains $(m-m')\,Q$, which is derived from work performed upon the combined engine from without, if A be more efficient than B. If B were the more efficient of the two, the refrigerator would lose heat to the amount $(m'-m)\,Q$, which would yield its full equivalent of external work, and thus a machine would be kept going and doing external work by means of heat drawn from the coldest body in its neighbourhood, a result which cannot be admitted to be possible.

213. Examples of Reversibility.—The following may be mentioned as examples of reversible operations.

When a gas expands at constant temperature, it must be supplied from without with a definite amount of heat; and when it returns, at the same temperature, to its original volume, it gives out the same amount of heat.

When a gas expands adiabatically (that is to say, without interchange of heat with other bodies), it falls in temperature; and when it is compressed adiabatically from the condition thus attained to its original volume, it regains its original temperature.

When water at 0° freezes, forming ice at 0°, under atmospheric pressure, it expands and does external work in pushing back the atmosphere. It also gives out a definite quantity of heat called the latent heat of liquefaction. This ice can be melted at the same pressure and temperature, and in this reverse process it must be supplied with heat equal to that which it formerly gave out. Also, since the shrinkage will be equal to the former expansion, the pressure of the surrounding atmosphere will do work equal to that formerly done against it.

On the other hand, conduction and radiation of heat are essentially irreversible, since in these operations heat always passes from the warmer to the colder body, and refuses to pass in the opposite direction.

214. **Second Law of Thermo-dynamics.**—It follows, from the principle thus established, that all *reversible engines* with the same temperatures of source and refrigerator have the same efficiency, whether the working substance employed in them be steam, air, or any other material, gaseous, liquid, or solid. Hence we can lay down the following law, which is called the second law of thermo-dynamics: *the efficiency of a completely reversible engine is independent of the nature of the working substance, and depends only on the temperatures at which the engine takes in and gives out heat; and the efficiency of such an engine is the limit of possible efficiency for any engine.*

As appendices to this law it has been further established:

1. That when one of the two temperatures is fixed, the efficiency is simply proportional to the difference between the two, provided this difference is very small. This holds good for all scales of temperature.

2. That the efficiency of a reversible engine is approximately $\frac{T-T'}{T}$, T denoting the upper and T′ the lower temperature between which the engine works, reckoned from absolute zero (§ 50), on the air-thermometer. This is more easily remembered when stated in the following more symmetrical form. Let Q denote the quantity of heat taken in at the absolute temperature T, Q′ the quantity given out at the absolute temperature T′, and consequently $Q - Q'$

the heat converted into mechanical effect, then we shall have approximately

$$\frac{Q}{T}=\frac{Q'}{T'}=\frac{Q-Q'}{T-T'}.$$

215. Proof of Formula for Efficiency.—This important proposition may be established as follows:—

Let the volume and pressure of a given portion of gas be represented by the rectangular co-ordinates of a movable point, which we will call "the indicating point," horizontal distance representing volume, and vertical distance pressure.

When the temperature is constant, the curve which is the locus of the indicating point is called an *isothermal*, and the relation between the co-ordinates is

$$vp = C\,,$$

where C is a constant, depending upon the given temperature, and in fact proportional to the absolute temperature by air-thermometer.

When the changes of volume and pressure are adiabatic (§ 219), a given change of volume will produce a greater change of pressure than when they are isothermal, and the curve traced by the indicating point is called an *adiabatic line*. Whenever the given gas gains or loses heat by interchange with surrounding bodies, the indicating point will be carried from one adiabatic line to another; and by successive additions or subtractions of small quantities of heat we can get any number of adiabatic lines as near together as we please. By drawing a number of adiabatic lines near together, and a number of isothermals near together, we shall cut up our diagram into a number of small quadrilaterals which will be ultimately parallelograms.

Let A B C D (Fig. 126) be one of these parallelograms, and let the gas be put through the series of changes represented by A B, B C, C D, D A, all of which, it will be observed, are reversible.

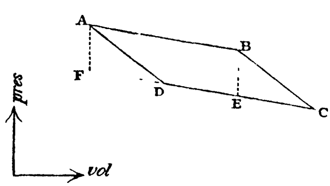


Fig. 126.

In A B the gas expands at constant temperature. Let this temperature, expressed on the absolute scale of the air or gas thermometer, be T, let the small increase of volume be v, and the mean pressure P, so that the work done against external resistances is Pv.

In B C the gas expands adiabatically and falls in temperature. Let the fall of temperature be τ.

In C D it is compressed at the constant temperature $T-\tau$.

In D A it is compressed adiabatically, and ends by being in the same state in which it was at the commencement of this cycle of four operations.

Since the external work done by a gas is equal to the algebraic sum of such terms as

$$\text{pressure} \times \text{increase of volume},$$

it is easily shown that the algebraic sum of external work done by the gas in the above cycle is represented by the area of the parallelogram A B C D.

Through A and B draw verticals A F, B E, which, by construction, represent diminution of pressure at constant volume; and produce C D to meet A F in F. Then the area A B C D is equal to the area A B E F (since the parallelograms are on the same base and between the same parallels), that is to A F multiplied by the perpendicular distance between A F and B E. But this perpendicular distance represents v, the increase of volume from A to B; and A F represents the difference (at constant volume) between the pressure at T and the pressure at $T+\tau$. This difference is

$$P\frac{\tau}{T},$$

hence the work done in the cycle is

$$P\frac{\tau}{T}v.$$

But the work done in the operation A B was

$$Pv,$$

and this work, being performed at constant temperature, is known (§ 208) to be sensibly equivalent to the whole heat supplied to the gas in the performance of it. This is the only operation in which heat is received from the source, and C D is the only operation in which heat is given out to the refrigerator. Hence we have

$$\frac{\text{heat converted}}{\text{heat from source}}=\frac{P\frac{\tau}{T}v}{Pv}=\frac{\tau}{T},$$

or, if Q_1 represent the heat received from the source, Q_2 the heat given to the refrigerator, T_1 the temperature of the source, and T_2 the temperature of the refrigerator,

$$\frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1} \text{ therefore } \frac{Q_2}{Q_1} = \frac{T_2}{T_1}.$$

This proves the law for any reversible engine with an indefinitely small difference of temperature between source and refrigerator.

Now, let there be a series of reversible engines, such that the first acts as source to the second, the second as source to the third, and so on; and let the notation be as follows:—

The first receives heat Q_1 at temperature T_1, and gives to the second heat Q_2 at temperature T_2. The second gives to the third heat Q_3 at temperature T_3, and so on.

Then supposing the excess of each of these temperatures above the succeeding one to be very small, we have, from above,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}, \ \frac{Q_2}{Q_3} = \frac{T_2}{T_3}, \ \ldots \ \frac{Q_{n-1}}{Q_n} = \frac{T_{n-1}}{T_n}.$$

Whence, by multiplying equals,

$$\frac{Q_1}{Q_n} = \frac{T_1}{T_n} \text{ therefore } \frac{Q_1 - Q_n}{Q_1} = \frac{T_1 - T_n}{T_1}.$$

The law is therefore proved for the engine formed by thus combining all the separate engines. But this engine is reversible, and therefore (§ 214) the law is true for all reversible engines.

216. **Thomson's Absolute Scale of Temperature.**—In ordinary thermometers, temperatures are measured by the apparent expansion of a liquid in a glass envelope. If two thermometers are constructed, one with mercury and the other with alcohol for its liquid, it is obviously possible to make their indications agree at two fixed temperatures. If, however, the volume of the tube intervening between the two fixed points thus determined be divided into the same number of equal parts in the two instruments, and the divisions be numbered as degrees of temperature, the two instruments will give different indications if plunged in the same bath at an intermediate temperature, and they will also differ at temperatures lying beyond the two fixed points. It is a simple matter to test equality of temperature, but it is far from simple to decide upon a test of equal differences of temperatures. Different liquids expand not only by different amounts but by amounts which are not proportional, no two liquids being in this respect in agreement.

In the case of permanent gases expanding under constant pressure, the discordances are much less, and may, in ordinary circumstances,

be neglected. Hence gases would seem to be indicated by nature as the proper substances by which to measure temperature, if differences of temperature are to be measured by differences of volume.

It is also possible to establish a scale of temperature by assuming that some one substance rises by equal increments of temperature on receiving successive equal additions of heat; in other words, by making some one substance the standard of reference for specific heat, and making its specific heat constant by definition at all temperatures. Here, again, the scale would be different according to the liquid chosen. A mixture of equal weights of water at 0° C. and 100° C. will not have precisely the same temperature as a mixture of equal weights of mercury at these temperatures. If, however, we resort to permanent gases, we find again a very close agreement, so that, if one gas be assumed to have the same specific heat at all temperatures (whether at constant volume or at constant pressure), the specific heat of any other permanent gas will also be sensibly independent of temperature. More than this;—the measurement of temperature by assuming the specific heats of permanent gases to be constant, agrees almost exactly with the measurement of temperature by the expansion of permanent gases. For, as we have seen (§ 69), a permanent gas under constant pressure has its volume increased by equal amounts on receiving successive equal additions of heat.

The air-thermometer, or gas-thermometer, then, has a greatly superior claim to the mercury thermometer to be considered as furnishing a natural standard of temperature.

But a scale which is not only sensibly but absolutely independent of the peculiarities of particular substances, is obtained by *defining temperature in such a sense as to make appendix* (2) *to the second law of thermo-dynamics rigorously exact.* According to this system, which was first proposed by Thomson (Lord Kelvin), the ratio of any two temperatures is the ratio of the two quantities of heat which would be drawn from the source and supplied to the refrigerator by a completely reversible thermo-dynamic engine working between these temperatures. This ratio will be rigorously the same, whatever the working substance in the engine may be, and whether it be solid, liquid, or gaseous.

217. Temperature estimated in this way is called *absolute thermo-dynamic temperature,* and is almost exclusively employed in thermo-dynamic investigations.

It is not difficult to prove (see a later section on Entropy) that when a reversible engine takes in and gives out heat at any number of different temperatures, the sum of all the quotients Q/T is zero, for any series of operations which end by leaving the body in the condition in which it was at first, Q denoting any one of the quantities of heat taken in or given out (reckoned positive if taken in and negative if given out), and T the temperature at which it is taken in or given out. This result, that the sum of the quotients *Q/T vanishes for any cyclic series of operations*, is often quoted as the *general statement of the second law of thermo-dynamics*, and is the most practically useful statement of the law.

218. **Heat required for Change of Volume and Temperature.**—The amount of heat which must be imparted to a body to enable it to pass from one condition, as regards volume and temperature, to another, is not a definite quantity, but depends upon the course by which the transition is effected. It is, in fact, the sum of two quantities, one of them being *the heat which would be required if the transition were made without external work*—as in Joule's experiment of the expansion of compressed air into a vacuous vessel—and the other being *the heat equivalent to the external work which the body performs in making the transition.* As regards the first of these quantities, its amount, in the case of permanent gases, depends almost entirely upon the difference between the initial and final temperatures, being sensibly independent of the change of volume, as Joule's experiment shows. In the case of liquids and solids, its amount depends, to a very large extent, upon the change of volume, so that, if the expansion which heat tends to produce is forcibly prevented, the quantity of heat required to produce a given rise of temperature is greatly diminished. This contrast is sometimes expressed by saying that expansion by heat involves a large amount of internal work in the case of liquids and solids, and an exceedingly small amount in the case of gases; but the phrase *internal work* has not as yet acquired any very precise meaning.

As an illustration of the different courses by which a transition may be effected, suppose a quantity of gas initially at 0° C. and a pressure of one atmosphere, and finally at 100° C. and the same pressure, the final volume being therefore 1·366 times the initial volume. Of the innumerable courses by which the transition may be made, we will specify two:—

1st. The gas may be raised, at its initial volume, to such a tem-

perature that, when afterwards allowed to expand against pressure gradually diminishing to one atmosphere, it falls to the temperature 100° C. Or,

2d. It may be first allowed to expand, under pressure diminishing from one atmosphere downwards, until its final volume is attained, and may then, at this constant volume, be heated up to 100°.

In both cases it is to be understood that no heat is allowed to enter or escape during expansion.

Obviously, the first course implies the performance of a greater amount of external work than the second, and it will require the communication to the gas of a greater quantity of heat,—greater by the heat-equivalent of the difference of works.

When a body passes through changes which end by leaving it in precisely the same condition in which it was at first, we are not entitled to assume that the amounts of heat which have entered and quitted it are equal. They are not equal unless the algebraic sum of external work done by the body during the changes amounts to zero. If the body has upon the whole done positive work, it must have taken in more heat than it has given out, otherwise there would be a creation of energy; and if it has upon the whole done negative work, it must have given out more heat than it has taken in, otherwise there would be a destruction of energy. In either case, *the difference between the heat taken in and given out must be the equivalent of the algebraic sum of external work.*

These principles are illustrated in the following sections.

219. Adiabatic Changes. Heating by Compression, and Cooling by Expansion.—When a gas is compressed in an absolutely non-conducting vessel, or, more generally, when a gas alters its volume without giving heat to other bodies or taking heat from them, its changes are called *adiabatic* [literally, *without passage across*].

Let unit volume of gas, at pressure P and absolute temperature T, receive heat which raises its temperature to $T+\tau$ at constant pressure. The increase of volume will be $\frac{\tau}{T}$, and the work done by the gas against external resistance will be $\frac{P\tau}{T}$.

Next let the gas be compressed to its original volume without entrance or escape of heat, and let the temperature at the end of this second operation be denoted by $T+\kappa\tau$, so that the elevation of temperature produced by the compression is $(\kappa-1)\tau$. The pressure will now be $P\,\frac{T+\kappa\tau}{T}$, as appears by comparing the final condition of the gas

with its original condition at the same volume. This may be written $P\left(1+\frac{\kappa\tau}{T}\right)$, and the mean pressure during the second operation may be taken as half the sum of the initial and final pressures, that is, as $P\left(1+\frac{1}{2}\frac{\kappa\tau}{T}\right)$. The work done *upon* the gas by the external compressing forces in the second operation is therefore

$$P\left(1+\tfrac{1}{2}\frac{\kappa\tau}{T}\right)\frac{\tau}{T};$$

or, to the first order of small quantities, $P\frac{\tau}{T}$, which is the same as the work done *by* the gas in the first operation. Hence, to the first order of small quantities, the heat which has been given to the gas is the same as if the gas had been brought without change of volume from its initial to its final condition. That is to say, the heat which produces an elevation τ of temperature, at constant pressure, would produce an elevation $\kappa\tau$ at constant volume. Hence

$$\frac{\text{Specific heat at constant pressure}}{\text{Specific heat at constant volume}}=\kappa,$$

where κ is defined by the condition that $\kappa-1$ is *the ratio of the elevation of temperature produced by a small adiabatic compression to the elevation of temperature which would be required to produce an equal expansion at constant pressure.*

220. Relations between Adiabatic Changes of Volume, Temperature, and Pressure.—For the sake of greater clearness, we will tabulate the values of volume, temperature, and pressure, at the beginning and end of the adiabatic compression above discussed.

	At beginning	At end.	Change
Volume, ...	$1+\frac{\tau}{T}$	1	$-\frac{\tau}{T}$
Temperature, . .	$T+\tau$	$T+\kappa\tau$	$(\kappa-1)\tau$
Pressure, . .	P	$P\left(1+\frac{\kappa\tau}{T}\right)$	$P\frac{\kappa\tau}{T}$

Denoting volume, temperature, and pressure by V, T, P, and their changes by dV, dT, dP, we have, to the first order of small quantities,

$$\frac{dV}{V}=-\frac{\tau}{T},\qquad \frac{dT}{T}=\frac{(\kappa-1)\tau}{T},\qquad \frac{dP}{P}=\frac{\kappa\tau}{T}.$$

$\frac{dV}{V}, \frac{dT}{T}, \frac{dP}{P}$ are therefore proportional to $-1, \kappa-1, \kappa$,

that is $$\frac{d\log V}{-1}=\frac{d\log T}{\kappa-1}=\frac{d\log P}{\kappa};$$

whence, if V_1, T_1, P_1 are one set of corresponding values, and V_2, T_2, P_2 another set, we have

$$\left(\frac{V_1}{V_2}\right)^{\kappa-1}=\frac{T_2}{T_1},$$

$$\left(\frac{V_1}{V_2}\right)^{\kappa}=\frac{P_2}{P_1}.$$

221. **Numerical Value of κ.**—Since $\frac{dP}{P}$ divided by $-\frac{dV}{V}$ is κ, dP divided by $-\frac{dV}{V}$ (which, by definition, is the coefficient of elasticity of the gas), is equal to $P\kappa$. Now the square of the velocity of sound in a gas can be proved to be equal to the coefficient of elasticity divided by the density, and hence from observations on the velocity of sound the value of κ can be determined. It is thus found that

$$\kappa=1{\cdot}408$$

for perfectly dry air; and its value is very nearly the same for all other gases which are difficult to liquefy.

222. **Rankine's Prediction of the Specific Heat of Air.**—Let S_1 denote the specific heat of air at constant pressure, and S_2 its specific heat at constant volume. Then (§§ 220, 221) we have

$$\frac{s_1}{s_2}=1{\cdot}408.$$

But we have proved (§ 210) by thermo-dynamic considerations, independent of any direct observation of specific heat, that

$$s_1-s_2={\cdot}0690.$$

From these two equations we have

$$s_2(1{\cdot}408-1)={\cdot}0690$$

$$s_2=\frac{{\cdot}069}{{\cdot}408}={\cdot}169$$

$$s_1={\cdot}169+{\cdot}069={\cdot}238.$$

In this way the correct values of the two specific heats of air were calculated by Rankine, before any accurate determinations of them had been made by direct experiment.

223. **Cooling of Air by Ascent. Convective Equilibrium.**—When a body of air ascends in the atmosphere it expands, in consequence of being relieved of a portion of its pressure, and the foregoing principles enable us to calculate the corresponding fall produced in its temperature. For we have

$$-\frac{dT}{T}=-\frac{\kappa-1}{\kappa}\frac{dP}{P}.$$

But if x denote height above a fixed level, and H "pressure height" or "height of homogeneous atmosphere," we have (see Part I.)

$$-\frac{dP}{P}=\frac{dx}{H};$$

also H is proportional to T, so that if H_0, T_0, denote the values of H, T at the freezing-point, we have $H=H_0\frac{T}{T_0}$. Thus we have

$$-\frac{dT}{T}=\frac{\kappa-1}{\kappa}\frac{dx}{H_0}\frac{T_0}{T}, \text{ or } -\frac{dT}{dx}=\frac{\kappa-1}{\kappa}\frac{T_0}{H_0}.$$

Expressing height in metres, the value of H_0 will be 7990, and $-\frac{dT}{dx}$ will denote the fall of temperature per metre of ascent. Thus, remembering that T_0 is 273, we have

$$-\frac{dT}{dx}=\frac{\cdot 408}{1\cdot 408}\frac{273}{7990}=\frac{1}{101};$$

that is, the temperature falls by $\frac{1}{101}$ of a degree Centigrade per metre of ascent, or falls 1° C. in ascending 101 metres. In descending air, elevation of temperature will be produced at the same rate. The calculation has been made on the supposition that the air is perfectly dry. The value of κ for superheated vapour is probably different from its value for dry air, and thus the presence of vapour may modify the above rate even when no liquid water is present. If ascending air contains vapour which is condensed into cloud by the cold of expansion, the latent heat thus evolved will retard the cooling; and if descending air contains cloud which is dissipated by the heat of compression, this dissipation retards the warming.

The ascent of warm air will not occur when the actual decrease of temperature upwards is slower than that due to cooling by ascent; for air will not rise if the process of rising would make it colder and heavier than the air through which it would have to pass. On the other hand, air is in an unstable condition, and tends to form convection currents, when the decrease of temperature upwards is more rapid than that due to cooling by ascent.

224. Adiabatic Compression of Liquids and Solids.—The following investigation, originally published by Lord Kelvin in the *Proceedings of the Royal Society of Edinburgh*, is applicable to liquids and solids as well as to gases.

Let unit volume of a substance be subjected to a cycle of four

small changes, two of them being adiabatic and the other two at constant pressure.

Representing volume by horizontal distance, and pressure by vertical distance, let the sides of the parallelogram ABCD (Fig. 127) represent the four operations, the base AD being very small compared with the altitude.

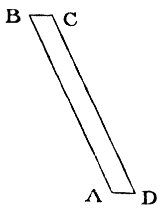


Fig. 127.

Let P denote the constant pressure in AD, and $P+p$ the constant pressure in BC, so that p is represented by the altitude.

AB and CD are adiabatics, and the difference of temperature between B and C will be sensibly equal to that between A and D; call it dt, and let e denote the expansion per degree at constant pressure. Then the increase of volume represented by BC or AD is $e\,dt$.

Let the cycle be performed in the direction ABCD. Then the work done by the substance is positive and is represented by the area of the parallelogram, which being the product of base by altitude is $pe\,dt$.

Heat is taken in during the operation BC, as the substance is then expanding at constant pressure, and is given out in DA, each of these quantities of heat being equal to $C\,dt$, where C denotes the thermal capacity of unit volume of the substance at constant pressure.

Let τ denote the excess of temperature of B above A, or of C above D, and T or $273+t$ the absolute temperature Then the "efficiency" is $\frac{\tau}{T}$ and is also $\frac{pe\,dt}{JC\,dt}$, that is $\frac{pe}{JC}$, where J denotes Joule's equivalent. Hence we deduce

$$\frac{\tau}{p}=\frac{Te}{JC};$$

where τ is the elevation of temperature produced by the adiabatic increase p of pressure.

For every substance which expands when heated at constant pressure, e is positive and therefore τ has the same sign as p, that is, increase of pressure produces elevation of temperature. On the other hand, substances which, like water below 4°, contract when heated, are cooled by adiabatic pressure, since e for such substances is negative.

225. Adiabatic Extension of a Wire.—In the above reasoning the

pressure is supposed to be of the nature of hydrostatic pressure, that is, to be equal in all directions. In order to treat the case of stress applied in one direction only, we have merely to modify the meaning of our symbols. The following is the form which the investigation takes when applied to the stretching of a wire.

Let a wire of unit length be subjected to a cycle of four small changes, two of them being adiabatic and the other two at constant tension.

Representing length by horizontal distance, and tension (that is, amount of stretching force,) by vertical distance, let the sides of the parallelogram A B C D (Fig 128) represent the four operations.

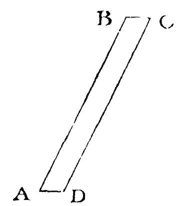


Fig 128

Let P denote the constant tension in A D, and $P+p$ the constant tension in B C, so that p is represented by the altitude.

Let dt denote the excess of temperature of C above B, or of D above A, and e the linear expansion per degree at constant tension. Then the increase of length represented by B C or A D is $e\,dt$, the figure being drawn on the supposition that e is positive. The area of the parallelogram is $pe\,dt$. If the cycle is performed in the direction A B C D, this area represents the work done by the external forces which tend to stretch the wire. As we wish it to represent the work done by the wire against these forces, we shall suppose the cycle to be performed in the opposite direction A D C B. Heat will then be taken in in the operation A D, and given out at a lower temperature in C B.

Let T be the absolute temperature in the middle of A D, and $T+\tau$ in the middle of B C, then τ will be negative, and by equating the two expressions for the efficiency, we have,

$$-\frac{\tau}{T}=\frac{pe\,dt}{JC\,dt}, \qquad \frac{\tau}{p}=-\frac{Te}{JC},$$

where C denotes the thermal capacity of the wire at constant tension (the length of the wire being unity), and τ is the elevation of temperature produced by the small increase p of the stretching force.

The signs of τ and p are opposite if e is positive. Hence every wire that is lengthened by heat will be cooled by the application of tension within its limits of elasticity. If the wire have

unit sectional area, C will denote the capacity of unit volume, and p the longitudinal stress in units of force per unit of area.

226. Adiabatic Coefficients of Elasticity.—Neglecting the exceptional cases of bodies which do not expand with heat, the resistance of a liquid to compression, and the resistance of a solid to both compression and extension, are greater under adiabatic conditions than under the condition of constancy of temperature. Thus, in the circumstances discussed in § 224 the pressure p produces an elevation of temperature τ, and the expansion due to this, namely $e\tau$, must be subtracted from the compression which would be produced at constant temperature. This latter is $\frac{p}{E}$, where E denotes the coefficient of elasticity at constant temperature; so that the compression will be only $\frac{p}{E} - e\tau$. The coefficient of elasticity is in the inverse ratio of the compression; hence, to find the adiabatic coefficient, we must multiply E by

$$\frac{\frac{p}{E}}{\frac{p}{E} - e\tau}, \text{ or by } \frac{1}{1 - \frac{Ee\tau}{p}}.$$

Substituting for τ its value $\frac{Tep}{JC}$, we find

$$\frac{Ee\tau}{p} = \frac{Ee^2T}{JC}.$$

In assigning the numerical values of E and J, it is to be remembered that if E is expressed in C.G.S. measure, as in the table of elasticities in Part I., the value of J will be 41·6 millions.

The factor for Young's modulus will be of the same form, E now denoting its value at constant temperature, and e the linear expansion for 1°, while C will still denote the thermal capacity of unit volume, which can be computed by multiplying the specific heat by the density.

227. Freezing of Water which has been Cooled below 0°.—We have seen in § 80 that when freezing begins in water which has been cooled below its normal freezing-point, a large quantity of ice is suddenly formed, and the temperature of the whole rises to 0°. In § 81 we have calculated the quantity of ice that will be formed, and we will now revise the calculation in the light of thermodynamics.

The same final condition would have been attained if the whole

mass (unity) of water at $-t^\circ$ had first been raised in the liquid state to 0°, and the mass x had then been frozen. The external work would also have been the same, being, in both cases, the product of atmospheric pressure by the excess of the final above the initial volume. Hence the algebraic sum of heat required is the same in both cases. But in the one case it is $t - 79{\cdot}25x$, and in the other case (that is, in the actual case) it is zero. Hence we have

$$t - 79{\cdot}25x = 0$$
$$x = \frac{t}{79{\cdot}25}.$$

The calculation in § 81 therefore requires no correction.

228. **Lowering of Freezing-point by Pressure.**—When a litre (or cubic decimetre) of water is frozen under atmospheric pressure, it forms 1·087 of a litre of ice, thus performing external work amounting to $\cdot 087 \times 103{\cdot}3 = 9$ kilogramme-decimetres $=$ ·9 of a kilogrammetre, since the pressure of one atmosphere or 760 mm. of mercury is 103·3 kilogrammes per square decimetre. Under a pressure of n atmospheres, the work done would be ·9 n kilogrammetres, neglecting the very slight compression due to the increase of pressure. If the ice is allowed to melt in vacuo, no external work is done upon it in the melting, and therefore, in the whole process, at the end of which the water is in the same state as at the beginning, heat to the amount of $\frac{\cdot 9\, n}{424} = \cdot 00212\, n$ of a kilogramme-degree is made to disappear. This process is *reversible*, for the water might be frozen in vacuo and melted under pressure; and hence, by appendix (2) to the second law of thermo-dynamics, we have

$$\cdot 00212n : Q :: T - T' : T;$$

where Q denotes the heat taken in in melting, which is 79·25 kilogramme-degrees, T the absolute temperature at which the melting occurs, about 273°, and T′ the absolute temperature of freezing under the pressure of n atmospheres. Hence we have

$$\cdot 00212n : 79{\cdot}25 :: T - T' : 273;$$

whence

$$T - T' = \cdot 0073n;$$

that is, the freezing-point is lowered by ·0073 of a degree Cent. for each atmosphere of pressure.

229. **Heat of Chemical Combination.**—There is potential energy between the particles of two substances which would combine chemi-

cally if the opportunity were afforded. When *combination* actually takes place, this potential energy runs down and yields an equivalent of heat. We may suppose that the particles rush together in virtue of their mutual attraction, and thus acquire motions which constitute heat.

In every case of *decomposition*, an amount either of heat or some other form of energy must be consumed equivalent to the heat of combination.

When the heat evolved in combination is so great as to produce incandescence, the process is usually called *combustion* or *explosion*, according as it is gradual or sudden. In combustion the action takes place at the surface of contact of the two combining bodies. In explosion they have been previously mingled mechanically, and combination takes place throughout the whole mass.

Chemical combination is often accompanied by diminution of volume, or by change of state from gas or solid to liquid or *vice versâ*. These changes sometimes tend to the evolution of heat, as

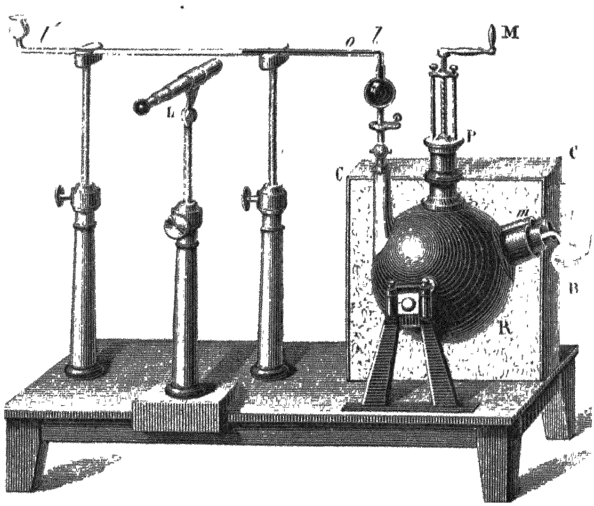


Fig 129.—Calorimeter of Favre and Silbermann

when oxygen and hydrogen unite to form liquid water; and sometimes to its absorption, as in freezing-mixtures. The observed thermal effect is therefore the sum or difference of two separate effects; and in general no attempt has been made to assign their respective proportions.

230. Observations on Heat of Combination.—Elaborate observa-

tions on the heats of combination of various substances have been made by Andrews, by Favre and Silbermann, and by Thomsen (of Copenhagen). The apparatus chiefly employed by Favre and Silbermann is represented in Fig. 129.

It is a kind of large mercurial thermometer, the reservoir R of which is made of iron, and contains one or more cylindrical openings similar to that shown at *m*. Into these are fitted tubes of glass or platinum, in which the chemical action takes place. One of the substances is introduced first, and the other, which is liquid, is then added by means of a pipette bent at B, and containing the liquid in a globe, as shown in the figure. This is effected by raising the pipette into the position indicated by the dotted lines in the figure.

In the upper part of the reservoir is an opening fitted with a tube containing a steel plunger P, which descends into the mass of mercury, and can be screwed down or up by turning the handle M. To prepare the apparatus for use, the plunger is so adjusted that the mercury stands at the zero-point of the graduated tube tt', the action is then allowed to take place, and the movement of the mercurial column is observed with the telescope L. In order to measure the quantity of heat corresponding to this displacement, a known weight of hot water is introduced into the reservoir, and allowed to give up its heat to the mercury; the displacement of the mercurial column is then observed, and since the quantity of heat corresponding to this displacement is known, that corresponding to any other displacement can easily be calculated. The iron reservoir is inclosed in a box filled with wadding or some other non-conducting material.[1]

When the chemical action takes the form of combustion, a different arrangement is necessary. The apparatus employed by Favre and Silbermann for this purpose is of too complex a construction to be described here. Fig. 130 represents the much simpler apparatus employed for the same purpose by Dulong.

It consists of a combustion-chamber C surrounded by the water contained in a calorimeter D, in which moves an agitator whose stem

[1] In the mode of experimentation adopted by Dr. Andrews, the combination takes place in a thin copper vessel inclosed in a calorimeter of water to which it gives up its heat; and the rise of temperature in the water is observed with a very delicate thermometer, the water being agitated either by stirring with a glass rod or by making the whole apparatus revolve about a horizontal axis.

In experimenting on the heat of combustion, the oxygen and the substance to be burned are introduced into the thin copper vessel, which is inclosed in the calorimeter as above, and ignition or explosion is produced by means of electricity.

held together by these forces are separated, and potential energy is thus obtained. When wood is burned, this potential energy is converted into heat.

We are not, however, to suppose that plants, any more than animals, have the power of *creating* energy. The forces which are peculiar to living plants are merely *directive.* They direct the energy of the solar rays to spend itself in separating the carbon and oxygen which exist united in the carbonic acid of the air; the carbon being taken up by the plant, and the oxygen left.

Coal is the remnant of vegetation which once existed on the earth. Thus all the substances which we are in the habit of employing as fuel, are indebted to the sun for the energy which they give out as heat in their combustion.

233. Solar Heat.—The amount of heat radiated from the sun is great almost beyond belief. The best measures of it have been obtained by two instruments which are alike in principle—Sir John Herschel's *actinometer* and Pouillet's *pyrheliometer.* We shall describe the latter, which is represented in Fig. 131. At the upper end, next the sun, is a shallow cylinder composed of very thin copper or silver, filled with water in which the bulb of a thermometer is inserted, the stem being partially inclosed in the hollow tube which supports the cylinder. At the lower end of the tube is a disc equal and parallel to the base of the cylinder. This is intended to receive the shadow of the cylinder, and thus assist the operator in pointing the instrument directly towards the sun. The cylinder is blackened, in order that its absorbing power may be as great as possible.

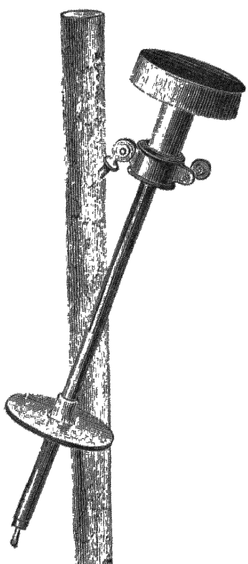


Fig. 131.—Pyrheliometer

The instrument, initially at the temperature of the atmosphere, is first placed for five minutes in a position where it is exposed to the sky, but shaded from the sun, and the increase or diminution of its temperature is observed; suppose it to be a fall of $\theta°$. The screen which shaded it from the sun is then withdrawn, and its rise of temperature is observed for five minutes with the sun shining upon it;

call this rise T°. Finally, it is again screened from the sun, and its fall in five minutes is noted;—call this θ'°. From these observations it is inferred, that the instrument, while exposed to the sun, lost $\frac{\theta+\theta'}{2}$ to the air and surrounding objects, and that the whole heat which it received from the sun was $T+\frac{\theta+\theta'}{2}$, or rather was the product of this difference of temperature by the thermal capacity of the cylinder and its contents. This is the heat which actually reaches the instrument from the sun, but a large additional amount has been intercepted by absorption in the atmosphere. The amount of this absorption can be roughly determined by comparing observations taken when the sun has different altitudes, and when the distance traversed in the air is accordingly different. Including the amount thus absorbed, Pouillet computes that *the heat sent yearly by the sun to the earth would be sufficient to melt a layer of ice* 30 *metres thick, spread over the surface of the earth;* and Sir John Herschel's estimate is not very different.

The earth occupies only a very small extent in space as viewed from the sun; and if we take into account the radiation in all directions, the whole amount of heat emitted by the sun will be found to be about 2100 million times that received by the earth, or sufficient to melt a thickness of two-fifths of a mile of ice per hour over the whole surface of the sun.

234. Sources of Solar Heat.—The only causes that appear at all adequate to produce such an enormous effect, are the energy of the celestial motions, and the potential energy of solar gravitation. The motion of the earth in its orbit is at the rate of about 96,500 feet per second. The kinetic energy of a pound of matter moving with this velocity is equivalent to about 104,000 pound-degrees Centigrade, whereas a pound of carbon produces by its combustion only 8080. The inferior planets travel with greater velocity, the square of the velocity being inversely as the distance from the sun's centre; and the energy of motion is proportional to the square of velocity. It follows that a pound of matter revolving in an orbit just outside the sun would have kinetic energy about 220 times greater than if it travelled with the earth. If this motion were arrested by the body plunging into the sun, the heat generated would be about 2800 times greater than that given out by the combustion of a pound of charcoal. We know that small bodies are travelling about in the celestial spaces; for they often become visible to us as meteors, their incan-

descence being due to the heat generated by their friction against the earth's atmosphere; and there is reason to believe that bodies of this kind compose the immense circumsolar nebula called the zodiacal light, and also, possibly, the solar corona which becomes visible in total eclipses. It is probable that these small bodies, being retarded by the resistance of an ethereal medium, which is too rare to interfere sensibly with the motion of such large bodies as the planets, are gradually sucked into the sun, and thus furnish some contribution towards the maintenance of solar heat. But the perturbations of the inferior planets and comets furnish an approximate indication of the quantity of matter circulating within the orbit of Mercury, and this quantity is found to be such that the heat which it could produce would only be equivalent to a few centuries of solar radiation.

Helmholtz has suggested that the smallness of the sun's density—only $\frac{1}{4}$ of that of the earth—may be due to the expanded condition consequent on the possession of a very high temperature, and that this high temperature may be kept up by a gradual contraction. Contraction involves approach towards the sun's centre, and therefore the performance of work by solar gravitation. By assuming that the work thus done yields an equivalent of heat, he brings out the result that, if the sun were of uniform density throughout, the heat developed by a contraction amounting to only one ten-thousandth of the solar diameter, would be as much as is emitted by the sun in 2100 years.

235. **Sources of Energy available to Man.**—Man cannot produce energy; he can only apply to his purposes the stores of energy which he finds ready to his hand. With some unimportant exceptions, these can all be traced to three sources:—

I. The solar rays.

II. The energy of the earth's rotation.

III. The energy of the relative motions of the moon, earth, and sun, combined with the potential energy of their mutual gravitation.

The fires which drive our steam-engines owe their energy, as we have seen, to the solar rays. The animals which work for us derive their energy from the food which they eat, and thus, indirectly, from the solar rays. Our water-mills are driven by the descent of water, which has fallen as rain from the clouds, to which it was raised in the form of vapour by means of heat derived from the solar rays.

The wind which propels our sailing-vessels, and turns our wind-

mills, is due to the joint action of heat derived from the sun, and the earth's rotation.

The tides, which are sometimes employed for driving mills, are due to sources II. and III. combined.

The work which man obtains, by his own appliances, from the winds and tides, is altogether insignificant when compared with the work done by these agents without his intervention, this work being chiefly spent in friction. It is certain that all the work which they do, involves the loss of so much energy from the original sources; a loss which is astronomically insignificant for such a period as a century, but may produce, and probably has produced, very sensible effects in long ages. In the case of tidal friction, great part of the loss must fall upon the energy of the earth's rotation; but the case is very different with winds. Neglecting the comparatively insignificant effect of aerial tides, due to the gravitation of the moon and sun, wind-friction cannot in the slightest degree affect the rate of the earth's rotation, for it is impossible for any action exerted between parts of a system to alter the angular momentum[1] of the system. The effect of easterly winds in checking the earth's rotation must therefore be exactly balanced by the effect of westerly winds in accelerating it. In applying this principle, it is to be remembered that the couple exerted by the wind is jointly proportional to the force of friction resolved in an easterly or westerly direction, and to the distance from the earth's axis.

236. Dissipation of Energy.—From the principles laid down in the present chapter it appears that, although mechanical work can be entirely spent in producing its equivalent of heat, heat cannot be entirely spent in producing mechanical work. Along with the conversion of heat into mechanical effect, there is always the transference of another and usually much larger quantity of heat from a body at a higher to another at a lower temperature. In conduction and radiation heat passes by a more direct process from a warmer to a colder body, usually without yielding any work at all. In these cases, though there is no loss of energy, there is a running to waste as far as regards convertibility; for a body must be hotter than neighbouring bodies, in order that its heat may be available for yielding work. This process of running down to less available forms has been variously styled *diffusion*, *degradation*, and *dissipation* of

[1] The angular momentum is measured by the product of the moment of inertia (see Part I.) and the angular velocity

energy, and it is not by any means confined to heat. We can assert of energy in general that it often runs down from a higher to a lower grade (that is to a form less available for yielding work), and that, if a quantity of energy is ever raised from a lower to a higher grade, it is only in virtue of the degradation of another quantity, in such sort that there is never a gain, and is generally a loss, of available energy.

This general tendency in nature was first pointed out by Lord Kelvin. It obviously leads to the conclusion that the earth is gradually approaching a condition in which it will no longer be habitable by man as at present constituted.

237. Kinetic Theory of Gases.—According to the theory of the constitution of gases which is now generally accepted and is called by the above name, a simple gas consists of a number of very small and exactly equal particles, called atoms or molecules, moving about with various velocities and continually coming into collision with one another and with the sides of the containing vessel. The total volume of the particles themselves is very small compared with the space in which they move, and consequently the time during which a particle is in collision with other particles is a very small part of its whole time.

Each particle is highly elastic. Its shape can be changed by the application of external forces; but it springs back when left to itself and executes vibrations, which we may compare to those of a tuning-fork or a bell. These are the cause of the peculiar features which are detected in the light of an incandescent gas when analysed by the spectroscope. It can also, like any other free body, have a rotatory or spinning motion. The kinetic energy of a particle is accordingly composed of three parts, one due to its vibration, another to its rotation, and a third to its translation. This third part, which is usually greater than the other two, is called the *energy of agitation*. The other two are included together under the name of *internal energy*, which may be defined as the energy of the relative motion of different parts of the same molecule.

In addition to these, we may have movement of the gas as a whole, which is what is meant when in ordinary language we speak of a gas in motion as distinguished from a gas at rest. In this sense, the velocity at any point of a gas is another name for the velocity of the centre of gravity of a small group of molecules surrounding the point. In what follows we leave such velocity out of account.

238. The ratio of the energy of agitation to the internal energy, though it may vary at a given instant from molecule to molecule, or may vary for the same particle from instant to instant, has a definite and permanent value for the aggregate of all the particles —a value independent of changes of pressure or temperature, but not the same for all gases. The symbol β is employed to denote the ratio of the whole kinetic energy of a gas to the energy of agitation, and the value of β for several of the more permanent gases is 1·634.

The heat of a gas is another name for its kinetic energy, that is, for $\Sigma\frac{1}{2}\beta mv^2$, or $\frac{1}{2}\beta m\,\Sigma v^2$, v denoting the velocity of a molecule, m its mass, and Σ indicating summation for all the molecules. To reduce the expression for this heat to ordinary thermal units we must divide by Joule's equivalent.

The absolute temperature of a given gas is proportional to the average kinetic energy of its molecules, that is, to the average value of $\frac{1}{2}\beta mv^2$, or, omitting constants, to the average value of v^2. We shall denote the average value of v^2 by V^2. Its square root V is called the *velocity of mean square.*

In a mixture of two simple gases the value of V^2 is not the same for them both, but varies inversely as m; in other words, mV^2 has the same value for both constituents. Accordingly, in comparing one gas with another mV^2 is taken as the proportional measure of absolute temperature.

239. The equality of the values of mV^2 for the two components of a mixture is not an arbitrary assumption, but a deduction obtained by a very elaborate mathematical investigation from the supposition of two sets of perfectly elastic balls flying about promiscuously amongst each other.

This and other similar calculations which form an important part of the kinetic theory of gases are conducted by what is called the *statistical method.* Large numbers give steadiness to statistics, and the number of molecules in a cubic centimetre of gas is more than a million of millions of millions. As long as a cubic centimetre of gas remains at the same pressure and temperature the statistics of the velocities of its molecules remain permanent. The velocity of each particle changes in the most irregular manner, but the number of its molecules that have velocities lying between given limits (which may be very close together) never changes by more than an infinitesimal part of itself.

Calculation shows that when we attend not merely to the actual velocities but to their components in a given direction, the statistics of such component velocities will be independent of the direction assumed, even when gravity is taken into account.

240. The pressure of a gas against the walls of the containing vessel is due to the impacts of its particles against the walls. To compute its amount, let u denote the component velocity of a molecule normal to one of the sides supposed plane, u being regarded as positive when the molecule is approaching the side and negative when receding. Let u_1 be a particular positive value of u, and let the number of molecules in unit volume that have approximately this velocity be n_1. The number of molecules of velocity u_1 that impinge on unit area of the side in unit time will be the number that occupy a volume u_1, and will therefore be $n_1 u_1$.

Their momentum before striking is their mass mn_1u_1 multiplied by their velocity u_1, and is therefore $mn_1u_1^2$. This is reversed by the collision, so that the change of momentum is $2mn_1u_1^2$. This, being the change of momentum produced in unit time by the reaction of unit area of the wall, is equal to the pressure on unit area due to the impacts of those molecules which we have been considering. But the number of molecules whose normal velocity is u_1 is, by symmetry, the same as the number whose normal velocity is $-u_1$, hence $2mn_1u_1^2$ is the sum of such terms as mu^2 for all the molecules for which the value of u^2 is u_1^2.

Thus the total pressure on unit area is the sum of such terms as mu^2 for all the particles in unit volume; that is, calling the pressure p,

$$p = \Sigma mu^2 = m\Sigma u^2. \qquad (1)$$

But, from the symmetry of the constitution of a gas, Σu^2 has the same value for all directions of u. Combining this principle with the principle that the square of a velocity is the sum of the squares of its three rectangular components, we easily deduce $\Sigma u^2 = \frac{1}{3}\Sigma v^2$.

Let N denote the whole number of molecules in unit volume, and ρ the density of the gas, which is Nm, then we have:

$$\Sigma u^2 = \tfrac{1}{3}\Sigma v^2 = \tfrac{1}{3}\mathrm{N}\,\mathrm{V}^2, \qquad (2)$$

$$p = m\Sigma u^2 = \tfrac{1}{3}\mathrm{N}m\mathrm{V}^2 = \tfrac{1}{3}\rho\mathrm{V}^2. \qquad (3)$$

241. This last result enables us to compute the value of V for any known gas, for it gives

$$\mathrm{V}^2 = \frac{3p}{\rho}. \qquad (4)$$

Thus in C.G.S. measure we have for hydrogen (see pp. 305, 306), $p = 1{\cdot}0136 \times 10^6$, $\rho = {\cdot}00008957$, whence $V = 184{,}300$ cm. per sec. This is about one nautical mile per second.

The value of V for any gas bears a constant ratio to the velocity of sound in the gas, namely, the ratio $\sqrt{\frac{3}{\kappa}}$, where κ denotes the ratio of the two specific heats.

Since the energy of agitation in unit volume is $\frac{1}{2}\rho V^2$, and p is $\frac{1}{3}\rho V^2$, these quantities have the same dimensions and are as 3 to 2.

The equation $p = \frac{1}{3}\rho V^2$ shows that when V^2 (and therefore the absolute temperature) is given, p varies as ρ. This is Boyle's law.

Again, it shows that when ρ is given, p varies as V^2, that is, as the absolute temperature; and that, when p is given, ρ varies inversely as V^2; that is, the volume varies directly as the absolute temperature.

Further, from the equation $p = \frac{1}{3}NmV^2$ we deduce that when two gases have the same pressure p and the same temperature (measured by mV^2), they have the same number of particles N in unit volume, and their densities (since $\rho = Nm$) are directly as m the mass of a single particle of each; that is, the densities (at the same pressure and temperature) are directly as the atomic weights. This is known as Avvogadro's law.

242. In questions relating to specific heat it is convenient to make the unit of heat equal to the unit of energy, so that the quantity of heat in a mass Σm will be not only proportional but equal to $\frac{1}{2}\beta\Sigma mv^2$, or to $\frac{1}{2}\beta V^2\Sigma m$, and to employ a unit of temperature such that absolute temperature shall be not only proportional but equal to mV^2. Then, denoting absolute temperature by θ, and quantity of heat or energy in unit volume by E, we have

$$V^2 = \frac{\theta}{m} \quad (5) \qquad\qquad E = \tfrac{1}{2}\beta V^2\Sigma m = \tfrac{1}{2}\beta V^2\rho = \tfrac{1}{2}\beta\frac{\rho}{m}\theta = \tfrac{1}{2}\beta N\theta. \quad (6)$$

N denoting, as before, the number of molecules in unit volume.

The thermal capacity at constant volume, for unit volume of the gas, is defined as $\frac{dE}{d\theta}$, and is $\frac{1}{2}\beta N$, it being assumed that β is constant. Since N is the same for all gases at the same temperature and pressure, the thermal capacity per unit volume is the same for all gases that have the same value of β.

The specific heat at constant volume is the thermal capacity of the volume $\frac{1}{\rho}$, and is therefore $\frac{1}{2}\beta N\frac{1}{\rho}$ or $\frac{1}{2}\frac{\beta}{m}$. Hence the specific heat

is inversely as the atomic weight, as asserted by the law of Dulong and Petit.

Again we have

$$p=\tfrac{1}{3}\rho V^2=\tfrac{1}{3}\rho\frac{\theta}{m}=\tfrac{1}{3}N\theta, \tag{7}$$

$$\frac{p}{\rho\theta}=\frac{1}{3m}=\frac{p\times\text{volume of unit mass}}{\theta}. \tag{8}$$

The work done by the gas (initially at unit volume) in expanding against constant pressure p when θ is increased by unity is $p\times$ increase of volume $=p\frac{1}{\theta}=\frac{\rho}{3m}$. If the original volume be $\frac{1}{\rho}$ (in which case the mass will be unity) the work in expanding will be $\frac{1}{3m}$. Hence, the ratio of the specific heat at constant pressure to that at constant volume is

$$\kappa=\frac{\frac{1}{3m}+\frac{\beta}{2m}}{\frac{\beta}{2m}}=\frac{2}{3\beta}+1, \tag{9}$$

giving

$$\beta=\frac{2}{3(\kappa-1)}. \tag{10}$$

If we assume $\kappa=1{\cdot}408$, we find $\beta=1{\cdot}634$.

243. The rate at which a gas escapes through a porous partition will be jointly as the number of molecules in unit volume and the mean value of the velocity resolved normal to the partition; or in our notation it will be jointly as N and the mean value of u. This latter, though not identical with the square root of the mean value of u^2, that is, with the square root of $\frac{1}{3}V^2$, can be shown to be in a fixed ratio to it. Hence the rate of diffusion will be proportional to NV At given temperature and pressure, N is the same for all gases, hence the rate of diffusion will be directly as V, that is inversely as $\sqrt{m}$, or inversely as the square root of the density Nm.

244. Van der Waals' Formula for correcting Boyle's Law.—In the calculation by which we have obtained the formula $p=\frac{1}{3}\rho V^2$, the molecules were treated as indefinitely small. Increased size of the molecules (for given V, m, and n) would involve more frequent collision and therefore increased pressure. Calculation shows that the value of p as corrected for the finite size of the molecules is

$$\frac{1}{3}\frac{\rho V^2}{1-b\rho}, \text{ or } \frac{1}{3} V^2/\left(\frac{1}{\rho}-b\right),$$

b being a small quantity which is constant for a given gas.

Again, the theory of capillarity as applied to liquids teaches that the mutual attraction of the molecules which compose the surface-layer of a liquid pulls the surface-layer inwards upon the rest of the liquid, and that the pressure at the outer boundary of the surface-layer is therefore less than the pressure at and within its inner boundary. The same reasoning which leads to this result in the case of liquids is applicable on a diminished scale to gases. Accordingly p and V are smallest at the boundary of a gas, and gradually increase for a very small distance inwards. The formula $p=\frac{1}{3}\rho V^2$ or the corrected formula $p=\frac{1}{3} V^2/\left(\frac{1}{\rho}-b\right)$, is applicable to the gas as a whole, but will not be true if we employ the value of V^2 for the gas as a whole in combination with the value of p at the boundary. In practical measurement of p it is the pressure at the boundary that is measured. This will be less than $\frac{1}{3}V^2/\left(\frac{1}{\rho}-b\right)$ by the pressure due to the skin attraction, which is easily shown to be proportional to ρ^2, and may be denoted by $a\rho^2$, a being constant for a given gas. Hence if we make p stand for the pressure at the bounding surface of the gas, we have

$$p+a\rho^2=\frac{1}{3} V^2/\left(\frac{1}{\rho}-b\right), \text{ or}$$

$$(p+a\rho^2)\left(\frac{1}{\rho}-b\right)=\frac{1}{3} V^2. \qquad (11)$$

This investigation is due to Van der Waals, who writes the first member of (11) in the form $(p+\frac{a}{v^2})(v-b)$, v denoting the volume of unit mass of the gas. According to his theory it is this product, and not the simple product pv, that is constant at given temperature.

CHAPTER XVI.

THERMO-DYNAMICS (CONTINUED).

245. To students familiar with the notation and elementary processes of the differential calculus, the deduction of many of the foregoing results can be presented in a more compact form, as follows:—

246. We define a *perfect gas* to be one which fulfils the three following conditions:—

First. Boyle's law: that vp is constant at constant temperature.

Let the temperature t of the gas be defined as proportional to the product vp and as increasing by 100 in passing from 0° C. to 100° C. Let v be taken as the volume of unit mass of the gas at pressure p and temperature t. Then our definition of the temperature is expressed by the equation

$$vp = Rt, \qquad (1)$$

R being independent of v, p, and t. This gives

$$v\,dp + p\,dv = R\,dt. \qquad (2)$$

Second. That the specific heat of the gas (*i.e.* the heat required to raise it through one degree of temperature as thus defined) at constant pressure is the same at all pressures and at all temperatures. We shall denote it by s.

Third. That the heat which must be given to the gas from without to enable it to expand at constant temperature is the equivalent of the external work done by the gas in its expansion.

We shall employ as our unit of heat the heat equivalent to the unit of work. The heat required for a small increase of volume dv at constant temperature will then be pdv. The heat required for a small rise of temperature dt at constant volume will be $s'\,dt$, if s' denote the specific heat at constant volume. Superposing these two small changes, we have as the general expression for the heat dQ required for any small change

$$dQ = s'\,dt + p\,dv. \qquad (3)$$

We may regard this equation as the expression of our third condition.

247. When the expansion is at constant pressure, (2) gives $p\,dv = \text{R}\,dt$; and we then have, by (3),

$$dQ = (s' + \text{R})\,dt.$$

But by the definition of s, $dQ = s\,dt$; hence we have

$$s' + \text{R} = s, \tag{4}$$

and as R and s are both constant, s' must be constant; that is to say, the specific heat of our gas at constant volume will be the same at all pressures and at all temperatures. The constant R may now be replaced by $s - s'$.

248. The condition of an adiabatic change is obtained by putting $dQ = 0$ in (3), and is

$$s'\,dt + p\,dv = 0. \tag{5}$$

Substituting for p its value $\text{R}t/v$ or $(s - s')\,t/v$, this reduces to

$$s' \frac{dt}{t} + (s - s') \frac{dv}{v} = 0. \tag{6}$$

Eliminating t and dt by the help of (1) and (2), this reduces to

$$s' \frac{dp}{p} + s \frac{dv}{v} = 0. \tag{7}$$

Thus $d \log p$, $d \log t$, and $-d \log v$ are proportional to

$$s,\ s - s',\ \text{and } s',$$

as proved in § 220.

249. The external work done by the gas in expanding from volume v_1 to volume v_2 is the integral of $p\,dv$ from v_1 to v_2. But for unit mass of the gas, $p = \text{R}t/v = (s - s')\,t/v$. Hence the expression for the work may be written $(s - s') \int \frac{t\,dv}{v}$.

When the expansion is isothermal, this becomes

$$(s - s')\,t \int \frac{dv}{v}, \text{ or } (s - s')\,t \log \frac{v_2}{v_1}. \tag{8}$$

When the expansion is adiabatic, $p\,dv$ is $-s'dt$ by (5). Hence the work in expanding is equal to s' multiplied by the fall of temperature.

250. In a cycle of four operations AB, BC, CD, DA (Fig. 132), of which AB, CD are isothermal, and BC, DA adiabatic; if t_2 denote the temperature in AB, and t_1 that in CD, the works in BC and DA are $\pm\, s'(t_2 - t_1)$, and destroy one another. The work in AB is

$(s-s')\,t_2(\log v_B - \log v_A)$, and that in CD is $(s-s')\,t_1(\log v_D - \log v_C)$. But by (6),

$$\log v_D - \log v_A = \frac{s'}{s-s'}(\log t_2 - \log t_1) = \log v_C - \log v_B. \quad (9)$$

Hence $\log v_B - \log v_A = \log v_C - \log v_D$, and the works in AB and CD are as t_2 to $-t_1$.

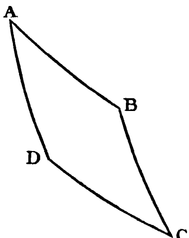


Fig. 132.

But by our definition of a perfect gas these amounts of work are equal to the amounts of heat given to the gas from without. Hence the ratio of the heat given to the gas in AB to the heat taken from it in CD is the ratio of the temperature in AB to that in CD when temperature is defined as proportional to vp for the gas in question. By the property of reversible engines this ratio is independent of the particular gas employed, and t as we have defined it is identical with temperature on the absolute thermo-dynamic scale. Thus all perfect gases must have the same coefficient of expansion.

251. We proceed to deal with working substances generally.

From the point of view of thermo-dynamics, two independent variables are just sufficient to specify the condition of a substance. Any two of the three elements—volume, pressure, temperature—will in general determine the third. In the case of a liquid and its vapour present together p and t determine each other without reference to v. Hence v and p, or v and t will suffice, but p and t will not suffice, not being independent; and the same remark applies to a liquid with its solid, or to a solid with its vapour.

252. Entropy.—Let AD, BC (Fig. 133) be two adiabatic lines for any substance, and AB, DC two isothermals crossing them. Then since ABCDA represents a Carnot's cycle of reversible operations, the heats taken in or given out by the substance in the two isothermal operations AB, DC are directly as the temperatures at which they are performed. Therefore the heat in AB, divided by the temperature in AB, is equal to the heat in DC, divided by the temperature in DC.

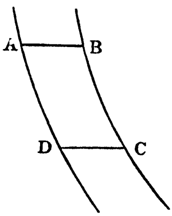


Fig 133

That is to say, *if the heat taken in when a substance passes at constant temperature from one adiabatic to another is divided by the temperature, the quotient is the same for all temperatures.* This quotient is called the *difference of entropy* of the two

adiabatics, and if a positive quantity of heat is taken in when the substance passes isothermally from the first of two adiabatics to the second, the second is said to have the greater entropy.

In general when a substance passes by any reversible course from one condition to another, the sum of such terms as dQ/t is defined to be its gain of entropy, dQ denoting the small quantity of heat taken in in any element of the course, and t the temperature at which it is taken in. In travelling along an adiabatic this gain vanishes because dQ is always zero; hence an adiabatic may also be called an *isentropic* or line of constant entropy. In travelling along an isothermal the definition clearly agrees with our definition of the difference of entropy of two adiabatics.

253. In travelling along an irregular course we may by the principle of superposition of small changes replace each small element ac of the course by two small changes ab, bc, or ad, dc (Fig. 134), one of them isothermal and the other adiabatic. In the adiabatic change the entropy remains constant. In the isothermal change it is increased by the excess of the entropy of the second adiabatic cd over that of the first ab. Thus the total gain of entropy in the whole course is the excess of entropy of an adiabatic drawn through the final point above the entropy of an adiabatic drawn through the initial point. It is accordingly independent of the course taken, and entropy, reckoned from an arbitrary condition as zero, is completely determined by the two independent variables which define the body's condition, or may itself be taken as one of the two independent variables. Entropy is usually denoted by the symbol ϕ, so that

$$d\phi = dQ/t, \text{ or } dQ = t\,d\phi. \qquad (10)$$

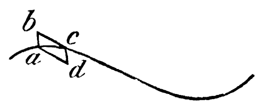


Fig 134.

254. When heat leaks by conduction or other irreversible process from a body at t_2 to a body at lower temperature t_1, the colder body gains more entropy than the warmer loses; for if dQ be the heat which passes, dQ/t_1 is greater than dQ/t_2. In general, irreversible thermal operations involve gain of entropy in the system as a whole; and the general tendency to "dissipation" of energy may be described as a tendency to increase of entropy. When heat passes by reversible operations from a source to a refrigerator the entropy lost by the source is equal to that gained by the refrigerator.

255. The work done in a cycle of two adiabatic and two iso-

thermal operations is equal to the difference of entropy of the two adiabatics multiplied by the difference of temperature of the two isothermals; for, with the notation previously employed, it is $Q_2 - Q_1 = t_2\, Q_2/t_2 - t_1\, Q_1/t_1$, and Q_2/t_2 or its equal Q_1/t_1 is the difference of entropy.

Hence, if we draw a series of adiabatics with a common difference of entropy, and cross them by a series of isothermals with a common difference of temperature, all the meshes of the diagram will be equal in area, for the area of each mesh is equal to the work done in going round it, and this is equal to the common difference of entropy multiplied by the common difference of temperature.

256. A number of relations between different thermal properties —relations which must be true for all substances—can be deduced from the well-known mathematical principle that when a function of two independent variables is differentiated first with respect to one of these variables and then with respect to the other, the order of differentiation is indifferent. We shall take four examples.

When a small quantity $dQ = t\, d\phi$ of heat is given to a body of unit mass which at the same time expands by the amount dv at the constant pressure p, the nett amount of energy given to the body from without (which we will call dE) is

$$\left.\begin{array}{lr} & dE = t\, d\phi - p\, dv \\ \text{again we have} & d\,(vp) = v\, dp + p\, dv \\ \text{and} & d\,(t\phi) = t\, d\phi + \phi\, dt \end{array}\right\} \qquad \text{(I.)}$$

By combining these equations we obtain—

$$d\,(E + vp) = t\, d\phi + v\, dp, \qquad (1)$$

$$d\,(t\phi - E) = p\, dv + \phi\, dt, \qquad (2)$$

$$d\,(E + vp - t\phi) = v\, dp - \phi\, dt, \qquad (3)$$

$$d\,E = t\, d\phi - p\, dv, \qquad (4)$$

in which the increments on the left hand, if summed through any course of change, give totals which depend only on the initial and final conditions. This entitles us to apply the principle that the order of differentiation is indifferent. Thus (1), in which ϕ and p are the two independent variables, gives

$$\frac{dt}{dp},\ \text{for constant } \phi, = \frac{dv}{d\phi},\ \text{for constant } p. \qquad (1\text{ A})$$

In like manner (2), (3), (4) give

$$\frac{dp}{dt},\ \text{for constant } v, = \frac{d\phi}{dv},\ \text{for constant } t. \qquad (2\text{ A})$$

$$\frac{dv}{dt},\ \text{for constant } p, = -\frac{d\phi}{dp},\ \text{for constant } t. \qquad (3\text{ A})$$

$$\frac{dt}{dv},\ \text{for constant } \phi, = -\frac{dp}{d\phi},\ \text{for constant } v. \qquad (4\text{ A})$$

By substituting dQ/t for $d\phi$ these can be reduced to

$$\frac{dt}{t\,dp}\ [\text{for constant } \phi] = \frac{dv}{dQ}\ [\text{for constant } p], \qquad (1\,\text{B})$$

$$\frac{dt}{t\,dp}\ [\quad ,, \quad v] = \frac{dv}{dQ}\ [\quad ,, \quad t], \qquad (2\,\text{B})$$

$$-\frac{dt}{t\,dv}\ [\quad ,, \quad p] = \frac{dp}{dQ}\ [\quad ,, \quad t], \qquad (3\,\text{B})$$

$$-\frac{dt}{t\,dv}\ [\quad ,, \quad \phi] = \frac{dp}{dQ}\ [\quad ,, \quad v]. \qquad (4\,\text{B})$$

In interpreting these results it is not necessary to retain the supposition that the mass acted on is unity, for if the mass were changed the change in dv would be balanced by the change in dQ. Constancy of ϕ means adiabatic change, and the student should verify that (1 B) is equivalent to the result obtained in § 224 for heating by adiabatic compression. It is to be remembered that Q is measured in units of work, and that t is absolute temperature.

257. When the substance is in two states which are present together (liquid and its solid, liquid and its vapour, or solid and its vapour) p depends on t only, and conversely t on p only, so that constancy of p is equivalent to constancy of t. The condition of the substance can still be specified by means of two independent variables, but these must not be p and t, though they may be any other two selected from the four v, p, t, ϕ.

Equation (1 B) as applied to this case becomes

$$\frac{dt}{t\,dp}\ [\text{unrestricted}] = \frac{dv}{dQ}\ [\text{constant } p \text{ and } t], \qquad (5)$$

and equation (2 B) becomes identical with it.

Equation (3 B) becomes nugatory, both sides vanishing.

Equation (4 B) is not specially modified.

The student should verify that (5) as applied to a mixture of ice and water leads to the result obtained in § 228 for the lowering of the freezing point by pressure. In making the application, using the C.G.S. system, dv may be taken as $-\cdot 087$, dQ as $80 \times 42 \times 10^6$, t as 273, and for an increase of one atmosphere dp may be taken as 10^6. These values will be found to give a lowering of $\cdot 0071$ of a degree Cent. per atmosphere.

258. **Specific Heat of Saturated Vapour.**—Suppose a liquid and its vapour to be present together, the total mass being unity, of which the part m is vapour, and therefore the part $1-m$ liquid. We may take t and m as the two independent variables. Then a small change of temperature without change of m would require the addition of heat

$$dQ = \{hm + c\,(1-m)\}\,dt,$$

h denoting what is called the *specific heat of the saturated vapour*, and c the specific heat of the liquid.

A change of m at constant t will require the addition

$$dQ = \lambda \, dm,$$

λ denoting the latent heat of evaporation at t. Hence the general expression for the heat required for a small change is

$$dQ = \{(h-c)\,m+c\}\,dt+\lambda\,dm; \qquad \text{whence}$$

$$d\phi = \frac{(h-c)\,m+c}{t}dt + \frac{\lambda}{t}dm.$$

The principle that the order of differentiation is indifferent gives, as applied to this expression for $d\phi$,

$$\frac{h-c}{t} = \frac{1}{t}\frac{d\lambda}{dt} - \frac{\lambda}{t^2}, \qquad \text{whence}$$

$$h = c + \frac{d\lambda}{dt} - \frac{\lambda}{t}.$$

This equation is true whatever unit of heat we employ. We may, therefore, employ for steam the value of λ given in (*b*) § 140, if we put $t-273$ for T. This gives approximately

$$\lambda = 800 - \cdot 7t.$$

In general if the latent heat of a vapour be expressible as

$$\lambda = a - bt,$$

we have $$\frac{d\lambda}{dt} - \frac{\lambda}{t} = -\frac{a}{t}; \text{ whence } h = c - \frac{a}{t}.$$

For water and steam this gives

$$h = 1 - \frac{800}{t}$$

as the approximate value of the specific heat of saturated steam. It is negative for all values of t that occur in the practical use of steam.

For saturated vapour by itself we must put $m=1$, and if the vapour is to continue just saturated while its volume and temperature change, we must put $dm=0$. The general expression for dQ will then be reduced to

$$dQ = h\,dt.$$

For every vapour the maximum density increases with the temperature, hence increase of temperature with continuance of saturation implies compression. When h is negative and dt positive, dQ will be negative; hence saturated steam when compressed must be allowed to give out heat if it is to remain just saturated. If compressed adiabatically it will be superheated.

CHAPTER XVII.

STEAM AND OTHER HEAT ENGINES.

259. By a heat engine we mean an engine which yields work in virtue of heat supplied to it. The principal heat engines in actual use are the steam-engine and the gas-engine; but air-engines, working by the expansion of air when heated, were at one time in use, and oil-engines in which a spray of oil is converted by heat into vapour are beginning to be employed.

260. Our limits will not permit us to discuss the mechanical details of the various kinds of steam-engine. We must content ourselves with describing the mode in which the pressure of the steam operates to produce mechanical work.

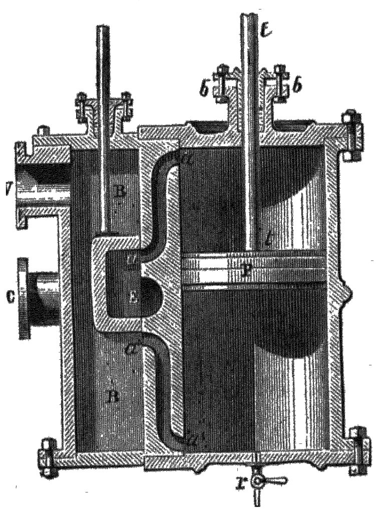


Fig. 135.—Cylinder and Connections.

Figs. 135, 136 illustrate the arrangements by which the steam in an ordinary double-acting engine is made to push the piston alternately in opposite directions. Fig. 135 shows the piston P, which works up and down in the cylinder, and is now nearly in the middle of its stroke, being pushed up by steam, which enters through the lower passage $a'a'$ leading from the steam-chest BB, which is in free communication with the steam-pipe V leading from the boiler. The steam on the upper side of the piston is escaping through the upper passage aa to the open air or to the condenser. E is the opening leading to the escape-pipe C.

In order to push the piston down again, it is necessary to let steam from the steam-chest enter above the piston, and to let the

steam below escape. The way in which this is done is exhibited in Fig. 136, which represents only the parts concerned in directing the course of the steam. There is a movable piece called the *slide-valve,* which slides up and down so as to alter the connections. The first figure shows the position which we have just been considering, the steam being admitted below the piston and allowed to escape

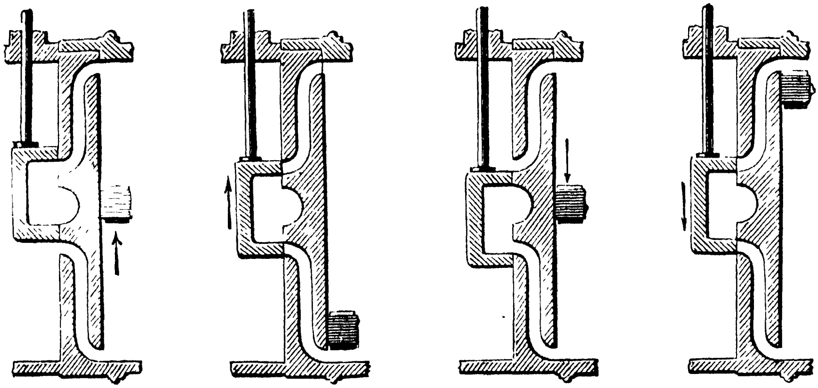


Fig. 136.—Movements of Slide-valve.

above. The second figure shows the slide-valve a little lower down, closing for the instant both passages. The third figure shows the slide-valve still lower; in this position the steam is admitted above the piston and escapes below. In the fourth figure it is again for the instant closing both passages. The slide-valve is in constant motion up and down, being driven by an arrangement (equivalent to a crank) called an *eccentric,* which is shown in duplicate at A, A′, Fig. 138, revolving about the point O as centre.

261. Working Expansively.—If the steam at its full pressure were discharged into the condenser, a great amount of expansive power would obviously be wasted. This power is utilized by what is called *expansive working.* When the piston has performed a part of its stroke, the steam is shut off (or in technical phrase *cut off*) from the cylinder, and the expansive force of the steam already admitted is left to urge the piston through the remainder of its course. The part of the stroke at which the cut-off occurs may be determined at pleasure. It is sometimes at half-stroke, sometimes at quarter-stroke, sometimes at one-fifth of stroke. In the latter case the piston describes the remaining four-fifths of the stroke under the gradually diminishing pressure of the steam which entered the

cylinder during the first fifth; and the work done during these four-fifths is so much work gained by working expansively.

The cutting off of the steam before the end of the stroke is usually effected by the contrivance represented in Fig. 137: *ad*, *a'd'*, are two plates forming part of the slide-valve and of much greater width than the openings L, L'. The excess of width is called *lap*. By this arrangement one of the apertures is kept closed for some time, so that the steam is shut off, and acts only by expansion. The expansion increases with the lap, but not in simple proportion, as equal movements of the slide-valve do not correspond to equal movements of the piston. The amount of expansion can also be regulated by the *link-motion*, which will be described in § 265.

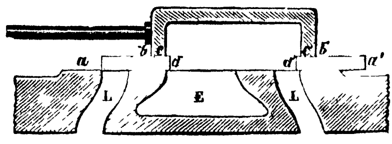


Fig 137 —Slide-valve for Expansive Working

262. Compound Engines.—This is the name given to engines in which the steam, instead of escaping from the cylinder into the condenser, or into the open air, escapes into a second cylinder of larger section than the first, in which it drives a second piston.

In triple-expansion engines, which are now in general use for ocean-going steamers, the steam, when it has driven the piston in the second cylinder, escapes into a third cylinder of still larger section, in which it drives a third piston before passing into the condenser. In the boiler and first cylinder the steam is at very high pressure and temperature (the boiler pressure being usually above 10 atmospheres). As it is worked expansively, it escapes into the second cylinder at a more moderate pressure and temperature, and into the third at a still lower pressure and temperature. If the same amount of total expansion occurred in a single cylinder, it would cool the cylinder too far below the temperature of the entering steam, which would thus be chilled on entering, and thereby deprived of a great portion of its pressure.

Expansive working is often combined with the *superheating* of steam, that is to say, heating the steam on its way from the boiler to the cylinder, so as to raise its temperature above the point of saturation.

263. In the construction of the boiler it is important to afford the greatest possible facility for the communication of heat from the furnace to the water. In the locomotive and in many other modern

engines this is effected by making the hot gases on their way from the furnace to the chimney pass through a large number of parallel tubes of copper or other good conductor, which traverse the boiler from end to end and are surrounded by the water. A very large heating surface is thus obtained, and the transmission of heat is proportionally rapid.

264. Surface Condensation.—In many modern engines, the condenser consists of a number of vertical tubes of about half an inch diameter, connected at their ends, and kept cool by the external contact of cold water. The steam, on escaping from the cylinder, enters these tubes at their upper ends, and becomes condensed in its passage through them, thus yielding distilled water, which is pumped back to feed the boiler. The same water can thus be put through the engine many times in succession, and the waste which occurs is usually repaired by adding from time to time a little distilled water prepared by a separate apparatus.

The old method of condensing is by a jet of cold water playing into the interior of the condenser.

265. Apparatus for Reversing: Link-motion.—The method usually employed for reversing engines is known as Stephenson's link-motion, having been first employed in locomotives constructed by Robert Stephenson, son of the maker of the "Rocket." The merit of the invention belongs to one or both of two workmen in his employ—Williams, a draughtsman, who first designed it, and Howe, a pattern-maker, who, being employed by Williams to construct a model of his invention, introduced some important improvements.

The link-motion, which is represented in Fig. 138, serves two purposes: first, to make the engine travel forwards or backwards at pleasure; and, secondly, to regulate the amount of expansion which shall take place in the cylinder. Two oppositely placed eccentrics, A and A′, have their connecting-rods jointed to the two extremities of the *link* B B′, which is a curved bar, having a slit, of uniform width, extending along nearly its whole length. In this slit travels a stud or button C, forming part of a lever, which turns about a fixed point E. The end D of the lever D E is jointed to the connecting-rod D N, which moves the rod P of the slide-valve. The link itself is connected with an arrangement of rods LIKH,[1] which enables the

[1] I is a fixed centre of motion, and the rods KI, ML are rigidly connected at right angles to each other. M is a heavy piece, serving to counterpoise the link and eccentric rods.

engine-driver to raise or lower it at pleasure by means of the handle G H F. When the link is lowered to the fullest extent, the end B of the connecting-rod, driven by the eccentric A, is very near the runner C which governs the movement of the slide-valve; this valve accordingly, which can only move in a straight line, obeys the eccentric A almost exclusively. When the link is raised as much as possible, the slide-valve obeys the other eccentric A′, and this change

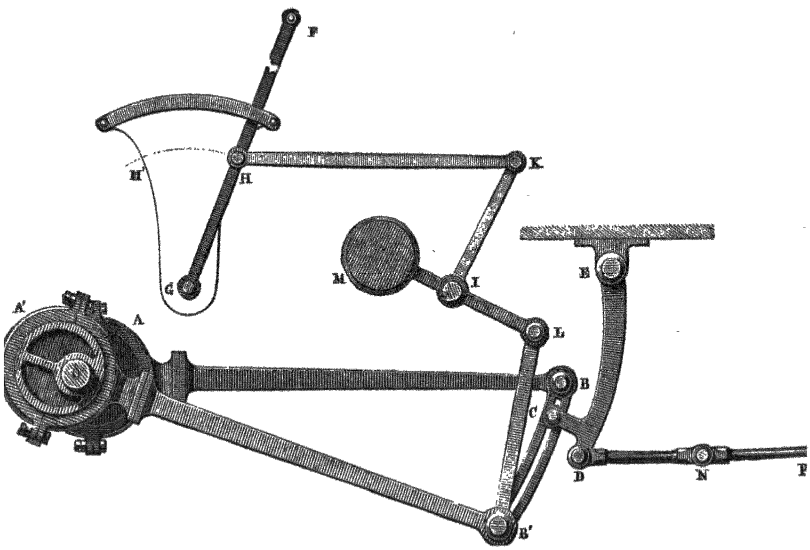


Fig 138.—Link-motion

reverses the engine. When the link is exactly midway between the two extreme positions, the slide-valve is influenced by both eccentrics equally, and consequently remains nearly stationary in its middle position, so that no steam is admitted to the cylinder, and the engine stops. By keeping the link near the middle position, steam is admitted during only a small part of the stroke, and consequently undergoes large expansion. By moving it nearer to one of its extreme positions, the travel of the slide-valve is increased, the ports are opened wider and kept open longer, and the engine will accordingly be driven faster, but with less expansion of the steam. As a means of regulating expansion, the link-motion is far from perfect, but its general advantages are such that it has come into very extensive use, not only for locomotives but for all engines which need reversal.

266. Gas-engines.—In Otto's "Silent Gas-engine," the earliest type of the class of engine now employed, a dilute mixture of gas and air (about one part in twelve being gas) is admitted into the cylinder, and, after being compressed to about three atmospheres, is ignited by instantaneous communication with a small jet of gas kept constantly burning. The effect is something intermediate between ignition and explosion; the maximum pressure in the early part of the stroke being 10 or 12 atmospheres, and the mean pressure in the whole stroke 4 or 5. In the return stroke, the products of combustion escape at atmospheric pressure, this return stroke being effected by the momentum of the fly-wheel, which also carries the piston through another forward stroke during which the charge of gas and air is admitted, and through another backward stroke in which it is compressed previous to ignition as above described.

This is the ordinary cycle of operations when the engine is working up to the full power for which it is intended; but a centrifugal governor is provided which prevents the gas from being admitted oftener than is necessary for keeping up the standard number of revolutions per minute; so that in working far below its full power the gas is only admitted at every third, fourth, or fifth stroke, the intervening strokes being maintained by the fly-wheel. The governor can be regulated to give any speed required, the most usual being 170 revolutions per minute; and the difference of speed between full work and running idle is only one or two revolutions.

The general appearance of the engine is shown in Fig. 139. A is the cylinder, with a jacket round it through which a convective circulation of water is maintained by means of two pipes, not shown in the figure, connecting it with a tank at a higher level. This is necessary to prevent overheating. C is the centrifugal governor. B, D are two vessels containing oil with automatic lubricators, B lubricates the piston, and D the slide which controls the ignition of the charge. E is a chimney, in the lower part of which the gas jet is kept burning. F is a spring fastening, which keeps the slide strongly pressed home so as to prevent leakage. The connecting-rod, crank, and heavy fly-wheel speak for themselves.

Gas-engines have a great advantage in being constantly ready for use without the tedious process of getting up steam. They are started by lighting the gas jet and giving a few quick turns to the fly-wheel; and are stopped by turning out the jet. The usual sizes are from $\frac{1}{2}$ to 20 horse-power. They are easily kept in order,

the principal trouble consisting in the removal of a hard deposit of carbon which forms in certain places.

Gas-engines and triple-expansion steam-engines resemble each other in having a very high initial temperature of the expanding

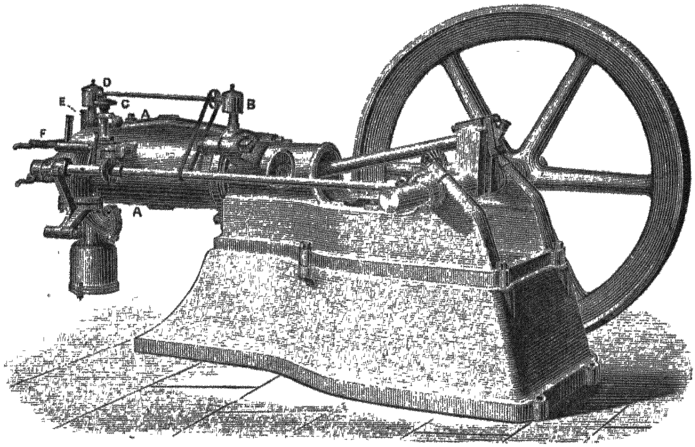


Fig 139.—Otto's Silent Gas-engine

fluid which drives the piston, and they resemble each other also in possessing a high degree of "efficiency;" that is to say, they convert a much larger fraction of the heat received into mechanical effect than the simpler forms of steam-engine. But even in the most favourable circumstances this fraction has never, we believe, exceeded one-seventh or thereabouts; so that, from a theoretical point of view, there is still large scope for improvement.

CHAPTER XVIII.

TERRESTRIAL TEMPERATURES AND WINDS.

267. **Temperature of the Air.**—By the *temperature of a place* meteorologists commonly understand the *temperature of the air* at a moderate distance (5 or 10 feet) from the ground. This element is easily determined when there is much wind; but in calm weather, and especially when the sun is shining powerfully, it is often difficult to avoid the disturbing effect of radiation. Thermometers for observing the temperature of the air must be sheltered from rain and sunshine, but exposed to a free circulation of air.

268. **Mean Temperature of a Place.**—The *mean temperature of a day* is obtained by making numerous observations at equal intervals of time throughout the day (24 hours), and dividing the sum of the observed temperatures by their number. The accuracy of the determination is increased by increasing the number of observations; as the mean temperature, properly speaking, is the mean of an infinite number of temperatures observed at infinitely short intervals.

If the curve of temperature for the day is given, temperature being represented by height of the curve above a horizontal datum line, the mean temperature is the height of a horizontal line which gives and takes equal areas; or is the height of the middle point of any straight line (terminated by the extreme ordinates of the curve) which gives and takes equal areas.

Attempts have been made to lay down rules for computing the mean temperature of a day from two, three, or four observations at stated hours; but such rules are of very limited application, owing to the different character of the diurnal variation at different places; and at best they cannot pretend to give the temperature of an individual day, but merely results which are correct in the long run. Observations at 9 A.M. and 9 P.M. are very usual in this country; and

the half-sum of the temperatures at these hours is in general a good approximation to the mean temperature of the day. The half-sum of the highest and the lowest temperature in the day, as indicated by maximum and minimum thermometers, is often adopted as the mean temperature. The result thus obtained is usually rather above the true mean temperature, owing to the circumstance that the extreme heat of the day is a more transient phenomenon than the extreme cold of the night. The employment of self-registering thermometers has, however, the great advantage of avoiding errors arising from want of punctuality in the observer. The correction which is to be added or subtracted in order to obtain the true mean from the mean of two observations is called a *correction for diurnal range.* Its amount differs for different places, being usually greatest where the diurnal range itself is greatest.

The *mean temperature of a calendar month* is computed by adding the mean temperatures of the days which compose it, and dividing by their number.

The *mean temperature of a year* is usually computed by adding the mean temperatures of the calendar months, and dividing by 12; but this process is not quite accurate, inasmuch as the calendar months are of unequal length. A more accurate result is obtained by adding the mean temperatures of all the days in the year, and dividing by 365 (or in leap-year by 366).

269. **Isothermals.**—The distribution of temperature over a large region is very clearly represented by drawing upon the map of this region a series of *isothermal lines;* that is, lines characterized by the property that *all places on the same line have the same temperature.* These lines are always understood to refer to mean annual temperture unless the contrary is stated; but isothermals for particular months, especially January and July, are frequently traced, one serving to show the distribution of temperature in winter, and the other in summer. The first extensive series of isothermals was drawn by Humboldt in 1817, on the basis of a large number of observations collected from all parts of the world; and the additional information which has since been collected has not materially altered the forms of the lines traced by him upon the terrestrial globe. They are in many places inclined at a very considerable angle to the parallels of latitude; and nowhere is this deviation from parallelism more observable than in the neighbourhood of Great Britain, Norway, and Iceland—places in this region having the same mean

annual temperature as places in Asia or America lying from 10° to 20° further south.

270. Insular and Continental Climates.—We have seen that the specific heat of water, the latent heat of liquid water, and the latent heat of aqueous vapour are all very large. The presence of water accordingly exerts a powerful effect in moderating the extremes both of heat and cold, and a moist climate will in general have a smaller range of temperature than a dry climate. Moreover, since earth and rock are opaque to radiant heat, while water is to a considerable extent diathermanous, the surface of the ground is much more quickly heated and cooled by radiation than the surface of water. This difference is increased by the continual agitation of the surface of the ocean. Large bodies of water thus act as equalizers of temperature, and the most equable climates are found on oceanic islands or on the ocean itself; while the greatest difference between summer and winter is found in the interior of large continents. It is common to distinguish in this sense between *continental* climates on the one hand, and *insular* or *marine* climates on the other.

Some examples of both kinds are given in the following table. The temperatures are Centigrade:—

MARINE CLIMATES.

	Winter	Summer.	Difference.
Faroe Islands,	3°·90	11°·60	7°·70
Isle of Unst (Shetland),	4 ·05	11 ·92	7 ·87
Isle of Man,	5 ·59	15 ·08	9 ·49
Penzance,	7 ·04	15 ·83	8 ·79
Helston,	6 ·19	16 ·00	9 ·81

CONTINENTAL CLIMATES.

	Winter	Summer.	Difference.
St. Petersburg,	− 8°·70	15°·96	24°·66
Moscow,	−10 ·22	17 ·55	27 ·77
Kasan,	−13 ·66	17 ·35	31 ·01
Slatoust,	−16 ·49	16 ·08	32 ·57
Irkutsk,	−17 ·88	16 ·00	33 ·88
Jakoutsk,	−38 ·90	17 ·20	56 ·10

271. Temperature of the Soil at Different Depths.—By employing thermometers with their bulbs buried in the earth, and their stems projecting above, numerous observations have been made of the temperature from day to day at different depths from 1 inch to 2 or 3 feet; and at a few places observations of the same kind have been made by means of gigantic spirit-thermometers with exceedingly strong

bulbs, at depths extending to about 25 feet. It is found that variations depending on the hour of the day are scarcely sensible at the depth of 2 or 3 feet, and that those which depend on the time of year decrease gradually as the depth increases, but still remain sensible at the depth of 25 feet, the range of temperature during a year at this depth being usually about 2° or 3° Fahrenheit.

It is also found that, as we descend from the surface, the seasons lag more and more behind those at the surface, the retardation amounting usually to something less than a week for each foot of descent; so that, at the depth of 25 feet in these latitudes, the lowest temperature occurs about June, and the highest about December.

Theory indicates that 1 foot of descent should have about the same effect on diurnal variations as $\sqrt{365}$ that is 19 feet on annual variations; understanding by *sameness of effect* equal *absolute amounts* of lagging and equal *ratios* of diminution.

As the annual range at the surface in Great Britain is usually about 3 times greater than the diurnal range, it follows that the diurnal range at the depth of a foot should be about one-third of the annual range at the depth of 19 feet.

The variations of temperature at the surface are, as every one knows, of a very irregular kind; so that the curve of surface temperature for any particular year is full of sinuosities depending on the accidents of that year. The deeper we go, the more regular does the curve become, and the more nearly does it approach to the character of a simple curve of sines, whose equation can be written

$$y = a \sin. x.$$

Neglecting the departures of the curve from this simple character, theory indicates that, if the soil be uniform, and the surface plane, the annual range (which is equal to $2\,a$) goes on diminishing in geometrical progression as the depth increases in arithmetical; and observation shows that, if 10 feet be the common difference of depth, the ratio of decrease for range is usually about $\frac{1}{2}$ or $\frac{1}{3}$.

To find a range of a tenth of a degree Fahrenheit, we must go to a depth of from 50 to 80 feet in this climate. At a station where the surface range is double what it is in Great Britain, we should find a range of about two-tenths of a degree at a depth and in a soil which would here give one-tenth.

These remarks show that the phrase "stratum of invariable temperature," which is frequently employed to denote the supposed

lower boundary of the region in which annual range is sensible, has no precise significance, inasmuch as the boundary in question will vary its depth according to the sensitiveness of the thermometer employed.

272. Increase of Temperature Downwards.—Observations in all parts of the world show that the temperature at considerable depths, such as are attained in mining and boring, is much above the surface temperature. In sinking a shaft at Rose Bridge Colliery, near Wigan, which is the deepest mine in Great Britain, the temperature of the rock was found to be 94° F. at the depth of 2440 feet. In cutting the Mont Cénis tunnel, the temperature of the deepest part, with 5280 feet of rock overhead, was found to be about 85° F.

The rate of increase downwards is by no means the same everywhere; but it is seldom so rapid as 1° F. in 40 feet, or so slow as 1° F. in 100 feet. The observations at Rose Bridge show a mean rate of increase of about 1° in 55 feet; and this is about the average of the results obtained at other places.

This state of things implies a continual escape of heat from the interior of the earth by conduction, and the amount of this loss per annum can be approximately calculated from the absolute values of conductivity of rock which we have given in Chap. xii.

There can be no reasonable doubt that the decrease of temperature upwards extends to the very surface, when we confine our attention to mean annual temperatures, for all the heat that is conducted up through a stratum at any given depth must also traverse all the strata above it, and heat can only be conducted from a warmer to a colder stratum. Professor Forbes found, at his three stations near Edinburgh, increases of 1°·38, 0°·96, and 0°·19 F. in mean temperature, in descending through about 22 feet, that is, from the depth of 3 to the depth of 24 French feet. The mean annual temperature of the surface of the ground is in Great Britain a little superior to that of the air above it, so far as present observations show. The excess appears to average about 1° F.

273. Decrease of Temperature Upwards in the Air.—In comparing the mean temperatures of places in the same neighbourhood at different altitudes, it is found that temperature diminishes as height increases, the rate of decrease for Great Britain, as regards mean annual temperature, being about 1° F. for every 300 feet. A decrease of temperature upwards is also usually experienced in balloon ascents, and numerous observations have been taken for the purpose of deter-

mining its rate. Mr. Glaisher's observations, which are the most numerous as well as the most recent, show that, upon the whole, the decrease becomes less rapid as we ascend higher; also, that it is less rapid with a cloudy than with a clear sky. The following table exhibits a few of Mr. Glaisher's averages:—

Height.	Decrease of Temperature Upwards With clear sky	With cloudy sky
From 0 to 1000 feet, . . .	1° F in 139 feet.	1° F. in 222 feet.
From 0 to 10,000 ft. . . .	1° F. in 288 feet.	1° F. in 331 feet.
From 0 to 20,000 ft. . . .	1° F. in 365 feet.	1° F. in 468 feet

These rates may be taken as representing the general law of decrease which prevails in the air over Great Britain in the daytime during the summer half of the year; but the results obtained on different days differ widely, and alternations of increase and decrease are by no means uncommon in passing upwards through successive strata of air. Still more recent observations by Mr. Glaisher, relating chiefly to the first 1000 feet of air, show that the law varies with the hour of the day. The decrease upwards is most rapid soon after midday, and is at this time, and during daytime generally, more rapid as the height is less. About sunset there is a uniform *decrease* at all heights if the sky is clouded, and a uniform *temperature* if the sky is clear. From a few observations which have been taken after sunset, it appears that, with a clear sky, there is an *increase* upwards at night.

That an extremely low temperature exists in the interplanetary spaces, may be inferred from the experimental fact recorded by Sir John Herschel, that a thermometer with its bulb in the focus of a reflector of sufficient size and curvature to screen it from lateral radiation, falls lower when the axis of the reflector is directed upwards to a clear sky than when it is directed either to a cloud or to the snow-clad summits of the Alps. The atmosphere serves as a protection against radiation to these cold spaces, and it is not surprising that, as we increase our elevation, and thus diminish the thickness of the coating of air above us, the protection should be found less complete. But probably the principal cause of the diminution of temperature upwards is the cooling of air by expansion, which we have discussed in § 223

274. **Causes of Winds.**—The influences which modify the direction and intensity of winds are so various and complicated that anything like a complete account of them can only find a place in treatises specially devoted to that subject. There is, however, one fundamental

principle which suffices to explain the origin of many well-known winds. This principle is plainly illustrated by the following experiment, due to Franklin. A door between two rooms, one heated, and the other cold (in winter), is opened, and two candles are placed, one at the top, and the other at the bottom of the doorway. It is found that the flame of the lower candle is blown towards the heated room, and that of the upper candle away from it.

The principle which this experiment illustrates may be stated as follows:—*When two neighbouring regions are at different temperatures, a current of air flows from the warmer to the colder in the upper strata of the atmosphere; and in the lower strata a current flows from the colder to the warmer.* The reason is that variation of pressure with height is greater in the cold than in the hot region; so that if there be one level at which the pressure is the same in both, the pressure in the cold region will preponderate at lower and that in the hot region at higher levels. We proceed to apply this principle to the land and sea breezes, the monsoons, and the trade-winds.

275. Land and Sea Breezes.—At the sea-side during calm weather a wind is generally observed to spring up at about eight or nine in the morning, blowing from the sea, and increasing in force until about two or three in the afternoon. It then begins gradually to die away, and shortly before sunset disappears altogether. A few hours afterwards, a wind springs up in the opposite direction, and lasts till nearly sunrise. These winds, which are called the sea-breeze and land-breeze, are exceedingly regular in their occurrence, though they may sometimes be masked by other winds blowing at the same time. Their origin is very easily explained. During the day the land grows warmer than the water; hence there results a wind blowing towards the warmer region, that is, towards the land. During the night the land and sea both grow colder, but the former more rapidly than the latter; and, accordingly, the relative temperatures of the two elements being now reversed, a breeze blowing from the land towards the sea is the consequence.

Monsoons.—The same cause which, on a small scale, produces the diurnal alternation of land and sea breezes, produces, on a larger scale, the annual alternation of monsoons in the Indian Ocean, and the seasonal winds which prevail in some other parts of the world. The general direction of these winds is towards continents in summer and away from them in winter.

276. Trade-winds: General Atmospheric Circulation.—The trade-

winds are winds which blow constantly from a north-easterly quarter over a zone of the northern hemisphere extending from a little north of the tropic of Cancer to within 9 or 10 degrees of the equator; and from a south-easterly quarter over a zone of the southern hemisphere extending from about the tropic of Capricorn to the equator. Their limits vary slightly according to the time of year, changing in the same direction as the sun's declination. Between them is a zone some 5° or 6° wide, over which calms and variable winds prevail.

The cause of the trade-winds was first correctly indicated by Hadley. The greater power of the sun over the equatorial regions causes a continual ascent of heated air from them. This flows over to both sides in the upper regions of the atmosphere, and its place is supplied by colder air flowing in from both sides below. If the earth were at rest, we should thus have a north wind sweeping over the earth's surface on the northern side of the equatorial regions, and a south wind on the southern side. But, in virtue of the earth's rotation, all points on the earth's surface are moving from west to east, with velocities proportional to their distances from the earth's axis. This velocity is nothing at the poles, and increases in approaching the equator. Hence, if a body on the earth's surface, and originally at rest relatively to the earth, be urged by a force acting along a meridian, it will not move along a meridian, but will outrun the earth, or fall behind it, according as its original rotational velocity was greater or less than those of the places to which it comes. That is to say, it will have a relative motion from the west if it be approaching the pole, and from the east if it be approaching the equator.

This would be true, even if the body merely tended to keep its original rotational velocity unchanged, and the reasoning becomes still more forcible when we apply the principle of conservation of angular momentum, in virtue of which the body tends to increase[1] its absolute rotational velocity in approaching the pole, and to diminish it in approaching the equator.

Thus the currents of air which flow in from both sides to the equatorial regions, do not blow from due north and due south, but from north-east and south-east. There can be little doubt that, notwithstanding the variable character of the winds in the temperate and frigid zones, there is, upon the whole, a continual interchange of air between them and the intertropical regions, brought about by the permanent excess of temperature of the latter. Such an interchange,

[1] The tendency is for velocity to vary inversely as distance from the axis of rotation

when considered in conjunction with the difference in the rotational velocities of these regions, implies that the mass of air over an equatorial zone some 50° or 60° wide, must, upon the whole, have a motion from the east as compared with the earth beneath it; and that the mass of air over all the rest of the earth must, upon the whole, have a relative motion from the west. This theoretical conclusion is corroborated by the distribution of barometric pressure. The barometer stands highest at the two parallels which, according to this theory, form the boundaries between easterly and westerly winds, while at the equator and poles it stands low. This difference may be accounted for by the excess of centrifugal force possessed by west winds, and the defect of centrifugal force in east winds. If the air simply turned with the earth, centrifugal force combined with gravity would not tend to produce accumulation of air over any particular zone, the ellipticity of the earth being precisely adapted to an equable distribution. But if a body of air or other fluid is moving with sensibly different rotational velocity from the earth, the difference in centrifugal force will give a tendency to move towards the equator, or from it, according as the differential motion is from the west or from the east. The easterly winds over the equatorial zone should therefore tend to remove air from the equator and heap it up at the limiting parallels; and the westerly winds over the remainder of the earth should tend to draw air away from the poles and heap it up at the same limiting parallels. This theoretical consequence exactly agrees with the following table of mean barometric heights in different zones given by Maury:[1]—

North Latitude.	Barometer.	South Latitude.	Barometer.
0° to 5°	29·915	0° to 5°	29·940
5° to 10°	29·922	5° to 10°	29·981
10° to 15°	29·964	10° to 15°	30·028
15° to 20°	30·018	15° to 20°	30·060
20° to 25°	30·081	20° to 25°	30·102
25° to 30°	30·149	25° to 30°	30·095
30° to 35°	30·210	30° to 36°	30·052
35° to 40°	30·124	42° 53′	29·90
40° to 45°	30·077	45° 0′	29·66
45° to 50°	30·060	49° 8′	29·47
51° 29′	29·99	51° 33′	29·50
59° 51′	29·88	54° 26′	29·35
78° 37′	29·759	55° 52′	29·36
		60° 0′	29·11
		66° 0′	29·08
		74° 0′	28·93

[1] *Physical Geography and Meteorology of the Sea*, p. 180, art. 362, edition 1860.

This table shows that the barometric height falls off regularly on both sides from the two limiting zones 30° to 35° N. and 20° to 25° S., the fall continuing towards both poles as far as the observations extend, and continuing inwards to a central minimum between 0° and 5° N.

If the bottom of a cylindrical vessel of water be covered with saw-dust, and the water made to rotate by stirring, the saw-dust will be drawn away from the edges, and heaped up in the middle, thus showing an indraught of water along the bottom towards the region of low barometer in the centre. It is probable that, from a similar cause (a central depression due to centrifugal force), there is an indraught of air along the earth's surface towards the poles, underneath the primary circulation which our theory supposes; the diminution of velocity by friction against the earth, rendering the lowest portion of the air obedient to this indraught, which the upper strata are enabled to resist by the centrifugal force of their more rapid motion. This, according to Professor James Thomson,[1] is the explanation of the prevalence of south-west winds in the north temperate zone; their southerly component being due to the barometric indraught and their westerly component to differential velocity of rotation. The indraught which also exists from the limiting parallels to the region of low barometer at the equator, coincides with the current due to difference of temperature; and this coincidence may be a main reason of the constancy of the trade-winds.

277. **Origin of Cyclones.**—In the northern hemisphere a wind which would blow towards the north if the earth were at rest, does actually blow towards the north-east; and a wind which would blow towards the south blows towards the south-west. In both cases, the earth's rotation introduces a component towards the right with reference to a person travelling with the wind. In the southern hemisphere it introduces a component towards the left.

Again, a west wind has an excess of centrifugal force which tends to carry it towards the equator, and an east wind has a tendency to move towards the pole; so that here again, in the northern hemi-

[1] The fullest and clearest account, historical and expository, of the theory of general atmospheric circulation is the Bakerian Lecture by Professor James Thomson, *Phil Trans*, 1892. Some calculations bearing on the subject will be found in a paper by the editor of the present work in *Phil. Mag.*, Sept. 1871.

sphere the deviation is in both cases to the right, and in the southern hemisphere to the left.

We have thus an explanation of cyclonic movements. In the northern hemisphere, if a sudden diminution of pressure occurs over any large area, the air all around for a considerable distance receives an impetus directed towards this area. But, before the converging streams can meet, they undergo deviation, each to its own right, so that, instead of arriving at their common centre, they blow tangentially to a closed curve surrounding it, and thus produce an eddy from right to left with respect to a person standing in the centre. This is the universal direction of cyclonic rotation in the northern hemisphere; and the opposite rule holds for the southern hemisphere. The former is opposite to, the latter the same as the direction of motion of the hands of a watch lying with its face up. In each case the motion is opposite to the apparent diurnal motion of the sun for the hemisphere in which it occurs.

278. Anemometers.—Instruments for measuring either the force or the velocity of the wind are called *anemometers*. Its force is usually measured by Osler's anemometer, in which the pressure of the wind is received upon a square plate attached to one end of a spiral spring (with its axis horizontal), which yields more or less according to the force of the wind, and transmits its motion to a pencil which leaves a trace upon paper moved by clock-work. It seems that the force received by the plate is not rigorously proportional to its size, and that a plate a yard square receives rather more than 9 times the pressure of a plate a foot square. The anemometer which has yielded the most satisfactory results is that invented by the Rev. Dr. Robinson of Armagh, which is represented in Fig. 140, and which indicates the velocity of the wind. It consists of four hemispherical cups attached to the ends of equal horizontal arms, forming a horizontal cross, which turns freely about a vertical axis. By means of an endless screw carried by the axis, a train of wheel-work is set in motion; and the indication is given by a hand which moves round a dial; or, in some instruments, by several hands moving round different dials like those of a gas-meter. The anemometer can also be made to leave a continuous record on paper, for which purpose various contrivances have been successfully employed. It was calculated by the inventor, and confirmed by his own experiments both in air and water, as well as by experiments conducted by Prof. C. Piazzi Smyth at Edinburgh, and more

recently by Sir George Airy at Greenwich, that the centre of each cup moves with a velocity which is almost exactly one-third of that of the wind. This is the only velocity - anemometer whose indications are exactly proportional to the velocity itself. Dr. Whewell's anemometer, which resembles a small windmill, is very far from fulfilling this condition, its variations of velocity being much less than those of the wind.

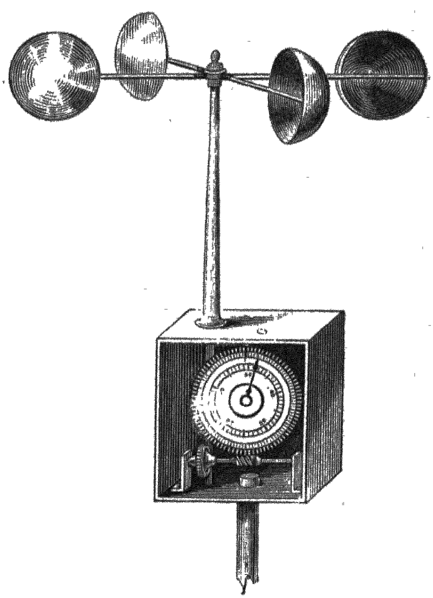


Fig 140 - Robinson's Anemometer

The direction of the wind, as indicated by a vane, can also be made to leave a continuous record by various contrivances; one of the most common being a pinion carried by the shaft of the vane, and driving a rack which carries a pencil. But perhaps the neatest arrangement for this purpose is a large screw with only one thread composed of a metal which will write on paper. A sheet of paper is moved by clock-work in a direction perpendicular to the axis of the screw, and is pressed against the thread, touching it of course only in one point, which travels parallel to the axis as the screw turns, and comes back to its original place after one revolution. When one end of the thread leaves the paper, the other end at the same instant comes on. The screw turns with the vane, so that a complete revolution of the screw corresponds to a complete revolution of the wind. This is one of the many ingenious contrivances devised and executed by Mr. Beckley, mechanical assistant in Kew Observatory.

279. **Oceanic Currents.**—The general principle of § 274 applies to liquids as well as to gases; though the effects are usually smaller, owing to their smaller expansibility.

The warm water in the equatorial regions overflows towards the poles, and an under-current of cold water which has descended in the polar regions flows towards the equator. Recent observations

have shown that a temperature not much above 0° C. prevails at the bottom of the ocean even between the tropics. A very gradual circulation is thus produced on a very large scale.

The rapid currents which are observed on some parts of the surface of the ocean are probably due to wind. Among these may be mentioned the Gulf Stream. This current of warm water forms a kind of immense river in the midst of the sea, differing in the temperature, saltness, and colour of its waters from the medium in which it flows. Its origin is in the Gulf of Mexico, whence it issues through the straits between the Bahamas and Florida, turns to the northwest, and splits into two branches, one of which goes to warm the coasts of Ireland and Norway, the other gradually turns southwards, traverses the Atlantic from north to south, and finally loses itself in the regions of the equator.

"The Gulf Stream is a river in the ocean; in the severest droughts it never fails, and in the mightiest floods it never overflows; its banks and its bottom are of cold water, while its current is of warm; it takes its rise in the Gulf of Mexico, and empties into Arctic seas. There is on earth no other such majestic flow of waters. Its current is more rapid than the Mississippi or the Amazon, and its volume more than a thousand times greater. Its waters, as far out from the Gulf as the Carolina coasts, are of indigo blue. They are so distinctly marked that their line of junction with the common sea-water may be traced by the eye. Often one-half of the vessel may be perceived floating in Gulf Stream water, while the other half is in common water of the sea, so sharp is the line."—(Maury, *Physical Geography of the Sea.*)

It would appear that an accumulation of water is produced in the Gulf of Mexico by the trade-wind which blows steadily towards it over the South Atlantic, and that the elevation of level thus occasioned is the principal cause of the Gulf Stream.

EXAMPLES.

[*The Centigrade Scale is employed, except where otherwise stated.*]

Scales of Temperature.

1. The difference between the temperatures of two bodies is 30° F. Express this difference in degrees Cent. and in degrees Réau.

2. The difference between the temperatures of two bodies is 12° C. Express this difference in degrees Réau. and in degrees Fahr.

3. The difference between the temperatures of two bodies is 25° R. Express this difference in the Cent. and Fahr. scales.

4. Express the temperature 70° F. in the Cent. and Réau. scales.

5. Express the temperature 60° C. in the Réau. and Fahr. scales.

6. Express the temperature 30° R. in the Cent. and Fahr. scales.

7. Air expands by ·00366 of its volume at the freezing-point of water for each degree Cent. By how much does it expand for each degree Fahr.?

8. The temperature of the earth increases by about one degree Fahr. for every 50 feet of descent. How many feet of descent will give an increase of 1° Cent., and how many centimetres of descent will give an increase of 1° Cent., the foot being 30·48 cm.?

9. The mean annual range of temperature at a certain place is 100° F. What is this in degrees Cent.?

10. Lead melts at 326° C., and in melting absorbs as much heat as would raise 5·37 times its mass of water 1° C. What numbers will take the place of 326 and 5·37 when the Fahrenheit scale is employed?

11. Show that the temperature $-40°$ C. and the temperature $-40°$ F. are identical.

12. What temperature is expressed by the same number in the Fahr. and Réau. scales?

Expansion.

The following coefficients of expansion can be used:—

Linear.		Cubical.	
Steel,	·0000116	Glass,	·000024
Copper,	0000172	Mercury,	·000179
Brass,	·0000188	Alcohol,	·001050
Glass,	0000080	Ether,	·001520

13. The correct length of a steel chain for land measuring is 66 ft. Express, as a decimal of an inch, the difference between the actual lengths of such a chain at 0° and 20°.

14. One brass yard-measure is correct at 0° and another at 20°. Find, as a decimal of an inch, the difference of their lengths at the same temperature.

15. A lump of copper has a volume 258 cc. at 0°. Find its volume at 100°.

16. A glass vessel has a capacity of 1000 cc. at 0°. What is its capacity at 10°?

17. A weight-thermometer contains 462 gm. of a certain liquid at 0° and only 454 gm. at 20°. Find the mean relative expansion per degree between these limits.

18. A weight-thermometer contains 325 gm. of a liquid at zero, and 5 gm. run out when the temperature is raised to 12°. Find the mean coefficient of apparent expansion.

19. If the coefficient of relative expansion of mercury in glass be $\frac{1}{6500}$, what mass of mercury will overflow from a weight-thermometer which contains 650 gm. of mercury at 0° when the temperature is raised to 100°?

20. The capacity of the bulb of a thermometer together with as much of the stem as is below zero is ·235 cc. at 0°, and the section of the tube is $\frac{1}{2000}$ sq. cm. Compute the length of a degree (1), if the fluid be mercury; (2), if it be ether.

21. The bulb, together with as much of the stem as is below the zero-point, contains 3·28 gm. of mercury at zero, and the length of a degree is ·1 cm. Compute the section of the tube, the density of mercury being about 13·6.

22. What will be the volume at 300° of a quantity of gas which occupies 1000 cc. at 0°, the pressure being the same?

23. What will be the volume at 400° of a quantity of gas which occupies 1000 cc. at 100°, the pressure being the same?

24. What will be the pressure at 30° of a quantity of gas which at 0° has a pressure of a million dynes per sq. cm., the gas being confined in a close vessel whose expansion may be neglected?

25. A thousand cc. of gas at 1·0136 million dynes per sq. cm. are allowed to expand till the pressure becomes a million dynes per sq. cm., and the temperature is at the same time raised from its initial value 0° to 100°. Find the final volume.

26. A gas initially at volume 4500 cc., temperature 100°, and a pressure represented by 75 cm. of mercury, has its pressure increased by 1 cm. of mercury and its temperature raised to 200°. Find its final volume.

27. At what temperature will the volume of a gas at a pressure of a million dynes per sq. cm. be 1000 cc., if its volume at temperature 0° and pressure 1·02 million dynes per sq. cm. be 1200 cc.?

28. What temperature on the Fahrenheit scale is the absolute zero of the air-thermometer?

29. Find the coefficient of expansion of air per degree Fahrenheit, when 0° F. is the starting-point.

30. Express the freezing-point and boiling-point of water as absolute temperatures Fahrenheit.

31. What is the interior volume at 0° C. of a glass bulb which at 25° C. is exactly filled by 53 grammes of mercury?

For Densities of Gases see p. 50.

32. At what temperature does a litre of dry air at 760 mm. weigh 1 gramme?

33. At what temperature will the density of oxygen at the pressure 0·20 m. be the same as that of hydrogen at 0° C., at the pressure 1·60 m.?

[The tabulated densities are proportional to the values of $\frac{DT}{P}$ for the different gases.]

34. What must be the pressure of air at 15°, that its density may be the same as that of hydrogen at 0° and 760 mm.?

35. A mercurial barometer with brass scale reads at one time 770 mm. with a temperature 85°, and at another time 760 mm. with a temperature 5°. Find the ratio of the former pressure to the latter.

36. The normal density of air being ·000154 of that of brass, what change is produced in the force required to sustain a kilogramme of brass in air, when the pressure and temperature change from 713 mm. and $-19°$ to 781 mm. and $+36°$?

37. A cylindrical tube of glass is divided into 300 equal parts. It is loaded with mercury, and sinks to the 50th division from the top in water at 10°. To what division will it sink in water at 50°, the volumes of a given mass of water at these temperatures being as 1·000268 to 1·01205?

38. A closed globe, whose external volume at 0° is 10 litres, is immersed in air at 15° and at a pressure of 0·77 m. Required (1) the loss of weight which it experiences from the action of the air; (2) the change which this loss would undergo if the pressure became 0·768 m. and the temperature 17°.

39. A brass tube contains mercury, with a piece of platinum immersed in it; and the level of the liquid is marked by a scratch on the inside of the tube. On applying heat, it is found that the liquid still stands at this mark. Deduce the ratio of the weight of the platinum to that of the mercury, assuming the density of platinum to be 21·5, and its linear expansion ·00001 per degree.

40. A glass tube, closed at one end and drawn out at the other, is filled with dry air, and raised to a temperature x at atmospheric pressure. It is then hermetically sealed. When it has been cooled to the temperature 100° C., it is inverted over mercury, and its pointed end is broken off beneath the surface of the liquid. The mercury rises to the height of 19 centimetres in the tube, the external pressure remaining at 76 cm. as at the commencement of the experiment. The tube is re-inverted, and weighed with the mercury which it contains. The weight of this mercury is found to be 200 grammes; when completely full it contains 300 grammes of mercury. Deduce the temperature x.

41. A glass tube, whose interior is a right circular cylinder, 2 millimetres in diameter at 0° C., contains a column of mercury, whose length at this temperature is 2 decim. What will be the length of this column of mercury when the temperature is 80° C.?

42. Some dry air is inclosed in a horizontal thermometric tube, by means of an index of mercury. At 0° C. and 0·760 m. the air occupies 720 divisions of the tube, the tube being divided into parts of equal capacity. At an unknown temperature and pressure the same air occupies 960 divisions. The tube being immersed in melting ice, and the latter pressure being still maintained, the air occupies 750 divisions. Required the temperature and pressure.

43. A Graham's compensating pendulum is formed of an iron rod, whose length at 0° C. is l, carrying a cylindrical vessel of glass, which at the same temperature has an internal radius r, and height h. Find the depth x of mercury at 0° C. which is necessary for compensation, supposing that the compensation consists in keeping the centre of gravity of the mercury at a constant distance from the axis of suspension.

Thermal Capacity.

The following values of specific heat can be used:—

Iron,	·1098	Mercury,	·033
Copper,	·0949	Alcohol,	·548
Platinum,	·0335	Ether,	·529
Sand,	·215	Air, at constant pressure,	·2375
Ice,	·504		

44. 17 parts by mass of water at 5° are mixed with 23 parts at 12°. Find the resulting temperature.

45. 200 gm. of iron at 300° are immersed in 1000 gm. of water at 0°. Find the resulting temperature.

46. Find the specific heat of a substance 80 gm. of which at 100°, when immersed in 200 gm. of water at 10° give a resulting temperature of 20°.

47. 16 parts by mass of sand at 75°, and 20 of iron at 45° are thrown into 50 of water at 4°. Find the temperature of the mixture.

48. 300 gm. of copper at 100° are immersed in 700 gm. of alcohol at 0°. Find the resulting temperature.

49. If the length, breadth, and height of a room are respectively 6, 5, and 3 metres, how many gramme-degrees of heat will be required to raise the temperature of the air which fills the room by 20°, the pressure of the air being constant, and its average density ·00128 gm. per cubic centimetre?

50. Find the thermal capacities of mercury, alcohol, and ether per unit volume, their densities being respectively 13·6, ·791, and ·716.

Latent Heat.

The following values of latent heat can be used:—

In Melting.		In Evaporation at Atmospheric Pressure.	
Water,	80	Steam,	536
Lead,	5·4		

51. Find the result of mixing 5 gm. of snow at 0° with 23 gm. of water at 20°.

52. Find the result of mixing 6 parts (by mass) of snow at 0° with 7 of water at 50°.

53. Find the result of mixing 3 parts by mass of snow at $-10°$ with 8 of water at 40°.

54. Find the result of mixing equal masses of snow at $-10°$ and water at 60°.

55. Find the temperature obtained by introducing 10 gm. of steam at 100° into 1000 gm. of water at 0°.

56. Lead melts at 326°. Its specific heat is ·0314 in the solid, and ·0402 in the liquid state. Find what mass of water at 0° will be raised one-tenth of a degree by dropping into it 100 gm. of melted lead at 350°.

57. What mass of mercury at 0° will be raised 1° by dropping into it 150 gm. of lead at 400°?

58. A litre of alcohol, measured at 0° C., is contained in a brass vessel weighing 100 grammes, and after being raised to 58° C., is immersed in a kilogramme

of water at 10° C., contained in a brass vessel weighing 200 grammes. The temperature of the water is thereby raised to 27°. What is the specific heat of alcohol? The specific gravity of alcohol is 0·8; the specific heat of brass is 0·1.

59. A copper vessel, weighing 1 kilogramme, contains 2 kilogr. of water. A thermometer composed of 100 grammes of glass and 200 gr. of mercury, is completely immersed in this water. All these bodies are at the same temperature, 0° C. If 100 grammes of steam at 100° C. are passed into the vessel, and condensed in it, what will be the temperature of the whole apparatus when equilibrium has been attained, supposing that there is no loss of heat externally. The specific heat of mercury is 0·033; of copper, 0·095; of glass, 0·177.

Various.

60. A truly conical vessel contains a certain quantity of mercury at 0° C. To what temperature must the vessel and its contents be raised that the depth of the liquid may be increased by $\frac{1}{180}$ of itself?

61. There is a bent tube, terminating at one end in a large bulb, and simply closed at the other. A column of mercury stands at the same height in the two branches, and thus separates two quantities of air at the same pressure. The air in the bulb is saturated with moisture; that in the opposite branch is perfectly dry. The length of the column of dry air is known, and also its initial pressure, the temperature of the whole being 0° C. Calculate the displacement of the mercurial column when the temperature of the apparatus is raised to 100° C. The bulb is supposed to have enough water in it to keep the air constantly saturated; and is also supposed to be so large that the volume of the moist air is not sensibly affected by the displacement of the mercurial column.

Conduction.

(*Units the centimetre, gramme, and second.*)

62. How many gramme-degrees of heat will be conducted in an hour through each sq. cm of an iron plate ·02 cm. thick, its two sides being kept at the respective temperatures 225° and 275°, and the mean conductivity of the iron between these temperatures being ·12?

63. Through what thickness of copper would the same amount of heat flow as through the ·02 cm. of iron in the preceding question, with the same temperatures of its two faces, the mean conductivity of the copper between these temperatures being unity?

64. How much heat will be conducted in an hour through each sq. cm. of a plate of ice 2 cm. thick, one side of the ice being at 0° and the other at −3°, and its conductivity being ·00223; and what volume of water at 0° would be converted into ice at 0° by the loss of this quantity of heat?

65. How much heat will escape in an hour from the walls of a building, if their area be 80 sq. metres, their thickness 20 cm., their material sandstone of conductivity ·01, and the difference of temperature between outside and inside 15°? What quantity of charcoal burned per hour would generate heat equal to this loss? [see p. 220.]

Hygrometry.

66. A cubic metre of air at 20° is found to contain 11·56 grammes of aqueous vapour. What is the relative humidity of this air, the maximum pressure of vapour at 20° being 17·39 mm.?

67. Calculate the weight of 15 litres of air saturated with aqueous vapour at 20° and 750 mm.

Thermodynamics.

For the value of Joule's equivalent see § 205.

For heats of combustion see § 230.

68. The labour of a horse is employed for 3 hours in raising the temperature of a million grammes of water by friction. What elevation of temperature will be produced, supposing the horse to work at the rate of 6×10^9 ergs per second?

69. From what height (in cm.) must mercury fall at a place where g is 980, in order to raise its own temperature 1° by the destruction of the velocity acquired, supposing no other body to receive any of the heat thus generated?

70. With what velocity (in cm. per sec.) must a leaden bullet strike a target that its temperature may be raised 100° by the collision, supposing all the energy of the motion which is destroyed to be spent in heating the bullet?

71. What is the greatest proportion of the heat received by an engine at 200° that can be converted into mechanical effect, if the heat which is given out from the engine is given out at the temperature 10°?

72. If a perfect engine gives out heat at 0°, at what temperature must it take in heat that half the heat received may be converted?

73. What mass of carbon burned per hour would produce the same quantity of heat as the work of one horse for the same time, a horse-power being taken as 75×10^8 ergs per second.

74. A specimen of good coal contains 88 per cent. of carbon and $4\frac{1}{2}$ per cent. of hydrogen not already combined with oxygen. How many gramme-degrees of heat are generated by the combustion of 1 gm. of this coal; and with what velocity must a gramme of matter move that the energy of its motion may be equal to the energy developed by the combustion of the said gramme of coal?

75. Find the form of the isothermals for steam in contact with water.

76. In the cycle ABCD of § 215 or § 250, show that the volumes and pressures at A and B are proportional to those at D and C.

Adiabatic Compression and Extension.

77. Find the rise of temperature produced in water at 10° C. by an atmosphere of additional pressure, an atmosphere being taken as a million dynes per sq. cm., and the coefficient of expansion at this temperature being ·000092.

78. Find the ratio of the adiabatic to the isothermal resistance of water at 10° to compression, the value of the latter being $2·1 \times 10^{10}$ dynes per sq. cm.

79. Find the fall of temperature produced in a wrought iron bar by applying a pull of a million dynes per sq. cm. of section, the coefficient of expansion being ·0000122.

80. Find the ratio of the adiabatic to the isothermal resistance of the bar to extension, the value of the latter being $1·96 \times 10^{12}$ dynes per sq. cm.

ANSWERS TO EXAMPLES.

Ex. 1. $16°\frac{2}{3}$ C., $13°\frac{1}{3}$ R. Ex. 2. $9°\frac{3}{5}$ C., $21°\frac{3}{5}$ F. Ex. 3. $31°\frac{1}{4}$ C., $56°\frac{1}{4}$ F. Ex. 4. $21°\frac{1}{9}$ C., $16°\frac{8}{9}$ R. Ex. 5. 48° R., 140° F. Ex. 6. $37°\frac{1}{2}$ C., $99°\frac{1}{2}$ F. Ex. 7. ·00203. Ex. 8. 90 ft., 2743 cm. Ex. 9. $55°\frac{5}{9}$ C. Ex. 10. 619°, 9·666. Ex. 12. $-25·6$.

Ex. 13. ·184 in. Ex. 14. ·0135 in. Ex. 15. 259·33 cc. Ex. 16. 1000·24 cc. Ex. 17. ·000881. Ex. 18. ·001302. Ex. 19. $\frac{650}{66} = 9·85$ gm. Ex. 20. (1) ·073 cm., (2) ·703 cm. Ex. 21. ·000374 sq. cm.

Ex. 22. 2098 cc. Ex. 23. 1804 cc. Ex. 24. 1·1098 million. Ex. 25. 1385 cc. Ex. 26. 5631 cc. Ex. 27. $-50°$.

Ex. 28. $-459°$. Ex. 29. $\frac{1}{459}$. Ex. 30. 491°, 671°. Ex. 31. 3·913 cc. Ex. 32. 80° C. Ex. 33. 272°. Ex. 34. 55·5 mm. Ex. 35. 759·7 : 759·4. Ex. 36. $·155 - ·140 = ·015$ grammes of increase in the apparent weight.

Ex 37. 47·3. Ex. 38 Loss of 12·42 gm, diminished by ·12 gm. Ex. 39. The ratio of the platinum to the mercury is 4·6 to 1 by volume, and 7·3 to 1 by weight. Ex. 40. 1219°, neglecting expansions of glass and mercury.

Ex. 41. 2·026 decim. Ex. 42. 76°·5, ·7296 m. Ex. 43. $·15l + ·1h$.

Ex. 44. 9°·02. Ex. 45. 6°·44. Ex. 46. $\frac{5}{16} = ·3125$. Ex. 47. 10°. Ex. 48. 6°·91. Ex. 49. 547200. Ex. 50. ·449, ·433, ·379.

Ex. 51. Water at $2°\frac{1}{7}$. Ex. 52. $1\frac{5}{8}$ part snow, $11\frac{3}{8}$ water, all at zero. Ex. 53. Water at 5·9. Ex. 54. ·313 snow, 1·687 water, all at zero. Ex. 55. Water at 6°·3. Ex. 56. 16600 gm. nearly. Ex. 57. 84400 gm. nearly.

Ex. 58. ·687. Ex. 59. 28·7. Ex. 60. 88°. Ex. 61. The displacement x is given by the equation $2x = 753·7 - \frac{373}{273}\frac{px}{l-x}$, p and l being the given pressure and length.

Ex. 62 1080000. Ex. 63 $\frac{1}{6}$ cm. $= 166\dot{6}$ cm. Ex. 64. 12·04 gm.-deg., ·15 cc. Ex. 65 21600000 gm.-deg., 2673 gm

Ex. 66. 67 per cent. Ex. 67. 17·68 gm.

Ex. 68. 1°·56. Ex. 69. 1401 cm. Ex. 70. 16240 cm. per sec. Ex. 71. $\frac{120}{273} =$ ·4 nearly. Ex. 72. 273°. Ex. 73. 80 32 gm. Ex. 74. 8661 gm.-deg., 849000 cm. per sec. nearly.

Ex 75. Straight lines, because pressure is constant.

Ex. 76. If m denote the ratio of the temperature in A B to that in C D, the ratio of the two pressures either at A and D or at B and C is m raised to the power $s/(s-s')$, and the ratio of the volumes is $1/m$ to the power $s'/(s-s')$; see §§ 220, 250.

Ex. 77. 0°·000626. Ex. 78. 1·0012. Ex. 79. 0°·00009. Ex 80. 1 002.

PART III.

ELECTRICITY AND MAGNETISM.

ELECTROSTATICS.

CHAPTER I.

INTRODUCTORY PHENOMENA.

1. **Fundamental Phenomena.**—If a glass tube be rubbed with a silk handkerchief, both tube and rubber being very dry, the tube will be found to have acquired the property of attracting light bodies. If the part rubbed be held near to small scraps of paper, pieces of

Fig. 1.—Attraction of Light Bodies by an Electrified Body.

cut straw, sawdust, &c., these objects will move to the tube; sometimes they remain in contact with it, sometimes they are alternately attracted and repelled, the intensity as well as the duration of these effects varying according to the amount of friction to which the tube has been subjected.

If the tube be brought near the face, the result is a sensation similar

to that produced by the contact of a cobweb. If the knuckle be held near the tube, a peculiar crackling noise is heard, and a bright *spark* passes between the tube and knuckle. The tube then has acquired peculiar properties by the application of friction. It is said to be *electrified*, and the name of *electricity* is given to the agent to which the various phenomena just described are attributed.

Glass is not the only substance which can be electrified by friction; the same property is possessed also by resin, sulphur, precious stones, amber, &c. The Greek name of this last substance (ἤλεκτρον) is the root from which the word *electricity* is derived.

At first sight it appears that this property of becoming electrified by friction is not common to all bodies; for if a bar of metal be held in the hand and rubbed with wool, it does not acquire the properties

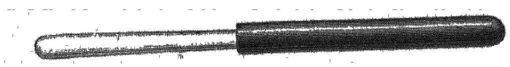


Fig 2.—Electrification of a Metal by Friction.

of an electrified body. But we should be wrong in concluding that metals cannot be electrified by friction; for if the bar be fitted on to a glass rod, and, while held by this handle, be struck with flannel or catskin, it may be very sensibly electrified. There is therefore no basis for the distinction formerly made between electrics and non-electrics, that is, between substances capable and incapable of being electrified by friction; for all bodies, as far as at present known, are capable of being thus excited. There is, however, an important difference of another kind between them, which was first pointed out by Stephen Grey in 1729.

2. **Conductors and Non-conductors.**—In certain bodies, such as glass and resin, electricity does not spread itself beyond the parts of the surface where it has been developed; while in other bodies, such as metals, the electricity developed at any point immediately spreads itself over the whole body. Thus, in the last-mentioned experiment, the signs of electricity are immediately manifested at the end of the metal bar which is farthest from the glass rod, if the end next the rod be submitted to friction. Bodies of the former kind, such as glass, resin, &c., are said to be *non-conductors*. Metals are said to be good *conductors*. A non-conductor is often called an *insulator*, and a conductor supported by a non-conductor is said to be *insulated*. The appropriateness of these expressions is evident. No substance is perfectly non-conducting, but the difference in conduct-

ing power between what are called non-conductors and good conductors, is enormous. The following are lists of conductors and non-conductors, arranged, at least approximately, in order of their conducting powers. In the list of conductors, the best conductors are put first; in the list of non-conductors, the worst conductors (or best insulators) are put first.

CONDUCTORS.

All metals.	Metallic ores.	Living vegetables.
Well-burned charcoal.	Animal fluids.	Flax.
Plumbago.	Sea water.	Hemp.
Concentrated acids.	Spring water	Living animals.
Dilute acids.	Rain water.	Flame.
Saline solutions.	Snow.	Moist earth and stones.

NON-CONDUCTORS.

Shellac.	Gems.	Leather.
Amber.	Ebonite.	Baked wood.
Resins.	Caoutchouc.	Porcelain.
Sulphur	Gutta-percha.	Marble.
Wax.	Silk.	Camphor.
Jet.	Wool.	Chalk.
Glass.	Feathers.	Lime.
Mica.	Dry paper.	Oils.
Diamond.	Parchment.	Metallic oxides.

The human body is a good conductor of electricity. If a person standing on a stool with glass legs be struck with a catskin, he becomes electrified in a very perceptible degree, and sparks may be drawn from any part of his body.

When an insulated and electrified conductor is allowed to touch another conductor insulated but not electrified, it is observed that, after the contact, both bodies possess electrical properties, electricity having been communicated to the second body at the expense of the first. If the second body be much the larger of the two, the electricity of the first is greatly diminished, and may become quite insensible. This explains the disappearance of electricity when a body is put in connection with the earth, which, together with most of the objects on its surface, may be regarded as constituting one enormous conductor. On account of its practically inexhaustible capacity for furnishing or absorbing electricity, the earth is often called *the common reservoir*.

It will now be easily understood why it is not possible to electrify a metal rod by rubbing it while it is held in the hand; since the

electricity, as fast as it is generated, passes off through the body into the earth.

Air, when thoroughly dry, is an excellent insulator; and electrified conductors exposed to it, and otherwise insulated, retain their charge with very little diminution for a considerable time. Dampness in the air is, however, a great obstacle to insulation, mainly, or (as it would appear from Sir W. Thomson's experiments) entirely, by reason of the moisture which condenses on the insulating supports. Electrical experiments are accordingly very difficult to perform in damp weather. The difficulty is sometimes met by employing a stove to heat the air in the neighbourhood of the supports, and thus diminish its relative humidity. Sir W. Snow Harris employed heating-irons, which were heated in a fire, and then fixed near the insulating supports; and thus succeeded in exhibiting electrical experiments to an audience in the most unfavourable weather. Sir W. Thomson, by keeping the air in the interior of his electrometers dry by means of sulphuric acid, causes them to retain their charge with only a small percentage of loss in twenty-four hours. Dry frosty days are the best for electrical experiments, and next perhaps to these, is the season of dry cutting winds in spring.

3. Duality of Electricity.—The elementary phenomena which we have mentioned in the beginning of this chapter may be more accurately studied by means of the electric pendulum, which consists of a pith-ball suspended by a silk fibre from an insulated support. When an electrified glass rod is brought near the insulated ball, the latter is attracted; but as soon as it touches the glass tube, the attraction is changed to repulsion, which lasts as long as the ball retains the electricity which it has acquired by the contact. A similar experiment can be shown by employing, instead of the glass tube, any other body which has been electrified by friction, for example, a piece of resin which has been rubbed with flannel.

If, while the pendulum exhibits repulsion for the glass, the electrified resin is brought near, it is attracted by the latter; and conversely, when it is repelled by the resin, it is attracted by the glass. These phenomena clearly show that the electricity developed on the resin is not of the same kind as that developed on the glass. They exhibit opposite forces towards any third electrified body, each attracting what the other repels. They have accordingly received names which indicate opposition. The electricity which glass acquires when rubbed with silk, is called *positive;* and that which resin acquires by friction

with flannel, *negative.* The former is also called *vitreous*, and the latter *resinous.* On repeating the experiment with other substances,

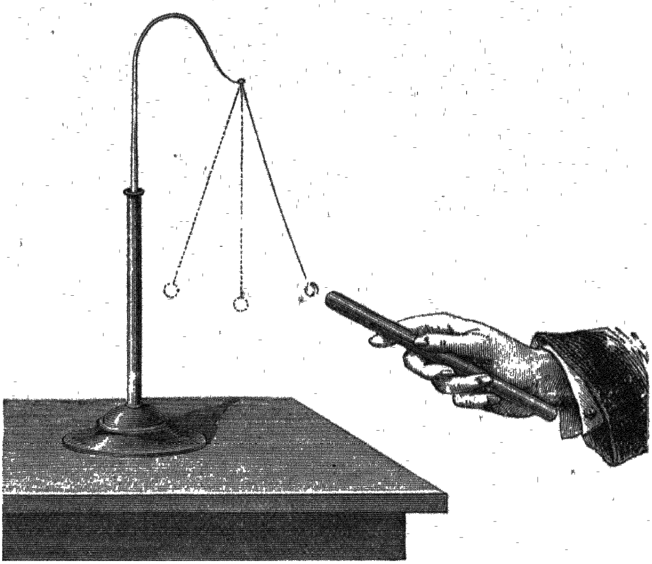


Fig 3—Electric Pendulum.

it is found that all electrified bodies behave either like the glass or like the resin.

4.—Without making any assumption as to what electricity is, we may speak of an electrified body as being *charged with electricity*, and we may compare quantities of electricity by means of the attractions and repulsions exerted. Bodies oppositely electrified must then be spoken of as charged with *electricities of opposite kind*, or *of opposite sign;* and experiment shows that, whenever electricity of the one kind is developed, whether by friction or by any other means, electricity of the opposite sign is always developed in exactly equal quantity. If a conductor receives two charges of electricity of equal quantity but opposite sign, it is found to exhibit no traces of electricity whatever.

Electricities of like sign repel one another and those of unlike sign attract one another.—The magnitude of the force exerted upon each other by two electrified bodies, is not altered in amount by reversing the sign of the electricity of one or both of them, provided that the quantities of electricity, and their distribution over the two

bodies, remain unchanged. If the sign of one only be changed, the mutual force is simply reversed, and if the signs of both be changed, the force is not changed at all.

5.—The simultaneous development of both kinds of electricity is illustrated by the following experiment:—Two persons stand on stools with glass legs, and one of them strikes the other with a catskin. Both of them are now found to be electrified, the striker positively, and the person struck negatively, and from both of them sparks may be drawn by presenting the knuckle.

The kind of electricity which a body obtains by friction with another body, evidently depends on the nature of their surfaces. If, for example, we take two discs, one of glass, and the other of metal, and, holding them by insulating handles, rub them briskly together, we shall find that the metal becomes negatively, and the glass positively electrified; but if the metal be covered with a catskin, and the experiment repeated, it will be the glass which will this time be negatively electrified. In the subjoined list, the substances are arranged in such order that, generally speaking, each of them becomes positively electrified by friction with those which follow it, and negatively with those which precede it.

Fur of cat.	Feathers.	Silk.
Polished glass.	Wood.	Shellac.
Woollen stuffs.	Paper.	Rough glass.

6. **Hypotheses regarding the Nature of Electricity.**—Two theories regarding the nature of electricity must be described on account of the historical interest attaching to them.

The two-fluid theory, originally propounded by Dufaye, reduced to a more exact form by Symmer, and still very extensively adopted, maintains that the opposite kinds of electricity are two fluids. Positive electricity is called the *vitreous fluid*, and negative electricity the *resinous fluid*. Fluids of like name repel, and those of unlike name attract each other. The union of equal quantities of the two fluids constitutes the neutral fluid which is supposed to exist in very large quantity in all unelectrified bodies. When a body is electrified, it gains an additional quantity of the one fluid, and loses an equal quantity of the other, so that the total amount of electric fluid in a body is never changed; and (as a consequence of this last condition) when a current of either fluid traverses a body in any direction, an equal current of the other fluid traverses it in the opposite direction.

This theory is in complete agreement with all electrical phenomena so far as at present known; but as it is conceivable that the two electricities, instead of being two kinds of matter, may be two kinds of motion, or, in some other way, may be opposite states of one and the same substance, it is more philosophical to avoid the assumption involved in speaking of *two electric fluids*, and to speak rather of *two opposite electricities*. They may be distinguished indifferently by the names *vitreous* and *resinous*, or *positive* and *negative*.

The *one-fluid theory*, as originally propounded by Franklin, maintained the existence of only one electric fluid, which unelectrified bodies possess in a certain normal amount. A positively electrified body has more, and a negatively electrified body less than its normal amount. The particles of this fluid repel one another, and attract the particles of other kinds of matter, at all distances. Æpinus, in developing this theory more accurately, found it necessary to introduce the additional hypothesis that the particles of matter repel one another. Thus, according to Æpinus, the absence of sensible force between two bodies in the neutral condition, is due to the equilibrium of four forces, two of which are attractive, and the other two repulsive. Calling the two bodies A and B, the electricity which A possesses in normal amount, is repelled by the electricity of B, and attracted by the matter of B. The matter of A is attracted by the electricity of B, and repelled by the matter of B. These four forces are all equal, and destroy one another; but, without the supplementary hypothesis of Æpinus, one of the four forces is wanting, and the equilibrium is not easily explained. To reconcile Æpinus's addition with the Newtonian theory of gravitation, it is necessary to suppose that the equality between the four forces is not exact, the attractions being greater by an infinitesimal amount than the repulsions.

The one-fluid theory in this form is, like the two-fluid theory, consistent with the explanation of all known phenomena. But it is to be remarked that there is no sufficient reason, except established usage, for deciding which of the two opposite electricities should be regarded as corresponding to an excess of the electric fluid.

Franklin was the author of the terms positive and negative to denote the two opposite kinds of electrification; but the names can legitimately be retained without accepting the one-fluid theory, understanding that opposite signs imply forces in opposite directions, and that the connection between the *positive* sign and the forces exhibited by *vitreous* electricity is merely conventional.

7.—In speaking of electric currents, the language of the one-fluid theory is almost invariably employed. Thus, if A is a conductor charged positively, and B a conductor charged negatively; when the two are put in connection by a wire, we say that the direction of the current is from A to B; whereas the language of the two-fluid theory would be, that a current of vitreous or positive electricity travels from A to B, and a current of resinous or negative from B to A.

CHAPTER II.

ELECTROSTATIC INDUCTION.

8. Induction.—In the preceding chapter we have spoken of movements of material bodies caused by electrical attractions and repulsions. We have now to treat of the movement of electricity itself in obedience to the attractions or repulsions exerted upon it by other electricity. This kind of action is called *induction.*

It may be illustrated by means of the arrangement shown in Fig. 4. The apparatus consists of a sphere C which is electrified positively, suppose, and of a conducting insulated cylinder A B placed near it. From this latter are suspended at equal distances a few pairs of pith-balls. When the cylinder is brought near the sphere, the balls are observed to diverge. The divergence of the different pairs is not the same, but goes on decreasing from the pair nearest the sphere until a point M is reached, where there is no divergence. Beyond this the divergence goes on increasing. The neutral point M does not exactly bisect the length of the cylinder, but is nearer the end A than the end B, and the former end is found to be more strongly electrified than the latter.

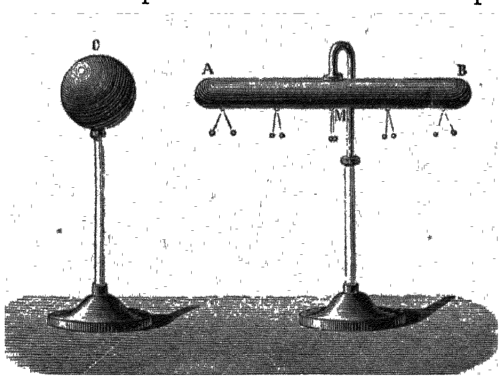


Fig. [illegible]

It is easy to show that the two ends of the cylinder are charged with opposite kinds of electricity; the end A being negatively, and

the end B positively electrified. We have only to bring an electrified stick of resin near the pith-balls at A, when these will be found to be repelled; if, on the contrary, it be held near those at B, they will be attracted.

The explanation is, that the positive electricity with which C is charged attracts the negative electricity of AB to the end A, and repels the positive to the end B. This action is more powerful at A than at B, on account of the greater proximity of the influencing body, and for the same reason the effect falls off more rapidly in the portion AM than in MB.

If the cylinder be brought closer to the sphere, the divergence of the balls increases; if it be removed farther from it, the divergence diminishes. Finally, all signs of electricity disappear if the sphere be taken away, or connected with the earth.

If, while the cylinder is under the influence of the electricity of C, the end B is connected with the earth, the pith-balls at this end

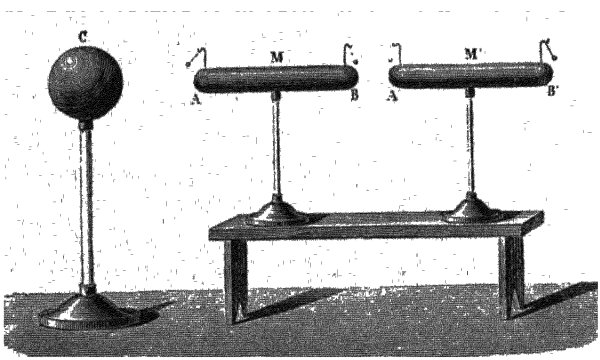


Fig 5.—Successive Induction

immediately collapse, while the divergence of those at A increases. The explanation is that the electricity which was repelled to the end B escapes to the earth, and thus affords an opportunity for a fresh exercise of induction on the part of the sphere, which increases the accumulation of negative electricity at A. We may also remark that the whole of the cylinder is now negatively electrified, the neutral line being pushed back to the earth. If the earth-connection be now broken, and the sphere C be then removed, the cylinder will remain negatively electrified, and will be in the same condition as if it had been touched by a negatively-electrified body. This mode

of giving a charge to a conductor is called *charging by induction*, and the charge thus given is always opposite to that of the *inducing body* C.

If a series of such conductors as A B be placed in line, without contact, and the positively-electrified body C be placed opposite to one end of the series, all the conductors will be affected in the same manner as the single conductor in the last experiment. They will all be charged with negative electricity at the end next C, and with positive electricity at the remote end, the effect, however, becoming feebler as we advance in the series. In this experiment each of the conductors acts inductively upon those next it; for example, if there be two conductors A B, A′B′, as in Fig. 5, the development of electricity at A′ and B′ is mainly due to the action of the positive electricity in MB. If the conductor AB be removed, the pith-balls at A′ and B′ will diminish their divergence.

The molecules of a body may be regarded as such a series of conductors, or rather as a number of such series. When an electrified body is brought near it, each molecule may thus become positive on one side and negative on the other. In the case of good conductors, this polarization is only instantaneous, being destroyed by the discharge of electricity from particle to particle. Good insulators are substances which are able to resist this tendency to discharge, and to maintain a high degree of polarization for a great length of time. This is Faraday's theory of " induction by contiguous particles."

9. **Electrical Attraction and Repulsion.**—The attraction which is observed when an electrified is brought near to an unelectrified body, is dependent upon induction. Suppose, for instance, that a body C, which is positively electrified, is brought near to an insulated and uncharged pith-ball. Negative electricity is induced on the near side of the pith-ball, and an equal quantity of positive on the further side. The former, being nearer to the body C, is more strongly attracted than the other is repelled. The ball is therefore upon the whole attracted.

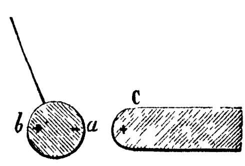


Fig 6.—Electrical Attraction

If the pith-ball, instead of being insulated, is suspended by a conducting thread from a support connected with the earth, it will be more strongly attracted than before, as it is now entirely charged with negative electricity.

In the case of any insulated conductor, the algebraic sum of the

electricities induced upon it by the presence of a neighbouring electrified body must be zero. If the pith-ball be insulated, and have an independent charge of either kind of electricity, the total force exerted on the pith-ball is the algebraic sum[1] of the two following quantities:—

(1) The force which the ball would experience, if it had no independent charge. This force, as we have just seen, is always attractive.

(2) The force due to the independent charge when distributed over the ball as it would be if C were removed. This second force is attractive or repulsive, according as the independent charge is of unlike or like sign to that of C. In the latter case, repulsion will generally be observed at distances exceeding a certain limit and attraction at nearer distances, the reason being that the force (1) due to the induced distribution increases more rapidly than the other as the distance is diminished.

It is important to remember this in testing, by the electric pendulum, or by any other electroscope, the kind of electricity with which a body is charged. In bringing the body towards the electroscope, the first movement produced is that which is to be observed, and repulsion is in general a more reliable test of kind of electricity than attraction.

10. **Electroscopes.**—An electroscope is an apparatus for detecting the presence of electricity, and determining its sign. The insulated electric pendulum is an electroscope. If the pith-ball, when itself uncharged, is attracted by a body brought near it, we know that the body is electrified. To determine the kind of electricity, the body is allowed to touch the pith-ball, which is then repelled. At this moment an excited glass tube is brought near. If it repels the ball, this latter, as well as the body which touched it, must be electrified positively. If the glass tube attracts it, or, still more decisively, if excited resin or sealing-wax repels it, the ball and the body which touched it are electrified negatively. The loss of electricity from the pith-ball is often so rapid as to render this test of sign somewhat uncertain.

The *gold-leaf electroscope* (Fig. 7) is constructed as follows:—

[1] We here suppose C to be a non-conductor, so that the distribution of its electricity is not affected by the presence of the pith-ball. If C be a conductor, the effect of induction upon it will be to favour attraction, so that an attractive force must be added to the two forces specified in the text.

Two small gold-leaves are attached to the lower end of a metallic rod, which passes through an opening in the top of a bell-glass, and terminates in a ball. The metallic rod is sometimes, for the sake of better insulation, inclosed in a glass tube secured by sealing-wax or some other non-conducting cement, and, for the same purpose. the upper part of the bell-glass is often varnished with shellac, which is less apt than glass to acquire a deposit of moisture from the air. The bell-glass is attached below to a metallic base, which excludes the external air. For the gold-leaves are sometimes substituted two straws, or two pith-balls suspended by linen threads; we have thus the *straw-electroscope* and the *pith-ball electroscope.*

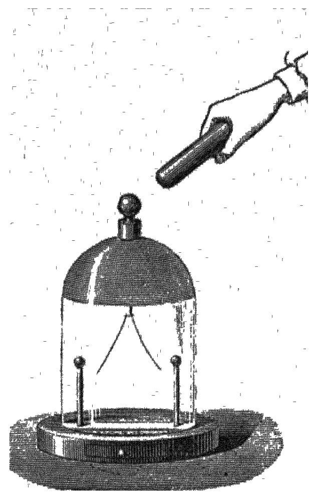


Fig. 7.—Gold-leaf Electroscope

To test whether a body is electrified, it is brought near the ball at the top of the electroscope. The like electricity is repelled into the leaves, and makes them diverge, while the unlike is attracted into the ball. The sign of the body's charge may be determined in the following manner:—While the leaves are divergent under the influence of the body, the operator touches the ball with his finger. This causes the leaves to collapse, and gives to the insulated conductor composed of leaves, rod, and ball, a charge opposite to that of the influencing body. The finger must be removed while the influencing body remains in position, as the amount of the induced charge depends upon the position of the influencing body at the instant of breaking connection. On now withdrawing the influencing body, the charge of unlike electricity is no longer attracted to the ball, but spreads over the whole of the conductor, and causes the leaves to diverge. If, while this divergence continues, an excited glass tube, when gradually brought towards the ball, diminishes the divergence, we know that the body in question was electrified positively. If it increases the divergence, the body was electrified negatively.

Great caution must be used in bringing electrified bodies near the gold-leaf electroscope, as the leaves are very apt to be ruptured by

quick movements. If they diverge so widely as to touch the sides of the bell-glass, it is often difficult to detach them from the glass without tearing. To prevent this contact, two metallic columns are interposed, communicating with the ground. If the leaves diverge too widely, they touch these columns and lose their electricity.

CHAPTER III.

COULOMB'S QUANTITATIVE LAWS.

11. Coulomb's Torsion-balance.—Coulomb, who was the first to make electricity an accurate science, employed in his researches an instrument which is often called after his name, and which is still extensively employed. It depends on the principle that the torsion of a wire is simply proportional to the twisting couple. We shall first describe it, and then point out some of its applications.

It consists of a cylindrical glass case AA (Fig. 8), from the upper end B of which rises another glass cylinder DD of much smaller diameter. This small cylinder is fitted at the top with a brass cap *a*, carrying an index C. Outside of this, and capable of turning round it, is another cap *b*, the top of which is divided into 360 equal parts. In the centre of the cap *b* is an opening through which passes a small metal cylinder *d*, capable of turning in the opening with moderate friction, and having at its lower end a notch or slit. When the cap *b* is turned, the cylinder *d* turns with it; but the latter can also be turned separately, so as not to change the reading. These parts compose the *torsion-head*. A very fine metallic wire is held by the notch, and supports a small piece of metal, through which passes a light needle of shellac *f*, carrying at one end a small gilt ball *g*. A circular

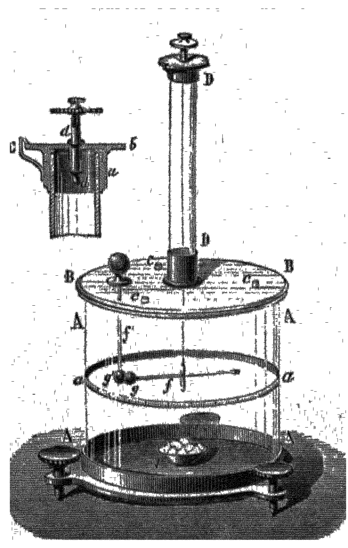


Fig 8.—Coulomb's Torsion-balance

scale runs round the outside of the large cylinder in the plane of the needle. Finally, opposite the zero of this scale, there is a fixed ball g' of some conducting material, supported by a rod f' of shellac, which passes through a hole in the cover of the cylindrical case.

12. **Laws of Electric Repulsion.**—To illustrate the mode of employing this apparatus for electrical measurements, we shall explain the course followed by Coulomb in investigating the law according to which electrical repulsions and attractions vary with the distance. The index is set to the zero of the scale. The inner cylinder d is then turned, until the movable ball just touches the fixed ball without any torsion of the wire. The fixed ball is then taken out, placed in communication with an electrified body, and replaced in the apparatus. The electricity with which it is charged is communicated to the movable ball, and causes the repulsion of this latter through a number of degrees indicated by the scale which surrounds the case. In this position the force of repulsion is in equilibrium with the force of torsion tending to bring back the ball to its original position. The graduated cap b is then turned so as to oppose the repulsion. The movable ball is thus brought nearer to the fixed ball, and at the same time the amount of torsion in the wire is increased. By repeating this process, we obtain a number of different positions in which repulsion is balanced by torsion. But we know, from the laws of elasticity, that the force (strictly the couple[1]) of torsion is proportional to the angle of torsion. Hence we have only to compare the total amounts of torsion with the distances of the two balls. By such comparisons Coulomb found that the force of electrical repulsion varies *inversely as the square of the distance.*

The following are the actual numbers obtained in one of the experiments. The original deviation of the movable ball being 36°, it was found that, in order to reduce this distance to 18°, it was necessary to turn the head through 126°, and, for a farther reduction of the deviation to 8°·5, an additional rotation through 441° was required. It will thus be perceived that at the distances of 36°, 18° and 8°·5, which may be practically considered as in the ratio of 1, $\frac{1}{2}$, and $\frac{1}{4}$, the forces of repulsion were equilibrated by torsions of 36°,

[1] The repulsive force on the movable ball is equivalent to an equal and parallel force acting at the centre of the needle (the point of attachment of the wire), and a couple whose arm is the perpendicular from this centre on the line joining the balls. This couple must be equal to the couple of torsion. The other component produces a small deviation of the suspending wire from the vertical.

$126° + 18° = 144°$, and $441 + 126 + 8·5 = 575°·5$ respectively. Now 144 is 36×4, and 575·5 may be considered as 576, or 36×16. Hence we perceive that, as the distance is divided by 2, or by 4, the force of repulsion is multiplied by 4 or by 16, which precisely agrees with the law enunciated above.

13. Equation of Equilibrium.—We must, however, observe that in this mode of reducing the observations two inaccurate assumptions are made. First, the distance between the balls is regarded as being equal to the arc which lies between them, whereas it is really the chord of that arc. Secondly, the force of repulsion is regarded as acting always at the same arm, whereas its arm, being the perpendicular from the centre on the chord, diminishes as the distance increases. The following investigation is more rigorous.

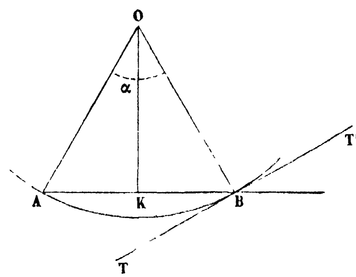


Fig. 9.

Let AOB (Fig. 9), the angular distance of the balls, be denoted by α, and let l be the length of the radius OA. Then the chord AB is $2l \sin \frac{1}{2} \alpha$, and the arm OK is $l \cos \frac{1}{2}\alpha$. Let f denote the force of repulsion at unit distance, and n the couple of torsion for 1°. Then the force of repulsion in the given position is $\frac{f}{4 l^2 \sin^2 \frac{1}{2} \alpha}$ if the law of inverse squares be true, and the moment of this about the centre is $\frac{f \cos \frac{1}{2} \alpha}{4 l \sin^2 \frac{1}{2} \alpha}$, which must be equal to nA, if A be the number of degrees of torsion. Hence we have

$$\frac{f}{4nl} = \text{A} \sin \tfrac{1}{2} \alpha \tan \tfrac{1}{2} \alpha,$$

and as the first member of this equation is constant, the second member must be constant also for different values of A and α, if the law of inverse squares be true. The degree of constancy is shown by the following table:—

	α	A	A $\sin \frac{1}{2} \alpha \tan \frac{1}{2} \alpha$.
1st experiment,	36	36	3·614
2d experiment,	18	144	3·568
3d experiment,	8·5	575·5	3·169
Supposed case,	9	576	3·557

The difference between the first and second numbers of the last

column is insignificant. That between the second and third is more considerable,[1] but in reality only corresponds to an error of half a degree in the measurement of the arc.

14. Case of Attraction.—The law of attractions may be investigated by a similar method. The index is set to zero, and the central piece is turned so as to place the movable ball at a known distance from the fixed ball. The two balls are then charged with electricity of different kinds. The movable ball is accordingly attracted towards the other, and settles in a position in which attraction is balanced by torsion. By altering the amount of torsion, different positions of the ball can be obtained. On comparing the distances with the corresponding torsions, it is found that the same law holds as in the case of repulsion. The experiment, however, is difficult, and is only possible when the balls are very feebly electrified. To prevent the contact of the two balls, Coulomb fixed a silk thread in the instrument, so as to stop the course of the movable ball.

15. Law of Attraction and Repulsion as depending on Amount of Charge.—We may assume as evident, that when an electrified ball is placed in contact with a precisely equal and similar ball, the charge will be divided equally between them, so that the first will retain only half the charge which it had before contact.

Suppose that an observation on repulsion has just been made with the torsion-balance, and that we touch the fixed ball with another exactly equal insulated ball, which we then remove. It will be found that the amount of torsion requisite for keeping the movable ball in its observed position is just half what it was before. The

[1] We have already seen that the mutual induction of two conductors tends to diminish their mutual repulsion, and that this inductive action becomes more important as the distance is diminished. Hence the repulsion at distance 9 should be less than a quarter of that at distance 18. The apparent error thus confirms the law.

Many persons have adduced, as tending to overthrow Coulomb's law of inverse squares, experimental results which really confirm it. Except when the dimensions of the charged bodies are very small in comparison with the distance, the observed attraction or repulsion is the resultant of an infinite number of forces acting along lines drawn from the different points of the one body to the different points of the other. The law of inverse squares applies directly to these several components, and not to the resultant which they yield. The latter can only be computed by elaborate mathematical processes.

It is incorrectly assumed in the text that the law ought to apply directly to two spheres, when by their distance we understand the distance between their nearest points. It is not obvious that the distance of the nearest points should give a better result than the distance between the centres.

The strongest evidence for the rigorous exactness of the law of inverse squares is indirect; see § 19.

same result will be obtained by touching the movable ball with a ball of its own size. We conclude that, if the charge of either body be altered, the attractive or repulsive force between the bodies at given distance will be altered in the same ratio. The law is not rigorously true for bodies of finite size, unless the distribution of the electricity on the two bodies remains unchanged. When the two bodies are very small in all their dimensions in comparison with the distance between them, their mutual force is represented by the expression

$$\frac{qq'}{D^2},$$

q and q' denoting their charges, and D the distance. If this expression has the positive sign, the force is repulsive, if negative attractive.

16. Electricity resides on the Surface.—Electricity (subject to the

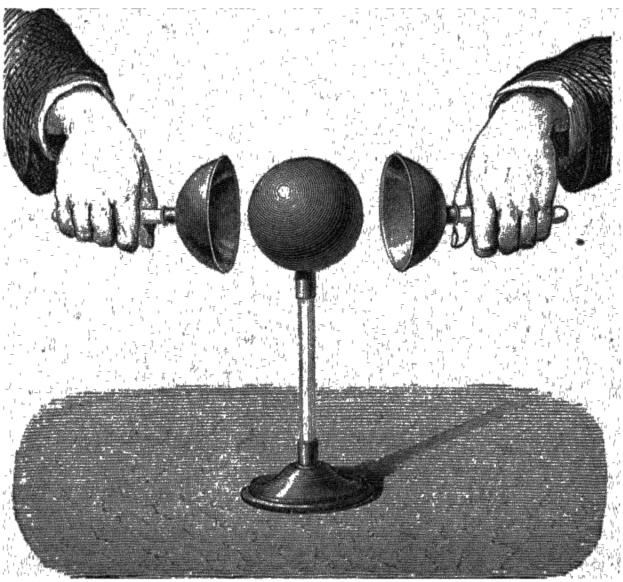


Fig. 10.—Biot's Experiment.

exceptions mentioned below) resides exclusively on the external surface of a conductor. This is perhaps implied in the experimental fact frequently observed by Coulomb, that when a solid and a hollow sphere of equal external diameter are allowed to touch each other, any charge possessed by either is divided equally between them. A

direct demonstration is afforded by the following experiment of Biot:—

We take an insulated sphere of metal, charge it with electricity, and cover it with two hemispheres furnished with insulating handles, which fit the sphere exactly (Fig. 10). If the two hemispheres be quickly removed, and presented to an electric pendulum, they will be found to be electrified, while the sphere itself will show hardly any traces of electricity. We must, however, remark that this experiment is rarely successful, and that generally the sphere remains very sensibly electrified. The reason of this is, that it is very difficult to remove the hemispheres so steadily, as not to permit their edges to touch the sphere after the first separation.

The following is a much surer form of the experiment:—

A hollow insulated sphere, with an orifice in the top, is charged with electricity (Fig. 11). A *proof-plane*, consisting of a small disc of gilt paper insulated by a thin handle of shellac, is then applied to the interior surface of the sphere, and, when tested by an electric pendulum or an electroscope, is found to exhibit no trace of electricity. But if, on the contrary, the disc be applied to the external surface of the sphere, it will be found to be electrified, and capable of attracting light bodies. Faraday varied this experiment, by substituting a cylinder of wire-gauze for the sphere. This cylinder rested on an insulated disc of metal. The disc was charged with electricity, and it was found that no trace of the electricity could be detected by applying the proof-plane to the interior surface of the cylinder.

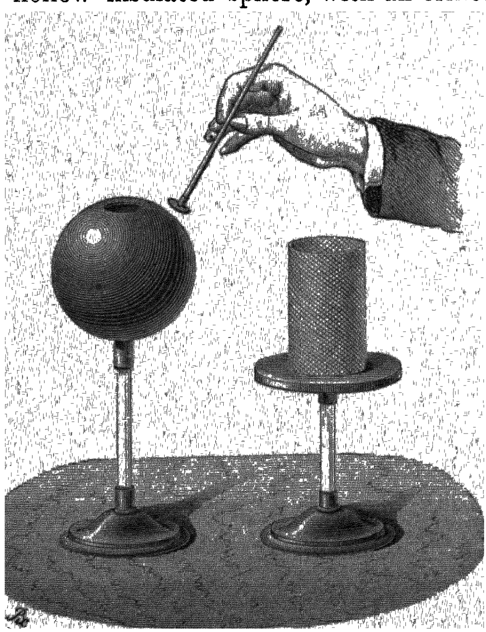
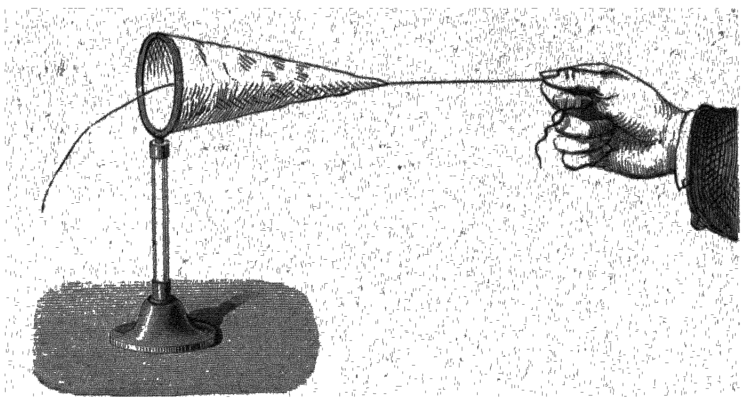


Fig. 11.—Proof-plane and Hollow Sphère.

The following experiment is also due to Faraday. A metal ring is fixed upon an insulating stand (Fig. 12). To this ring is attached a cone-shaped bag of fine linen, which is a conductor of electricity. A silk thread, attached to the apex of the cone, and extending both



ways, enables the operator to turn the bag inside out as often as required, without discharging it. When the bag is electrified, the application of the proof-plane always shows that there is electricity on the outer, but not on the inner surface. When the bag is turned inside out, the electricity therefore passes from one surface of the linen to the other.

17. **Limitations of the Rule.**—There are two exceptions to the rule that electricity is confined to the external surface of a conductor.

1. It does not hold for electric currents. We shall see hereafter in connection with galvanic electricity, that the resistance which a wire of given length opposes to the passage of electricity through it, depends not upon its circumference but upon its sectional area. A hollow wire will not conduct electricity so well as a solid wire of the same external diameter.

2. Electricity may be induced on the inner surface of a hollow conductor by the presence of an electrified body insulated from the conductor itself. If an insulated body charged with electricity be introduced into the interior of a hollow conductor, so as to be completely surrounded by it, but still insulated from it, it induces upon the inner surface a quantity equal to its own charge, but of opposite sign. If the conductor is insulated, an equal quantity, but of the same sign as the charge of the inclosed body, is repelled to the outside, and

this is true whether the conductor has an independent charge of its own or not. In this case, then, we have electricity residing on both the external and the internal surfaces of a hollow conductor, but it still resides only on the surfaces.

If a conducting body connected with the earth be introduced into the interior of a hollow charged conductor, so as to be partially surrounded by it, the body thus introduced will acquire an opposite charge by induction, and, by the reciprocal action of this charge, electricity will be induced on the inner at the expense of the outer surface of the hollow conductor, just as in the preceding case.

18. Ice-pail Experiment.—The effect of introducing a charged body within a hollow conductor is well illustrated by the following experiments of Faraday. Let A (Fig. 13) represent an insulated pewter ice-pail, ten and a half inches high and seven inches in diameter, connected by a wire with a delicate gold-leaf electroscope E, and let C be a round brass ball insulated by a dry thread of white silk, three or four feet in length, so as to remove the influence of the hand holding it from the ice-pail below. Let A be perfectly discharged, and let C, after being charged at a distance, be introduced into A as in the figure. If C be positive, E also will diverge positively; if C be taken away, E will collapse perfectly, the apparatus being in good order. As C enters the vessel A, the divergence of E will increase until C is about three inches below the edge of the vessel, and will remain quite steady and unchanged for any greater depression. If C be made to touch the bottom of A, all its charge is communicated to A, and C, upon being withdrawn and examined, is found perfectly discharged.

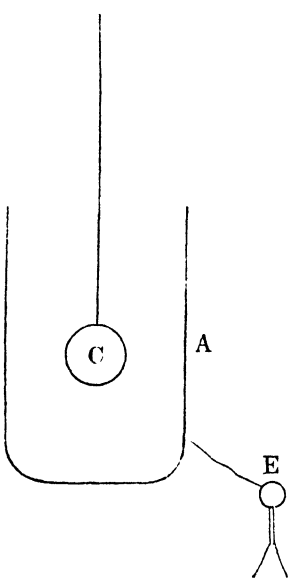


Fig. 13.—Ice-pail Experiment.

Now Faraday found that at the moment of contact of C with the bottom of A, not the slightest change took place in the divergence of the gold-leaves. Hence the charge previously developed by induction on the outside of A must have been precisely equal to that acquired by the contact, that is, must have been equal to the charge of C.

He then employed four ice-pails (Fig. 14), arranged one within the other, the smallest innermost, insulated from each other by plates of shellac at the bottom, the outermost pail being connected with the electroscope. When the charged carrier-ball C was introduced within the innermost pail, and lowered until it touched the bottom, the electrometer gave precisely the same indications as when the outermost pail was employed alone. When the innermost was lifted out by a silk thread after being touched by C, the gold-leaves collapsed perfectly. When it was introduced again, they opened out to the same extent as before. When 4 and 3 were connected by a wire let down between them by a silk thread, the leaves remained unchanged, and so they still remained when 3 and 2 were connected, and finally when all four pails were connected.

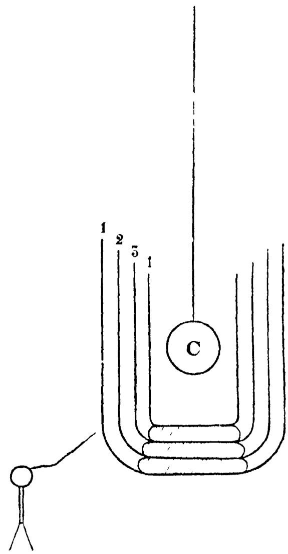


Fig 14.—Experiment with Four Ice-pails

19. No Force within a Conductor.—When a hollow conductor is electrified, however strongly, no effect is produced upon pith-balls, gold-leaves, or any other electroscopic apparatus in the interior, whether connected with the hollow conductor, or insulated from it, provided, in the latter case, that they have no communication with bodies external to the hollow conductor. Faraday constructed a cubical box, measuring 12 feet each way, covered externally with copper wire and tin-foil, and insulated from the earth. He charged this box very strongly by outside communication with a powerful electrical machine; but a gold-leaf electrometer within showed no effect. He says, "I went into the cube and lived in it, using lighted candles, electrometers, and all other tests of electrical states. I could not find the least influence upon them, or indication of anything particular given by them, though all the time the outside of the cube was powerfully charged, and large sparks and brushes were darting off from every part of its outer surface."

The fact that electricity resides only on the external surface of a conductor, combined with the fact that there is no electrical force in the space inclosed by this surface, affords a rigorous proof of the law

of inverse squares. For if the conductor be a sphere removed from the influence of external bodies, its charge must be distributed uniformly over its surface. Now it admits of proof, and is well known to mathematicians, that a uniform spherical shell exerts no attraction at any point of the interior space, if the law of attraction be that of inverse squares, and that the internal attraction does not vanish for any other law.

20. Electrical Density and Distribution.—When the proof-plane is applied to different parts of the surface of a conductor, the quantities of electricity which it carries off are not usually equal. But the electricity carried off by the proof-plane is simply the electricity which resided on the part of the surface covered by it, for the proof-plane during the time of its contact is virtually part of the surface of the conductor. We must therefore conclude that equal areas on different parts of the surface of a conductor have not equal amounts of electricity upon them. It is also found that if the charge of the conductor be varied, the electricity resident upon any specified portion of the surface is changed in the same ratio. The ratio of the quantities of electricity on two specified portions of the surface is in fact independent of the charge, and depends only on the form of the conductor. This is expressed by saying that *distribution* is independent of charge, and that the distribution of electricity on the surface of a conductor depends on its form.

By the *average electrical density* on the whole or any specified portion of the surface of a conductor, is meant the quantity of electricity upon it, divided by its area. By the *electrical density at a specified point* on the surface of a conductor, is meant the average electrical density on an exceedingly small area surrounding it, in other words, the *quantity of electricity per unit area* at the point. The name is appropriate, from the analogy of ordinary material density, which is mass per unit volume, and is not intended to imply any hypothesis as to the nature of electricity. The name was introduced by Coulomb, who first investigated the subject in question, and is generally employed by the best electricians in this country. The term *thickness of electrical stratum*, which was introduced by Poisson, is much used in France, but is more open to objection from the coarse assumptions which it seems to involve.

The following are some of Coulomb's results. The dotted line in each of the figures is intended to represent, by its distance from the outline of the conductor, the electric density at each point of the

latter. In all cases it is to be understood that the conductor is so far removed from external bodies as not to be influenced by them:—

1. *Sphere* (Fig. 15). The electric density is the same for all points on the surface of a spherical conductor.

2. *Ellipsoid* (Fig. 16). The density is greatest at the ends of the

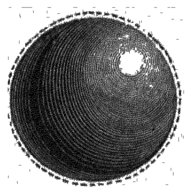


Fig 15.—Distribution on Sphere.

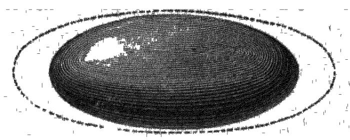


Fig 16.—Distribution on Ellipsoid

longest, and least at the ends of the shortest axis; and the densities at these points are simply proportional to the axes themselves.[1]

3. *Flat Disc* (Fig. 17). The density is almost inappreciable over the whole of both faces, except close to the edges, where it increases almost *per saltum*.

4. *Cylinder with Hemispherical Ends* (Fig. 18). The density is

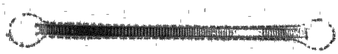


Fig 17.—Distribution on Disc.

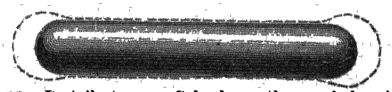


Fig 18.—Distribution on Cylinder with rounded ends.

a minimum, and nearly uniform, at parts remote from the ends, and attains a maximum at the ends. The ratio of the density at the ends to that at the sides increases as the radius of the cylinder diminishes, the length of the cylinder remaining the same.

5. *Spheres in Contact.*—In the case of equal spheres, the charge, which is nothing at the point of contact, and very feeble up to 30° from that point, increases very rapidly from 30° to 60°, less rapidly from 60° to 90°, and almost insensibly from 90° to 180°. When the spheres are of unequal size, the charge at any point on the smaller

[1] More generally, the density at any point on the surface of an ellipsoid is proportional to the length of a perpendicular from the centre of the ellipsoid on a tangent plane at the point.

If an ellipsoid, similar and nearly equal to the given one, be placed so that the corresponding axes of the two are coincident, we shall have a thin ellipsoidal shell, whose thickness at any point exactly represents the electric density at that point.

Such a shell, if composed of homogeneous matter attracting inversely as the square of the distance, would exercise no force at points in its interior.

sphere is greater than at the corresponding point on the larger one; and as the smaller sphere is continually diminished, the other remaining the same, the ratio of the densities at the extremities of the line of centres tends to become 2 : 1.

21. Method of Experiment.—The preceding results were obtained by Coulomb in the following manner. He touched the electrified body at a known point with the proof-plane, and then put the plane in the place of the fixed ball of the torsion-balance, the movable ball having previously been charged with electricity of the same sign. Repulsion was thus produced, and the amount of torsion necessary to keep the balls at a certain distance asunder was observed. He then repeated the experiment with electricity taken from a different point of the body under examination, and the ratio of the densities at the two points was given by the ratio of the torsions necessary to keep the balls at the same distance.

By way of checking the accuracy of this mode of experimentation, Coulomb electrified an insulated sphere, and measured the electric density on its surface by the method described above. He then touched the sphere with another precisely equal sphere, and on again applying the proof-plane he found that the charge carried off by the plane was just half what it had been before.

22. Alternate Contact.—The above experiments naturally require some time, during which the body under investigation is gradually losing its charge. The consequence is, that the densities indicated by the balance, if taken singly, do not correctly represent the electric distribution. This source of error was avoided by Coulomb in the following manner. He touched two points on the body successively, and determined the electric density at each; and then, after an interval equal to that between the two experiments, he touched the first point again, and obtained a second measure of its density, which was less than the first, on account of the dissipation of electricity. If the densities thus observed be denoted by A and A′, and the density observed at the second point by B, it is evident that $\frac{A}{B}$ is greater, and $\frac{A'}{B}$ less than the ratio required. Coulomb adopted, as the correct value, their arithmetic mean $\frac{1}{2}\,\frac{A+A'}{B}$.

23. Power of Points.—The distribution of electricity on a conductor of any form may be roughly described, by saying that the density is greatest on those parts of the surface which project most,

or which have the sharpest convexity, and that in depressions or concavities it is small or altogether insensible. Theory shows that at a perfectly sharp edge, such, for example, as is formed by two planes meeting at any angle however obtuse, but *not rounded off*, the density must be infinite, and *a fortiori* it must be infinite at a perfectly sharp point, for example at the apex of a cone, however obtuse, *if not rounded off.* Practically, the points and edges of bodies are always rounded off; the microscope shows them merely as places of very sharp convexity (that is, of very small radius of curvature), and hence the electric density at those places is really finite; but it is exceedingly great in comparison with the density at other parts, and this is especially true of very acute points, such as the point of a fine needle. The consequence is, that if a pointed conductor is insulated and charged, the concentration of a large amount of repulsive force within an exceedingly small area produces very rapid escape of electricity at the points. Conductors intended to retain a charge of electricity must have no points or edges, and must be very smooth. If of considerable length in proportion to their breadth, they are usually made to terminate in large knobs.

24. Dissipation of Charge.—When an insulated conductor is charged and left to itself, its charge is gradually dissipated, and at length completely disappears. This loss takes place partly through the supports, and partly through the air.

As regards the supports, the loss occurs chiefly at their surface, especially if (as is usually the case) they are not perfectly dry. It is diminished by diminishing their perimeter, and by increasing their length; for example, a long fibre of glass or raw silk is an excellent insulator.

As regards the air, we must distinguish between conduction and convection. Very hot air and highly rarefied air probably act as conductors; but air in the ordinary condition acts chiefly by contact and convection. Successive layers of air become electrified by contact with the conductor, and are then repelled, carrying off the electricity which they have acquired. It is by an action of this kind that electricity escapes into the air from points, as is proved by the wind which passes off from them (§ 43). Particles of dust present in the air, in like manner, act as carriers, being attracted to the conductor, charged by contact with it, and then repelled. They also frequently adhere by one end to the conductor,

and thus constitute pointed projections through which electricity is discharged into the air.

Coulomb deduced from his observations on dissipation of charge a law precisely analogous to Newton's law of cooling, namely, that when all other circumstances remain the same, *the rate of loss is simply proportional to the charge*, so that the charges at equal intervals of time form a decreasing geometric series. Subsequent experience has confirmed this law, as approximately true for moderate charges of the same sign. Negative charges are, however, dissipated more rapidly than positive.

CHAPTER IV.

ELECTROSTATIC MACHINES

25. Electrical Machines.—The first electrical machine was invented by Otto Guericke, to whom, as we have already seen, science is indebted for the invention of the air-pump. It consisted of a ball of sulphur which was turned upon its axis by one person, while another held his hands upon the ball, thus causing the friction necessary for the production of electricity. The result was that the globe was negatively electrified, and the positive electricity escaped into the earth through the hands of the operator. This machine, however, was capable of producing only very feeble effects, and the sparks obtained from it were visible only in the dark. An English philosopher, Hawksbee, substituted a globe of glass for the globe of sulphur; the electricity thus obtained was positive, and the sparks obtained by the new machine were of considerable brightness. The machine, however, was for the time superseded by the use of glass tubes, which continued to be the favourite instruments for generating electricity until the middle of the eighteenth century, when a German philosopher, Boze, professor of physics at Wittemberg, revived and perfected Hawksbee's machine, which became universally adopted.

26. Ramsden's Machine.—The kind of friction machine most commonly employed at present is the plate-machine, invented by Ramsden about 1768, and only slightly changed and improved since.

The most usual form of this machine is shown in Fig. 19. It has a circular plate of glass, which turns on an axis supported by two wooden uprights. On each side of the plate, at the upper and lower parts of the uprights, are two cushions, which act as rubbers when the plate is turned. In front of the plate are two metallic conductors supported on glass legs, and terminating in branches

which are bent round the plate at the middle of its height, and are studded with points projecting towards it. The plate becomes charged with positive electricity by friction against the cushions, and gives off its electricity through the points to the two conductors, or, what amounts to the same thing, the conductors give off negative electricity through the points to the positively-electrified plate. In order to avoid loss of electricity from that portion of the plate which

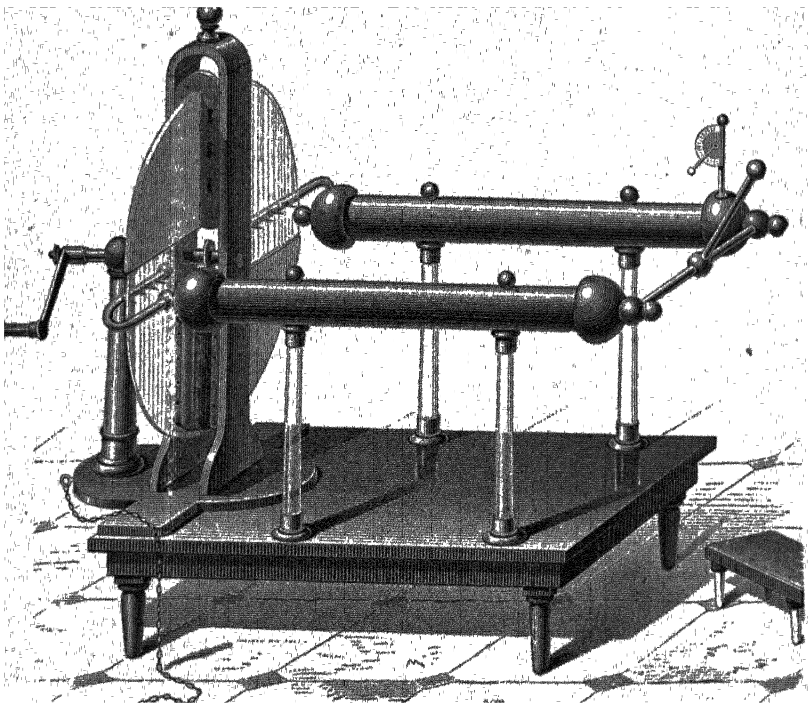


Fig. 10.—Ramsden's Electrical Machine.

is passing from the cushions to the points, sector-shaped pieces of oiled silk are placed so as to cover it on both sides. The cushions become negatively electrified by the friction; and the machine will not continue working unless this negative electricity is allowed to escape. The cushions are accordingly connected with the earth by means of metal plates let into their supports.

27. Limit of Charge.—As the conductors become more highly charged, they lose electricity to the air more rapidly, and a time soon arrives when they lose electricity as fast as they receive it from the

plate. After this, if the machine continues to be worked uniformly, their charge remains nearly constant. This limiting amount of charge depends very much upon the condition of the air; and in damp weather the machine often refuses to work unless special means are employed to keep it dry.

The rubbers are covered with a metallic preparation, of which several different kinds are employed. Sometimes it is the compound called *aurum musivum* (bisulphide of tin), but more frequently an amalgam. Kienmeier's amalgam consists of one part of zinc, one of tin, and two of mercury. The amalgam is mixed with grease to make it adhere to the leather or silk which forms the face of the cushion.

Before using the machine, the glass legs which support the conductors should be wiped with a warm dry cloth. The plate must also be cleaned from any dust or portions of amalgam which may adhere to it, and lastly, dried with a hot cloth or paper. When these precautions are taken, the machine, if standing near a fire, will always work: but the charging of Leyden jars, and especially of batteries, may be rendered impossible by bad weather.

The variations of charge are indicated by the *quadrant electroscope* (Fig. 20), which is attached to one of the conductors. It consists of an upright conducting stem, supporting a quadrant, or more commonly a semicircle of ivory, at whose centre a light needle of ivory is jointed, carrying a pith-ball at its end. When there is no charge in the conductor, this pendulum hangs vertically, and as the charge increases it is repelled further and further from the stem. In damp weather it will be observed to return to the vertical position almost immediately on ceasing to turn the machine, while in very favourable circumstances it gives a sensible indication of charge after two or three minutes.

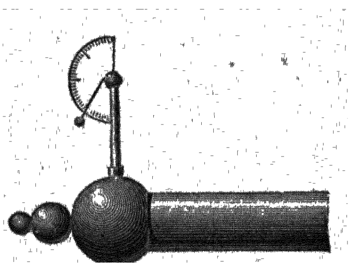


Fig. 20.—Quadrant Electroscope

28. Nairne's Machine.—Ramsden's machine furnishes only positive electricity. In order to obtain negative electricity, it is necessary to insulate the cushions from the ground, and to place them in communication with an insulated conductor. An arrangement of this kind is adopted in Nairne's machine.

In this machine a large cylinder of glass revolves between two

separately insulated conductors. One of these has a row of points projecting towards the glass, and collects positive electricity. The other is connected with the rubber, and collects negative. If one

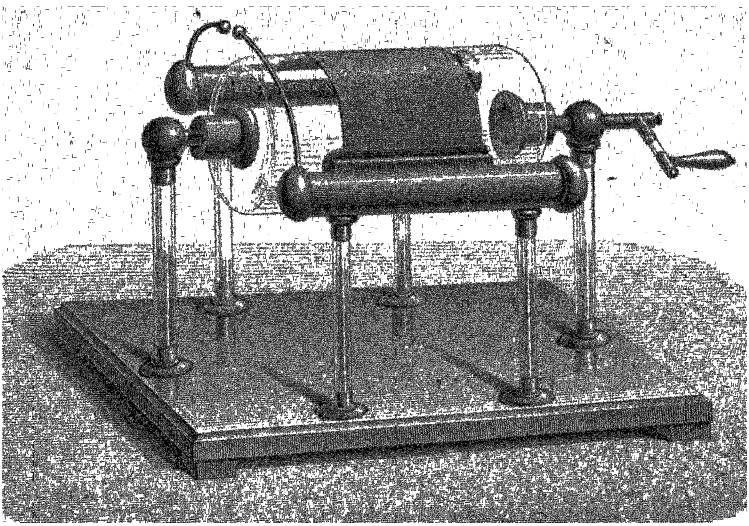


Fig. 21. Nairne's Electrical Machine.

kind of electricity only is required, the conductor which furnishes the other must be connected with the ground.

29. Classification of Machines.—The machines above described employ friction as the means of generating electricity; and the hydro-electric machine invented in 1840 by Mr. Wm. Armstrong (now Lord Armstrong) depends on the same principle. It consists of an insulated high-pressure boiler, from which partially condensed steam rushes through a row of circuitous exits, opposite to which is an insulated metal comb, to which it gives up its electricity. A fuller description will be found in previous editions of this work.

The machines which will be described in the remainder of this chapter depend upon a very different principle.

We may remark that the name "electrical machine" is now of somewhat doubtful signification. In a general sense it must include machines which depend on magneto-electric principles and those which depend upon the rapid interruption of electric currents; but these bear so little resemblance to the friction machines which monopolized the name for a hundred years that it is not usually applied to them. It is usually confined to machines which depend

upon electrostatical principles, under which head are included both friction and electrostatic induction.

30. Electrophorus.—When electricity is required in comparatively small quantities, it is readily supplied by the simple apparatus called the *electrophorus*. This consists (Fig. 22) of a disc of resin, or some other material easily excited by friction, and of a polished metal disc B with an insulating handle C D. The resin disc is electrified by striking or rubbing it with catskin or flannel, and the metal plate is then laid upon it. In these circumstances the upper plate does not receive a direct charge from the lower, but, if touched with the finger (to connect it with the earth), receives an opposite charge by induction. On lifting it away by its insulating handle, it is found to be charged, and will give a spark. It may then be replaced on the lower plate (touching it at the same time with the finger), and the process repeated an indefinite number of times, without any fresh excitation, if the weather is favourable.

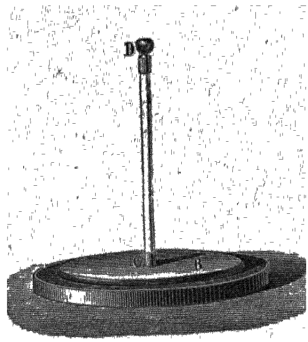


Fig 22 —Electrophorus

The resinous plate has usually a base or *sole* of metal, which is in connection with the earth while the electrophorus is being worked. When the cover receives its positive charge on being connected with the earth, the sole at the same time receives from the earth a negative charge, and as the cover is gradually lifted this negative charge gradually returns to the earth.

The most convenient form of the electrophorus is that of Professor Phillips, in which the cover, when placed upon the resinous plate, comes into metallic connection with the metal plate below. That this arrangement is allowable is evident, when we reflect that, when the upper plate is touched with the finger, it is in fact connected with the lower plate, since both are connected with the earth; and it effects a great saving of time when many sparks are required in quick succession, for the cover may be raised and lowered as fast as we please, coming alternately into contact with the resinous plate and the body which we wish to charge.

A machine which has been called a rotatory electrophorus was invented by Bertsch, and is described in previous editions of the present work.

31. Voss' Machine.—In Voss' machine, which is a modification of an earlier form invented by Holtz, the inducing charge may be indefinitely small at first, and is rapidly increased. It gives much more powerful effects than the friction machine, and is much easier to manage and keep in order. It is represented in Fig. 23.

There are two glass plates, a small distance asunder. The larger one is fixed, and the smaller one is made to revolve rapidly by means of a driving band passing over two grooved wheels, the larger one

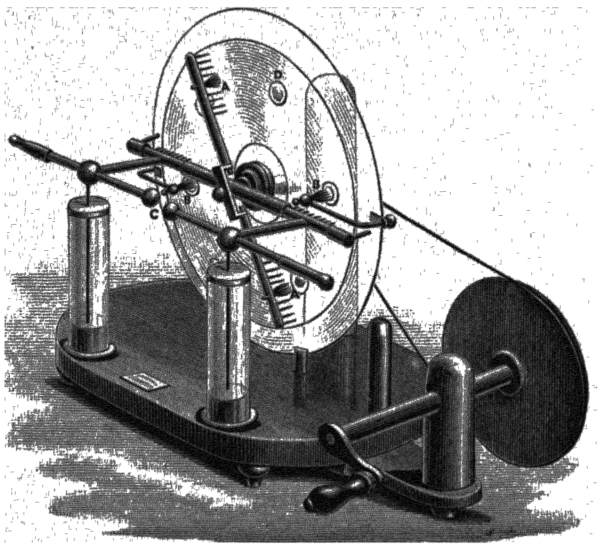


Fig. 23.—Voss' Machine.

being turned by hand. This plate has six metallic studs (like that at D) set in it at equal distances. The sloping bar which is seen in front of it is of brass, and carries two little brushes A A of thin brass wire, against which the studs rub as they pass by, and this happens at the same moment for both brushes. If we suppose that the fixed plate is charged with positive electricity in its upper part, and negative in its lower part, the upper stud will acquire a negative and the lower stud a positive charge, by induction, at the moment that the two contacts occur. When the studs have advanced about a quarter of a revolution, they come in contact with another pair of brushes B B which collect their charges.

These collecting brushes are in communication with two patches of tin-foil on the back of the fixed plate, which are not shown in

the figure. Thus the right-hand patch will be continually replenished with negative, and the left-hand patch with positive electricity. This left-hand patch extends to the top of the fixed plate, and acts as the influencing body to draw negative electricity to the upper brush and stud. The right-hand patch in like manner extends to the bottom, and attracts positive to the lower stud.

The action which we have described produces rapid increase of any slight charges that the two patches of tin-foil may possess at starting, and when the machine is dry there is generally a sufficient trace of electricity remaining in it to furnish a basis for this rapid process of multiplication. In unfavourable weather it may be necessary at the outset to employ a flat piece of vulcanite (or other suitable substance), which has been electrified by friction, and hold it at the back of the fixed plate opposite the highest or lowest brush, till the machine begins to work.

When the two patches of tin-foil have acquired their charges, a great deal more electricity is produced than is necessary for keeping them up. The surplus is collected from the revolving plate by rows of brass points, just as in the friction machine. They are ranged along the two horizontal radii of the plate, one row collecting positive and the other negative. They are in connection with the two knobs C which are seen in front of the machine, and a brilliant discharge of electricity takes place between these knobs. In the above description we have supposed the right-hand patch of tin-foil to be negative. It will accordingly attract positive electricity from the right-hand conductor to the points, through which the positive electricity will stream off on to the face of the plate, leaving the conductor with a strong negative charge. The right-hand knob will therefore be negative, and the left-hand knob positive. The knobs are at the ends of sliding rods with insulating handles, and can either be placed in contact or separated to a distance of several inches. They should be about half an inch apart at starting, and be gradually opened wider as the discharge becomes stronger.

In the original Holtz machine, in place of the brushes and studs for replenishing the charges of the *armatures* (that is, the patches of tin-foil, or paper patches answering the same purpose), these armatures are furnished with projecting points of cardboard which collect electricity from the revolving plate by discharge through the air. The plate passes these cardboard points just before it passes the brass points which supply the conductors.

32. Wimshurst's Machine.—The best of all the electrostatic machines of the present day is that of Mr. Wimshurst, Fig. 24. It has two glass plates revolving with equal speeds in opposite directions, each plate carrying on its outer face a number of equidistant

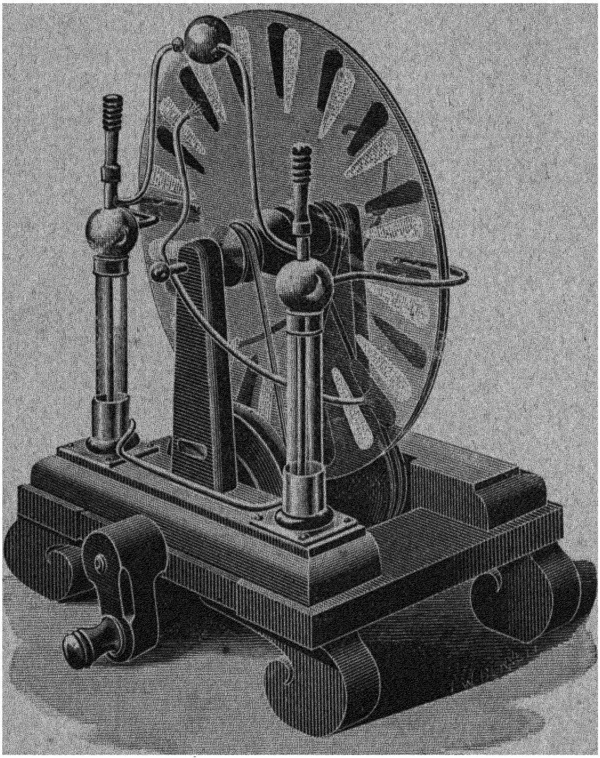


Fig 24 —Wimshurst's Electrical Machine

patches of tin-foil. The number should not be less than sixteen, and must be the same for both plates. Sometimes each patch is furnished with a projecting stud to make contact with the brushes, but this is of doubtful benefit. The curved bar of metal seen in front of the machine carries brass brushes at its two ends which make simultaneous contact with a pair of diametrically opposite patches. This bar is adjustable and is usually set to a slope of about 45°. At the back of the machine, just visible in the figure, is a similar bar sloping the opposite way, so that the two bars are nearly at right angles to one another.

The electricity furnished by the machine is collected by brass combs embracing the plates at the ends of a horizontal diameter and connected with the two conductors, one positive and the other negative. In the figure (which has been kindly supplied by Mr. Wimshurst expressly for this work) each of the two conductors consists of a fixed sphere of brass, in which turns, by means of an insulating handle, a curved arm of brass terminating in a ball. The two balls are of unequal sizes, and can be interchanged so as to make either of them positive at pleasure. Their distance apart is regulated by rotating the insulating handles.

As shown in the figure the conductors are connected with the inner coatings of two Leyden jars, whose outer coatings are connected with each other. To disconnect the conductors from the jars, the vertical rods seen in the centres of the jars are lifted out.

33. The working of the machine will be understood from the accompanying diagram, Fig. 25, which indicates the directions of rotation of the two plates, and the signs of the charges on their tin-foil patches when the right-hand conductor is the positive one. At the moment when the brushes at A and B make contact with a pair of tin-foil patches on the front plate, these two patches, together with the connecting rod AB, form one conductor, which is influenced by the negative electricity on the portion of the back plate opposite to A, and the positive on the portion opposite to B. Thus the patch at A acquires a positive and that at B a negative charge, and these charges are carried on till they are partly delivered up to the combs. Similar reasoning applies to the patches on the back plate. In passing C they become negatively and in passing D positively charged by the inductive action of the charges on the front plate. Thus the right-hand combs are supplied with positive electricity by both plates and the left-hand combs with negative.

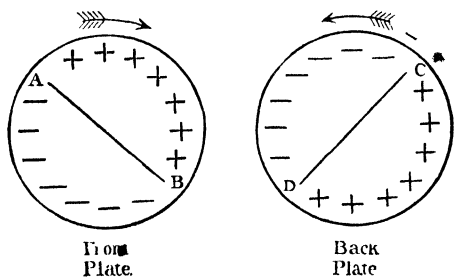


Fig 25.

Quite recently (1892) Mr. Wimshurst has exhibited a modification of his machine, in which each of the two conductors becomes alternately positive and negative, the reversals succeeding each other with extreme rapidity.

CHAPTER V.

EXPERIMENTS ON ELECTRIC DISCHARGE.

34. Spark, Brush, and Glow.—Luminous discharges of electricity are usually classed as of three kinds: the spark, the brush, and the glow. The spark, as furnished by ordinary electrical apparatus, may be of any length, from a small fraction of an inch to several inches. When short it is usually straight, and accompanied by an instantaneous sound which may be described as a crack. The crack becomes louder in proportion as the length of the spark is increased, and still more as the capacity of the discharging conductor is increased, the discharge of a battery of Leyden jars even across a distance of an inch being of deafening loudness. When the spark is long and loud it is usually crooked, resembling on a small scale a flash of forked lightning. Such sparks are easily obtained from an induction machine when its Leyden jars are in connection with the terminals. When the jars are out of connection there is no loud crack, but the discharges succeed each other so rapidly as to produce a sizzling sound, and a continuous stream of fire (often consisting of several nearly parallel threads) plays across the interval. As the knobs are drawn further apart, a dark interval is left near the negative knob. Lines of fire extend from the positive knob to a certain distance, where they terminate in mid air; the negative knob exhibits a luminous glow, and the intervening region is dark. When the room is darkened, the direct discharge is seen to be surrounded by a nebulous luminosity of purple tint, shaped like an egg, with its ends at the two knobs, and an interruption or defect of brightness at the dark space above-mentioned.

35. When a conductor is giving off electricity from points or sharp edges into the air, it exhibits in the dark an appearance of

lines of fire issuing from the points and spreading out into broader streams of fainter luminosity. This is called the *brush* discharge. It is best seen when the electricity which is escaping from the conductor is positive.

The glow discharge as seen in the dark renders the surface of the conductor luminous, but the luminosity does not extend into the surrounding air.

When an induction machine is working in the dark, the discharges which take place between the combs and the revolving glass-plates appear very different at the two combs. The positive conductor is receiving positive or giving off negative electricity through its comb. The points of its comb are accordingly covered with a glow, and some luminosity is seen on the surface of the plate. The negative conductor is receiving negative or giving off positive electricity through its comb. Lines of fire proceed from its points and spread out into a large sheet of light on the surface of the plate.

Very interesting variations in the appearance of the discharge between the terminal knobs can be obtained by making the knobs very unequal in size and interchangeable, as in the pattern of Wimshurst machine, shown in Fig. 24. The description given in § 34 ceases to apply when the small knob is negative.

It is probable that the passage of a spark is always preceded by a state of stress in the intervening air, the stress being most intense along the path of least resistance, which at last gives way, the spark passing along the line of rupture.

36. Duration of the Spark.—We can form no judgment of the duration of the electric spark from what we see with the unaided eye, for impressions made upon the retina remain uneffaced for something like $\frac{1}{10}$ of a second, and the duration of the spark is incomparably less than this. The discharges of a Wimshurst or Voss machine with Leyden jars cause the revolving plates with their tin-foil sectors to be seen as if stationary. As this appearance can be seen even in full daylight, it shows that the spark, though very brief, is an extremely powerful illuminant while it lasts.

Wheatstone, in a classical experiment, measured the duration of the spark of a Leyden jar by means of a revolving mirror; an expedient which has since been employed with great advantage in many other researches, especially in determining the velocity of light.

Let mn (Fig. 26) be a mirror revolving with great velocity about an axis passing through c, and suppose that, during the rotation, an electric spark is produced at a. An eye stationed at o will see an image in the symmetrical position a'. If the spark is strictly instantaneous, its image will be seen as a luminous point at a', notwithstanding the rotation of the mirror; but if it has a finite duration, the image will move from a' to a'', while the mirror moves from ee' to tt', the latter being its position when the spark ceases. What is actually seen in the mirror will therefore not be a point, but a luminous track $a'a''$.

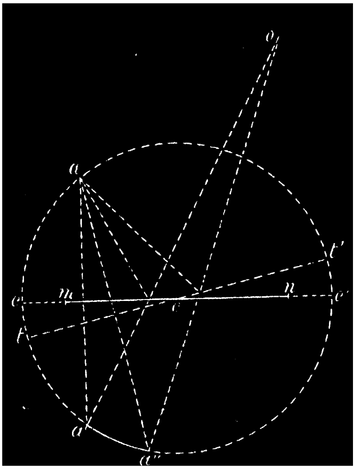


Fig. 26.—Duration of Spark.

The length of this image will be double of the arc et; for the angle ect at the centre is equal to the angle $a'aa''$ at the circumference, the sides of the one being perpendicular to those of the other. In Wheatstone's experiment, the mirror made 800 turns in a second, and the image $a'a''$ was an arc of 24°; the mirror therefore turned through 12°, or $\frac{1}{30}$ of a revolution, while the spark lasted. The duration of the spark was therefore $\frac{1}{30}$ of $\frac{1}{800}$, that is, $\frac{1}{24000}$ of a second.

By examining the brush in the same way, Wheatstone found it to consist of a succession of sparks.

37. Oscillatory Discharge.—Feddersen, by taking instantaneous photographs of the spark of a Leyden jar as seen in a revolving mirror, has shown that it consists not of one discharge only, but of a number of successive discharges, a result which was predicted by Sir Wm. Thomson from theory many years before. The successive discharges are in opposite directions, and are analogous to the successive swings of a pendulum, first to one side and then to the other, in a medium whose resistance produces a rapid decrease of amplitude.

The appearance of the spark is greatly modified by rarefying the air in which it is taken. We shall return to this subject in a later chapter.

38. Velocity of Electricity.—Soon after the invention of the Leyden jar, various attempts were made to determine the velocity with which the discharge travels through a conductor connecting the two coatings. Watson, about 1748, took two iron wires, each more than a mile long, which he arranged on insulating supports in such a way that all four ends were near together. He held one end of each wire in his hands, while the other ends were connected with the two coatings of a charged jar. Although the electricity had more than a mile to travel along each wire before it could reach his hands, he could never detect any interval of time between the passage of the spark from the knob of the jar and the shock which he felt. The velocity was in fact far too great to be thus measured.

Wheatstone, about 1836, investigated the subject with the aid of the revolving mirror of which we have spoken above (§ 36). He connected the two coatings of a Leyden jar by means of a conductor which had breaks in three places, thus giving rise to three sparks. When the sparks were taken in front of the revolving mirror, the positions of the images indicated a retardation of the middle spark, as compared with the other two, which were taken near the two coatings of the jar, and were strictly simultaneous. The middle break was separated from each of the other two by a quarter of a mile of copper wire. He calculated that the retardation of the middle spark was $\frac{1}{1,152,000}$ of a second, which was therefore the time occupied in travelling through a quarter of mile of copper wire. This is at the rate of 288,000 miles per second, a greater velocity than that of light, which is only about 186,000 miles per second.

Since the introduction of electric telegraphs, several observations have been taken on the time required for the transmission of a signal. For instance, trials in Queenstown harbour, in July, 1856, when the two portions of the first Atlantic cable, on board the *Agamemnon* and *Niagara*, were for the first time joined into one conductor, 2500 miles long, gave about $1\frac{3}{4}$ seconds as the time of transmission of a signal from induction coils, corresponding to a velocity of only 1400 miles per second. In 1858, before again pro-

ceeding to sea, a quicker and more sensitive receiving instrument—Thomson's mirror galvanometer—gave a sensible indication of rising current at one end of 3000 miles of cable about a second after the application of a Daniell's battery at the other.

It seems to be fully established by experiment that electricity has no definite velocity, and that its apparent velocity depends upon various circumstances, being greater through a short than through a long line, greater (in a long line) with the greater intensity and suddenness of the source, greater with a copper than with an iron wire, and much greater in a wire suspended in air on poles than in one surrounded by gutta-percha and iron sheathing, and buried under ground or under water. In a long submarine line, a short sharp signal sent in at one end, comes out at the other as a signal gradually increasing from nothing to a maximum and then gradually dying away.

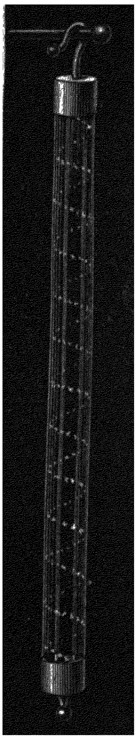


Fig 27.
Spangled Tube

39. Multiplication of the Electric Spark.—The old electricians contrived several pieces of apparatus for multiplying the electric spark. The principle of all is the same. Small squares of tin-foil are arranged in series at a small distance from each other on an insulating surface. The first of the series is connected with a metallic knob which can be brought near the electrical machine; and the last of them is connected with another knob which is in communication with the earth. By allowing a discharge to pass through the series, sparks can be simultaneously obtained at all the intervals between the successive squares.

In the *spangled tube* (Fig. 27) the squares of tin-foil are arranged spirally along a cylindrical glass tube which has a brass cap at each end. One cap is put in communication with the machine, and the other with the earth.

Sometimes a glass globe is substituted for the cylinder. We have thus the *spangled globe* (Fig. 28).

In the *sparkling pane* a long strip of tin-foil is disposed in one continuous crooked line (consisting of parallel strips connected at alternate ends) from a knob at the top to another knob at the bottom of the pane. A pattern is then traced by scratching away the tin-foil in numerous places with a point, and when the spark

passes, it is seen at all these places, so as to render the pattern luminous (Fig. 29).

40. Physiological Effects of the Spark: Electric Shock.—When the finger is held between the knobs of an induction machine, with the jars disconnected, the sparks produce a sharp pricking sensation.

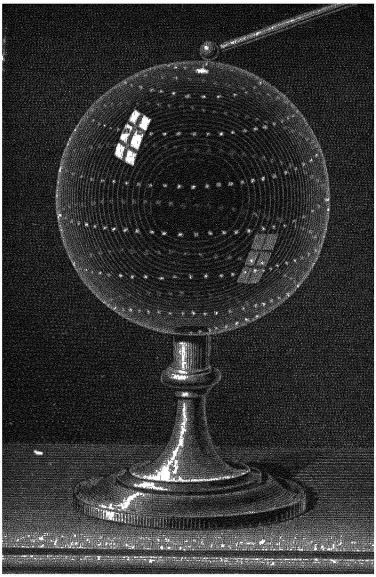


Fig. 28.– Spangled Globe.

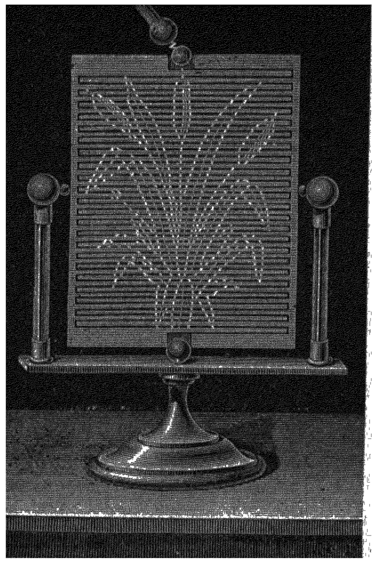


Fig 29.– Sparkling Pane

In general a pricking sensation more or less strong is felt at the place where a spark enters or leaves the body.

When the spark is taken from a Leyden jar, a shock is felt, especially if one hand is kept in contact with the outer coating of the jar while the other hand is brought to the knob. The sensation experienced is altogether unique, and is accompanied by spasmodic contraction of the muscles.

At the distance of a few feet from a machine in powerful action, a tickling sensation is felt on the exposed parts of the body, due to the movement of the hairs in obedience to electrical force. These phenomena are exhibited in a still more marked manner when a person stands on a stool with glass legs, and keeps his hand upon the conductor. He thus becomes highly charged with electricity. His hair stands on end, and is luminous if seen in the dark. If a

conductor connected with the earth is presented to him, a spark passes, and his hair falls again.

Electricity has frequently been resorted to for medical purposes. The electrical machine was first employed, and afterwards the Leyden jar, but both have now been abandoned in favour of magneto-electric machines and other apparatus for obtaining induced currents, which we shall describe in a later chapter.

41. Heating and Mechanical Effects.—The heating power of the electric spark may be conveniently shown by taking the discharge of an induction machine (with or without jars attached) across gas escaping from a burner. The gas is instantly ignited.

If a person standing on a stool with glass legs keeps one hand in permanent contact with one conductor of the machine, while the other conductor is connected to the earth, he can light gas by giving a spark from his other hand to the burner. The luminosity of the spark is doubtless due, either wholly or in part, to the very high temperature which is produced in the particles of air traversed by the discharge.

The commotion produced in the air by the spark can be exhibited

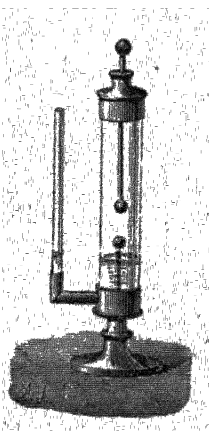


Fig. 30.—Kinnersley's Thermometer.

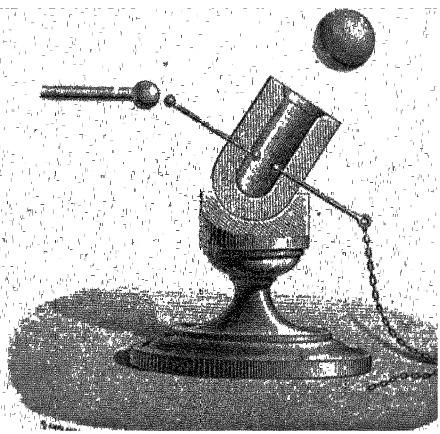


Fig. 31.—Electric Mortar.

by means of Kinnersley's (so called) thermometer (Fig. 30), which consists of two glass tubes of unequal diameters, the smaller being open at the top, while the larger is completely closed, with the exception of a side passage, by which it communicates with the smaller. The caps which close the ends of the large tube are traversed by rods terminating in knobs, and the upper one can be

raised and lowered to vary the distance between the knobs. Both tubes are filled, to a height a little below the lower knob, with a very mobile liquid such as alcohol. When the spark passes between the knobs, the liquid is projected with great violence, and may rise to a height of several yards if the spark is very strong. The same property of the spark is exhibited in the experiment of the electric mortar, which is sufficiently explained by the figure (Fig. 31).

The spark may be obtained in the interior of a non-conducting liquid, which it agitates in a similar manner. If the liquid is contained in a closed vessel, this is often broken.

The spark of an induction machine with jars attached will puncture card, the hole being larger or smaller according to the strength of the spark. Thin glass can be punctured by the discharge of a battery of jars.

42. Chemical Properties of the Spark.—Discharges of electricity, whether in the form of a single strong spark, or a succession of sparks, or a continuous discharge like that obtained between the conductors of an induction machine without the jars, are capable of producing on a very small scale the effects of a voltaic battery. Electrolytes can be very slowly decomposed and metallic thread can be heated. The former result can be easily obtained by moistening a piece of blotting-paper with solution of iodide of potassium and placing it close to the two knobs of the machine. Iodine is set free at the positive knob and produces discoloration of the paper.

43. Wind from Points.—If a metallic rod terminating in a point be attached to the conductor of the electrical machine, electricity escapes in large quantity from the point, which, accordingly, when viewed in the dark, is seen to be crowned with a tuft of light. A layer of air in front of the point is electrified by contact, and then repelled, to make way for other portions of air, which are in their turn repelled. A continuous current of air is thus kept up, which is quite perceptible to the hand, and produces a very visible effect on the flame of a taper.

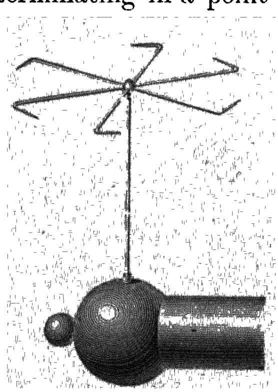


Fig. 32.—Electric Whirl.

The *electric whirl* (Fig. 32) consists of a set of metallic arms, radiating horizontally from a common centre about which they can turn freely, and bent, all in the same direction,

at the ends, which are pointed. When the central support is mounted on the conductor of the machine, the arms revolve in a direction opposite to that in which their ends point. This effect is due to the mutual repulsion between the pointed ends and the electrified air which flows off from them.

It is instructive to remark that if, by a special arrangement, the rotating part be inclosed in a well-insulating glass case, the rotation soon ceases, because, in these circumstances, the inclosed air quickly attains a state of permanent electrification.

44. Electric Watering-pot.—Let a vessel containing a liquid, and furnished with very fine discharge tubes, be suspended from the conductor of the machine. When the vessel is not electrified, the liquid comes out drop by drop; but when the machine is turned, it issues in continuous fine streams. It has, however, been observed that the quantity discharged in a given time is sensibly the same in both cases. This must be owing to the equality of action and reaction between different parts of the issuing stream.

It is found that the drops of a small fountain exhibit a marked tendency to coalesce when the jet from which they issue is electrified even to a very feeble degree, or when an electrified body is held near it so as to give an induced charge to the issuing water.

CHAPTER VI.

ELECTRICAL POTENTIAL, AND LINES OF ELECTRICAL FORCE.

45. The object of the present chapter is to give a brief outline of the methods by which mathematicians have succeeded in bringing numerous electrical problems within the range of accurate reasoning.

The fundamental conception in the mathematical theory of electricity is that of attraction and repulsion, acting according to the law of inverse squares; and the unit quantity of electricity is defined to be that quantity which would attract or repel an equal quantity at unit distance with unit force.

The influence which an electrified body exercises in the region around it can be specified by stating the force of attraction or repulsion which the body would exert upon a small charged body placed in various parts of this region or field (which is accordingly called a field of electrical force), the force being stated not only in magnitude but also in direction. In this sense we speak of *the electrical force at a point*, meaning the force which would be exerted upon a unit of electricity placed at the point; and in any such specification the unit of electricity is supposed not to disturb by its presence the previously existing distribution of electricity.

There can be electrical force at points in the air, or in the substance of any non-conductor, without disturbance of equilibrium; but electrical force in a conductor instantly produces a current of electricity in the direction of the force. At the *surface* of a conductor electrical force can exist, but it must always be normal to the surface; for if there were any tangential component, a current would be produced along the surface.

46. **Definition of Difference of Potential.**—We know, by the principle of the conservation of energy, that the work done upon a unit

of electricity in its passage from one point to another, must be independent of the path pursued; and we agree to call this work the difference of potential of the two points.

47. **Relation between Potential and Force.**—If V denote the potential at a point, and $V-\delta V$ the potential at a neighbouring point, δV is the work which electrical attractions and repulsions do upon a unit of positive electricity in its passage from the first point to the second; and since work is equal to force multiplied by distance, the average force along the joining line can be computed by dividing δV by the distance, which we will call δs. Hence the limiting value of $\frac{\delta V}{\delta s}$ as the two points are taken nearer together, is the component force in the direction δs; that is, *the rate of variation of potential in any direction is equal to the component force in that direction.*

The direction in which the variation is most rapid will be the direction of the resultant force; and when δs is measured in this direction $\frac{\delta V}{\delta s}$ will be equal to the resultant force.

48. **Lines of Force.**—If a line be traced such that every small portion of it (small enough to be regarded as straight) is the direction of resultant force at the points which lie upon it, it is called a line of force; in other words, a line of force is a line whose tangent at any point is the direction of the force at that point. We may express this briefly by saying that lines of force are the lines along which resultant force acts.

49. **Equipotential Surfaces.**—An equipotential surface is a surface over the whole of which there is the same value of potential. When δs lies in such a surface, the value of $\frac{\delta V}{\delta s}$ is zero; and therefore there is no component force along any line lying in the surface. The resultant force must therefore be normal; that is, *lines of force cut equipotential surfaces* at right angles.

When we are dealing with gravitational forces instead of with electrical attractions and repulsions, equipotential surfaces are called *level* surfaces, and lines of force are called *verticals.*

If two equipotential surfaces are given, their potentials being V_1 and V_2, the work done in carrying a unit of electricity from any point of the one to any point of the other, is constant, and equal to the difference of V_1 and V_2.

If we consider two equipotential surfaces very near one another, so that the portions which they intercept on the lines of force may

be regarded as straight, the intensity of force at different points of the intermediate space will vary inversely as the distance between the two equipotential surfaces; for, when equal amounts of work are done in travelling unequal distances, the forces must be inversely as the distances.

50. Potential of a Conductor.—When electrical potential is constant throughout a given space, there is no electrical force in that space; and conversely, if there be an absence of electrical force in a given space, the potential throughout that space must be uniform. These propositions apply to the space within a hollow conductor. They also apply to the whole substance of a solid conductor, and to the whole space inclosed within the outer surface of a hollow conductor. Whenever a conductor is in electrical equilibrium, it has the same potential throughout the whole of its substance, and also through any completely inclosed hollows which it may contain.

When a conductor is not in electrical equilibrium, currents set in, tending to restore equilibrium; and the direction of such currents is always from places of higher to places of lower potential.

It is usual to assume as the zero of potential the potential of the earth; but this assumption is not consistent with itself, since the existence of earth currents proves that there are differences of potential between different parts of the earth. The absolute zero of potential is the potential of places infinitely distant from all electricity.

51. Energy of a Charged Conductor.—When positive electricity is allowed to run down from a conductor of higher to one of lower potential, there is a loss of potential energy, just as there is a loss of potential energy in the running down of a heavy body from a higher to a lower level; and on the other hand, to make positive electricity pass from a conductor of lower to one of higher potential, work must be expended from some external source, just as work must be expended to raise a heavy body. In the case of the heavy body, the work expended in the latter case, or the potential energy which runs down in the former, is equal to its weight multiplied by the difference of levels; and in the analogous case of the electrical operation, the work or the energy is the product of the quantity of electricity which passes from one conductor to the other by the difference of potentials of the two conductors,[1] provided that these potentials remain sensibly constant during the operation.

[1] The closeness of the analogy will be better understood when it is remembered that if

When a conductor is charged in the ordinary way, its charge is drawn from the earth, the potential of which is unaffected. If we suppose the charge to be communicated in a numerous succession of small equal parts, the potential of the conductor, which is originally zero, is increased by a succession of equal steps, till it attains its final value. Hence it is only the last part that is raised through the full difference of potential, and the mean value of the difference of potential through which the successive parts are raised is the half of this. Hence the work done in charging a conductor, or the energy which runs down in discharging it into the earth, is half the product of its potential and its charge.

52. **Tubes of Force.**—If we conceive a narrow tube bounded on all sides by lines of force, and call it a *tube of force*, we can lay down the following remarkable rules[1] for the comparison of the forces which exist at different points in its length. (1) *In any portion of a tube of force not containing electricity, the intensity of force varies inversely as the cross-section of the tube,* or the product of *intensity of force by section of tube* is constant.[2] (2) *When a tube of force cuts through electricity, this product changes, from one side of this electricity to the other, by the amount* $4\pi q$, where q denotes the quantity of the electricity inclosed by the tube.

The following are particular cases of (1):—

When the electricity to which the force is due is collected in a point, the lines of force are straight, the tubes of force are cones (in the most general sense), and the law of force becomes the law of inverse squares, since the section of a cone varies as the square of the

a series of level surfaces are described completely surrounding the earth, and one foot apart at the equator, they will be less than a foot apart at the poles, for the distance between them will, by the reasoning in the text, be inversely as the intensity of gravity. The work done in lifting a body from any one of these surfaces to any other, will be proportional to the product of its mass (not its weight) by the number of intervals crossed.

If one of the surfaces passes through the top of Mount Everest, and another through a point on the Indian coast, the distance between them will be greater at the coast than at the mountain. Hence the height of the mountain above the coast is an ambiguous quantity.

[1] For the proof of these rules, as mathematical deductions from the law of inverse squares, the student may refer to Everett's edition of Todhunter's *Analytical Statics*, articles 228, 235.

[2] This is obviously analogous to the law which applies to the comparison of the velocities of a liquid in different parts of a tube through which it flows, since the product of area by velocity is the volume of liquid which flows past any section in unit time. The tube may be an imaginary one, bounded by lines of flow in a large body of liquid flowing steadily. Lines of flow are thus the analogues of lines of force.

distance from the vertex. These results also apply to the case of electricity uniformly distributed over the surface of a sphere, the common vertex in which the cones would meet if produced being now at the centre of the sphere.

When the electricity consists of the charges of two oppositely electrified parallel plates, whose length and breadth exceed the distance between them (the plates being conductors, and placed opposite to each other), the lines of force between their central portions are sensibly straight and parallel, the tubes of force are therefore cylinders (in the most general sense), and the force is constant, being equal to the difference of the potentials of the plates divided by the distance between them. The same thing holds if, instead of being oppositely electrified, the plates are similarly electrified, but not to the same potential.

53. Force Proportional to Number of Tubes which cut Unit Area.—The cross-sections of tubes of force are portions of equipotential surfaces. If one equipotential surface be divided into portions, such that the product of *area by force-intensity* shall be the same for all, then, if all neighbouring space not containing electricity be cut up into tubes, springing from these portions as their respective bases, the product of any cross-section of any one of these tubes by the force-intensity over it will be constant. The force-intensities at any points in this space are therefore inversely as the cross-sections of the tubes at these points, or are directly as the number of tubes per unit area of equipotential surfaces at the points.

54. Force just Outside an Electrified Conductor.—Since there is no force in the interior of a conductor, the lines and tubes of force become indeterminate; but proposition (2) of § 52 can be shown to hold when we give them any shape not discontinuous. Let ρ denote the electric density at a point on the surface, and a a small area around this point, which area we shall regard as a section of a tube of force cutting through the surface. Let F denote the intensity of force just outside the surface opposite this point, then, since the intensity inside is zero, we have

$$\mathrm{F}a = 4\pi q = 4\pi\rho a \quad , \quad \mathrm{F} = 4\pi\rho ;$$

that is, *the intensity of force just outside any part of the surface of a charged conductor, is equal to the product of* 4π *into the density at the nearest part of the surface.*

55. Relation of Induction to Lines and Tubes of Force.—Lines of

force are also the lines along which induction takes place. On Faraday's theory of induction by contiguous particles, the line of poles, for any particle, is coincident with the line of force which passes through the particle. Apart from all theory, it is matter of fact that *a tube of force extending from an influencing to an influenced conductor, and not containing any electricity in the interval between, has equal quantities of electricity on its two ends, these quantities being of opposite sign.* This equality follows at once from § 52 (2), if we consider the tube as penetrating the two conductors; for the product of force by section, which is constant for the portion of the tube in air, is zero in both conductors; and the quantity of electricity on *either* end of the tube must be the quotient of this constant product by 4π. In connection with this reasoning, it is to be remarked that the surface of a conductor is an equipotential surface, and is cut at right angles by lines of force.

In Faraday's ice-pail experiment, a tube of force springing from the upper side of the charged ball, and of such small section at its origin as to inclose only an insensible fraction of the charge of the ball, opens out so fast, as it advances, that it fills the whole opening at the top of the pail.

In every case of induction, therefore, *the total quantities of inducing and induced electricity are equal, and of opposite sign.*

When the inducing electricity resides in or upon a non-conductor, for example on the surface of a glass rod, or in the substance of a mass of air, the quantity of electricity induced on the base of a tube of force is equal and opposite to the quantity contained within the tube. In the simplest case, all the tubes will have a common apex, which will be a point of maximum or minimum potential.

56. Potential defined as $\Sigma \frac{q}{r}$.

Let a quantity q of electricity be collected at a point O, and let A, B be any two points very near together. The forces at A and B due to q will be $\frac{q}{\mathrm{OA}^2}$ and $\frac{q}{\mathrm{OB}^2}$, and these will be nearly equal to each other or to $\frac{q}{\mathrm{OA}\cdot\mathrm{OB}}$. When a unit of electricity is carried from A to B its motion can be resolved into two components, one of them in the direction of the force and equal to OB − OA, and the other perpendicular to the direction of the force. Hence the work done will be $\frac{q(\mathrm{OB}-\mathrm{OA})}{\mathrm{OA}\cdot\mathrm{OB}}$, that is, $\frac{q}{\mathrm{OA}}-\frac{q}{\mathrm{OB}}$; or if r denote the distance of any point from O, the *work done in a* small *movement is equal to*

the change in the value of $\frac{q}{r}$. Since any movement can be resolved into a succession of small movements, we may omit the word *small*, and the proposition will still be true. As r increases to infinity, $\frac{q}{r}$ will diminish to zero. Hence, $\frac{q}{r}$ denotes the work from distance r to infinite distance.

As regards sign, $\frac{q}{r}$ is the work done by electrical force when a unit of positive electricity is carried from distance r to infinite distance.

Next suppose several quantities, q_1, q_2, &c., to be collected at different points, O_1, O_2, &c. Let P be any other point, and let $O_1P=r_1$, $O_2P=r_2$, &c. Then in the passage of a unit of electricity from P to infinite distance, the electrical work is, by the preceding section,

$$\frac{q_1}{r_1}+\frac{q_2}{r_2}+ \quad \&c.,$$

which we will denote by $\Sigma\frac{q}{r}$, the symbol Σ being read "the sum of such terms as."

$\Sigma\frac{q}{r}$ is therefore the general expression for the potential at a point due to any quantity of electricity distributed in any manner; in other words, the potential is equal to the sum of the quotients obtained by dividing each element of electricity by its distance from the point. The distances are essentially positive. If the electricity is not all of one sign, some of the quotients, $\frac{q}{r}$, will be positive and others negative, and their algebraical sum is to be taken.

57. Application to Sphere.—In the case of a charged conducting sphere, all the elements q are equally distant from the centre of the sphere, and the sum of the quotients $\frac{q}{r}$, when we are computing the potential at the centre, will be $\frac{Q}{R}$, Q denoting the charge, and R the radius of the sphere. But the potential is the same at all points in a conductor. $\frac{Q}{R}$ is therefore the potential of a sphere of radius R, with charge Q, when uninfluenced by any other electricity than its own.

58. Capacity.—The electrical capacity of a conductor is *the quantity of electricity required to charge it to unit potential*, when it is not influenced by any other electricity besides its own charge and the electricity which this induces in neighbouring conductors. Or,

since, in these circumstances, potential varies directly as charge, capacity may be defined as the *quotient of charge by potential.* Let C denote capacity, V potential, and Q charge, then we have

$$C = \frac{Q}{V} \quad ; \quad V = \frac{Q}{C} \quad ; \quad Q = VC.$$

But we have seen that, for a sphere of radius R, at a distance from other conductors or charged bodies, $V = \frac{Q}{R}$. Hence $C = R$; that is, the *capacity of a sphere is numerically equal to its radius.*

This is a particular instance of the general proposition that the capacities of similar conductors are as their linear dimensions; which may be proved as follows:—

Let the linear dimensions of two similar conductors be as $1 : n$. Divide their surfaces *similarly* into very small elements, which will of course be equal in number. Then the areas of corresponding elements will be as $1 : n^2$, and, if the electrical densities at corresponding points be as $1 : x$, the charges on corresponding elements are as $1 : n^2x$. The potential at any selected point of either conductor is the sum of such terms as $\frac{q}{r}$ (§ 56). Selecting the corresponding point in the other conductor, and comparing potentials, the values of q are as $1 : n^2x$, and the values of r are as $1 : n$; therefore the values of $\frac{q}{r}$ are as $1 : nx$. Hence the potentials of the two conductors are as $1 : nx$. If they are equal, we have $nx = 1$, and therefore $n^2x = n$; that is, the charges on corresponding elements, and therefore also on the whole surfaces, are as $1 : n$.

We shall see, in the next chapter, that the capacity of a conductor may be greatly increased by bringing it near to another conductor connected with the earth.

59. Connection between Potential and Induced Distribution.—In the circumstances represented in Fig. 4 (§ 8), if we suppose the influencing body C to be positively charged, the potential due to this charge will be algebraically greater at the near end A of the influence conductor than at the remote end B. The induced electricity on A B must be so distributed as to balance this difference, in fact the potential due to this induced electricity is negative at A and positive at B. All cases of induced electricity upon conductors fall under the rule that *the potential at all parts of a conductor must be the same, and hence, wherever the potential due to the influencing*

electricity is algebraically greatest, the potential due to the electricity on the influenced conductor must be algebraically least.

As there can be no force in the interior of a conductor, the force at any point in the interior, due to the influencing electricity, must be equal and opposite to the force due to the electricity on the surface of the conductor. This holds, whether the conductor be solid or hollow. A hollow conductor thus completely screens from external electrical forces all bodies placed in its interior.

60. Electrical Images.—If a very large plane sheet of conducting material be connected with the earth, and an electrified body be placed in front of it near its middle, the plate will completely screen all bodies behind it from the force due to the electrified body. The induced electricity on the plate therefore exerts, at all points behind the plate, a force equal and opposite to that of the electrified body, or, what is the same thing, a force identical with that which the electrified body would exert if its electricity were reversed in sign. But electricity distributed over a plane surface must act symmetrically towards both sides. Hence the force which the indu[illegible] electricity exerts in front, is identical with that which would b[illegible] a body precisely similar to the given electrical body, sy[illegible] placed behind the plane, and charged with the opposit[illegible] The total force at any point in front of the plane is th[illegible] the force due to the given electrified body, and the for[illegible] imaginary image. The name and the idea of *electrica*[illegible] which this is one of the simplest examples, are due to S[illegible] son.

The mathematical student will have no difficulty in [illegible] the joint effect of a small electrified body in front of [illegible] its image behind the plane is to produce at any point [illegible] of the plane a force which varies inversely as the cub[illegible] tance of this point from the electrified body; whence, by § 52, the distribution of induced electricity on the plane is inversely as the cube of the distance.

CHAPTER VII.

ELECTRICAL CONDENSERS.

61. Condensers.—The process called *condensation of electricity* consists in increasing the capacity of a conductor by bringing near it another conductor connected with the earth, or with some other source fr m which electricity can be supplied. The two conductors are thin plates or sheets of metal, placed parallel to one an- a larger plate of non-conducting material between them. d B (Fig. 33) be the two conducting plates, of which A, *ollecting plate*, is connected ductor of the machine, and e *condensing plate*, with the et C be the non-conducting ectric) which separates them. machine has been turned it of charge is attained, the 3 which faces towards A is n negative electricity, drawn from the earth, and held by the attraction of the positive electricity of A; and, conversely, the surface of A which faces towards B, is covered with positive electricity, held there by the attraction of the negative of B, in addition to the charge which would reside upon it if the conductor were at the existing potential, and B and C were absent. In fact, the electrical density on the face of A, as well as the whole charge of A, would, in this latter case, be almost inappreciable, in comparison with those which exist in the actual circumstances. By condensation of electricity, then,

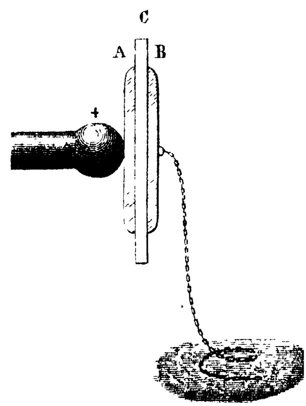


Fig. 33.—Theoretical Condenser.

we are to understand *increase*—usually enormous increase—*of electrical density on a given surface, attained without increase of potential.* If two conducting plates, in other respects alike, but one with, and the other without a condensing plate, be connected by a wire, and the whole system be electrified, the two plates will have the same potential, but nearly the whole of the charge will reside upon the face of that which is accompanied by a condensing plate.

62. **Calculation of Capacity of Condenser.**—The lines of force between the two plates A and B are everywhere sensibly straight and perpendicular to the plates, with the exception of a very small space round the edge, which may be neglected. The tubes of force (§ 52) are therefore cylinders, and the intensity of force is constant at all parts of their length. Also, since the potential of the plate B is zero, if we take V to denote the potential of the plate A, which is the same as the potential of the conductor, and t to denote the thickness of the intervening plate C, the rate at which potential varies along a line of force is $\frac{V}{t}$, which is therefore the expression for the force at any point between the plates whole space between the plates may be regarded as on tube of force of cross-section S equal to the area of eith two ends of the tube being the inner faces of the plates d tities of electricity $\pm$ Q residing on these faces are th but of opposite sign (§ 55); and as the force changes to $\frac{V}{t}$ in passing from one side to the other of the elec resides on either of these surfaces, we have (§ 52)

$$\frac{V}{t} . S = 4\pi Q.$$

Hence the capacity of the plate A, being, by definition, equal to $\frac{Q}{V}$, is equal to

$$\frac{S}{4\pi t}.$$

We should, however, explain that, if the intervening plate C is a solid or liquid, we are to understand by t not the simple thickness, but the thickness reduced to an equivalent of air, in a sense which will be explained further on (§ 69). This reduced thickness is, in the case of glass, about half the actual thickness.

If s denote an element of area of A, and q the charge residing

upon it, it is evident, from considering the tube of force which has s for one of its ends, that

$$\frac{V}{t} \cdot s = 4 \pi q;$$

and the electric density $\frac{q}{s}$ on the element is equal to $\frac{V}{4 \pi t}$, which is constant over the whole face of the plate.

To give a rough idea of the increase of capacity obtained by the employment of a condensing plate, let us compare the capacity of a circular disc of 10 inches diameter, accompanied by a condensing plate at a reduced distance of $\frac{1}{20}$ of an inch, with the capacity of a globe of the same diameter as the disc. The capacity of the globe is equal to its radius, and may therefore be denoted by 5. The capacity of the disc is $\frac{25 \pi}{4 \pi \times \frac{1}{20}} = 125$, or 25 times the capacity of the globe. It is, in fact, the same as the capacity of a globe 250 inches (or 20 ft. 10 in.) in diameter.

63. **Discharge of Condenser.**—If, by means of a jointed brass discharger (Fig. 34) with knobs M N at the ends, and with glass

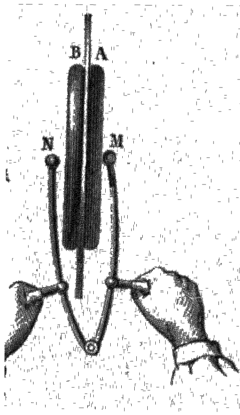


Fig. 34.—Discharge of Condenser.

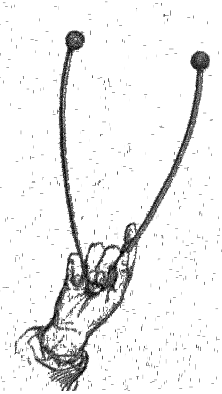


Fig. 35.—Discharger without Handles

handles, we put the two plates A and B in communication, a brilliant spark is obtained, resulting from the combination of the positive charge of A with the negative of B, and the condenser is discharged. When the quantity of electricity is small, the glass handles are unnecessary, and the simpler apparatus represented in Fig. 35 may be employed, consisting simply of two brass rods jointed together, and with knobs at their ends, care being taken to touch the plate B, which is in communication with the earth, before the other. The

operator will then experience no shock, as the electricity passes in preference through the brass rods, which are much better conductors than the human body. If, however, the operator discharges the condenser with his hands by touching first the plate B, and then also the plate A, the whole discharge takes place through his arms and chest, and he experiences a severe shock. If he simply touches the plate A, while B remains connected with the earth by a chain, as in Fig. 33, he receives a shock, but less violent than before, because the discharge has now to pass through external bodies which consume a portion of its energy. If, instead of a chain, B is connected with the earth by the hand of an assistant touching it, he too will receive a shock when the operator touches A.

64. Discovery of Cuneus.—The invention of the Leyden jar was brought about by a shock accidentally obtained. Some time in the

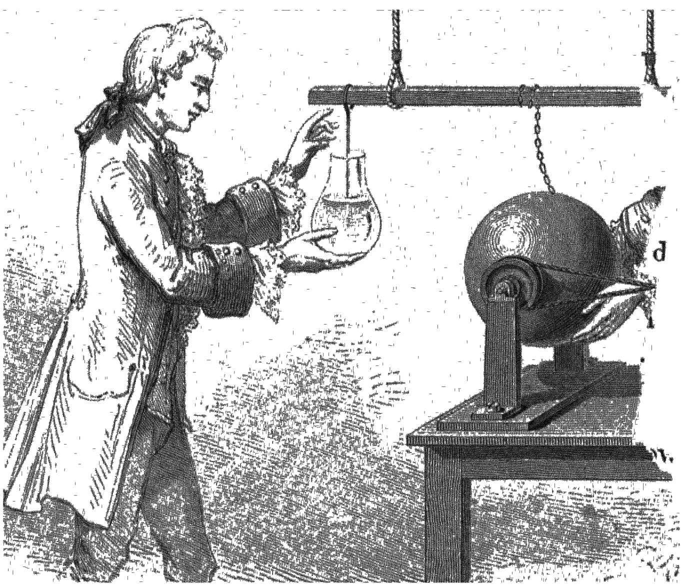


Fig 36 —Experiment of Cuneus.

year 1746, Cuneus, a pupil of Muschenbroeck, an eminent philosopher of Leyden, wishing to electrify water, employing for this purpose a wide-mouthed flask, which he held in his hand, while a chain from the conductor of the machine dipped in the water (Fig. 36). When the experiment had been going on for some time, he wished to disconnect the water from the machine, and for this purpose was about

to lift out the chain; but, on touching the chain, he experienced a shock, which gave him the utmost consternation, and made him let fall the flask. He took two days to recover himself, and wrote to Réaumur that he would not expose himself to a second shock for the crown of France. The news of this extraordinary experiment spread over Europe with the rapidity of lightning, and it was eagerly repeated everywhere. Improvements were soon introduced in the arrangement of the flask and its contents, until it took the present form of the *Leyden Phial* or *Leyden Jar*. It is easy to see that the effect obtained by Cuneus depended on condensation of electricity, the water in the vessel serving as the collecting plate, the hand as condensing plate, and the vessel itself as the dielectric. When he

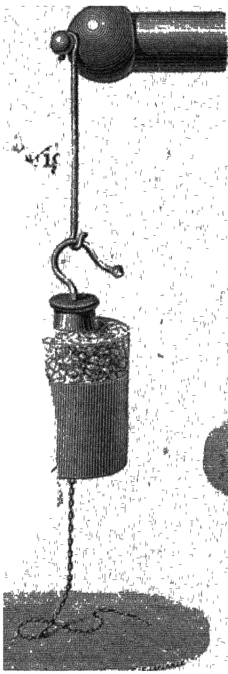


Fig. 37.—Leyden Jar.

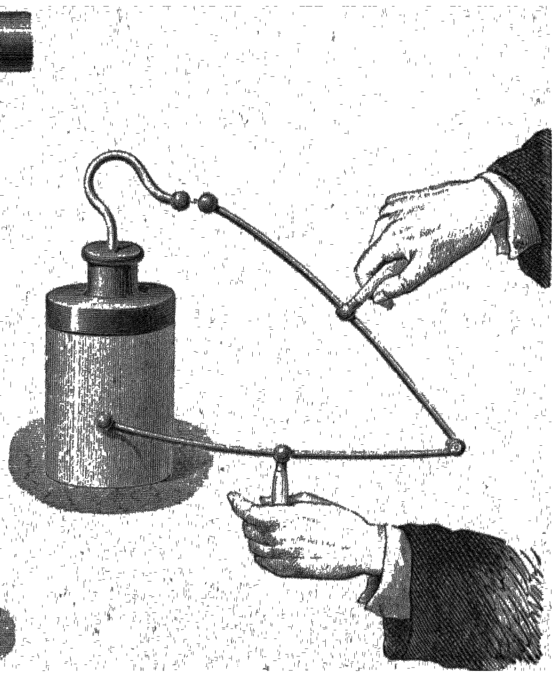


Fig. 38 —Discharge of Leyden Jar.

touched the chain, the two oppositely charged conductors were put in communication through the operator's body, and he received a shock.

65. Leyden Jar.—The Leyden Jar, as now usually constructed, consists of a glass jar coated, both inside and out, with tin-foil, for

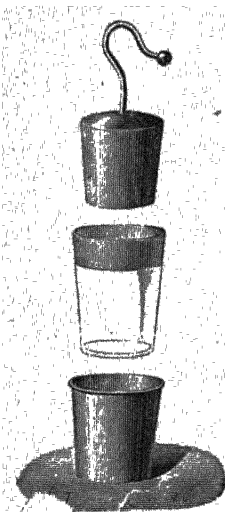
about four-fifths of its height. The mouth is closed by a cork, through which passes a metallic rod, terminating above in a knob, and connected below with the inner coating, either by a chain depending from it, or by pieces of metallic foil with which the jar is filled. The interior of the jar must be thoroughly dry before it is closed, and the cork and neck are usually covered with sealing-wax, and shellac varnish, which is less hygroscopic than glass. The Leyden jar is obviously a condenser, its two coatings of tin-foil performing the parts of a collecting plate and a condensing plate. If the inner coating is connected with the electrical machine, and the outer coating with the earth, the former acquires a positive, and the latter a negative charge. On connecting them by a discharger, as in Fig. 38, a spark is obtained, whose power depends on the potential of the inner coating, and on its electrical capacity. If these be denoted respectively by V and C, and if Q denote the quantity of electricity residing on either coating, the amount of electrical energy which runs down and undergoes transformation when the jar is discharged, is $\frac{1}{2} Q V = \frac{1}{2} C V^2 = \frac{1}{2} \frac{Q^2}{C}$. (§ 51).

The quantities Q, V, C, which are, properly speaking, th
potential, and capacity of the *inner coating*, are usually (
charge, potential, and capacity of the jar.

66. Residual Charge.—When a Leyden jar has been d
by connecting its two coatings, if we wait a short time we
another but much smaller spark by again connecting them
sparks may sometimes be obtained after further intervals.
called secondary discharges, and the electricity which tl
after the first discharge is called the *residual charge.*
to arise from a state of strain into which the glass is thro
charge, and from which it takes some time to recover.

The whole charge of the outer coating, and all except an insignificant portion of the charge of the inner coating, resides on the side of the foil which is in contact with the glass, or, more probably, on the surfaces of the glass itself, the mutual attraction of the two opposite electricities causing them to approach as near to each other as the glass will permit. This is illustrated by Franklin's experiment of the *jar with movable coatings* (Fig. 39). The jar is charged in the ordinary way, and placed on an insulating stand. The inner coating is then lifted out by a glass hook, and touched with the hand to discharge it of any electricity which it may retain. The

glass is then lifted out, and the outer coating also discharged. The jar is then put together again, and is found to give nearly as strong a spark as it would have given originally.



Jar with Mov-
Coatings

67. Discharge by Alternate Contacts.—Instead of discharging a Leyden jar at once by connecting its two coatings, we may gradually discharge it by alternate contacts. To do this we must set it on an insulating stand (or otherwise insulate both coatings from the earth), and then touch the two coatings alternately. At every contact a small spark will be drawn. The coating last touched has always rather less electricity upon it than the other, but the difference is an exceedingly small fraction of the whole charge, and, after a great number of sparks have been drawn by these alternate contacts, we may still obtain a powerful discharge by connecting the two coatings.

The quantities of electricity thus alternately discharged from the two coatings form two ɹg geometric series, one for each coating. In fact, if m 'e two proper fractions such that, when the outer coating is with the earth, the ratio of its charge to that of the inner ıd, when the inner coating is connected with the earth, ɔf its charge to that of the outer is $-m'$, we have the ;eries of values:—

	On inner coating.		On outer coating
ıal charges, ...	$+ Q$	...	$- m\ Q$
ıter 1st contact, ...	$+ m'm\ Q$	...	$- m\ Q$
2d "	$+ m'm\ Q$	..	$- m'm^2\ Q$
3d "	$+ m'^2m^2\ Q$	...	$- m'm^2\ Q$
	&c.		&c.

The quantities discharged from the inner coating are, successively $(1-m'm)\ Q$, $m'm\ (1-m'm)\ Q$, $m'^2m^2\ (1-m'm)\ Q$, &c.; and the quantities successively discharged from the outer, neglecting sign, are $m\ (1-m'm)\ Q$, $m'm^2\ (1-m'm)\ Q$, $m'^2m^3\ (1-m'm)\ Q$, &c.

The quantity $(1-m'm)\ Q$ discharged at the first contact represents that portion of the charge[1] which is not due to condensation; so

[1] This portion of the original charge is said to be *free*, and the remaining portion to be *bound*, *dissimulated*, or *latent*. These terms are applicable to all cases of condensation.

that the actual capacity of the Leyden jar is to the capacity of the inner coating if left to itself as $1 : 1 - m'm$.

The discharge by alternate contacts can be effected by means of a carrier suspended between two bells, as in Fig. 40. The rod from the inner coating terminates in a bell, and the outer coating is connected, by means of an arm of tin, with another bell supported on a metallic column. An insulated metallic ball is suspended between the two. This is first attracted by the positive bell. Then, being repelled by this and attracted by the other, it carries its charge of positive electricity to the negative bell, and receives a charge of negative, which it carries to the positive bell, and so on alternately. The whole apparatus stands upon an insulating support. It is not, however, necessary that the carrier should be insulated from the earth, but it must be insulated from both coatings.

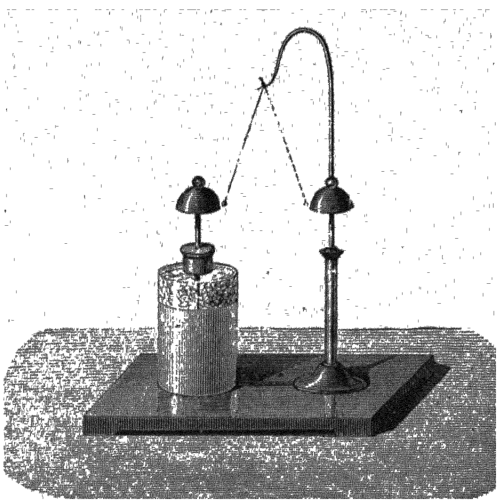


Fig. 40.—Alternate Discharge.

68. **Condensing Power.**—By the condensing power of a given arrangement is meant the ratio in which the capacity of the collecting plate is increased by the presence of the condensing plate, which ratio, as we have seen in last section, is equal to the fraction $\frac{1}{1-m'm}$. Riess has investigated its amount experimentally under varying conditions, by means of the apparatus represented in Fig. 41, which is a modification of the condenser of Æpinus. It consists of two metallic plates A and B, supported on glass pillars, and travelling on a rail, so that they can be adjusted at different distances. Between them is a large glass plate C. A is charged from the machine, B being at the same time touched to connect it with the ground. The electrical density on the anterior face of A was observed by means of Coulomb's proof-plane and torsion-balance.

Riess' experiments are completely in agreement with the theory laid down in the preceding sections of this chapter; for example, he found, among other results, that the condensing power was independent of

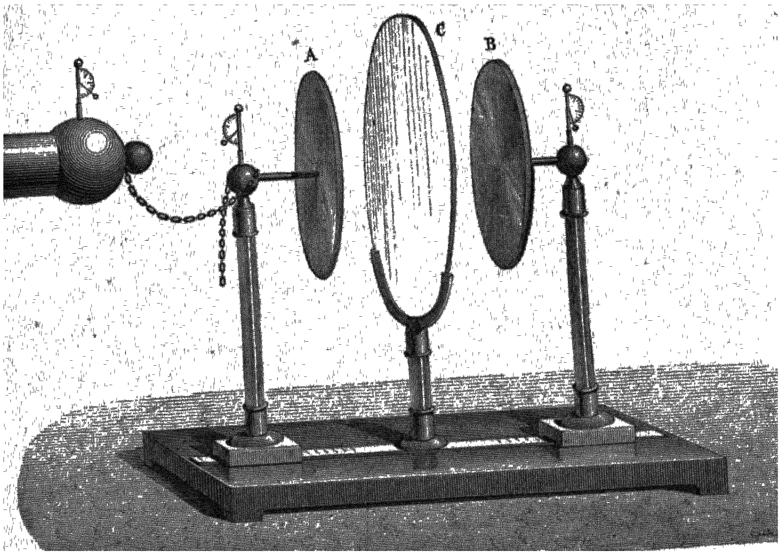


Fig. 41.—Condenser of Æpinus.

the absolute charge, and that it varied nearly in the inverse ratio of the distance.

69. Influence of the Dielectric.—Faraday discovered that the amount of condensation obtained in given positions of the two conducting plates depended upon the material of the *intervening non-conductor* or *dielectric.* Fig. 42 represents a modification of one of Faraday's experiments. A is an insulated metallic disc, with a charge, which we will suppose to be positive. B and C are two other insulated metallic discs at equal distances from A, each having a small electric pendulum suspended at its back. Let B and C be touched with the hand; they will become negatively electrified by induction, but their negative electricity will reside only on their sides which face towards A, and the pendulums will hang vertically. If, while matters are in this condition, we move B nearer to A, we shall see both the pendulums diverge, and on testing, we shall find that the pendulum B diverges with positive, and C with negative electricity. The reason is obvious. The approach of B to

A causes increased induction between them, so that more negative is drawn to the face of B, and positive is driven to its back; at the same time the symmetrical distribution of electricity on A is disturbed, a portion being accumulated on the side next B at the expense of the side next C. The inductive action of A upon C is thus diminished, and a portion of the negative charge of C is left free to spread itself over the back, and affect the pith-ball.

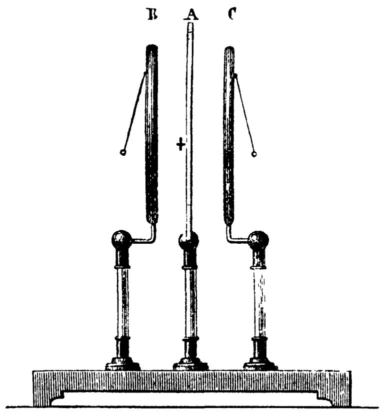


Fig. 42 —Change of Distance.

If, while the discs are in their initial position, B and C being equidistant from A, and the pendulums vertical, we interpose between B and A a plate of sulphur, shellac, or any other good insulator, the same effect will be produced as if B had been brought nearer to A. We see, then, that the insulating plate of a condensing arrangement serves not only to prevent discharge, but also to increase the inductive action and consequent condensation, as compared with a layer of air of the same thickness; inductive action through a plate of sulphur or shellac of given thickness, is the same as through a thinner plate of air. The numbers in the subjoined table (which contains Faraday's results) denote the thickness of each material which is equivalent to unit thickness of air. For example, the mutual induction through 2·24 inches of sulphur is the same as through 1 inch of air. These numbers are called

SPECIFIC INDUCTIVE CAPACITIES

Air or any gas,	1·00	Pitch,	1·80
Spermaceti,	1·45	Wax,	1·86
Glass,	1·76	Shellac,	2·00
Resin,	1·77	Sulphur,	2·24

The quotient of the actual thickness of the plate by the specific inductive capacity of its material may appropriately be called the *thickness reduced to its equivalent of air*, or simply the *reduced thickness.*

When the comparisons are made by very rapid charges and dis-

charges (so as to minimize the residual charge), larger values are found; for example,—

Glass,	7 to 9	India-rubber,	2
Shellac,	3·4	,, vulcanized,	2·8
Sulphur,	3	Ebonite,	2·6
Solid paraffin,	2	Turpentine,	2·2

70. Faraday's Determinations.—Faraday, to whom the name and discovery of specific inductive capacity are due, operated by comparing the capacities of condensers, alike in all other respects, but differing in the materials employed as dielectrics. One of his condensers is represented in Fig. 43. It is a kind of Leyden jar, containing a metallic sphere A, attached to the rod M, and forming with it the inner conductor. The outer conductor consists of the hollow sphere B divided into two hemispheres which can be detached from each other. The interval between the outer and inner conductor can be filled, either with a cake of solid non-conducting material, or with gas, which can be introduced by means of the cock R. The method of observation and reduction will be best understood from an example.

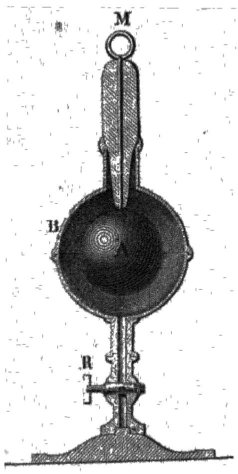


Fig. 43.—Apparatus for Specific Inductive Capacity.

The interval being occupied by air, the apparatus was charged, and a carrier-ball, having been made to touch the summit of the knob M, was introduced into a Coulomb's torsion-balance, and found to be charged with a quantity of electricity represented by 250° of torsion. When the second apparatus was precisely similar to the first, it was found that, on contact of the two knobs, the charge divided itself equally, and the carrier-ball, if applied to either knob, took a charge represented very nearly by 125°.

The conditions were then changed in the following way. The first jar still containing air, the interval between the two conductors in the second was filled with shellac. It was then found that the air-jar, being charged to 290°, was reduced, by contact of its knob with that of the shellac-jar, to 114°, thus losing 176°. If no allowance be made for dissipation, the capacities of the air-jar and shellac-jar would

therefore be as 114 : 176, or as 1 : 1·54, and the specific inductive capacity of shellac would be 1·54.

71. **Polarization of the Dielectric.**—As the interposed non-conductor, or dielectric, modifies the mutual action of the two electricities which it separates, and does not play the mere passive part which was attributed to it before Faraday's experiments, it is natural to conclude that the dielectric must itself experience a peculiar modification. According to Faraday, this modification consists in a polarization of its particles, which act inductively upon each other along the lines of force, and have each a positive and a negative side, the positive side of each facing the negative side of the next. This polarization is capable of being sustained for a great length of time in good non-conductors; but in good conductors it instantly leads to discharge between successive particles, and the opposite electricities appear only at the two surfaces.

The polarization of dielectrics is clearly shown in the following experiment. In a glass vessel (Fig. 44) is placed oil of turpentine,

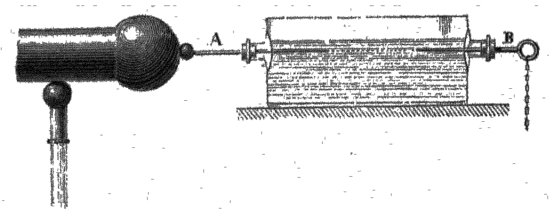


Fig. 44.—Polarization of Dielectric.

containing filaments of silk 2 or 3 millimetres long. Two metallic rods, A, B, each terminating within in a point, are connected, one with the ground, and the other with an electric machine. On working the machine, the little filaments are seen to arrange themselves in a line between the points, and, on endeavouring to break the line with a glass rod, it will be found that they return to this position with considerable pertinacity. On stopping the machine, they immediately fall to the bottom.

An experiment of Matteucci's demonstrates this polarization still more directly. A number of thin plates of mica are pressed strongly together between two metallic plates. One of the metallic plates is charged, while the other is connected with the ground; and, on removing the metallic plates by insulating handles, it is found that all the mica plates are polarized, the face turned towards the positive

metal plate being covered with positive electricity, and the other face with negative.

Dr. Kerr has recently shown that glass and other transparent insulators, when subjected to strong dielectric action, become for the time doubly-refracting, a property which is also producible in such substances as glass by longitudinal extension or compression. In some substances, including glass itself, the dielectric effect is identical with the effect of *compression* along the lines of force. In others it is identical with the effect of *extension* along these lines. Liquid as well as solid dielectrics are thus affected, the only difference being that in solids the effect takes about half a minute to attain its maximum, and dies away gradually when the electrical forces are removed; whereas in liquids, the full effect is attained instantaneously, and the disappearance is also instantaneous. The direction of vision, in the experiments, was at right angles to the lines of force; and the optical effect, per unit of thickness in this direction, was found to vary, in any given liquid, directly as the square of the electric force.

72. Limit to Thinness of Interposed Plate.—We have seen (§ 62) that the capacity of a condenser varies inversely as the distance between the collecting and the condensing plate. But if this distance is very small, the resistance of the interposed dielectric (which varies directly as its thickness) may be insufficient to prevent discharge, and it will not be practicable to establish a great difference of potential between the two plates. We may practically distinguish two sorts of condensers, one sort having a very thin dielectric and very great condensing power, but only capable of being charged to feeble[1] potential; the other having a dielectric thick enough to resist the highest tensions attainable by the electrical machine. The Leyden jar comes under the second category. The first includes the electrophorus (except in so far as its action is aided by the metallic sole), and the condenser of Volta's electroscope.

73. Condensers for Galvanic Electricity.—Condensers of very large surface are used for certain applications of galvanic electricity, especially in connection with telegraphy. They are constructed by arranging a number of sheets of tin-foil in a pile, with either thin

[1] *Strong* potential is potential differing very much from zero either positively or negatively. *Feeble* potential is potential not differing much from zero. *Tension* is measured by difference of potential; and when the earth is one of the terms of the comparison, tension becomes identical with potential.

mica or paper saturated with paraffin wax between them. The 1st, 3d, 5th, &c., sheets of tin-foil are connected together and correspond to one coating of a Leyden jar, and the 2d, 4th, 6th, &c., sheets are connected and correspond to the other coating. They are charged by connecting one of these coatings to one pole of a battery and the other coating to the other pole.

74. Volta's Condensing Electroscope.—This instrument, which has rendered very important services to the science of electricity, differs from the simple gold-leaf electroscope previously described (§ 10), in having at its top two metal plates, of which the lower one is connected with the gold-leaves, and is covered on its upper face with insulating varnish, while the upper is varnished on its lower face, and furnished with a glass handle. These two plates constitute the condenser. In using the instrument, one of the two plates (it matters not which) is charged by means of the body to be tested, while the other is connected with the earth. They thus receive opposite and sensibly equal charges. The upper plate is then lifted off, and the higher it is raised the wider do the gold-leaves diverge. The separation of the plates diminishes the capacity, and strengthens the potential of both, one becoming more strongly positive, and the other more strongly negative. This involves increase of potential energy, which is represented by the amount of work done against electrical attraction in separating the plates. No increase in quantity of electricity is produced by the separation; hence the instrument is chiefly serviceable in detecting the presence of electricity which is available in large quantity but at weak potential. The glass handle of the upper plate is by no means essential, as it is only necessary that the lower plate should be insulated. The charge may be given by induction; in which case one plate must be connected with the earth while the inducing body is held near it, and the other plate must be kept connected with the earth while the influencing body is withdrawn. The plates will then be left charged with opposite electricities, that

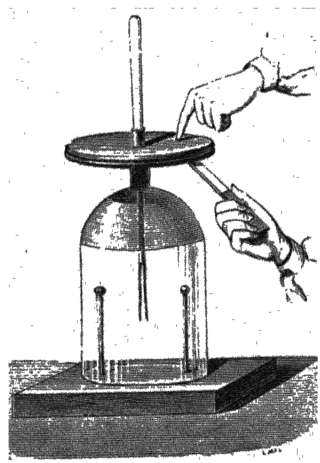


Fig. 45.—Condensing Electroscope.

which was more remote from the influencing body having acquired a charge similar to that of the body. For inductive charges, however, the condensing arrangement serves no useful purpose, beyond enabling the electroscope to retain its charge for a longer time, the effect finally obtained on separating the plates being no greater than would have been obtained by employing only the lower plate.

75. Leyden Battery.—The Leyden battery consists of a number of Leyden jars, placed in compartments of a box lined with tin-foil, which serves to establish good connection between their outer coat-

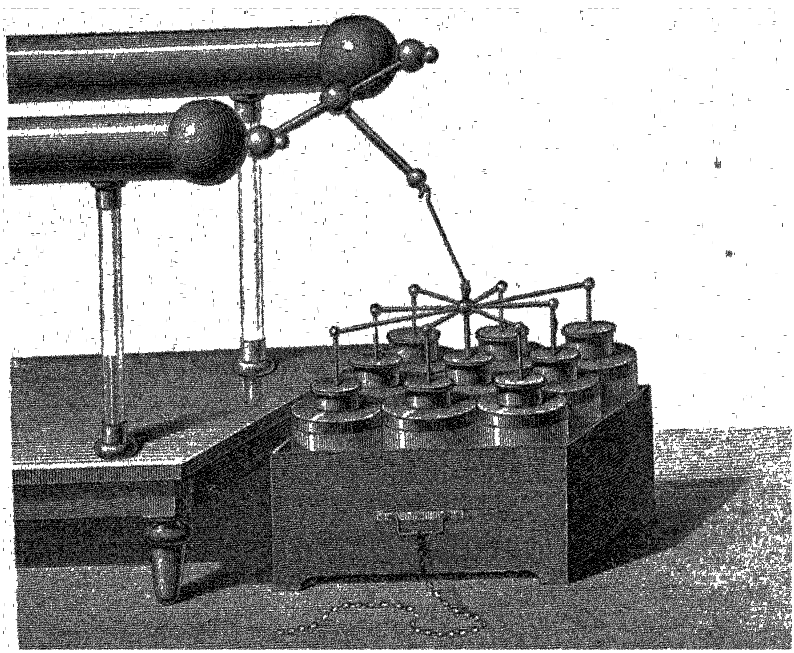


Fig. 46.—Battery of Leyden Jars.

ings, while their inner coatings are connected by brass rods. It is advisable that the outer coatings should have very free communication with the earth. For this purpose a metallic handle, which is in metallic communication with the lining of the box, should be connected, by means of a chain, with the gas or water pipes of the building.

The capacity of a Leyden battery is the sum of the capacities of the jars which compose it. The charge is given in the ordinary way,

by connecting the inner coatings with the conductor of the machine. In bad weather this is usually a very difficult operation, on account of the large quantity of electricity required for a full charge, and the large surface from which dissipation goes on.

Holtz's machine can be very advantageously employed for charging a battery, one of its poles being connected with the inner, and the other with the outer coatings. In dry weather it gives the charge with surprising quickness.

76. Lichtenberg's Figures.—An interesting experiment devised by

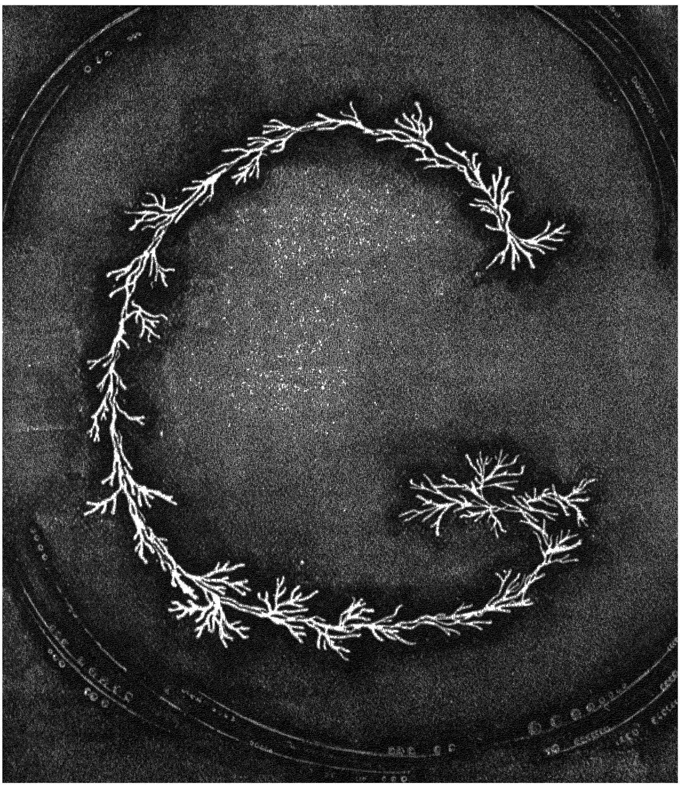


Fig 47 --Lichtenberg's Figures.

Lichtenberg serves to illustrate the difference between the physical properties of positive and negative electricity.

A Leyden jar is charged, and the operator, holding it by the outer

coating, traces a design with the knob on a plate of shellac or vulcanite. He then places the jar on an insulating stand, to enable him to transfer his hold to the knob, and traces another pattern on the cake with the outer coating. A mixture of flowers of sulphur and red-lead, which has previously been well dried and shaken together, is then sprinkled over the cake. The sulphur, having become negatively electrified by friction with the red-lead, adheres to the pattern which was traced with positive electricity, while the red-lead adheres to the other. The yellow and red colours render the patterns very conspicuous. The particles of sulphur (represented by the inner pattern in Fig. 47) arrange themselves in branching lines, while the red-lead (shown in the outer pattern) forms circular spots; whence it would appear that positive electricity travels along the surface more easily than negative. A similar difference has already been pointed out between positive and negative brushes.

77. **Charge by Cascade.**—Instead of connecting all the inner coatings together, and all the outer coatings together, as in the Leyden battery, we may connect a number of jars in series. The inner coating of the first jar is to be connected with the prime conductor of the machine; and its outer coating, which must be insulated from the earth, is to be connected with the inner coating of the second jar. The outer coating of this is in like manner to be connected with the inner coating of the next, and so on to the last jar, the outer coating of which must be connected with the earth. When the machine is worked, a positive charge is given to the inner coating of the first, and a sensibly equal negative charge is induced upon its outer coating, this negative charge being drawn from the inner coating of the second, which accordingly acquires a positive charge sensibly equal to that given from the machine to the first jar. This reasoning can be extended through the whole chain. Hence if we denote by Q the charge given to the inner coating of the first jar, the inner coating of each jar in the series has a charge $+Q$, and the outer coating a charge $-Q$. If we further suppose all the jars to have the same capacity C, and if we denote the potentials of the inner coatings by $V_1, V_2, V_3, \ldots V_n$, we shall have

$$\frac{Q}{C} = V_1 - V_2 = V_2 - V_3 =, \text{ \&c.}, = V^n - 0,$$

since the quotient of the charge of a jar by its capacity is equal to the difference of potential of the two coatings, and the potentials of the outer coatings are $V_2, V_3, \ldots V_n, 0$. By adding the n differ-

ences V_1-V_2, $V_2-V_3 \ldots V_n-0$, we obtain V_1, which is accordingly equal to n times $\frac{Q}{C}$, and we have

$$\frac{Q}{C}=V_1-V_2=V_2-V_3=, \text{\&c.}, =\frac{V_1}{n}.$$

If we compare the charges of these jars with the charge which the first jar would have received if its outer coating had been connected to earth in the ordinary way, the prime conductor being supposed to attain the same potential V_1 in both cases, we have

$$Q=\frac{1}{n}CV_1$$

for each jar in the series, whereas we should have had $Q'=CV_1$ for the single jar. The charge of each jar in the series is therefore $\frac{1}{n}$ of its ordinary charge.

As regards energy; for the single jar the energy would be $\frac{1}{2}Q'V_1=\frac{1}{2}CV_1^2$, while for any one jar in the series the energy would be

$$\tfrac{1}{2}Q(V_1-V_2)=\tfrac{1}{2}\frac{1}{n}CV_1\frac{V_1}{n}=\tfrac{1}{2}\frac{1}{n^2}CV_1^2,$$

which is $\frac{1}{n^2}$ of the energy of the single jar.

Jars thus arranged are said to be charged *by cascade*, the name being suggested by the successive falls of potential from jar to jar. They can either be discharged in succession by connecting the two coatings of each, or all together by connecting the inner coating of the first with the outer coating of the last. In the former case the energy of each spark is $\frac{1}{2}\frac{1}{n^2}CV_1^2$, as appears from the above calculation. In the latter case the energy of the single spark is $\frac{1}{2}\frac{1}{n}CV_1^2$.

78. Force modified by value of K.—The following modifications must be made in the foregoing statements when specific inductive capacity is taken into account.

The mutual force between two charges q_1 and q_2 collected at points at distance r in a medium of specific capacity K is $q_1 q_2 / Kr^2$.

The intensity of electrical force at a point due to a charge q at distance r is $q / K r^2$.

The work done on unit charge which moves from distance r_1 to distance r_2 by the repulsion of a charge q is $q / K r_1 - q / K r_2$.

When there are several repelling or attracting charges, the work done by their forces in a given displacement of a unit charge is $\Sigma \frac{q}{K r_1} - \Sigma \frac{q}{K r_2}$. Hence the potential at a point is $\Sigma \frac{q}{K r}$.

When a tube of force incloses a quantity q of electricity, the product

$$\text{intensity of force} \times \text{section} \times K$$

changes by the amount $4\pi q$ in passing from one side of the inclosed electricity to the other. This product is called the *quantity of induction* within the tube. It is independent of the medium, and does not change its value when the tube passes out of one medium into another; but the force does change, being inversely as K. Maxwell accordingly preferred the name "tubes of induction" to "tubes of force."

The force just outside a conductor with a surface-density ρ is $4\pi\rho / K$.

The force at a point between two parallel plates with potentials V_1 and V_2, at a small distance x apart, is $\frac{V_2 - V_1}{x}$, and is also $\frac{4\pi\rho}{K}$ or $\frac{4\pi Q}{KS}$, S denoting the area of either plate, and Q the charge upon it. Hence the capacity of the condenser formed by the plates is $C = \frac{Q}{V_2 - V_1} = \frac{KS}{4\pi x}$.

79. Analogy between Specific Inductive Capacity and Thermal Conductivity.—Suppose steady flow of heat to be taking place uniformly in all directions from a small source in a medium of conductivity k. Let the quantity of heat emitted by the source in unit time be denoted by $4\pi H$. Then, since the flow is steady, this quantity crosses in unit time every spherical surface $4\pi r^2$ that can be described about the source, and the flow across unit area of such a surface is H / r^2, r denoting the radius of the sphere. This flow must be equal to k multiplied by the gradient, that is, to $-k\,d\theta / dr$, θ denoting the temperature at distance r from the source. Thus we have

$$d\theta = -\frac{H}{kr^2}dr = d\frac{H}{kr};$$

whence it can be shown that, when there are any number of sources, the difference of the values of $\Sigma\frac{H}{kr}$ at any two points in the medium is the difference of the temperatures at these points.

Thus, in the analogy between Electrostatics and Heat, potential corresponds to temperature, q to H, K to k, and intensity of electrical force at a point to intensity of flow of heat at a point.

[At this stage the student should turn to the Appendix and read to the end of the account of Electrostatic Units.]

CHAPTER VIII.

ELECTROMETERS.

80. Object of Electrometers.—Electrometers are instruments for the measurement of differences of electrical potential. The gold-leaf electroscope, the straw-electroscope, and other instruments of the same type, afford rough indications of the difference of potential between the diverging bodies and the air of the apartment, and more measurable indications are given by the electrometers of Peltier and Dellmann, but none of these instruments are at all comparable in precision to the various electrometers which have been invented from time to time by Sir Wm. Thomson.

81. Attracted-disc Electrometers, or Trap-door Electrometers.[1]—We shall first describe what Sir Wm. Thomson calls "Attracted-disc Electrometers." These instruments, one of which is represented in Figs. 48, 49, contain two parallel discs of brass, g, h, with an aperture in the centre of one of them, nearly filled up by a light trap-door of aluminium f, which is supported in such a way as to admit of its electrical attraction towards the other disc being resisted by a mechanical force which can be varied at pleasure. The trap-door and the perforated plate surrounding it must have their faces as nearly as possible in one plane when the observation is taken, and, as they are electrically connected, they may then be regarded as forming *one disc of which a small central area is movable.* There is always attraction between the two parallel discs, except when they are at the same potential.

Let their potentials be denoted by V and V′, the electrical densities on their faces by ρ and ρ', and their mutual distance by D. We have seen (§ 62) that, in such circumstances, ρ and ρ' are constant

[1] Sometimes called "guard-ring" or "guard-plate" electrometers, the trap-door being "guarded" by the fixed plate which surrounds it.

(except near the edges of the discs), opposite in sign, and equal, and that the intensity of force in the space between them is everywhere the same, and equal at once to $\frac{V-V'}{D}$ and to $4\pi\rho$. This force is jointly due to attraction by one plate and repulsion by the other, each of these having the intensity $2\pi\rho$, or half the total intensity.

Let A denote the area of the trap-door. The quantity of electricity upon it will be ρ A, and the force of attraction which this experiences will be $\rho A \times 2\pi\rho = 2\pi\rho^2 A$, which we shall denote by F. Then from the equations

$$F = 2\pi\rho^2 A \quad , \quad \frac{V-V'}{D} = 4\pi\rho, \tag{1}$$

we find, by eliminating ρ,

$$F = \frac{A}{8\pi}\left(\frac{V-V'}{D}\right)^2, \text{ or } V-V' = \pm D\sqrt{\frac{8\pi F}{A}}. \tag{2}$$

82. Absolute Electrometer.—In the *absolute electrometer*, which somewhat resembles Fig. 49 turned upside down, the force of electrical attraction on the trap-door is measured by direct comparison with the gravitating force of known weights. This is done by first observing what weights must be placed on the trap-door to bring it into position when no electrical force acts (the plates being electrically connected), and by then removing the weights, allowing electrical force to act, and adjusting the plates at such a distance from one another, by the aid of a micrometer screw, that the trap-door shall again be brought into position. Then, in equation (2), F, A, and D are known, and the difference of potentials $V-V'$ can be determined. In the absolute electrometer, the perforated disc h is uppermost, so that the direction of electrical attraction on the trap-door is similar to the direction of the gravitating force of the weights. The reverse arrangement is usually adopted in the portable electrometer, which we shall next describe. In both instruments, the trap-door constitutes one end of a very light lever $f\,i\,l$ of aluminium, balanced on a horizontal axis.

83. Portable Electrometer.—In the *portable electrometer* (Figs. 48, 49) this axis passes very accurately through the centre of gravity of the lever, the suspension being effected by means of a fine platinum wire $w\,w$ tightly stretched, which is secured at its centre to the lever in such a manner that, when the trap-door comes into position, the wire is under torsion tending to draw back the disc from the attracting plate g. This torsion (except in so far as it is

affected by causes of error such as temperature and gradual loss of elasticity), is always the same when the disc is in position, and as it is to be balanced in every observation by electrical attraction, the latter must also be always the same; that is to say, the quantity F in equations (2) is constant for all observations with the same instrument; whence it is obvious that $V - V'$ is directly proportional to D, the distance between the plates. The observation for difference of

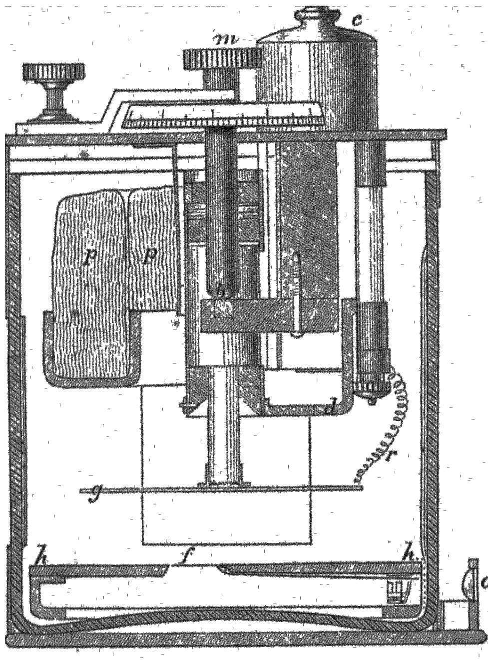


Fig 48.—Portable Electrometer.

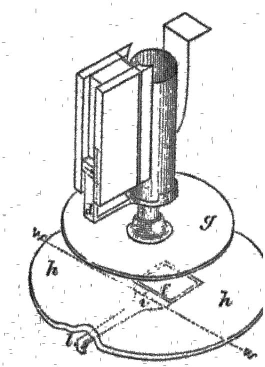


Fig 49 —Parallel Discs

potential therefore consists in altering this distance until the trap-door comes into position. This is done by turning the micrometer screw, by means of the milled head *m*. The divided circle of the micrometer indicates the amount of turning for small distances, and whole revolutions are read off on the vertical scale traversed by the index carried by the arm *d*. The correct position is very accurately identified by means of two sights, one of them being attached to a fixed portion of the instrument, and the other to one end *l* of the lever. One of these sights moves up and down close in front of the other, and they are viewed through a lens *o* in

front of both. This arrangement is also adopted in the absolute electrometer.

One of the two parallel plates h is connected with the inner coating of a Leyden jar,[1] which, being kept dry within by means of pumice p wetted with sulphuric acid, retains a sufficient charge for some weeks. The other plate g is in communication, by means of

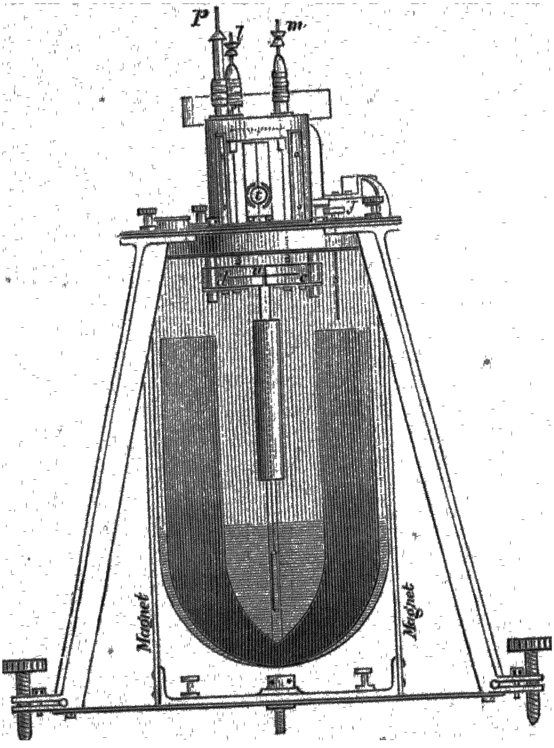


Fig. 50.—Quadrant Electrometer.

the spiral wire r, with the insulated umbrella c, which can be connected with any external conductor; and, in order to determine the potential of any conductor which we wish to examine, two observations are taken, one of them giving the difference of potential

[1] The use of the Leyden jar is to give constancy of potential. Its capacity is so much greater than that of the disc with which it is connected that the electricity which enters or leaves the latter in consequence of the inductive action of the other disc is no sensible fraction of the whole charge of the jar, and produces no sensible change in its potential. Its great capacity in comparison with the extent of surface exposed likewise tends to prevent rapid loss of potential by dissipation of charge.

between this conductor and the Leyden jar, and the other the difference between the earth and the jar. We thus obtain, by subtraction, the difference of potential between the conductor in question and the earth.

84. Quadrant Electrometer.—The most sensitive instrument yet invented for the measurement of electrical potential is the *quadrant electrometer*, which is represented in front view in Fig. 50, some of its principal parts being shown on a larger scale in Figs. 51, 52.

In this instrument, the part whose movements give the indications is a thin flat piece of aluminium *u*, narrow in the middle and broader towards the ends, but with all corners rounded off. This piece, which is called the *needle*, and is represented by the dotted line in Fig. 51, is inclosed almost completely in what may be described as a shallow cylindrical box of brass, cut into four quadrants, *c*, *d*, *c′*, *d′*. These parts are shown in plan in Fig. 51, and in front view in Fig. 50. The needle *u* is attached to a stiff platinum wire, which is supported by a silk fibre hanging vertically. The same wire carries a small concave mirror *t* (Fig. 50) for reflecting the light from an illuminated vertical slit. An image of the slit is thus formed at the distance of about a yard, and is received upon a paper scale of equal parts, by reference to which the movements of the image can be measured. The movements of the image depend upon the movements of the mirror, which are precisely the same as those of the needle. We have now to explain how the movements of the needle are produced.

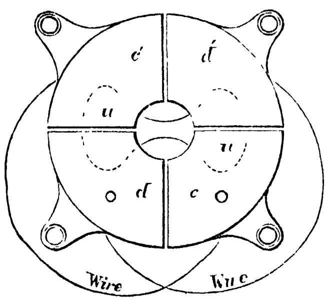


Fig 51.—Needle and Quadrants

One pair of opposite quadrants *c c′* are connected with each other, and with a stiff wire *l* projecting above the case of the instrument. The other quadrants *d d′* are in like manner connected with the other projecting wire *m*. The projecting parts *l m* are called the *chief electrodes*, and are to be connected respectively with the two conductors whose difference of potential is required, one of which is usually the earth. Suppose the needle to have a positive charge of its own, then if the potential of *c* and *c′* be higher (algebraically) than that of *d* and *d′*, one end of the needle will experience a force urging it from *c* to *d*, and the other end will experience a force urging it

from c' to d'. These two forces constitute a couple tending to turn the needle about a vertical axis. If the potential of c and c' be lower than that of d and d', the couple will be in the opposite direction. To prevent the needle from deviating too far under the action of this couple, and to give it a definite position when there is no electrical couple acting upon it, a small light magnet is attached to the back of the mirror, and by means of controlling magnets outside the case the earth's magnetism is overpowered, so that, whatever position be chosen for the instrument, the needle can be made to assume the proper zero position. In more recent instruments the magnets are dispensed with, and a bifilar suspension is substituted for the single silk fibre. The permanent electrification of the needle is attained by connecting it, by means of a descending platinum wire, with a quantity of strong sulphuric acid, which occupies the lower part of the containing glass jar. The acid, being an excellent conductor, serves as the inner coating of a Leyden jar, the outside of the glass opposite to it being coated with tin-foil, and connected with the earth. The acid at the same time serves the purpose of keeping the interior of the apparatus very dry. The charge is given to the jar through the *charging electrode* p, which can be thrown into or out of connection at pleasure. As the sensibility of the instrument increases with the potential of the jar, a *gauge* and *replenisher* are provided for keeping this potential constant. The *gauge* is simply an "attracted-disc electrometer," in which the distance between the parallel discs is never altered, so that the aluminium square only comes into position when the potential of one of the discs, which is connected with the acid in the jar, differs by a certain definite amount from the potential of the other, which is connected with the earth. A glance at the gauge shows, at any moment, whether the potential of the jar has the normal strength. If it has fallen below this point, the *replenisher* is employed to increase the charge.

This apparatus, which is separately represented, dissected, in Fig. 52, and is for simplicity omitted in Fig. 50, consists of a vertical stem of ebonite s, which can be rapidly twirled with the finger by means of a milled head y, and which carries two metal wings or *carriers*, b, b, insulated from each other. In one part of their revolution, these come in contact with two light steel springs $f\,f$, which simply serve to connect them for the instant with each other. In another part of their revolution, they come in contact with two other springs $e\,e$, connected respectively with the acid of the jar and

with the earth. The first of these contacts takes place just before the wings emerge from the shelter of the larger metallic sectors or *inductors a a,* of which one is connected with the acid, and the other with the earth. Suppose the acid to have a positive charge. Then, at the instant of contact, an inductive movement of electricity takes place, producing an accumulation of negative electricity in the carrier which is next the positive inductor, and an accumulation of positive in the other. The next contacts are effected when the carrier which has thus acquired a positive charge is well under cover of the positive inductor, to which accordingly it gives up its electricity; for, being in great part surrounded by this inductor, and being connected with it by the spring, the carrier may be regarded as forming a portion of the interior of a concave conductor, and the electricity accordingly passes from it to the external surface, that is to the inductor, and to the acid connected with it, which forms the lining of the jar. The negative electricity on the other carrier is, in like manner, given off to the other inductor, and so to the earth.

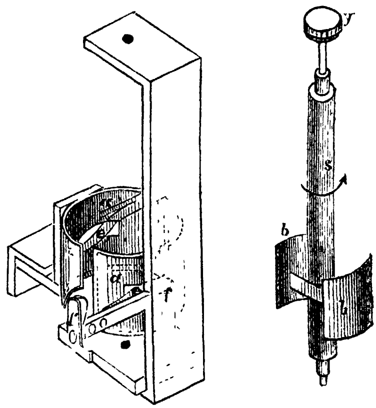


Fig. 52 —Replenisher

The jar thus receives an addition to its charge once in every half-revolution of the replenisher; and, as these increments are very small, it is easy to regulate the charge so that the gauge shall indicate exactly the normal potential. If the charge is too strong, it can be diminished by turning the replenisher in the reverse direction.

NOTE ON THE ENERGY OF A SYSTEM OF CHARGED CONDUCTORS, WITH APPLICATION TO THOMSON'S QUADRANT ELECTROMETER.

(1.) By the energy of a system of charged conductors is meant the work which must have been spent in charging them, or, what is the same thing, the energy which will run down when the conductors are connected with the earth. We shall investigate its amount in terms of the charges Q_1, Q_2, &c., of the conductors, and their potentials V_1, V_2, &c., these latter being supposed to depend only on the charges of the system itself.

Let the conductors be charged gradually all at the same time, and let

their charges at any time be xQ_1, xQ_2, &c., the value of x being the same for all the conductors. The potentials at the same time will be xV_1, xV_2, &c. By § 51, the work required to bring the small quantity of electricity $Q_1\, dx$ from the earth to the conductor of potential xV_1 is $xV_1\, Q_1\, dx$; thus, when x receives the small increase dx, the whole addition of energy to the system is

$$(V_1 Q_1 + V_2 Q_2 + \&c.)\, x\, dx.$$

If this operation is repeated time after time, beginning with $x = 0$, and ending with $x = 1$, the conductors will begin with being uncharged, and will end by having the given charges. Since the integral of $x\, dx$ between these limits is $\frac{1}{2}$, *the whole energy acquired by the system is* $\frac{1}{2}(V_1 Q_1 + V_2 Q_2 +$ &c.), which may be written $\frac{1}{2}\,\Sigma\, V\, Q$.

(2.) Hence when any small changes dQ_1, dQ_2, &c., are made in the charges, and any small changes dV_1, dV_2, &c., in the potentials, either with or without displacement of the conductors, the increase of energy is $\frac{1}{2}\,\Sigma\,(V\, d\, Q + Q\, d\, V)$.

(3.) If the conductors are *stationary*, another simple expression can be found for the increase of energy; for the work required to bring the electricity dQ_1 to the conductor of potential V_1 is $V_1\, dQ_1$; thus the whole increase of energy is $\Sigma\, V\, d\, Q$; and by comparing this with the expression for the same thing in (2.) we see that a third expression for the increase of energy will be $\Sigma\, Q\, d\, V$.

(4.) If the conductors are insulated, so that their *charges remain constant*, the increase of energy when they are displaced will be the difference between the initial energy $\frac{1}{2}\,\Sigma\, Q\, V$ and the final energy $\frac{1}{2}\,\Sigma\, Q\, V'$, that is, will be $\frac{1}{2}\,\Sigma\, Q\,(V' - V)$, where V' denotes the final potential of the conductor whose initial potential is V. If the charges are small the increase of energy will be $\frac{1}{2}\,\Sigma\, Q\, d\, V$. This, it will be noticed, is exactly half the increase in (3.), the changes of potential being supposed the same in both cases.

When insulated charged conductors are allowed to move under the influence of their own mutual forces, these forces will do positive work, and the system will lose electrical energy of the same amount. On the other hand, if external forces move the conductors in opposition to their mutual forces, there will be a gain of electrical energy equal to the work done by external forces against the forces of the system. These consequences follow immediately from the principle of the conservation of energy.

(5.) If the conductors while displaced are kept *at constant potentials*, their charges must change, and we cannot make the same direct application of the principle of conservation of energy which we have made above, unless we include in our reasonings the external sources from which electricity

comes or to which it goes in making these alterations in the charges. We can, however, arrive at the relation between the change of energy in the system and the work done in the following way.

Divide the whole displacement into a series of small steps. In each step let the conductors be insulated, so that the potentials will change slightly, and then let the potentials be restored to their original values before the next step. The forces between the conductors will thus be sensibly the same as if the potentials were absolutely constant, and the work done by these forces will be the same.

In any one of the steps, the increase of energy, by (4.), is $\frac{1}{2}\Sigma Q\,dV$, and in the restoration of the potentials to what they were at the beginning of this step the increase of energy, by (3.), is $-\Sigma Q\,dV$, the minus sign being rendered necessary by the fact that the change from $V+dV$ back to V is $-dV$.

In the two operations combined the whole increase of energy is $-\frac{1}{2}\Sigma Q\,dV$, and the mechanical work done by the forces of the system is, by (4.), equal to the loss of energy in the displacement which constitutes the first operation, that is to $-\frac{1}{2}\Sigma Q\,dV$ also. Hence in each step combined with its following restoration of potential, the change of energy is the same both in amount and in sign as the work which the forces of the system do in the movements. As this equality holds through all the steps, it holds for the complete result; that is, the *gain of energy* in a system of conductors which are *displaced at constant potentials* is equal to the *work which the forces of the system do* in the displacement. In any system cut off from external supplies of energy the work done is equal to the energy lost, but here it is equal to the energy gained. Hence the external sources which supply the electricity for keeping the conductors at constant potentials furnish an amount of energy double of the work done by the forces of the system. On the other hand, if the conductors are moved in opposition to the forces of the system, the external sources will draw energy from the system to double the amount of the work done against the forces of the system.

(6.) In Thomson's quadrant electrometer, let V denote the potential of the needle and sulphuric acid, V_1 the potential of one pair of quadrants which we will call the first pair, and V_2 the potential of the other pair. The needle and quadrants form two condensers, the inner coatings of both being at the same potential V, and the outer coatings at the respective potentials V_1 and V_2. When the needle turns through an angle θ from the first pair of quadrants towards the second, the capacity of the first condenser is diminished by a quantity proportional to θ, say $c\theta$, and the capacity of the second condenser is increased by the same amount. Hence, supposing V to be higher than V_1, and V_1 than V_2, the charge of

the first condenser is diminished by $c\theta$ $(V-V_1)$, and its energy by $\frac{1}{2}c\theta$ $(V-V_1)^2$, (§ 620). The energy of the second condenser is increased by $\frac{1}{2}$ $c\theta$ $(V-V_2)^2$. The total increase of energy of the system of two condensers is therefore—

$$\begin{aligned}&\tfrac{1}{2}\,c\theta\left\{(V-V_2)^2-(V-V_1)^2\right\}\\ &=\tfrac{1}{2}\,c\theta\left\{2\,VV_1-2\,VV_2-V_1^2+V_2^2\right\}\\ &=c\theta(V_1-V_2)\left\{V-\tfrac{1}{2}(V_1+V_2)\right\}\end{aligned}$$

This is the increase of energy produced by the displacement of the needle through the angle θ while the three potentials remain unchanged, and is equal, by (5.), to the work done by the electrical forces against the mechanical forces of the bifilar suspension. Dividing the work by the angle, we get the average working couple. This quotient

$$c(V_1-V_2)\left(V-\frac{V_1+V_2}{2}\right)$$

is independent of θ, and is therefore the value of the working couple itself. This couple is balanced by the couple due to the suspension, which is proportional to θ. Hence the deflection θ is proportional to

$$(V_1-V_2)\left(V-\frac{V_1+V_2}{2}\right).$$

(7.) In the ordinary use of the instrument, V is very large compared with V_1 and V_2. Hence θ is sensibly proportional to V_1-V_2.

(8.) If the needle is connected with the first pair of quadrants, V_1 may be substituted for V, and the deflection is proportional to $(V_1-V_2)^2$. The direction of the deflection will be from the first pair towards the second, whether V_1-V_2 be positive or negative. Joubert has taken advantage of this circumstance to use the instrument for measuring the difference of potential between the two terminals of an alternate-current dynamo. He connects these terminals with the electrodes of the quadrant electrometer, having first discharged the needle and sulphuric acid and connected them with one pair of quadrants. The difference of potential is reversed in sign, as well as changed in amount, some hundreds of times per second, and the needle gives a steady deflection which is proportional to the mean square of the difference of potential.

CHAPTER IX.

ATMOSPHERIC ELECTRICITY.

85. Resemblance of Lightning to the Electric Spark.—The resemblance of the effects of lightning to those of the electric spark struck the minds of many of the early electricians. Lightning, in fact, ruptures and scatters non-conducting substances, inflaming those which are combustible; heats, reddens, melts, and volatilizes metals; and gives shocks, more or less severe, and frequently fatal, to men and animals, all of these being precisely the effects of the electric spark with merely a difference of intensity. We may add that lightning leaves behind it a characteristic odour precisely similar to that which is observed near an electrical machine when it is working, and which we now know to be due to the presence of ozone. Moreover, the form of the spark, its brilliancy, and the detonation which attends it, all remind one forcibly of lightning.

To Franklin, however, belongs the credit of putting the identity of the two phenomena beyond all question, and proving experimentally that the clouds in a thunder-storm are charged with electricity. This he did by sending up a kite, armed with an iron point with which the hempen string of the kite was connected. To the lower end of the string a key was fastened, and to this again was attached a silk ribbon intended to insulate the kite and string from the hand of the person holding it. Having sent up the kite on the approach of a storm, he waited in vain for some time even after a heavy cloud had passed directly over the kite. At length the fibres of the string began to bristle, and he was able to draw a strong spark by presenting his knuckle to the key. A shower now fell, and, by wetting the string, improved its conducting power, the silk ribbon being still kept dry by standing under a shed. Sparks in rapid succession were drawn from the key, a Leyden jar was charged by it, and a shock given.

Shortly before this occurrence, Dalibard, acting upon a published suggestion of Franklin, had erected a pointed iron rod on the top of a house near Paris. The rod was insulated from the earth, and could be connected with various electrical apparatus. A thunder-storm having occurred, a great number of sparks, some of them of great power, were drawn from the lower end of the rod.

These experiments were repeated in various places, and Richmann of St. Petersburg, while conducting an investigation with an apparatus somewhat resembling that of Dalibard, received a spark which killed him on the spot.

86. Duration of Lightning.—It appears that thunder-clouds must be regarded as charged masses of considerable conducting power. The discharges which produce lightning and thunder occur sometimes between two clouds, and sometimes between a cloud and the earth. The duration of the illumination produced by lightning is certainly less than the ten-thousandth of a second. This has been established by observing a rapidly rotating disc (Fig. 53) divided into sectors alternately black and white. If viewed by daylight, the disc appears of a uniform gray; and if lightning, occurring in the dark, renders the separate sectors visible, the duration of its light must be less than the time of revolving through the breadth of one sector. The experiment has been tried with a disc divided into 60 sectors, and making 180 revolutions per second, so that the time of turning through the space occupied by one sector is $\frac{1}{60}$ of $\frac{1}{180}$ of a second, that is, $\frac{1}{10800}$. When the disc, turning with this velocity, is rendered visible by lightning, the observer sees black and white sectors with gray ones between them. For the black and white sectors to be seen sharply defined, without intermediate gray, it would be necessary that the illumination should be absolutely instantaneous.

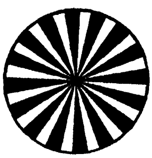


Fig. 53.
Duration of Flash.

87. Thunder.—Thunder frequently consists of a number of reports heard in succession. This can be explained by supposing that (as in the experiment of the spangled tube, § 39) discharge occurs at several places at once, or, what amounts nearly to the same thing, that the line of discharge is crooked. The reports of these explosions will be heard in the order of their distance from the observer. If, for example, the lines of discharge form the zigzag M N (Fig. 54), an observer at O will hear first the explosion at *a*, then, a little later, the five explosions at *m*, *n*, *r*, *s*, *t*; he will consequently observe an

increase of loudness. When any considerable portion of the path of discharge is at a uniform distance from the observer, the simultaneous arrival of the disturbances propagated from all this portion will produce a specially loud burst of sound.

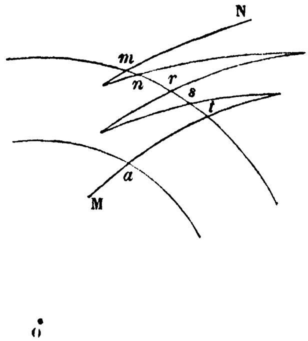


Fig. 54.—Simultaneous Explosions.

88. Shock by Influence.—Persons near whom a flash of lightning passes, frequently experience a severe shock by induction. This is analogous to the phenomenon, first observed by Galvani, that a skinned frog in the neighbourhood of an electrical machine, although dead, exhibits convulsive movements every time a spark is drawn from the conductor.

89. Lightning Conductors.—Experience having shown that electricity travels in preference through the best conductors, it is easy to understand that, if a building be fitted with metallic rods terminating in the earth, lightning will travel through these instead of striking the building. But further, if these rods terminate above in a point, they may exercise a preventive influence by enabling the earth and clouds to exchange their opposite electricities in a gradual way, just as the conductor of a machine is prevented from giving powerful sparks by presenting to it a sharp point connected with the earth. The following conditions should always be complied with:—

1. The connection with the ground should be continuous.

2. The conductor must be everywhere of so large a section that it will not be melted by lightning passing through it.

3. The earth contact must be good. The conductor may be connected at its base with the iron pipes which supply the neighbourhood with water; or it may terminate in the water of a well or pond. Failing these, it should be provided with branches traversing the soil in different directions and surrounded by coke, which is a good conductor.

4. At no part of its course above ground should it come near to the metal pipes which supply the house with water or gas, nor to any large masses of metal in the house. All large masses of metal on the outside of the house, such as lead roofing, should be well connected with the conductor.

5. It should terminate above in a sharp point or points, which should project above the highest parts of the building.

90. Ordinary Electricity of the Atmosphere.—The presence of electricity in the upper regions of the air is not confined to thunder-clouds, but can be detected at all times. In fine weather this electricity is almost invariably positive, but in showery or stormy weather negative electricity is as frequently met with as positive; and it is in such weather that the indications of electricity, whether positive or negative, are usually the strongest.

91. Methods of obtaining Indications.—One of the early methods of observing atmospheric electricity consisted in shooting up an arrow, attached to a conducting thread, having at its lower end a ring, which was laid upon the top of a gold-leaf electroscope As the arrow ascends higher, the leaves diverge more and more with electricity of the same sign as that overhead; and they remain divergent after the ring has been lifted off by the movement of the arrow.

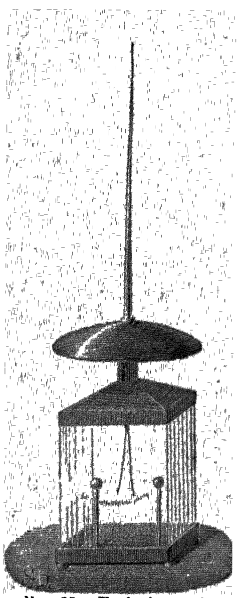


Fig. 55.—Early form of Electrometer.

Sometimes, instead of the arrow, a point on the top of the electroscope is employed to collect electricity from the air, as in Fig. 55. Both these methods are very uncertain in their action.

A better method of collecting electricity from the air was long ago devised by Volta, who employed for this purpose a burning match attached to the top of a rod connected with the gold-leaves or straws of his electroscope. If there is positive electricity overhead, its influence causes negative electricity to collect at the upper end of the rod, whence it passes off by convection in the products of combustion of the match, leaving the whole conducting system positively electrified. In like manner, if the electricity overhead be negative, the system will be left negatively electrified.

Another method which, in the hands of Peltier, Quetelet, and Dellmann, has yielded good results, consists in first exposing, in an elevated position such as the top of a house, a conducting ball supported on an insulating stand, and, while exposed, connecting it with the earth; then insulating it, and examining the charge which it has

acquired. This charge, being acquired from the earth by the inductive action of the electricity overhead, is opposite in sign to the inducing electricity.

Another method, which in principle resembles that of Volta, but is speedier in its action, has been introduced by Sir W. Thomson. It consists in allowing a fine stream of water to flow from an insulated metallic vessel, through a pipe, which projects through an open window or other aperture in the wall of a house, so that the nozzle from which the water flows is in the open air. The apparatus for this purpose, called the water-dropping collector, is represented in Fig. 56. *a* is a copper can, containing water, which can be discharged through the brass pipe *b* by turning a tap. The mode of insulation is worthy of notice. The can is supported on a glass stem *c*, which is surrounded, without contact, by a ring or rings of pumice *d d*, moistened with sulphuric acid. These are protected by an outer case of brass *e e*, having a hole in its top rather larger than the glass stem, the brass being separated from the moist pumice by an inner case of gutta percha. The acid needs renewal about once in two months.

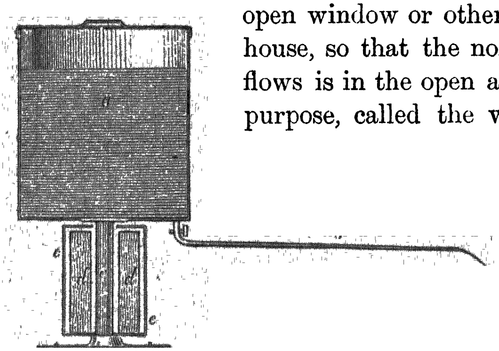


Fig 56.—Water-dropping Collector.

In severe frost, burning matches can be used instead of water, and are found to give identical indications. Whether water or match be used, the principle of action[1] is that, as long as any differ-

[1] The following quotation from an article by Sir W. Thomson puts the matter very clearly:—"If, now, we conceive an elevated conductor, first belonging to the earth, to become insulated, and to be made to throw off, and to continue throwing off, portions from an exposed part of its surface, this part of its surface will quickly be reduced to a state of no electrification, and the whole conductor will be brought to such a potential as will allow it to remain in electrical equilibrium in the air, with that portion of its surface neutral. In other words, the potential throughout the insulated conductor is brought to be the same as that of the particular equi-potential surface in the air, which passes through the point of it from which matter breaks away. A flame, or the heated gas passing from a burning match, does precisely this: the flame itself, or the highly heated gas close to the match, being a conductor which is constantly extending out, and gradually becoming a non-conductor. The drops [into which the jet from the water-dropper breaks] produce the same effects, with more pointed decision, and with more of dynamical energy to remove the rejected matter, with the electricity which it carries, from the neighbourhood of the fixed conductor."—*Nichol's Cyclopædia*, second edition, art. "Electricity, Atmospheric."

ence of potential exists between the insulated conductor and the point of the air where the issuing stream (whether of water or smoke) ceases to be one continuous conductor, and begins to be a non-conductor or a succession of detached drops, so long will each drop or portion that detaches itself carry off either positive or negative electricity, and thus diminish the difference of potential. The time required to reduce the system to the potential which exists at the point above specified, is practically about half a minute with the water-jet, and from half a minute to a minute or more, according to the strength of the wind, with a match.

The water-dropper is the most convenient collecting apparatus when the observations are taken always in the same place. For portable service, Sir Wm. Thomson employs blotting-paper, steeped in solution of nitrate of lead, dried, and rolled into matches. The portable electrometer carries a light brass rod or wire projecting upwards, to the top of which the matches can be fixed.

92. Interpretation of Indications.—We have seen that the collecting apparatus, whether armed with water-jet or burning match, is merely an arrangement for reducing an insulated conductor to the potential which exists at a particular point in the air. An electrometer will then show us the difference between this potential and that of any other given conductor, for example the earth. The earth offers so little resistance to the passage of electricity, that any temporary difference of potential which may exist between different parts of its surface, must be very slight in comparison with the differences of potential which exist between different points in the non-conducting atmosphere above it. As there is no possible method of determining absolute potential, since all electric phenomena would remain unchanged by an equal addition to the potentials of all points, it is convenient to assume, as the zero of potential, that of the most constant body to which we have access, namely the earth; and under the name earth we include trees, buildings, animals, and all other conductors in electrical communication with the soil.

Now we find that, as we proceed further from the earth's surface, whether upwards from a level part of it, or horizontally from a vertical part of it, such as an outer wall of a house, the potential of points in the air becomes more and more different from that of the earth, the difference being, in a broad sense, simply proportional to the distance. Hence we can infer[1] that there is electricity residing on the

[1] By § 54, if ρ denote the quantity of electricity per unit area on an even part of the

surface of the earth, the density of this electricity, at any moment, in the locality of observation, being measured by the difference of potential which we find to exist between the earth and a given point in the air near it. Observations of so-called atmospheric electricity[1] made in the manner we have described, are in fact simply determinations of the quantity of electricity residing on the earth's surface at the place of observation. The results of observations so made are however amply sufficient to show that electricity residing in the atmosphere is really the main cause of the variations observed. A charged cloud or body of air induces electricity of the opposite kind to its own on the parts of the earth's surface over which it passes; and the variations which we find to occur in the electrical density at the parts of the surface where we observe, are so rapid and considerable, that no other cause but this seems at all adequate to account for them. We may therefore safely assume that the difference of potential which we find, in increasing our distance from the earth, is mainly due to electricity induced on the surface of the earth by opposite electricity in the air overhead.

As electrical density is greater on projecting parts of a surface than on those which are plane or concave, we shall obtain stronger indications on hills than in valleys, if our collecting apparatus be at the same distance from the ground in both cases. Under a tree, or in any position excluded from view of the sky, we shall obtain little or no effect.

93. Results of Observation.—The first regular series of observations taken with Sir Wm. Thomson's instruments which were published[2] consisted of two years' continuous observations with self-recording apparatus at Kew Observatory, and two years' observations, at three stated times daily, and at other irregular times, at Windsor in Nova Scotia (lat. 45° N.). The electrometer used at Kew was an earlier

earth's surface, the force in the neighbouring air is $4\pi\rho$. This must be equal to the change of potential in going unit distance (§ 47). If potential increases positively, ρ is negative

[1] No good electrical observations have yet been made in balloons, and very little is known regarding the distribution of electricity at different heights in the air. A method of gauging this distribution by balloon observations is suggested by the principles of § 52, which show that, when the lines of force are vertical, and the tubes of force consequently cylindrical, the difference of electrical force at different heights is proportional to the quantity of electricity which lies between them.

[2] The observations at Windsor, N.S., and at Kew, are described in three papers by the editor of this work, *Proc. R. S.*, June 1863, January 1865, and *Trans. R. S.*, December 1867. Dellmann's observations at Kreuznach, which were taken with apparatus devised by himself, are described in *Phil. Mag.*, June 1858. Quetelet's observations (taken with Peltier's apparatus) are described in his volume *Sur le Climat de la Belgique* (Brussels, 1849).

form of the quadrant electrometer already described; and the autographic registration was effected by throwing the image of a bright point (a small hole with a lamp behind it) upon a sheet of photographic paper drawn upwards by clock-work, whereas the movements of the image, formed by means of the mirror attached to the needle, were horizontal. The curves thus obtained give very accurate information respecting the potential of the air at the point of observation, when of moderate strength; but fail to record it when of excessive strength, as the image on these occasions passed out of range. The Windsor observations were taken with the cage-electrometer, of which two forms were employed, one being much more sensitive than the other. The more sensitive form was usually employed. When the potential became inconveniently strong, the first step was to shorten the discharging pipe by screwing off some of its joints. This reduced the strength of potential in about the ratio of 3:1; but even this reduction was often not enough for the more sensitive instrument, and on such occasions the other (which was intended as a portable electrometer) was employed instead. As the ratio of the indications of the two instruments was known, a complete comparison of potentials in all weathers was thus obtained. The results are as follows:

Employing a unit in terms of which the average fine-weather potential for the year was $+4$, the potential was seldom so weak as 1, though on rare occasions it was for a few minutes as low as 0·1. In wet weather, especially with sudden heavy showers, the potential was often as strong as ± 20 to ± 30, and it was fully as strong during hail. With snow, the average strength was about the same as with heavy rain, but it was less variable, and the sign was almost always positive. Occasionally, with high wind accompanying snow, during very severe frost, it was from $+80$ to $+100$, or even higher. With fog, it was always positive, averaging about $+10$. In thunderstorms it frequently exceeded ± 100, and on a few occasions exceeded -200. There was usually a great predominance of negative potential in thunder-storms. Change of sign was a frequent accompaniment of a flash of lightning or a sudden downpour of rain. At all times, there was a remarkable absence of steadiness as compared with most meteorological phenomena, wind-pressure being the only element whose fluctuations are at all comparable, in magnitude and suddenness, with those of electrical potential. Even in fine weather, its variations during two or three minutes usually amount to as much

as 20 per cent. In changeable and stormy weather they are much greater; and on some rare occasions it changes so much from second to second that, notwithstanding the mitigating effect of the collecting process, which eases off all sudden changes, the needle of the electrometer is kept in a continual state of agitation.

94. Annual and Diurnal Variations.—Observations everywhere[1] concur in showing that the average strength of potential is greater in winter than in summer; but the months of maxima and minima appear to differ considerably at different places. The chief maximum occurs in one of the winter months, varying at different places from the beginning to the end of winter; and the chief minimum occurs everywhere in May or June. Both Kew and Windsor show distinctly two maxima in the year, but Brussels, and apparently Kreuznach, show only one. The ratio of the highest monthly average to the lowest is at Kew about 2·5, at Windsor 1·9, and at Kreuznach 2·0.

The Kew observations, being continuous, are specially adapted to throw light on the subject of diurnal variation. They distinctly indicate for each month two maxima, which in July occur at about 8 A.M. and 10 P.M., in January about 10 A.M. and 7 P.M., and in spring and autumn about 9 and 9. The result of the Brussels observations is about the same.

95 Causes of Atmospheric Electricity.—Various conjectures have been hazarded regarding the sources of atmospheric electricity. but little or no certain knowledge has yet been obtained on this subject. Evaporation has been put forward as a cause, but, as far as laboratory experiments show, whenever electricity has been generated in connection with evaporation, the real source has been friction, as in Armstrong's hydro-electric machine. The chemical processes involved in vegetation have also been adduced as causes, but without any sufficient evidence. It is perhaps not too much to say that the only natural agent which we know to be capable of electrifying the air is the friction of solid and liquid particles against the earth and against each other by wind. The excessively strong indications of electricity observed during snow accompanied by high wind, favour the idea that this may be an important source.

Without knowing the origin of atmospheric electricity, we may, however, give some explanation of the electrical phenomena which

[1] The remarks in this section express the results of observation at places all of which are in the north temperate zone.

occur both in showers and in thunder-storms. Very dry air is an excellent non-conductor; very moist air has, on the other hand, considerable conducting power. When condensation of vapour takes place at several centres, a number of masses of non-conducting matter are transformed into conductors, and the electricity which was diffused through their substance passes to their surfaces. These separate conductors influence one another. If one of them is torn asunder while under influence, its two portions may be oppositely charged; and if rain falls from the under surface of a cloud which is under the influence of electricity above it, the rain which falls may have an opposite charge to the portion which is left suspended.

Lord Rayleigh has found that drops of water which are slightly electrified have a tendency to coalesce, whereas unelectrified drops usually rebound after collision.[1] It is probable that electricity may in this way give rise to showers.

The coalescence of small drops to form large ones, though it increases the electrical density on the surfaces of the drops, does not increase the total quantity of electricity, and therefore (§ 56) cannot directly influence the observed potential.[2]

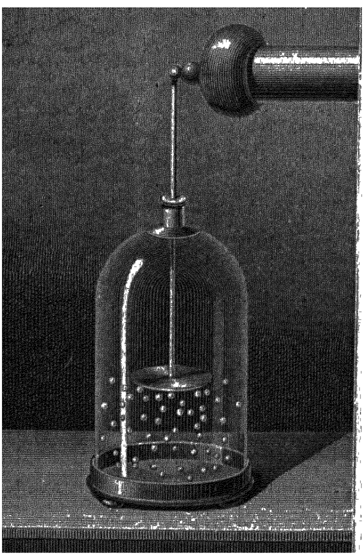


Fig. 57.—Electric Hail.

Thunder-storms and other powerful manifestations of atmospheric electricity seem to be accompaniments of very sudden and complete condensation which gives unusually free scope to the causes of irregular distribution just indicated.

96. Hail.—Hail has sometimes been ascribed to an electrical origin, and a singular theory was devised by Volta to account for the supposed fact that hailstones are sustained in the air. He imagined

[1] The observation was made upon jets discharged from a nozzle directed upwards, and it was found that connecting the discharging vessel with one pole of a single Grove's cell (the other pole being to earth) was sufficient to produce coalescence of the drops. *Proc. Royal Society*, Feb. 27, 1879.

[2] In several modern works there is an attempt to prove that the coalescence of small

that two layers of cloud, one above the other, charged with opposite electricities, kept the hailstones continually moving up and down by alternate attraction and repulsion. An experiment called *electric hail* is sometimes employed to illustrate this idea. Two metallic plates are employed (Fig. 57), the lower one connected with the earth, and the upper one with the conductor of the electrical machine; and pith-balls are placed between them. As the machine is turned, the balls fly rapidly backwards and forwards from one plate to the other.

97. Waterspouts. — Waterspouts, being often accompanied by

Fig. 58.—Waterspouts.

strong manifestations of electricity, have been ascribed by Peltier and others to an electrical origin; but the account of them given in the subjoined note appears more probable.[1]

drops to form large ones must increase their potential. The reasoning employed regards the potential of a conductor as depending only on its own charge. But it is easy to show that, in computing the potential of one drop in a group of millions, the part depending on its own charge is negligible; and as regards the terms depending on the charges of the other drops, they involve volume-densities, not surface-densities, and are unchanged by coalescence

[1] "On account of the centrifugal force arising from the rapid gyrations near the centre of a tornado, it must frequently be nearly a vacuum. Hence when a tornado passes over a building, the external pressure, in a great measure, is suddenly removed, when the atmosphere within, not being able to escape at once, exerts a pressure upon the interior, of perhaps nearly fifteen pounds to the square inch, which causes the parts to be thrown in

MAGNETISM.

CHAPTER X.

GENERAL STATEMENT OF FACTS AND LAWS.

98. Magnets, Natural and Artificial.—Natural magnets, or *lodestones*, are exceedingly rare, although a closely allied ore of iron, capable of being strongly acted on by magnetic forces, and hence called *magnetic iron-ore*, is found in large quantity in Sweden and elsewhere. Artificial magnets are usually pieces of steel, which have been permanently endowed with magnetism by methods which we shall hereafter describe. Magnets are chiefly characterized by the property of attracting iron, and by the tendency to assume a particular orientation when freely suspended.

99. Force Greatest at the Ends.—The property of attracting iron is very unequally manifested at different points of the surface of a magnet. If, for example, an ordinary bar-magnet be plunged in iron-filings, these cling in large quantities to the terminal portions, and leave the middle bare, as in the lower diagram of Fig. 59. If

every direction to a great distance. For the same reason, also, the corks fly from empty bottles, and everything with air confined within explodes. When a tornado happens at sea, it generally produces a waterspout. This is generally first formed above, in the form of a cloud shaped like a funnel or inverted cone. As there is less resistance to the motions in the upper strata than near the earth's surface, the rapid gyratory motion commences there first. . . . This draws down the strata of cold air above, which, coming in contact with the warm and moist atmosphere ascending in the middle of the tornado, condenses the vapour and forms the funnel-shaped cloud. As the gyratory motion becomes more violent, it gradually overcomes the resistances nearer the surface of the sea, and the vertex of the funnel-shaped cloud gradually descends lower, and the imperfect vacuum of the centre of the tornado reaches the sea, up which the water has a tendency to ascend to a certain height, and thence the rapidly ascending spiral motion of the atmosphere carries the spray upward, until it joins the cloud above, when the waterspout is complete. The upper part of a waterspout is frequently formed in tornadoes on land. When tornadoes happen on sandy plains, instead of waterspouts they produce the moving pillars of sand which are often seen on sandy deserts."—W. Ferrel, in *Mathematical Monthly*.

the magnet is very thick in proportion to its length, we may have filings adhering to all parts of it, but the quantity diminishes rapidly towards the middle. The name *poles* is used, in a somewhat loose sense, to denote the two terminal portions of a magnet, or to denote two points, not very accurately defined, situated in these portions. The middle portion, to which the filings refuse to adhere, is called *neutral*.

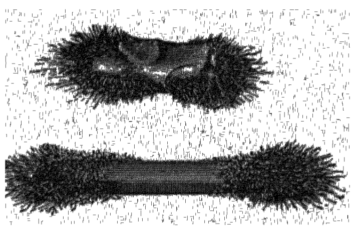


Fig 59 —Magnets dipped in Filings

100. Lines Formed by Filings.—If a sheet of card is laid horizontally upon a magnet, and wrought-iron filings are sifted over it, these will, with the assistance of a few taps given to the card, arrange

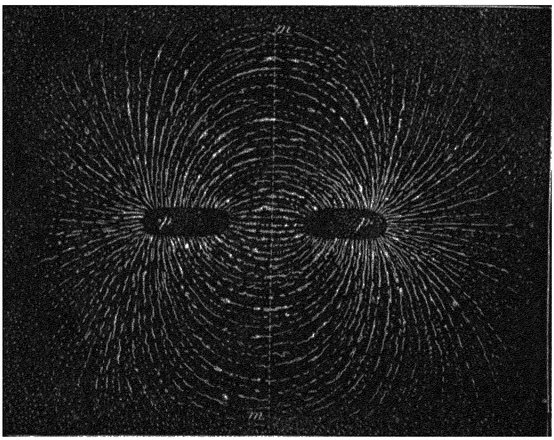


Fig 60 —Magnetic Curves

themselves in a system of curved lines, as shown in Fig. 60. These lines give very important indications both of the direction and intensity of the force produced by the magnet at different points of the space around it.[1] They cluster very closely about the two poles $p\ p$, and thus indicate the places where the force is most intense.

101. Curve of Intensities.—Some idea may be obtained of the relative intensities of magnetic force at different points in the length of

[1] The lines formed by the filings may be called the lines of *effective force for particles only free to move in the plane of the card.* The lines of total force cut the card at various angles, and are at some places perpendicular to it, as shown by the filings standing on end. For the definition of lines of magnetic force, see § 109.

a magnet, by measuring the force required to detach an iron ball at various points. Better determinations can be obtained by counting the number of vibrations made by a small magnetized needle when suspended opposite different parts of the bar, the bar being in a vertical position, and the vibrations of the needle being horizontal. The intensity of the force is nearly as the square of the number of vibrations; on the same principle that the force of gravity at different places is proportional to the square of the number of vibrations of a pendulum.

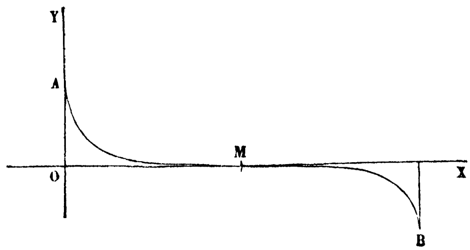


Fig. 61.—Curve of Intensities.

Both these methods of determination were employed by Coulomb, who was the first to make magnetism an accurate science; and the results which he obtained are represented by the curve of intensities A M B (Fig. 61). M is the middle of the bar, O one end of it, and the ordinates of the curve (that is, the distance of its points from the line O X) represent the intensities of force at the different points in its length. The curve was constructed from observations of the force at several points in the length; but in dealing with the observation made opposite the very end, the force actually observed was multiplied by 2. Perfect symmetry was found between the intensities over the two halves of the length. In the figure we have inverted the curve for one-half, in order to indicate an opposition of properties, which we shall shortly have to describe. The curves of intensities for two magnets of different sizes but of the same form are usually similar.

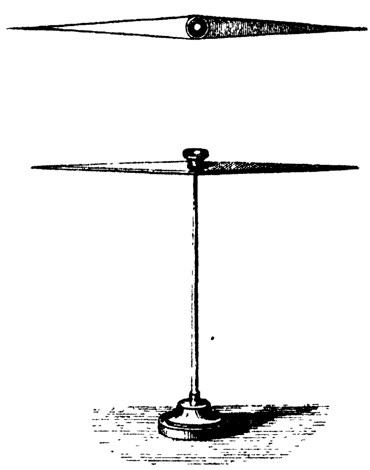


Fig. 62.—Magnetized Needle.

102. Magnetic Needle.—Any magnet freely suspended near its centre is usually called a *magnetic needle*, or more properly a *magnetized needle.* One of its most usual forms is that of a very elon-

gated rhombus of thin steel, having, very near its centre, a concavity or *cup* by means of which it can be balanced on a point. When it is thus balanced horizontally, it does not, like a piece of ordinary matter, remain in equilibrium in all azimuths[1] but assumes one particular direction, to which it always comes back after displacement. In this position of stable equilibrium, one of its ends points to magnetic north, and the other to magnetic south, which differ in general by several degrees from geographical (or true) north and south. This is the principle on which compasses are constructed.

103. Declination.—The difference between magnetic and true north, or the angle between the magnetic meridian and the geographical meridian, is called *magnetic declination.*[2] It is very different at different places, and at a given place undergoes a gradual change from year to year, besides smaller changes, backwards and forwards, which are continually taking place. At Greenwich, at the present time, its value is about 17° W., that is, magnetic north is west of true north by this amount.

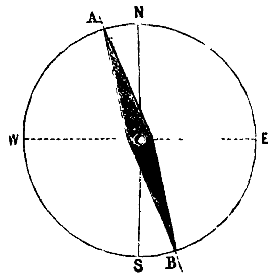


Fig 63 —Declination.

104. Inclination or Dip.—If, before magnetizing a needle, we mount it on an axis passing through its centre of gravity, and support the ends of the axis, as in Fig. 64, by a thread without torsion, the needle will remain in equilibrium in any position in which it may be placed. If it be then magnetized, it will no longer be indifferent, but will place itself in a particular vertical plane called the magnetic meridian, and will take a particular direction in this plane. This direction is not horizontal, but inclined, generally at a considerable angle, to the horizon; and this angle is called *dip* or

[1] All lines in the same vertical plane are said to have the same *azimuth.* Azimuthal angles are angles between vertical planes, or between horizontal lines. The azimuth of a line when stated numerically, is the angle which the vertical plane containing it makes with a vertical plane of reference, and this latter is usually the plane of the meridian. Some readers may be glad to be reminded that by the plane of the *meridian* is meant a vertical plane passing through the place of observation, and through or parallel to the earth's axis. A horizontal line in this plane is a meridian line. The *magnetic meridian* is the vertical plane in which a magnetized needle, when freely suspended, tends to place itself.

[2] The nautical name for magnetic declination is *variation;* but it is most inconvenient and confusing to denote the element itself by the same name as the variations of the element.

inclination. Its value at Greenwich is about $67\frac{1}{2}°$, the end which points to the north pointing at the same time downwards. In the northern hemisphere generally, it is the north end of the needle which dips, and in the southern hemisphere it is the end which points south.

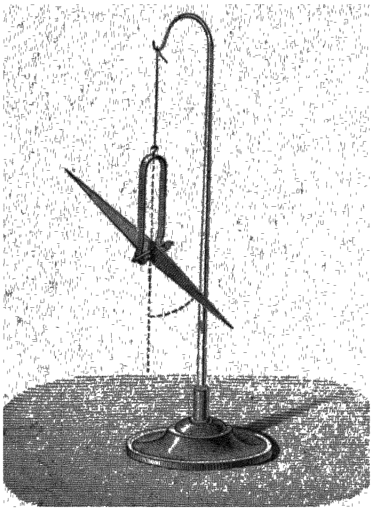


Fig. 64.—Dip

It follows that, if a magnetized needle is to be balanced in a horizontal position, the point or axis of support must not be in the same vertical with the centre of gravity, but must be between the centre of gravity and the end which tends to dip. Needles thus balanced, as in the ordinary mariner's compass, are called *declination needles.*

105. Mutual Action of Poles.—On presenting one end of a magnet to one end of a needle thus balanced, we obtain either repulsion or attraction, according as the pole which is presented is similar or dissimilar to that to which it is presented. *Poles of contrary name attract each other; poles of the same name repel each other.*

This property furnishes the means of distinguishing a body which is merely magnetic (that is, capable of temporary magnetization) from a permanent magnet. The former, a piece of soft iron for example, is attracted by either pole of a magnet; while a body which has received permanent magnetization has, in ordinary cases, two poles, of which one is attracted where the other is repelled. Magnetic attractions and repulsions are exerted without modification through any body which may be interposed, provided it be not magnetic.

106. Names of Poles.—The phenomena of declination and inclination above described, evidently require us to regard the earth, in a broad sense, as a magnet, having one pole in the northern and the other in the southern hemisphere. Now since poles which attract one another are dissimilar, it follows that the magnetic pole of the earth which is situated in the northern hemisphere is *dissimilar* to that end of a magnetized needle which points to the north. Hence

there is some doubt as to which end of a needle or magnet is to be called the *north* pole. We shall avoid this ambiguity by calling that end of it which seeks the north, the *north-seeking* end or pole, and the other the *south-seeking* end or pole. Sir Wm. Thomson calls the north-seeking pole the *south* pole, and the other the *north* pole, because the former is similar to the south, and the latter to the north pole of the earth. In like manner most French writers call the north-seeking pole of a needle the *austral*, and the other the *boreal* pole. Popular usage in this country calls the north-seeking end the *north*, and the other the *south* pole, a nomenclature which introduces great confusion whenever we have to reason respecting the earth regarded as a magnet. Faraday, to avoid the ambiguity which has attached itself to the names north and south pole, calls the north-seeking end the *marked*, and the other the *unmarked* pole. Airy, for a similar reason, employed, in his *Treatise on Magnetism*, the distinctive names *red* and *blue* to denote respectively the north-seeking and south-seeking ends, these names, as well as those employed by Faraday, being purely conventional, and founded on the custom of marking the north-seeking end of a magnet with a transverse notch or a spot of red paint.

107. Magnetic Induction.—When a piece of iron is in contact with a magnet, or even when a magnet is simply brought near it, it becomes

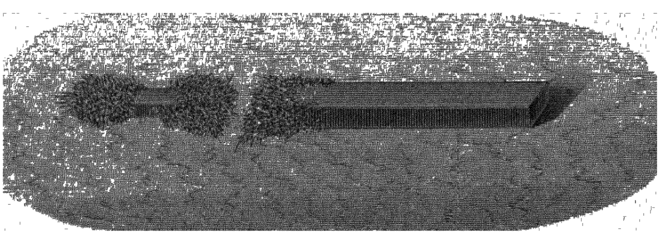


Fig. 65.—I[illegible]m.

itself for the time a magnet, with two poles and a neutral portion between them. If we scatter filings over the iron, they will adhere to its ends, as shown in Fig. 65. If we take away the influencing magnet, the filings will fall off, and the iron will retain either no traces at all or only very faint ones of its magnetization. If we apply similar treatment to a piece of steel, we obtain a result similar in some respects, but with very important differences in degree. The steel, while under the influence of the magnet, exhibits much weaker

effects than the iron; it is much more difficult to magnetize than iron, and does not admit of being so powerfully magnetized; but, on the other hand, it retains its magnetization after the influencing magnet has been withdrawn. This property of retaining magnetism when once imparted has been (somewhat awkwardly) named *coercive force.* Steel, especially when very hard, possesses great coercive force; iron, especially when very pure and soft, scarcely any.

In magnetization by influence, which is also called *magnetic induction*, it will be found, on examination, that the pole which is next the inducing pole is of contrary name to it; and it is on account of the mutual attraction of dissimilar poles that the iron is attracted by the magnet. The iron can, in its turn, support a second piece of iron; this again can support a third, and so on through many steps. A magnetic chain can thus be formed, each of the component pieces having two poles. An action of this kind takes place in the clusters of filings which attach themselves to one end of a magnetized bar, these clusters being composed of numerous chains of filings.

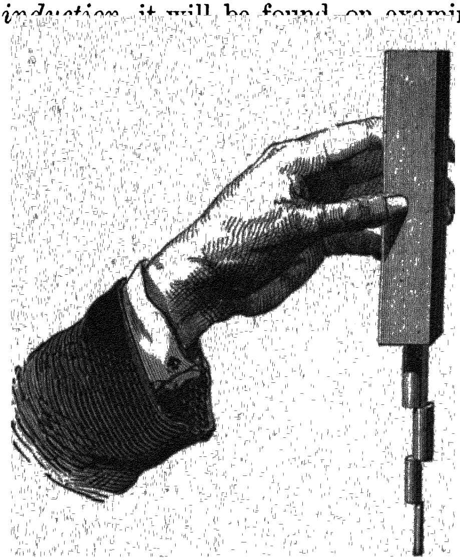


Fig. 66.—Magnetic Chain

In comparing the phenomena of magnetic induction with those of electrical induction, we find both points of resemblance and points of difference. In the case of electricity, if the influencing and influenced body are allowed to come in contact, the former loses some of its own charge to the latter. In the case of magnetism there is no such loss, a magnet after touching soft iron is found to be as strongly magnetized as it was before.

108. Effect of Rupture on a Magnet.—If a magnet is broken into any number of pieces, every piece will be a complete magnet with poles of its own. In the case of an ordinary bar-magnet or needle, the similar poles of the pieces will all be turned the same way, as

in Fig. 67, which represents a magnet A B broken into four pieces. The ends *a, a, a, a* are of one name, and the ends *b, b, b, b* of the opposite name.

109. Imaginary Magnetic Fluids: Magnetic Potential.—All mutual forces between magnets can be reduced to attractions and repulsions

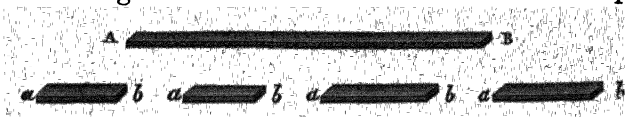


Fig 67.—Broken Magnet

between different portions of two imaginary fluids, called *magnetisms*,[1] one of which may be called *positive* and the other *negative*. Neither magnetism can exist apart from the other; every magnet possesses equal quantities of both; quantity being measured by force of attraction or repulsion at given distance, just as in the case of electricity, like portions repelling, and unlike portions attracting each other inversely as the square of the distance. Equal quantities of the two magnetisms, when coexisting at the same place, produce no resultant effect, and may be regarded as destroying each other. Any magnetism which is not thus cancelled is called *free*.

With reference to these imaginary fluids, *magnetic potential* can be defined in the same way as electrical potential (§§ 46, 47, 56), and magnetic *lines of force* possess the same properties as electrical lines of force (§§ 48, 49, 52, 53). The direction of magnetic force at a point can either be defined as the direction in which a pole of a magnet would be urged if brought to the point, or as the direction in which a small magnetized needle, if brought to the point and balanced at its centre of gravity, would place its line of poles; and lines of magnetic force are lines to which this direction is everywhere tangential. It is important to remark that a linear piece of soft iron, though it sets its length along a line of force, does not travel along a line of force, but deviates towards the concave side. On tapping the card represented in Fig. 60, it will be found that filings placed on the line *m m* move along that line, and therefore at right angles to the lines of force.

The force which is specified by magnetic "lines of force" is the force which *one pole* of a permanent magnet would experience; and

[1] Poisson, following Coulomb, spoke of *two magnetic fluids*, and laid down a theory of their action Sir W. Thomson, avoiding the hypothetical parts of Poisson's theory, speaks of *imaginary magnetic matter* of two dissimilar kinds.

it is the same in intensity, but opposite in direction, for dissimilar poles. The two poles of a small magnet (temporary or permanent) in the position of stable equilibrium as regards rotation, are pulled in nearly opposite directions; and the force which tends to produce movement of translation is the resultant of these two nearly opposite pulls. The direction of this resultant for a small sphere is the direction in which the intensity of the field increases most rapidly.[1]

110. Specification of Magnetization.—A piece of steel is said to be *uniformly magnetized,* if equal and similar portions, cut in parallel directions from all parts of it, are precisely alike in their magnetic properties.

If a piece of magnetized steel be suspended at its centre of gravity, so as to be free to turn all ways about it, the effect of the earth's magnetism upon it consists in a tendency for a particular line through this centre of gravity to take a determinate direction, which is the direction of terrestrial magnetic force. When the line is placed in any other position, the couple tending to bring it back is proportional to the sine of the angle between the two positions, and is the same for all directions of deviation. The line which possesses this property is the *magnetic axis* of the body, and the name is sometimes given to all lines parallel to it. If the piece of steel be uniformly magnetized, this axis is the direction of magnetization; or the *direction of magnetization is the common direction of all those lines which tend to place themselves along lines of force* in a field[2] where the lines of force are parallel.

111. Ideal Simple Magnet: Thin Bar, uniformly and longitudinally Magnetized.—The mutual actions of magnets admit of very accurate expression when the magnets are very thin in comparison with their length, uniform in section, and uniformly magnetized in the direction of their length. Such bars, which may be called *simple magnets,* behave as if their forces resided solely in their ends, which may

[1] The following considerations will help to explain the direction of motion :—

Suppose a small piece of iron wire bent to coincide with a portion of a line of force. It will be pulled at both ends, and will therefore move towards the concave side.

Also if the force is stronger at one end of the piece than at the other, the stronger pull will prevail.

Both these actions combine to move the iron from weaker to stronger parts of the field.

[2] A *field of force* is any region of space traversed by lines of force; or, in other words, any region pervaded by force of attraction or repulsion. A *magnetic field* is any region pervaded by magnetic force. All space in the neighbourhood of the earth is a magnetic field, and within moderate distances the lines of force in it may be regarded as parallel, unless artificial magnets, pieces of iron, or other sources of disturbance are present.

therefore in the strictest sense be called their poles. The two poles of any one such bar are equal in strength; that is to say, one of them attracts a pole of another simple magnet with the same force with which the other repels it at the same distance. They have no free magnetism except at the two ends, and the quantities at the two ends are equal but of opposite sign. The same number which denotes the quantity of free magnetism at either pole, denotes the *strength of the pole,* or, as it is often called, the *strength of the magnet.* Its definition is best expressed by saying that the force between a pole of one simple magnet and a pole of another, is the product of their strengths divided by the square of the distance between them.[1]

The force which a pole of a simple magnet experiences in a magnetic field, is the *product of the strength of the pole and the intensity of the field.* This rule applies to the force which a pole experiences from the earth's magnetism, the intensity of the field being in this case the intensity of terrestrial magnetic force, and, from the uniformity of the field, the forces on the two poles are in this case equal, constituting a couple, whose arm is the line joining the poles multiplied by the sine of the angle which this line makes with the lines of force.

The product of the line joining the two poles by the strength of either pole is called the *moment of the magnet,* and it is evident, from what has just been said, that the continued product of *the moment of the magnet, the intensity of terrestrial magnetic force, and the sine of the angle between the length of the magnet and the lines of force,* is equal to the moment of the couple which the earth's magnetism exerts upon the magnet.

112. Compound Magnet of Uniform Magnetization.—Any magnet which is not a simple magnet in the sense defined in § 111 may be called a *compound magnet.* It is convenient to define the moment of a compound magnet by the condition stated in the concluding words of that section, so that the moments of different magnets, whether simple or compound, may be compared by comparing the couples exerted on them by terrestrial magnetism when their axes are equally inclined to the lines of force.

If a number of simple magnets of equal strength be joined end to

[1] We here, and throughout the remainder of this chapter, ignore the existence of induction, which, however, is not altogether absent even in the hardest steel. The effect of induction is always to favour attraction. The attractions will therefore be somewhat stronger, and the repulsions somewhat weaker, than our theory supposes.

end, with their similar poles pointing the same way, there will be mutual destruction of the two magnetisms at every junction, and the system will constitute one simple magnet of the same strength as any one of its components; but its moment will evidently be the sum of their moments.

If any number of simple magnets be united, either end to end or side to side, provided only that they are parallel, and have their similar poles turned the same way, the resultant couple exerted upon the whole system by terrestrial magnetism will be the sum of the separate couples exerted by it on each magnet, and the moment of the system will be the sum of the moments of its parts. But any piece of uniformly magnetized material may be regarded as being thus built up, and hence, if different portions be cut from the same uniformly magnetized mass, their moments will be simply proportional to their volumes. The quotient of moment by volume, for any uniformly magnetized mass, is called *intensity of magnetization.*

113. Actual Magnets.—The definitions and laws of simple magnets are approximately applicable to actual magnets, when magnetized in the usual manner.

If an actual bar-magnet in the form of a rectangular parallelepiped were magnetized with perfect uniformity, and in the direction of its length, it might be regarded as made up of a number of simple magnets laid side by side, and its behaviour would be represented by supposing a complete absence of magnetic fluid from all parts of it except its *ends* (in the strict mathematical sense). One of these terminal faces would be covered with positive, and the other with negative fluid, and if the magnet were broken across at any part of its length, the quantities of positive and negative fluid on the broken ends would be the same as on the ends of the complete magnet. The observed fact that magnets behave as if the fluids were distributed through a portion of their substance in the neighbourhood of the ends, and not confined to the ends strictly so called, indicates a falling off in magnetization towards the extremities, and is approximately represented by conceiving of a number of short magnets laid end to end, and falling off in strength towards the two extremities of the series.[1]

[1] Thus the last magnet at the positive end being weaker than its neighbour, its negative pole will be weaker than its neighbour's positive pole, so that there will be an excess of positive fluid at this junction. Similar reasoning applies to all the junctions near the ends. There will be an excess of positive fluid at all junctions near the positive end, and an excess of negative at all junctions near the negative end.

The resultant force due to the imaginary magnetic fluids which are distributed through the terminal portions of an actual bar-magnet is, in the case of actions at a great distance, sensibly the same as if the two portions of fluid were collected at their respective centres of gravity. These two centres of gravity are the poles of the magnet for all actions between the magnet and other magnets at a great distance, and more especially between the magnet and the earth.

CHAPTER XI.

EXPERIMENTAL DETAILS.

114. The Earth's Force simply Directive.—The forces which produce the orientation of a magnet depend upon causes of which very little is known. They are evidently connected in some way with the earth, and are accordingly referred to TERRESTRIAL MAGNETISM. We have already stated (§ 110) that the combined effect of the forces exerted by terrestrial magnetism upon a magnetized needle is equivalent to a couple tending to turn the needle into a particular direction, and (§ 113) that in the case of needles magnetized in the ordinary way, there are two definite points or poles (near the two ends of the needle) which may be regarded as the points of application of the two equal forces which constitute the couple.

The fact that terrestrial magnetic force simply tends to turn the needle, and not to give it a movement of translation, in other words, that the resultant *force* (as distinguished from *couple*) is zero, is completely proved by the two following experiments:—

(1) If a bar of steel is weighed before and after magnetization, no change is found in its weight. This proves that the vertical component is zero.

(2) If a bar of steel, not magnetized, is suspended by a long and fine thread, the direction of the thread is of course vertical. If the bar is then magnetized, the direction of the thread still remains vertical. The most rigorous tests fail to show any change of its position. This proves that the horizontal component is zero, a conclusion which may be verified by floating a magnet on water by means of a cork. It will be found that there is no tendency to move across the water in any particular direction.

115. Horizontal, Vertical, and Total Intensities.—If S denote the strength of a magnet, and I the intensity of terrestrial magnetic force,

each pole of the magnet experiences a force S I, and if L denote the distance between the poles (often called the length of the magnet), the distance between the lines of action of these two parallel and opposite forces may have any value intermediate between L and zero, according to the position in which the needle is held. It will be zero when the line of poles is that of the dipping-needle; it will be L when the line of poles is perpendicular to the dipping-needle; and will be L sin α when the line of poles is inclined at any angle α to the dipping-needle.

The force SI upon either pole of the magnet acts in the direction of the dipping-needle; in other words, in the direction of the lines of force due to terrestrial magnetism. Let δ denote the dip, that is the inclination of the lines of force to the horizon, then the force SI can be resolved into SI cos δ horizontal, and SI sin δ vertical. Hence the horizontal and vertical intensities H and V are connected with the total intensity and dip I and δ by the two equations

$$H = I\cos\delta \quad , \quad V = I\sin\delta \qquad (1)$$

which are equivalent to the following two

$$\frac{V}{H} = \tan\delta \quad , \quad V^2 + H^2 = I^2. \qquad (2)$$

116. Torsion-balance.—Coulomb, in investigating the laws of the mutual action of magnets, employed a torsion-balance scarcely differing from that which he used in his electrical researches. The suspending thread carried, at its lower end, a stirrup on which a magnetized bar was laid horizontally. The torsion-head was so adjusted that one end of the magnet was opposite the zero of the divisions on the glass case when the supporting thread was without torsion. In order to effect this adjustment, the magnet was first suspended by a thread whose torsional power was inconsiderable, so that the magnet placed itself in the magnetic meridian. The case was then turned till its zero came to this position. The torsionless thread was then replaced by a fine metallic wire, and the magnet was replaced by a copper bar of the same weight. The head was then turned till this bar came into the magnetic meridian, and lastly the magnet was put in the place of the bar.

Fig. 68 shows the arrangement adopted for observing the repulsion or attraction between one pole of the suspended magnet and one pole of another magnet placed vertically. Before the insertion of the latter, the suspended magnet was acted on by no horizontal

forces except the horizontal component of terrestrial magnetism and the torsion of the wire. It was then found that the torsion requisite for keeping the magnet in any position was proportional to the sine of the displacement from the meridian.

This result is evidently in accordance with the principles stated above, for the two equal horizontal forces on the two poles being

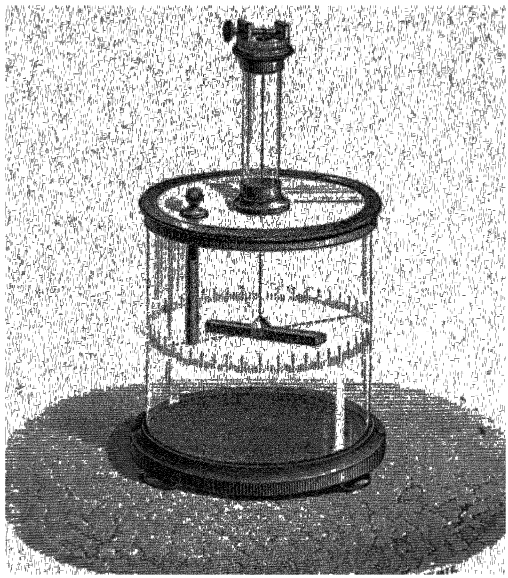


Fig. 68. Torsion balance

constant for all positions, the couple which they compose is proportional to the distance between their lines of action, and this distance is evidently L sin θ, L denoting the constant distance between the poles, and θ the deviation of the needle from the meridian.

117. Measurement of Declination.—Magnetic declination has been observed with several different forms of apparatus.

At sea, the most common method of determining it has consisted in observing the magnetic bearing of the rising or setting sun, and comparing this with its true bearing as calculated by a well-known astronomical method.

At fixed observatories more accurate methods of observation are employed. Fig. 69. shows the arrangement adopted at Greenwich. A bar-magnet B carries at one end a cross of fine threads C, and at the other a lens D, the distance between them being equal to the

focal length of the lens, thus forming a kind of inverted telescope, whose line of collimation is the line joining the cross to the optical centre of the lens. The bar is suspended by means of a stirrup from a torsionless thread, and sets its magnetic axis in the magnetic meridian. The telescope E, with theodolite mounting,[1] is stationed opposite the end which carries the lens, and is so adjusted at each observation that its line of collimation is parallel to that of the

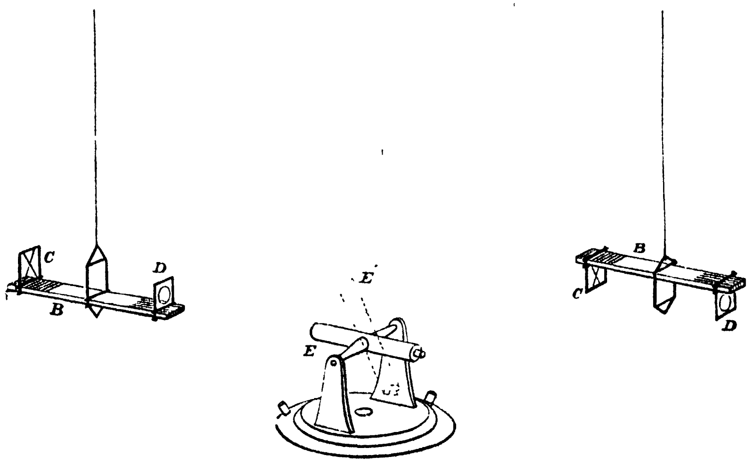


Fig. 69.—Declination Magnet

inverted telescope carried by the magnet, an adjustment which is identified by seeing the cross C coincident with a similar cross fixed in the interior of the telescope E. When the observation has been made with the magnet in one position, it must be repeated with the magnet turned upside down as shown in the figure. Error of parallelism between the magnetic axis of the bar and the line of collimation of the inverted telescope which it carries, will affect these two observations to the same extent in opposite directions, and will therefore disappear from their mean. The readings are taken on the horizontal circle at the base of the instrument, and astronomical observations must be made once for all to determine what reading corresponds to the geographical meridian.

Another very accurate method consists in rigidly attaching to the

[1] A *theodolite* consists of a telescope mounted so as to have independent motions in azimuth and altitude, the amounts of these motions being indicated by divided circles or arcs of circles. It does not differ essentially from the larger instrument called the *altazimuth*.

bar, instead of the lens and cross, a small vertical mirror. This can either be viewed through a telescope, so as to show the reflection of a horizontal scale of equal parts, which will appear to travel across the field of view of the telescope as the magnet turns, or it can be employed to throw the image of a spot of light either upon a screen viewed by the observer, or still better upon photographic paper drawn by clock-work, which leaves a permanent record of continuous changes. Both these methods of employing mirrors for the observation of small movements of rotation are now extensively employed in many applications. They appear to have been first introduced by Gauss, who employed them for the purpose which we are now considering.

118. Measurement of Dip.—The dip-circle or inclination compass is represented in Fig. 70. It consists essentially of a magnetized needle, very accurately and delicately mounted on a horizontal axis through its centre of gravity, in the centre of a vertical circle on which the positions of the two ends of the needle can be read off. This circle can be turned with the needle into any azimuth, the amount of rotation being indicated by a horizontal circle. It is obvious that, if the vertical circle is placed in the plane of the magnetic meridian, the needle, being free to move in this plane, will directly indicate the dip. On the other hand, if the vertical circle is placed in a plane perpendicular to the magnetic meridian, the horizontal component of terrestrial magnetism is prevented from moving the needle, which, accordingly, obeys the vertical component only, and takes a vertical position. In intermediate positions of the vertical circle, the needle will assume positions intermediate between the vertical and the true angle of dip. In fact, if θ be the angle which the plane of the vertical circle makes with the magnetic meridian, the component H sin θ of terrestrial magnetism, being perpendicular to this plane, merely tends to produce pressure against the supports, and the horizontal component influencing the position

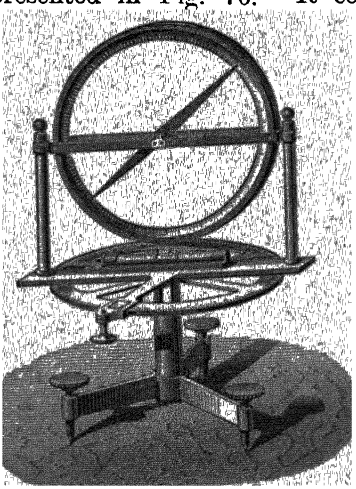


Fig. 70.—Dip-circle.

of the needle is only H cos θ, which lies in the plane of the circle. As none of the vertical force is destroyed, the tangent of the apparent dip will be $\frac{V}{H \cos \theta} = \frac{\tan \delta}{\cos \theta}$. The most accurate method of setting the vertical circle in the magnetic meridian consists in first adjusting it so that the needle takes a vertical position, and then turning it through 90°.

The instrument having thus been set, and a reading taken at each end of the needle, it should be turned in azimuth through 180°, and another pair of readings taken. By employing the mean of these two pairs of readings, several sources of error are eliminated, including non-coincidence of the axis of magnetization with the line joining the ends of the needle. One important source of error—deviation of the centre of gravity from the axis of suspension in a direction parallel to the length of the needle, is, however, not thus corrected. It can only be eliminated by remagnetizing the needle in the reverse direction so as to interchange its poles. The mean of the results obtained before and after the reversal of its magnetization will be the true dip.

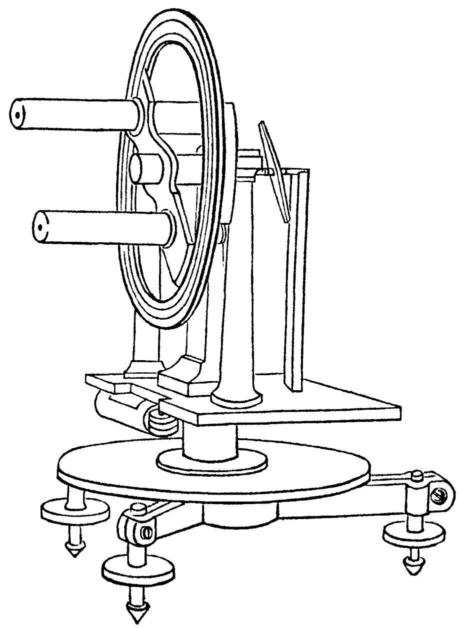


Fig 71.—Kew Dip-circle.

A better form of instrument, known as the Kew dip-circle, is now employed. Its essential parts are represented in Fig. **71.** There is no metal near the needle, and the readings are taken on a circle round which two telescopes travel. In each observation the telescopes are directed to the two ends of the needle.

119. Measurement of Intensity of Terrestrial Magnetic Force.—The complete specification of the earth's magnetic force at any place involves three independent elements. For example, if declination,

dip, and horizontal force are determined by observation, vertical force and total force can be calculated by the formulæ of § 115.

Observations of magnetic force are made either by counting the number of vibrations executed in a given time, or by statical measurements. If a magnet executes small horizontal vibrations under the influence of the earth's magnetism, the square of the number of vibrations in a given time is proportional to $\frac{HM}{\mu}$, H denoting the horizontal intensity, M the moment of the magnet, and μ its moment of inertia about the centre of suspension. Hence it is easy to observe the *variations* of horizontal intensity which occur from time to time, if we can ensure that our magnet itself shall undergo no change, or if we have the means of correcting for such changes as it undergoes. To obtain absolute determinations of horizontal intensity, the following method is employed.

First, observe the time of vibration of a freely-suspended horizontal magnet under the influence of the earth alone,—this will give the *product* of the earth's horizontal intensity and the moment of the magnet.

Secondly, employ this same magnet to act upon another also freely suspended, and thus compare its influence with that of the earth,—this will give the ratio of the same two quantities whose product was found before. Hence the two quantities themselves can easily be computed. (See § 144.)

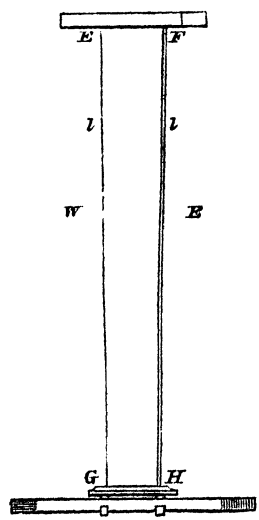


Fig. 72.—Bifilar Magnetometer.

120. Bifilar and Balance Magnetometers.—The changes of horizontal intensity are measured statically by means of the bifilar magnetometer. This consists of a bar-magnet (Fig. 72) suspended by two equal threads which would be in one vertical plane if the bar were unmagnetized; but matters are so arranged that, under the combined action of the pull of the threads, the weight of the bar, and the earth's magnetism, the bar is kept in a position nearly perpendicular to the magnetic meridian. The changes which occur in its position from time to time are due only to changes in the *intensity* of the earth's horizontal force; changes in the direction

of this force, to the extent of a few minutes of angle, having no sensible effect, on account of the near approach to perpendicularity.

Let the distance E F of the upper points of attachment of the threads be $2a$ and the distance G H of the lower points $2b$, and let the angle between the directions of E F and G H be ϕ. Also let W be the weight of the magnet, l the length of each thread, T its tension, and θ its inclination to the vertical. In Fig. 73 E′, F′ are the projections of E, F upon the horizontal plane which contains G, H; and the two lines G H, E′ F′ bisect each other at O, so that we have $OE' = a$, $OG = b$, $E'OG = \phi$.

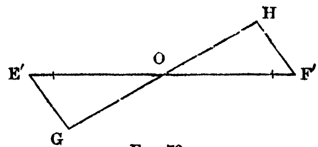


Fig 73

G E′ is the projection of one of the threads; we have therefore $GE' = l \sin\theta$, and the tension of this thread can be resolved into a vertical component $T\cos\theta$ and a horizontal component which acts along G E′, and is $T\sin\theta$ or $T\frac{GE'}{l}$. The moment of the latter round O is found by multiplying by the perpendicular dropped from O upon G E′. But G E′ multiplied by this perpendicular is double the area of the triangle E′ O G, or is $ab\sin\phi$, hence the moment in question is equal to $\frac{T}{l}ab\sin\phi$, and as this is due to one thread only, the couple due to the two threads is $\frac{2T}{l}ab\sin\phi$, which is practically equal to $\frac{W}{l}ab\sin\phi$, since θ is practically so small that $\cos\theta$ may be taken as unity.

If M be the magnetic moment of the magnet, and H the earth's horizontal intensity, M H will be the horizontal magnetic couple acting on the magnet, if the axis of the latter is perpendicular to the magnetic meridian. If the deviation from perpendicularity be β the couple will be $MH\cos\beta$, and β is practically so small that $\cos\beta$ may be taken as unity. Since in the position of equilibrium the two couples balance each other, we have the equation

$$W\frac{ab}{l}\sin\phi = MH,$$

which shows that H varies as $\sin\phi$.

The changes of vertical intensity are measured by the *balance-magnetometer*, which consists of a bar-magnet placed in the magnetic meridian, and suspended on knife-edges like the beam of an ordinary balance. Its deviations from horizontality are measures of the changes of vertical intensity.

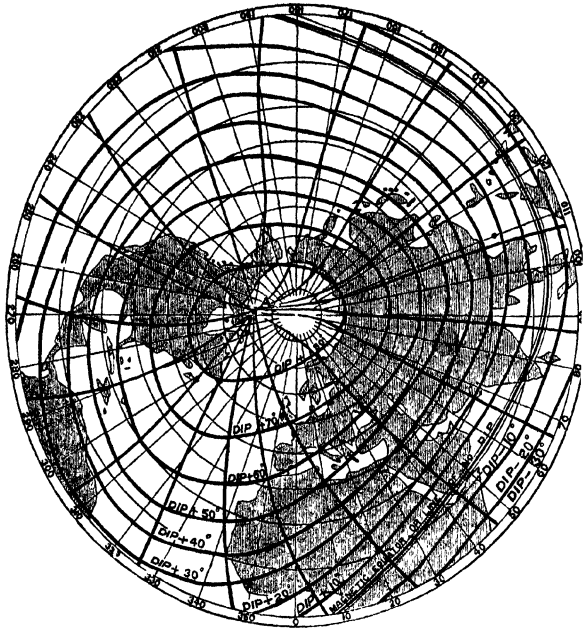


Fig. 74.—Northern Hemisphere.

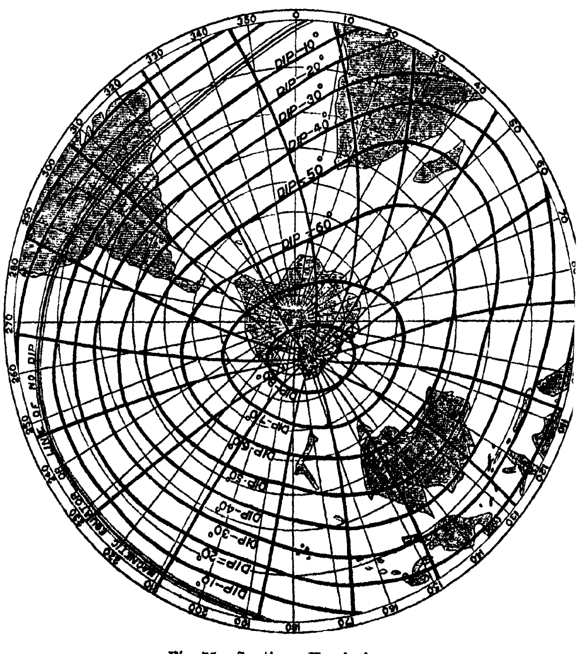


Fig. 75.—Southern Hemisphere.

MAGNETIC MERIDIANS AND LINES OF EQUAL DIP.

Both these instruments have mirrors attached to the magnet, which produce a photographic record of the movements of the magnet, on principles above explained.

The moment of a magnet varies with temperature, being diminished by something like one ten-thousandth part of itself for each degree Fahr. of increase, and increasing again at the same rate when the temperature falls. Hence magnetic observatories must be kept at a nearly uniform temperature. They must also be completely free from iron. No iron nails are allowed to be used in their construction, copper being employed instead.

121. Results of Observation.—Figs. 74, 75[1] contain an approximate representation of the magnetic meridians and lines of equal dip over both hemispheres of the earth. These two systems of lines combined, furnish a complete specification of the *direction* of magnetic force at all parts of the earth's surface; but they indicate nothing as to *intensity*. The curves of equal total intensity have a general resemblance to the lines of equal dip, the intensity being greatest near the poles, and least near the equator, but their arrangement is somewhat more complicated, there being two north poles of greatest intensity, one in Canada, and the other in the northern part of Siberia. Speaking roughly, the intensity near the poles is about double of the intensity near the equator. Curves of equal total intensity are often called *isodynamic* lines; curves of equal dip are often called *isoclinic* lines; curves of equal declination are often called *isogonic* lines; curves cutting the magnetic meridians at right angles are often called *magnetic parallels*. They are the lines which would be traced by continually travelling in the direction of magnetic east or west.

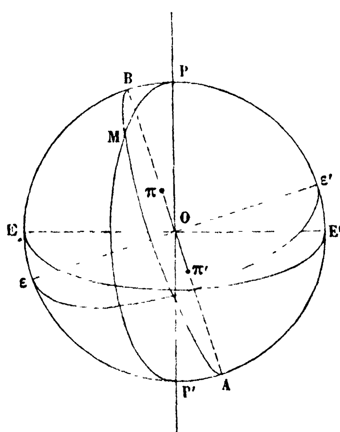


Fig 76 —Biot s Hypothesis.

122. The Earth as a Magnet.—The intensity and direction of terrestrial magnetic force at different places may be *roughly* represented by supposing that there is a magnet $\pi\pi'$ (Fig. 76) at the earth's centre, having a length very small in compari-

[1] For Figs. 69, 71, 72, 74, 75 we are indebted to the publishers of Airy's *Treatise on Magnetism.*

son with the earth's radius, and making an angle of about 20° with the earth's axis of rotation. The points A and B obtained by producing this magnet longitudinally to meet the surface, would be the magnetic poles, and at any other place the magnetic meridian would be the vertical plane containing the magnetic axis A B. At places situated on the great circle whose plane contains both the axis of rotation and the magnetic axis, the magnetic meridian would coincide with the geographical meridian, and the declination would be zero. At any other place M, the two meridians would cut each other at an angle which would be the angle of declination. At all places on the great circle $\epsilon \epsilon'$ whose plane is perpendicular to the magnetic axis, a needle suspended at its centre of gravity would place itself parallel to this axis, and consequently the dip would be zero. This circle would be the magnetic equator.[1] It would cut the geographical equator at an angle of 20°. Proceeding from the magnetic equator towards the north magnetic pole B, the needle would dip more and more, until at B it became vertical. A declination needle at B would remain indifferently in all positions. Similar phenomena would be observed at the other magnetic pole A. The end of the needle which would dip at B, and which at other parts of the earth would point to magnetic north, is that which is similar to the southern pole π' of the terrestrial magnet $\pi \pi'$, and the pole which is similar to π would dip at A.

The supposition of such a central magnet is known as *Biot's hypothesis*. It leads to the same results as the supposition that the earth is a uniformly magnetized sphere. For if we have a sphere built up of a number of equal and similar small magnets with their poles pointing the same way, we may suppose all the imaginary fluid at their northern ends to be collected at one central point, and all the imaginary fluid at their southern ends at another central point, the distance between these two points being equal to the common length of the small magnets. Hence the small central magnet will have the same moment as the uniformly magnetized earth.

The actual phenomena of terrestrial magnetism are much more irregular than the results to which this hypothesis leads. It would

[1] If latitude reckoned from the magnetic equator be called magnetic latitude, and denoted by λ, it can be shown that we should have, on this theory,

$$\tan \delta = 2 \tan \lambda ; \ I = E \sqrt{\cos^2 \lambda + 4 \sin^2 \lambda},$$

E denoting the intensity at the magnetic equator.

appear that the earth's magnetism is distributed in a manner not reducible to any simple expression.

123. Secular Changes.—Declination and dip vary greatly not only from place to place, but also from time to time. Thus at the date of the earliest recorded observations at Paris, 1580, the declination was about 11° 30′ E. In 1663 the needle pointed due north and south, so that Paris was on the line of no declination. Since that time the declination has been west, increasing to a maximum of 22° 34′, which it attained in 1814. Since then it has gone on diminishing to the present time, its present value being about 19° W.

As to dip, its amount at Paris has continued to diminish ever since it was first observed in 1671. From 75° it has fallen to 66°, its present value. As its variations since 1863 have been scarcely sensible, it would seem to have now attained a minimum, to be followed by a gradual increase.

124. Periodic and Irregular Fluctuations.—Besides the gradual changes which occur in terrestrial magnetism, both as regards direction and intensity of force, in the course of long periods of time, there are minute fluctuations continually traceable. To a certain extent these are dependent on the varying position of the sun, and, to a much smaller extent, of the moon, with respect to the place of observation; but over and above all regular and periodic changes, there is a large amount of irregular fluctuation, which occasionally becomes so great as to constitute what is called a *magnetic storm*. Magnetic storms "are not connected with thunder-storms, or any other known disturbance of the atmosphere; but they are invariably connected with exhibitions of aurora borealis, and with spontaneous galvanic currents in the ordinary telegraph wires; and this connection is found to be so certain, that, upon remarking the display of one of the three classes of phenomena, we can at once assert

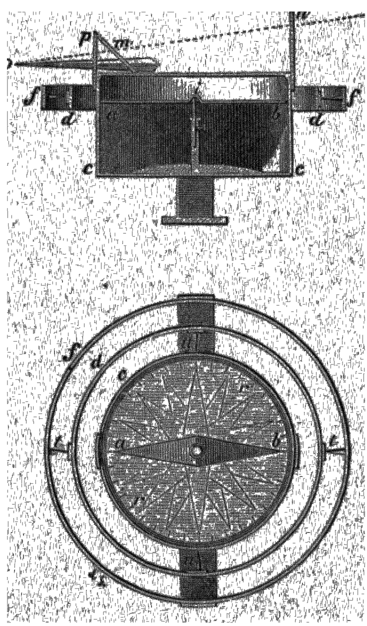


Fig. 77.—Ship's Compass.

that the other two are observable (the aurora borealis sometimes not visible here, but certainly visible in a more northern latitude)."[1] They are sensibly the same at stations many miles apart, for example at Greenwich and Kew, and they affect the direction and amount of horizontal much more than of vertical force.

125. Ship's Compass.—In a ship's compass, the box *c c* (Fig. 77) which contains the needle is weighted below, and hung on gimbals, which consist of two rings so arranged as to admit of motion about two independent horizontal axes *t t*, *u u*, at right angles to each other. This arrangement prevents it from being tilted by the pitching and rolling of the ship. The needle *a b* is firmly attached to the compass card, which is a circular card marked with the 32 points of the compass, as in Fig. 78, and also usually divided at its circumference into 360 degrees. The card with its attached needle is accurately balanced on a point at its centre. The needle, which, in actual use, is concealed from view, lies along the line N S. The box contains a vertical mark in its interior on the side next the ship's bow; and this mark serves as an index for reading off on the card the direction to which the ship's head is turned. Sometimes a reflector is employed, as *m* in the first part of Fig. 77, in such a position that an observer looking in from behind can read off the indicated direction by reflection, and can at the same time sight a distant object whose magnetic bearing is required. The origin of the compass is very obscure. The ancients were aware that the loadstone attracted iron, but were ignorant of its directive property. The instrument came into use in Europe some time in the course of the thirteenth century.

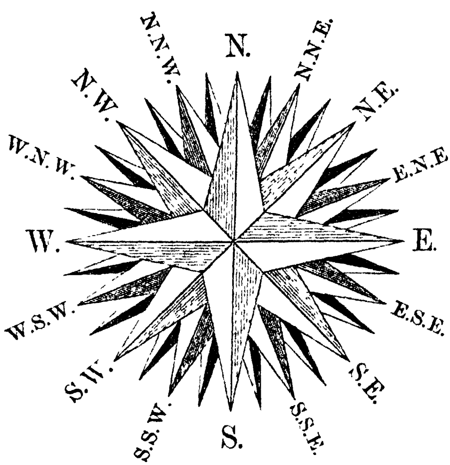


Fig. 78.—Compass Card.

126. Methods of Magnetization.—The usual process of magnetizing a bar consists in rubbing it with or against a bar already magnetized.

[1] Airy on *Magnetism*, p. 204.

Different methods of doing this, called single touch, double touch, &c., have been devised, in which magnetized bars of steel were the magnetizing agents. Much greater power can, however, be obtained by means of electro-magnetism; and the two following methods are now almost exclusively employed by the makers of magnets.

1. A fixed electro-magnet (Fig. 79) is employed, and the bar to be magnetized is drawn in opposite directions over its two poles.

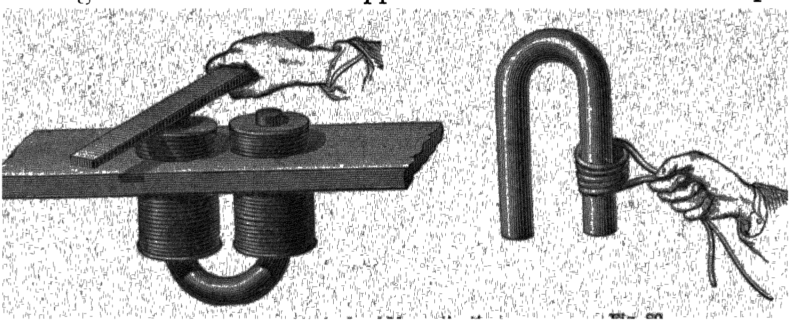


Fig 79 **Methods of Magnetization.** Fig 80.

Each stroke tends to develop at the end of the bar at which the motion ceases, the opposite magnetism to that of the pole which is in contact with it. Hence strokes in opposite directions over the two contrary poles tend to magnetize the bar the same way.

2. When very intense magnetization is to be produced, the electro-magnet must be very powerful, and the bar then adheres to it so strongly that the operation above described, becomes difficult of execution, besides scratching the bar. Hence it is more convenient to move along the bar, as in Fig 80, a coil of wire through which a current is passing This was the method employed by Arago and Ampère.

A bar of steel is said to be magnetized to *saturation*, when its magnetization is as intense as it is able to retain without sensible loss It is possible, by means of a powerful magnet, to magnetize a bar considerably above saturation, but in this case it rapidly loses intensity

Pieces of iron and steel frequently become magnetized temporarily or permanently by the influence of the earth's magnetism and this action is the more powerful as the direction of their length more nearly coincides with that of the dipping-needle. If fire-irons which have usually stood in a nearly vertical position be examined by their

influence on a needle, they will generally be found to have acquired some permanent magnetism, the lower end being that which seeks the north.

It sometimes happens that, either from some peculiarity in the structure of a bar, or from some irregularity in the magnetizing

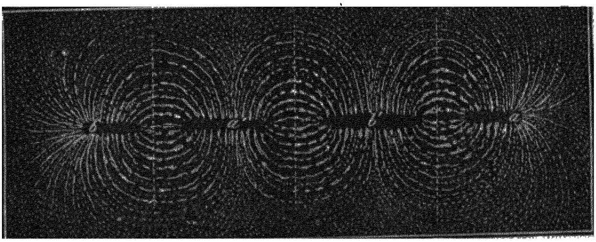


Fig 81.—Consequent Points.

process, a reversal of the direction of magnetization occurs in some part or parts of the length as compared with the rest. In this case the magnet will have not only a pole at each end, but also a pole at each point where the reversal occurs. These intermediate poles are called *consequent points*. Fig. 81 represents the arrangement of iron-filings about a bar-magnet which has two consequent points $a' b'$. The whole bar may be regarded as consisting of three magnets laid end to end, the ends which are in contact being similar poles. Thus the two poles at a' and the one pole at a are of one kind, while the two poles at b' and the one pole at b are of the opposite kind.

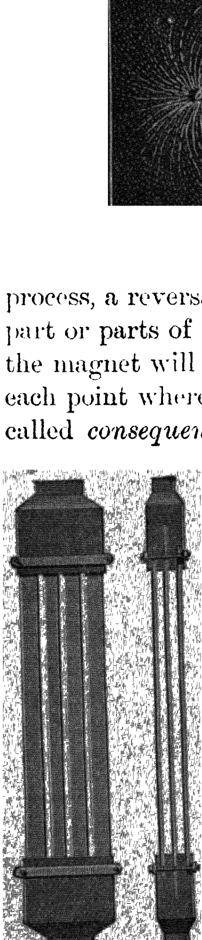


Fig 82.—Compound Magnet.

The lifting power (or *portative* force) of a magnet of given shape and intensity of magnetization is proportional, not to its volume, but its superficial dimensions. Thin bars can be more thoroughly magnetized than thick ones; and hence it has been found advantageous to construct compound magnets, consisting of a number of thin bars laid side by side, with their similar poles all pointing the same way. Fig. 82 represents such a compound magnet composed of twelve elementary bars, arranged 4×3. Their ends are inserted in masses of soft iron, the extremities of which constitute the poles of the system

Fig 83 represents a compound horse-shoe magnet, whose poles N and S support a keeper of soft iron, from which is hung a bucket for holding weights By continually adding fresh weights day after day, the magnet may be made to carry a much greater load than it could have supported originally; but if the keeper is torn away from the magnet, the additional power is instantly lost, and the magnet is only able to sustain its original load.

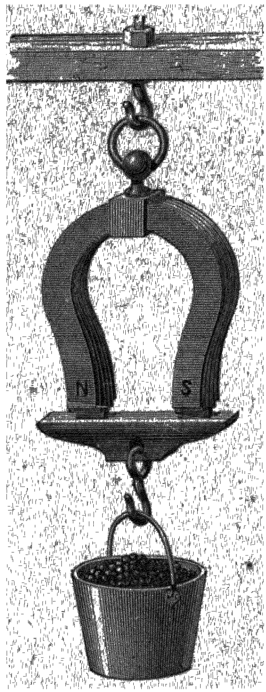


Fig 83.—Compound Horse-shoe Magnet

Much attention was at one time given to methods of obtaining steel magnets of great power. These researches have now been superseded by electro-magnetism, which affords the means of obtaining temporary magnets of almost any power we please.

127. Molecular Changes accompanying Magnetization.—Joule has shown that, when a bar of iron is magnetized longitudinally, it acquires a slight increase of length, compensated, however, by transverse contraction, so that its volume undergoes no change.

If the magnetization is effected suddenly, by completing an electric circuit, an ear close to the bar hears a clink, and another clink is heard when the current is stopped.

These phenomena have been accounted for by the hypothesis that, when iron is magnetized, its molecules place their longest dimensions in the direction of magnetization.

The effect of heat in diminishing the strength of a magnet is another instance of the connection between magnetism and other molecular conditions In ordinary cases, this diminution is merely transient; but if a steel magnet is raised to a white-heat, it is permanently demagnetized.

128. Action of Magnetism on all Bodies.—It has long been known that iron and steel are not the only substances which can be acted on by magnetism. Nickel and cobalt especially were known to be attracted by a magnet, though very much more feebly than iron, while bismuth and antimony were repelled. Faraday, by means of a powerful electro-magnet, showed that all or nearly all sub-

stances in nature, whether solid, liquid, or gaseous, were susceptible of magnetic influence, and that they could all be arranged in one or the other of two classes, characterized by opposite qualities. This opposition of quality is manifested in two ways.

1. As regards attraction and repulsion, iron and other *paramagnetic* bodies are attracted by either pole of a magnet, or more generally they tend to move from places of weaker to places of stronger force. On the other hand, bismuth and other *diamagnetic* bodies are repelled by either pole of a magnet, and in general tend to move from places of stronger to places of weaker force.[1]

2. As regards orientation, a paramagnetic[2] body, when suspended between the poles of a magnet, tends to set *axially;* that is to say, tends to place its length along the line joining the poles; whereas a diamagnetic body tends to set *equatorially,* that is, to place its length at right angles to the line joining the poles.

The fundamental difference is, that a piece of bismuth (or other diamagnetic substance) when it becomes a temporary magnet from the inductive influence of the field in which it is placed, has its poles opposite, end for end, to those of a piece of iron (or other paramagnetic substance) similarly placed. From this reversal of the poles, it follows that the resultant force upon the bismuth is opposite in direction to the resultant force upon the iron; and as the iron is urged from places of weaker to places of stronger force, the bismuth is urged from stronger to weaker. This is the cause of the equatorial setting of a diamagnetic bar when suspended between the poles of a magnet. It is merely a result of the tendency of the particles to move outwards into the regions of weaker magnetic action. In a uniform field, with parallel lines of force, the equatorial setting would not occur.

The axial setting of an iron bar between the poles of a magnet is jointly due to two causes, one being the tendency of its ends to move to places of stronger force, while the other cause, which we will now proceed to explain, tends to produce axial setting even in a uniform field.

[1] The motion of iron from weaker to stronger parts of the field is roughly explained in the second foot-note to § 109. The same explanation serves for bismuth if we substitute "push" for "pull" and "convex" for "concave."

[2] The nomenclature here adopted was proposed by Faraday in 1850 (*Researches*, § 2790), and is eminently worthy of acceptance. Many writers, however, continue to employ *magnetic* in the exclusive sense of *para*magnetic. To be consistent, they should call the other class *anti*magnetic, not *dia*magnetic. "The word *magnetic* ought to be general, and include *all* the phenomena and effects produced by the power."

129. Reason of Setting in a Uniform Field.—Suppose a row of iron balls placed axially, as in Fig. 84, either between the poles of a magnet, or along a line of force in uniform field; the force of the field being such as to urge a north pole from left to right. Each ball will, by induction, become a magnet with its north pole to the right, and the force which each ball experiences from its neighbours will be in the same direction as the force of the field. The mutual action of the balls, therefore, increases the induction due to the field.

Fig 84.

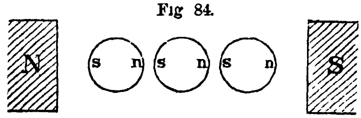
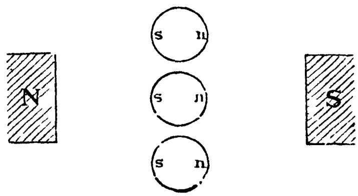


Fig 85



Next, suppose a row of iron balls placed equatorially in the same field (Fig. 85). The north pole of each ball will be attracted to the left by the south poles of its neighbours, and the induction due to the field will therefore be diminished.

Fig. 86

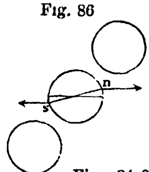
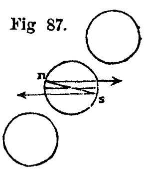


Fig 87.



Figs. 84–87 —Reason of Setting in Uniform Field.

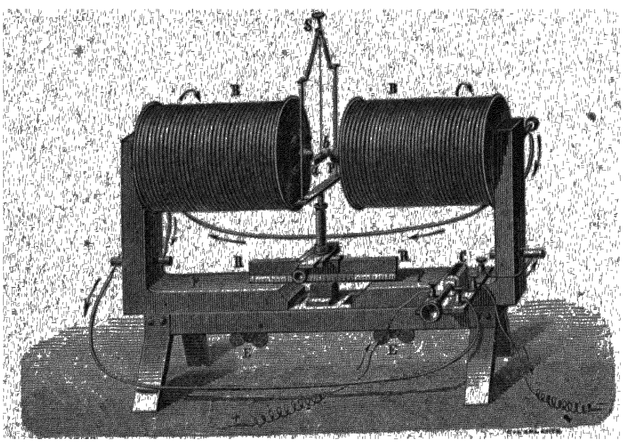
Thirdly, let a row of iron balls be placed in a line inclined to the lines of force (Fig. 86.) The force of the field can be resolved into two components, one (which we shall call the longitudinal component) along the line of balls, and the other (which we shall call the transverse component) normal to the line of balls. Mutual induction will, as above shown, augment the longitudinal and diminish the transverse component. It will, therefore, alter the direction of the total induction, so as to make it more nearly longitudinal. The poles of any one of the balls will therefore have such positions as are shown at n and s in the figure, n being above and s below the horizontal line through the centre.

In estimating the forces which tend to turn the row of balls as a whole when they are rigidly connected together, we must remember that mutual actions between different parts of a rigid body do not tend to move the body as a whole. Such motion can only be produced by forces from without, that is, in the present case, by the original force of the field, which urges north poles from left to right, and south poles from right to left. The forces on each ball will constitute a couple as shown by the two arrows in the

figure, and these couples tend to turn the body into the axial position.

If we apply similar reasoning to a row of balls of bismuth, we shall find that mutual induction diminishes the longitudinal component, increases the transverse component, and in the case of the oblique row, gives the poles of each ball positions such as are shown in Fig. 87. The couple due to the external forces of the field is represented by the two arrows in the figure, and tends (just as in the case of iron) to turn the body into the axial position. This directive action is, however, excessively feeble, the forces due to mutual induction in bismuth being insensible in comparison with the external forces of the field.

130. Experimental Arrangements. Faraday's Apparatus.— Fig. 88 represents the apparatus commonly employed for experiments on



this subject. B, B are two large coils of stout copper wire, wound on massive hollow cylinders of soft iron. The latter form portions of the heavy frames F, F, which can be slid to or from each other, and fixed firmly at any distance by means of the screws E, E. The pole pieces P, which can be screwed on or off, have the form of round[illegible] and produce a [illegible]oncentration of force at their extremities.

The action of magnetism upon a solid can be examined by suspending a small bar of it $a\,b$ by means of a special support R S, between the poles P. When a [illegible]ed through the coils, the bar

immediately exhibits a preference either for the axial or the equatorial position. Attraction and repulsion are most easily tested by suspending a small ball of the substance at the level of the central line of poles, but a little beside it, the poles having first been brought very near together. On passing the current through the coil, the ball will move inwards towards the line of poles if paramagnetic, and outwards if diamagnetic.

It is important, however, to remark, that experiments of this kind, unless performed *in vacuo*, are merely differential—they merely indicate that the suspended body is, in the one case, more paramagnetic or less diamagnetic, in the other case more diamagnetic or less paramagnetic, than the medium in which it moves, the comparison being made between equal volumes. Oxygen is paramagnetic, and nitrogen is nearly or quite indifferent. Air is accordingly paramagnetic, and a body suspended in air appears less paramagnetic or more diamagnetic than it really is. If more feebly paramagnetic than air, it will appear to be diamagnetic. Thus heated air, in consequence probably of its rarefaction, appears diamagnetic when surrounded by cold air, and the flame of a taper is repelled downwards and outwards from the axial line.

If, on the other hand, the body under examination is suspended in water, it will appear more paramagnetic than it really is, by reason of the diamagnetism of water.

The following metals are paramagnetic: iron, nickel, cobalt, manganese, chromium, titanium, cerium, paladium, platinum. osmium.

The following are diamagnetic: bismuth, antimony, lead, tin, mercury, gold, silver, zinc, copper.

The following substances are also diamagnetic: water, alcohol, flint, glass, phosphorus, sulphur, resin, wax, sugar, starch, wood, ivory, beef (whether fresh or dried), blood (whether fresh or dried), leather, apple, bread.

131. **Magneto-crystallic Action.**—The orientation of crystals in a magnetic field presents some remarkable peculiarities which were extremely perplexing to investigators until Tyndall and Knoblauch discovered the principle on which they depend. This principle is, that crystals are susceptible of magnetic induction to different degrees in different directions. Every crystal (except those belonging to the cubic system) has either one line or one plane along which induction takes place more powerfully than in any other direction; and it is this line or plane which tends to place itself axially or

equatorially according as the crystal is paramagnetic or diamagnetic. The directions of most powerful and least powerful induction are found to be closely related to the optic axes of crystals, and also to their planes of cleavage. When a sphere cut from a crystal is brought near to one pole of a magnet, it is attracted or repelled (according as it is para- or dia-magnetic) with the greatest force when the direction of most powerful induction coincides with the direction of the force.

Directions of unequal induction can be produced artificially in non-crystalline substances by applying pressure. "Bismuth is a brittle metal, and can readily be reduced to a fine powder in a mortar. Let a tea-spoonful of the powdered metal be wetted with gum-water, kneaded into a paste, and made into a little roll, say an inch long and a quarter of an inch across. Hung between the excited poles, it will set itself like a little bar of bismuth—equatorial. Place the roll, protected by bits of pasteboard, within the jaws of a vice, squeeze it flat, and suspend the plate thus formed between the poles. On exciting the magnet, the plate will turn, with the energy of a magnetic substance, into the axial position, though its length may be ten times its breadth.

"Pound a piece of carbonate of iron into fine powder, and form it into a roll in the manner described. Hung between the excited poles, it will stand as an ordinary [para]magnetic substance—axial. Squeeze it in the vice, and suspend it edgeways, its position will be immediately reversed. On the development of the magnetic force, the plate thus formed will recoil from the poles *as if violently repelled*, and take up the equatorial position."[1]

In these experiments the direction of most powerful induction is a line transverse to the thickness, and this is also the direction in which pressure has been applied. Tyndall accordingly concludes that "if the arrangement of the component particles of any body be such as to present different degrees of proximity, in different directions, then the line of closest proximity, other circumstances being equal, will be that chosen by the respective forces for the exhibition of their greatest energy. If the mass be [para]magnetic, this line will stand axial; if diamagnetic, equatorial."[2]

[The account of Magnetic Units in the Appendix should be read before commencing the next chapter.]

[1] Tyndall on *Diamagnetism*, p. 18. [2] Ibid. p. 23.

magnetization and the normal to the surface. Our conventions as to sign are:—

North-seeking magnetism is positive.

The direction of the magnetic force H is the direction in which a north-seeking pole would be urged.

The direction of the magnetization I is the direction in which the north-seeking end of the elementary magnet points.

Accordingly, the surface density will be positive if the direction of magnetization makes an acute angle with the outward-drawn normal.

134. Volume-Density of Free Magnetism.—When the magnetization is parallel to a fixed direction, the relation between the volume-density of free magnetism in the interior and the magnetization can be investigated as follows:—

Consider a long narrow prismatic filament in the direction of the magnetization. Let a be its cross section, and let it be divided into elementary prisms of common length l. Regard each element as uniformly magnetized with the magnetization actually existing at its centre. This will give a surface-density $+\mathrm{I}$ at the forward end, and $-\mathrm{I}$ at the backward end of the element. For the consecutive element on the forward side, the surface-density will be $\mathrm{I}+\frac{d\mathrm{I}}{ds}l$ on the forward end, and $-\mathrm{I}-\frac{d\mathrm{I}}{ds}l$ on the backward end, s denoting distance measured in the forward direction. At the junction of the two elements we have to combine the two surface-densities I and $-\mathrm{I}-\frac{d\mathrm{I}}{ds}l$, and the resulting surface-density is $-\frac{d\mathrm{I}}{ds}l$. Thus we have a quantity of free magnetism $-\frac{d\mathrm{I}}{ds}la$, which we may regard as belonging to a volume la, consisting of the second half of the first element and the first half of the second. Thus the expression for the volume-density is $-\frac{d\mathrm{I}}{ds}$.

135.—Abolishing the restriction as to parallelism of magnetization, the student should verify for himself, by dividing the substance into elementary prisms by three sets of planes parallel to rectangular co-ordinate planes, and applying reasoning similar to the above, that the volume-density is $-\frac{d\mathrm{A}}{dx}-\frac{d\mathrm{B}}{dy}-\frac{d\mathrm{C}}{dz}$, A, B, C denoting the components of I parallel to the axes of x, y, z.

When the magnetization is such that the volume-density at all

internal points is zero, it is said to be *solenoidal*.[1] A solenoidal magnet may either be ring-shaped and have no free magnetism anywhere, or it may have free magnetism residing on some portions of its surface.

136. Force in a Crevasse; H and B.—When a piece of iron is in a magnetic field, the magnetic force at the centre of a small cavity in the iron will depend partly on the shape and direction of the cavity. It will be a maximum for a narrow cavity with plane parallel sides perpendicular to the direction of magnetization of the iron, and will be a minimum for a similar cavity with its sides parallel to the direction of magnetization. Such cavities are briefly designated a *transverse crevasse* and a *longitudinal crevasse.*

The surface density on the sides of a transverse crevasse is I, and the repulsion of one surface layer and attraction of the other are both in the same direction as the magnetizing force to which I is due. Each of them amounts, by a well-known theorem in attractions, to $2\pi I$. Hence if we denote the magnetizing force by H, and the force in the transverse crevasse by B, we have

$$B = H + 4\pi I,$$

whereas in a longitudinal crevasse the force is simply H. The modern name for B is *induction*, or more fully "induction at a point," or "intensity of induction." We shall call H the *magnetizing force*. It is often called the *magnetic force*. In non-magnetic matter, such as air or copper, B and H are the same, and I is zero.

When we speak of the values of B and H at a point in any medium, we mean the forces which would exist in crevasses excavated at that point. H may be more simply defined as the *force due to the actual free magnetism.*

The ratio of I to H is called magnetic *susceptibility*, and is denoted by the Greek letter κ. The ratio of B to H is called magnetic *permeability*, and is denoted by the Greek letter μ. The above equation for the value of B in terms of H and I may accordingly be written

$$\mu = 1 + 4\pi\kappa.$$

The following table gives a good idea of the values of these various quantities for very soft iron. The values will differ considerably for different specimens.

[1] Because in this case lines drawn in the direction of magnetization are analogous to lines of steady flow in a liquid, and the magnet can be divided into filaments which are analogous to *tubes* of flow. Compare § 52.

H	I	B	κ	μ
0.3	3	41	10	128
1·4	32	413	23	299
2·2	117	1460	53	670
3·5	574	7230	164	2070
4·9	917	11540	187	2350
6·7	1078	13520	161	2020
10·2	1173	14840	115	1450
22·3	1249	15710	56	705
78	1337	16900	17	215
208	1452	18500	7	89
585	1530	19800	2·6	34
24500	1660	45300	0·067	1·9

137.—In passing from the substance of a magnet of any kind into the external air at a part where the direction of magnetization is normal to the surface, there is no abrupt change in B either as regards magnitude or direction. To prove this, we shall consider the case in which H and I in the magnet are directed outwards at the surface in question, and shall regard this outward direction as the positive direction.

On account of the repulsion $2\pi I$ due to the surface layer, which reverses its direction as we pass through the surface, the value of H just outside is equal to the value of $H+4\pi I$ just inside. But H outside is the same as B outside, since I outside is zero; and $H+4\pi I$ inside is B inside. Hence B outside is equal to B inside.

138.—The student may verify for himself the more general statement that in passing out of a magnet into air at any point of the surface (which will generally be oblique to the direction of magnetization) there is *no abrupt change in the normal component of B*. This property, as we shall see hereafter, is of great importance.

139. Application to Steel Magnet.—In an ordinary steel bar-magnet, although the direction of the magnetization is everywhere from the negative to the positive (that is north-seeking) end, the magnetizing force (which is due to the free magnetism of the bar) is in the opposite direction. A compass needle held over the bar points in the opposite direction to the bar itself, and it would do the same if it could be placed in a longitudinal crevasse inside the bar. The lines of magnetizing (or magnetic) force both inside and outside the bar run from the positive to the negative end. The lines of induction, on the other hand, are closed curves running outside from positive to negative, and returning inside from negative to positive.

140. Self-Demagnetizing Force.—The opposition of direction between magnetization and magnetizing force is the chief cause of the

tendency of steel magnets to get weaker with time; and it does not exist in a ring, such as would be formed if a uniformly magnetized bar could be bent into a circle and its ends joined. There is no free magnetism in a ring thus circularly magnetized, and though the force in a transverse crevasse may be very strong, the force in a longitudinal crevasse would be evanescent. It is accordingly found that rings possess much greater retentive power than bars or open horse-shoes. The soft iron keeper which is usually kept in contact with the two ends serves to convert the horse-shoe into a temporary ring, cancelling most of the free magnetism on the terminal surfaces by the development of opposite magnetism at the surfaces of the soft iron, and thus checking the self-demagnetizing tendency.

141.—A demagnetizing tendency also exists in soft iron temporarily magnetized. When a long cylinder of iron is brought into a uniform magnetic field and placed with its axis parallel to the lines of the field, the free magnetism called out at either end exerts upon the adjoining parts a force opposite to that of the undisturbed field, but not sufficiently strong to reverse it.

The difference between H just inside and H just outside is $4\pi I$, and this difference is not sufficient to change the sign of H; hence I at the ends is numerically smaller than the fraction $1/4\pi$ of either the internal or the external H. In the middle of the cylinder, I may be some hundreds of times greater than at the ends. As the length of the cylinder is increased the intensity of magnetization in the middle is increased also. The demagnetizing tendency is nevertheless sensible even in the middle portions of long cylinders. The form which is most amenable to calculation is the ellipsoid, and Ewing calculates that for an ellipsoid of very soft iron 200 times as long as it is broad, in a field of moderate strength, the demagnetizing force in the centre is half the undisturbed force of the field, so that the value of H in the centre is only half the value of H in very distant parts of the field.

142. Small Magnet at Earth's Centre.—The following is the working out of Biot's hypothesis described in § 122.

Let r be the distance of any point at or near the earth's surface from the centre, and θ its angular distance from the earth's north magnetic pole, which is determined by producing the axis of the small magnet through its south-seeking end. The southern end of the small magnet will be north-seeking; let its strength be m, and that of the northern end $-m$, their distance apart being $2l$. The

magnetic potential at the point in question will be $m/(r+l\cos\theta)-m/(r-l\cos\theta)$, which to a first approximation equals $-2lm\cos\theta/r^2$. But $2lm$ is the moment of the small magnet. Denoting this by M, and the potential by V, we have $V = -\frac{M}{r^2}\cos\theta$.

The force on a unit north-seeking pole along any short line ds is $-dV/ds$. Vertically downwards, we have $ds = -dr$. Horizontally northwards, we have $ds = -rd\theta$. These values give for the vertical force $\frac{2M}{r^3}\cos\theta$, and for the horizontal force $\frac{M}{r^3}\sin\theta$. Hence the total force is $\frac{M}{r^3}\sqrt{(1+3\cos^2\theta)}$, and tan dip $=2\cot\theta$.

We shall return to the subject of magnetic potentials in a later chapter, in connection with electro-magnetism.

143.—The vibrations of a horizontally balanced magnet under the action of the earth's horizontal magnetic force are closely analogous to the vibrations of a pendulum under the action of gravity. In both cases the couple acting on the body is proportional to $\sin\theta$, where θ denotes the deviation from the position of equilibrium. In the case of the pendulum this couple is $mgh\sin\theta$, m denoting the mass of the pendulum, and h the distance of the centre of gravity from the point of suspension. In the case of the magnet it is $MH\sin\theta$, M denoting the magnetic moment of the magnet. The angular acceleration in both cases is the quotient of this couple by the moment of inertia about the point of suspension, which we shall denote by mk^2. For the magnet this acceleration will be $\frac{MH}{mk^2}\sin\theta$, and for the pendulum it is $\frac{mgh\sin\theta}{mk^2}$ or $\frac{gh}{k^2}\sin\theta$ or $\frac{g}{l}\sin\theta$, l denoting the length of the equivalent simple pendulum. Thus, the time of one to-and-fro vibration, which is $2\pi\sqrt{\frac{l}{g}}$ for the pendulum, will be $2\pi\sqrt{\frac{mk^2}{MH}}$ for the magnet.

144.—To determine experimentally the value of H at a given time and place, two observations must be made. One of them consists in timing the horizontal vibrations of a hard steel magnet, and the other consists in observing the deflection which this magnet produces in a magnetic needle.

In the deflection observation the deflecting magnet is either placed "end on" or "broadside on."

In the former case it is magnetic east or west of the centre of the needle, and points directly towards it. Let r be the distance from centre to centre, and $2a$ the distance between the poles of the

magnet, the strength of each pole being P. Then the force at the centre of the needle due to the magnet is $\frac{P}{(r-a)^2} - \frac{P}{(r+a)^2} = \frac{4Pra}{(r^2-a^2)^2}$ $= \frac{4Pa}{r^3}$ nearly, since a is small compared with r.

But $2Pa$ is the moment of the magnet; denoting this by M, the force is $\frac{2M}{r^3}$, and its direction is perpendicular to the earth's force, H. If we regard the field as uniform, the tangent of the deflection will be the ratio of these two forces, say

$$\tan\delta = \frac{2M}{Hr^3}.$$

By combining this equation with $T^2 = 4\pi^2 \frac{mk^2}{MH}$, T denoting the time of a complete vibration, we find

$$H^2 = \frac{8\pi^2}{T^2} \frac{mk^2}{r^3 \tan\delta}.$$

In the "broadside on" position, the line of centres is magnetic north and south, and the deflecting magnet points magnetic east and west. With the same notation as above, the force at the centre of the needle due to either pole is approximately $\frac{P}{r^2}$, and its east and west component is approximately $\frac{a}{r}$ of this, or is $\frac{Pa}{r^3}$. This is in the same direction for both poles of the magnet, hence the total east and west force is $\frac{2Pa}{r^3}$ or $\frac{M}{r^3}$. This is the whole disturbing force, for the north and south components destroy one another. Accordingly we have now

$$\tan\delta = \frac{M}{Hr^3}, \text{ and } H^2 = \frac{4\pi^2}{T^2} \frac{mk^2}{r^3 \tan\delta}.$$

To eliminate errors arising from want of symmetry in the magnetization, the position of the disturbing magnet should be reversed in three ways, viz. by turning it upside down, by turning it end for end, and by shifting it to the opposite side of the needle, keeping the distance r unchanged. This makes 8 positions in all, and the mean of the 8 deflections obtained is to be adopted. These remarks apply alike to the "end on" and the "broadside on" positions.

The above formulæ may be regarded as the first approximations; for closer approximations we must refer to fuller treatises.

As regards the moment of inertia, mk^2, if the magnet is a rectangular bar, k^2 is one-twelfth of the square of the diagonal of its horizontal face. This diagonal and the distance r should be ex-

pressed in centimetres, and the mass m in grammes, the time T being in seconds. The value of H will thus be obtained in C.G.S. measure. Its average value for the British Isles is about ·17.

The observations must be made at a distance from all other magnets and from all large masses of iron.

145. Field Observations.—We shall conclude this chapter with an account of the methods of observation usually employed in a magnetic survey. The measurement of declination is made in the following manner:—

A "magnetic theodolite" is employed, having a tripod stand and a horizontal divided circle like an ordinary theodolite; but its telescope, instead of occupying a central position, is supported on a projecting arm, movable round the circumference, and is directed towards the centre of the instrument. In the centre hangs the magnet, suspended by a bundle of silk fibres and suitably protected from wind. This magnet is a hollow steel cylinder, having an achromatic lens at the near end, and a glass scale in its focus at the other. Thus the observer sees in the telescope, when properly placed, the divisions of the glass scale.

The line joining the centre of the lens to the middle division of the scale is called the line of collimation of the magnet.

The telescope is moved along the divided circle till its line of collimation coincides with that of the magnet; in other words, till its wire is seen in coincidence with the middle point of the scale. The magnet is then turned upside down and the observation repeated. The mean of the two readings will be the reading when the telescope is pointed to magnetic south. The reading which corresponds to true south is then determined by observing the sun's azimuth as indicated on the horizontal circle at a noted time. For making this observation a mirror is provided, which travels round the instrument on the opposite side from the telescope, and can be turned about a horizontal axis which is at right angles to the telescope, when it reflects the sun's rays into the telescope, the telescope is in the same vertical plane with the sun. Two observations are made, in one of which the sun is in front of the telescope, and in the other behind it.

146.—The same theodolite serves for the observations of horizontal force; the deflecting magnet (which is usually the same magnet that was employed in the observation of declination) being supported on a lateral bar attached for the purpose, and the deflection of a

second and smaller magnet of the same construction being observed in a telescope, which is moved round the divided circle till its wire coincides with the deflected position of the line of collimation of this small magnet. In a complete series of observations, the deflecting magnet is placed successively at two certain distances from the suspended magnet, on each side of it, and in each position two deflections are found—one with the N. end pointing E., and one with the N. end pointing W. The vibration observation is made by dismounting the small magnet from the centre of the instrument, and suspending the deflecting magnet in its place.

147.—For observing dip, the theodolite is dismounted, and in its place is erected an instrument with two microscopes and verniers, travelling round a vertical circle, as in Fig. 71.

The plane of the vertical circle is first made approximately perpendicular to the meridian, by setting the lower microscope and vernier to 90° (the lowest point of the circle), and turning the instrument in azimuth till the lower end of the needle is bisected by the wire of the microscope. A reading of the horizontal circle is taken. The needle is then reversed in its bearings and a second reading taken. A similar adjustment is made with the upper microscope and the upper end of the needle, giving a third and fourth reading of the horizontal circle, which will be nearly the same as the first and second. The instrument is then turned in azimuth through about 180°, and the adjustments are repeated, giving four additional readings, which will differ by about 180° from the first four. If these eight readings are added, and the sum is divided by 8, the quotient will differ by about 90° from each of the readings, and will correspond to the magnetic meridian, as will also another reading differing by 180° from this.

The instrument is now set to one of these two readings, so that the plane of the vertical circle is in the magnetic meridian, the lower microscope is set so that its wire bisects the lower end of the needle, and a first determination of dip is obtained by reading the verniers. A similar adjustment is then made with the upper microscope and the upper end of the needle, and fresh readings are taken, which will not differ much from the preceding. The needle is then reversed in its bearings so as to turn its other face to the microscopes, and the observations are repeated. The instrument is then turned in azimuth through 180°, and similar observations are made in this position. Finally, the needle is dismounted, fixed on

a wooden support provided for the purpose, and stroked with two magnets from the centre to the ends in such a way as to reverse its magnetization, and all the above observations of dip are repeated with the needle in this condition. The mean of all the readings for dip is adopted as the true dip.

CURRENT ELECTRICITY.

CHAPTER XIII.

GALVANIC BATTERY.

148. Voltaic Electricity.—Towards the close of last century, when the discovery of the various phenomena of frictional electricity had been followed by Coulomb's investigations, which first reduced them to an accurate theory, a new instrument was brought to light destined to effect a complete revolution in electrical science. In place of an element difficult to manage, capricious and uncertain in its behaviour, and constantly baffling investigation by the rapidity of its dissipation, the galvanic battery furnished a steady source of electricity, constantly available in all weathers, and requiring no special precautions to prevent its escape. Moreover, the electricity thus developed exhibited an entirely new set of phenomena, and opened up the way to such various and important applications, that frictional electricity at once fell into the second place, and the new agent became the main object of interest with all electrical investigators.

149. Galvanic Element.—If two plates, one of zinc and the other of copper (Fig. 89), are immersed in water acidulated by the addition of sulphuric acid, and are not allowed to touch each other within the acid, but are connected outside it, either by direct contact, or by a metallic wire M and binding screws, as in the figure, a continuous current of electricity flows round the circuit thus formed, the direction of the positive current being from copper to zinc in the portion external to the liquid, and from zinc to copper through the liquid. Chemical action at the same time takes place, the zinc being gradually dissolved by the acid, and hydrogen being given out at the copper plate.

If, instead of employing two metals and a liquid, we form a circuit with any number of metals alone, no current will be gene-

rated, provided that the whole circuit be kept at one temperature. If, however, some of the junctions be kept hot and others cold, a current will in general be produced.

To explain these phenomena it is necessary to suppose that there is an abrupt difference of potential in crossing one or more of the surfaces of junction of dissimilar substances; and this seems to imply the presence of two layers of opposite electricities facing each other at a very small distance. It can be shown that two layers, of surface density ρ, at distance l, would give a difference of potential $4\pi\rho l$. There has been much discussion of late years as to whether the abrupt change occurs at the junction of the zinc and copper, or at the junction of the zinc and the liquid.

If the latter view is adopted, we may suppose that the chemical action which takes place at this surface involves a continual generation of equal and opposite electricities, which refuse to combine across the junction at which the chemical action occurs, but are able to combine by flowing round in opposite directions through the circuit. On this theory the zinc is negative relative to the liquid, in other words the potential falls abruptly as we cross the boundary from the liquid to the zinc.

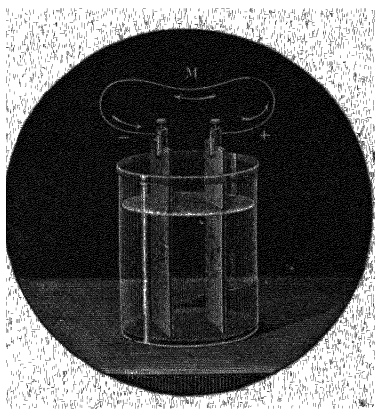


Fig 89 —Voltaic Element

On the opposite theory, zinc and copper in direct contact are supposed to have an abrupt difference of potential, zinc being the higher; but when these metals are immersed in an acid solution, without being connected in any other way, they are supposed to have the same or nearly the same potential. Accordingly when the circuit of the cell is completed, the highest and lowest potentials are those of the zinc and copper respectively at their junction.

Whichever theory is held, it is agreed that for keeping up the current there must be a continual running down of the potential energy of chemical affinity by the formation of chemical compounds, this energy being the source of the energy of the current.

Every electric current may be regarded as a flow of positive electricity in one direction, and of negative electricity in the opposite

direction. The direction in which the positive electricity flows is always spoken of as the *direction of the current.*

150. Galvanic Battery.—By connecting the plates of successive elements in the manner represented in Fig. 90, we obtain a battery. The copper of the first cell on the left hand is connected with the

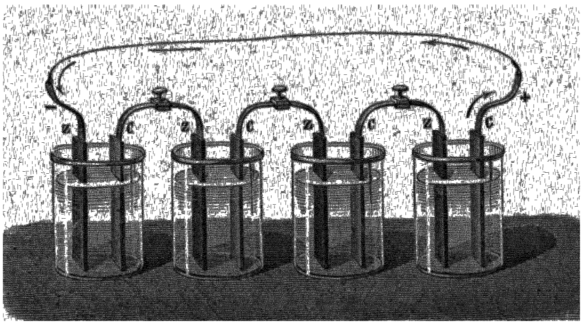


Fig. 90 —Battery of Four Elements

zinc of the second; the copper of the second with the zinc of the third; and so on to the end of the series.

If two wires of the same metal be connected, one with the first zinc and the other with the last copper, the difference of potential between these wires is independent of the particular metal of which they are composed, and is called the *electro-motive force* of the battery. Its amount can be measured by means of Thomson's quadrant electrometer; and in applying this test, it is not necessary that the wires which connect the battery with the electrometer should be of the same metal. Whatever metals these wires may be composed of, the quadrants of the electrometer will assume the same potentials as if in direct contact with the plates of the battery.[1]

The zinc of the first and the copper of the last cell (or wires proceeding from them) are called the *electrodes* or *poles* of the battery, the zinc being the negative and the copper the positive electrode. The current flows through the connecting wire from the positive to the negative electrode, and is forced through the battery from the negative to the positive.

151. Galvani's Discoveries.—About the year 1780, Galvani, professor

[1] The difference of potentials of two metals must be the same whether they are in direct contact or are connected by a third metal; otherwise a circuit of the three metals would give a current, which is contrary to experimental fact. The temperature is supposed uniform.

of anatomy at Bologna, had his attention called to the circumstance that some recently skinned frogs, lying on a table near an electrical machine, moved as if alive, on sparks being drawn from the machine. Struck with the apparent connection thus manifested between electricity and vital action, he commenced a series of experiments on the effects of electricity upon the animal system. In the course of these experiments, it so happened that, on one occasion, several dead frogs were hung on an iron balcony by means of copper hooks which were in contact with the lumbar nerves, and the legs of some of them were observed to move convulsively. He succeeded in obtaining a repetition of these movements by placing one of the frogs on a plate of iron, and touching the lumbar nerves with one end of a copper wire, the other end of which was in contact with the iron plate. Another mode of obtaining the result is represented in Fig. 91, two wires of different metals being employed which touch each other at one end, while their other ends touch respectively the lumbar nerves and the crural muscles. Every time the contact is completed, the limb is convulsed.

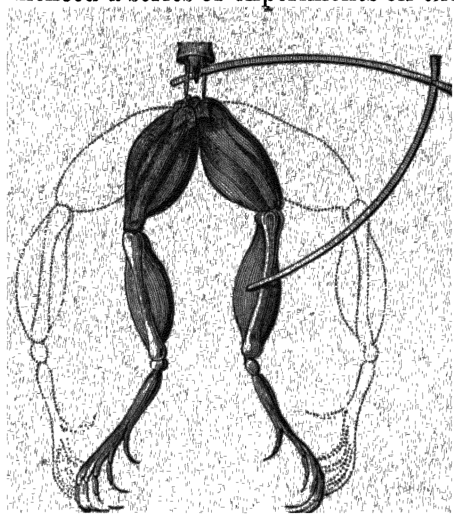


Fig 91. - Experiment with Frog.

Galvani's explanation was, that at the junction of the nerves and muscles there is a separation of the two electricities, the nerve being positively, and the muscle negatively electrified, and that the convulsive movements are due to the establishment of communication between these two electricities by means of the connecting metals.

Volta, professor of physics at Pavia, disproved this explanation by showing that the movements could be produced by merely connecting two parts of a muscle by means of an arc of two metals; and he referred the source of electricity not to the junction of nerve and muscle, but to the junction of the two metals. Acting on this belief, he constructed in the year 1800 a voltaic pile.

152. Voltaic Pile.—This consisted of a series of discs of copper, zinc, and wet cloth, *c*, *z*, *d*, Fig. 92, arranged in uniform order, thus—

copper, zinc, cloth, copper, zinc, cloth . . . the lowest plate of all being copper and the highest zinc. The wet cloth was intended merely to serve as a conductor, and prevent contact between each zinc and the copper above it. All the contacts between zinc and copper were between a copper below and a zinc above, so that they all tended, according to Volta's theory, to produce a current of electricity in the

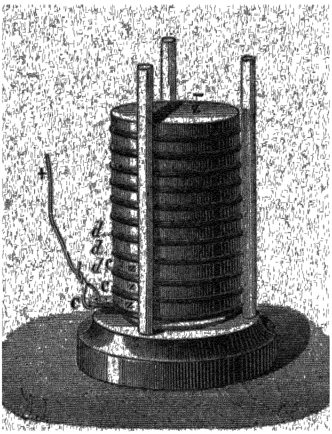


Fig. 92.—Structure of Pile

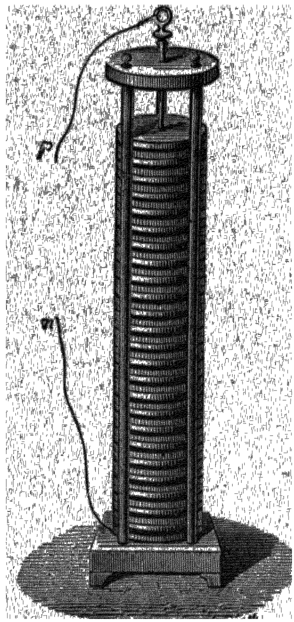


Fig. 93.—Complete Pile

same direction. The effects obtained from the pile were so powerful as to excite extraordinary interest in the scientific world.

A *dry pile,* built up on the general plan of Volta's moist pile, was subsequently devised by De Luc, and improved by Zamboni. In Zamboni's construction, sheets of paper are prepared by pasting finely laminated zinc or tin on one side, and rubbing black oxide of manganese on the other. Discs are punched out of this paper, and piled up into a column, with their similar sides all facing the same way, to the number of a thousand or upwards, and are well pressed together. The difference of potential between the two ends is sufficient to produce sensible divergence of the gold-leaves of an electroscope, but the quantity of electricity which can be developed in a given time is exceedingly small. *No pile or battery can generate a sensible current, except by a sensible consumption of its materials in the shape of chemical action.*

A very delicate gold-leaf electroscope was devised by Bohnen-

berger, consisting of a single leaf suspended between the two poles of a dry pile, which for this purpose is arranged in two columns connected below, so that the poles are at the summits. If their lower ends, which form the middle of the series, be connected with the earth, one pole will always have positive, and the other negative potential. A very slight charge, positive or negative, given to the gold-leaf by means of the knob at the top of the case, suffices to make it move to the negative or the positive pole.

153. Couronne de Tasses.—Volta improved upon his invention of the pile by constructing the *couronne de tasses* (crown of cups),

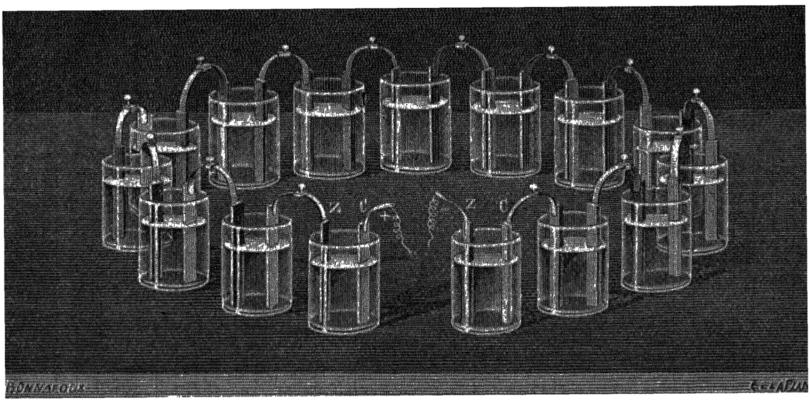
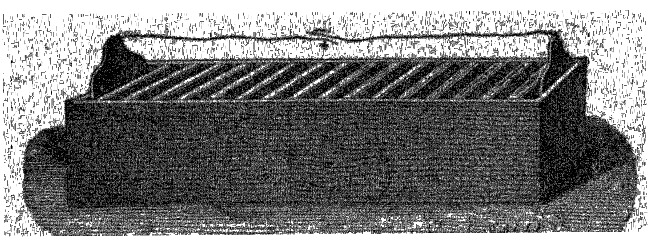


Fig 94.—Couronne de Tasses

consisting of a series of cups arranged in a circle, each containing salt water with a plate of silver or copper and a plate of zinc immersed in it, the silver or copper of each cup being connected with the zinc of the next

154. Trough Battery.—More convenient arrangements, equivalent



to the couronne de tasses, were soon introduced. One of these, devised by Cruickshank, is represented in Fig. 95, consisting of a

rectangular box, called a trough, of baked wood, which is a non-conductor of electricity, divided into compartments by partitions each consisting of a plate of zinc and a plate of copper soldered together. Dilute acid is poured into these compartments.

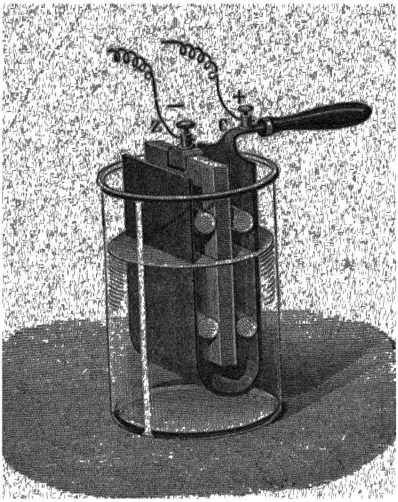


Fig. 96.—Woll...

155. Wollaston's Battery.—In Wollaston's battery, the plates were suspended from a single horizontal bar, by means of which they could all be let down into the acid, or lifted out of it together. The liquid was contained either in compartments of a trough of glazed earthenware, with partitions of the same material, or in separate vessels as shown in Fig. 97. The plates were double-coppered; that is to say, they consisted of a zinc plate with a copper

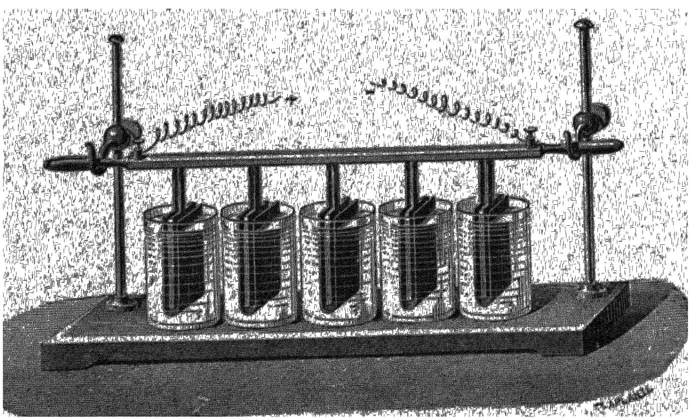


Fig. 97.—Wollaston Battery.

plate bent round it on both sides (Fig. 96), contact between them being prevented by pieces of wood or cork.

156. Hare's Deflagrator.—For some purposes it is more important to diminish the resistance of a cell, or, in other words, to facilitate the conduction of electricity between the zinc and the copper plate, than to increase the electromotive force by multiplying cells. The

spiral arrangement devised by Hare of Philadelphia (Fig. 98) is specially adapted to such purposes. It consists of two very large plates of zinc and copper rolled upon a central cylinder of wood, and prevented from touching each other by pieces of cloth or twine inserted between them It is plunged in a tub of acidulated water, as represented in the figure. From the remarkably powerful heating effects which can be obtained by the use of this cell, it is called Hare's *deflagrator*.

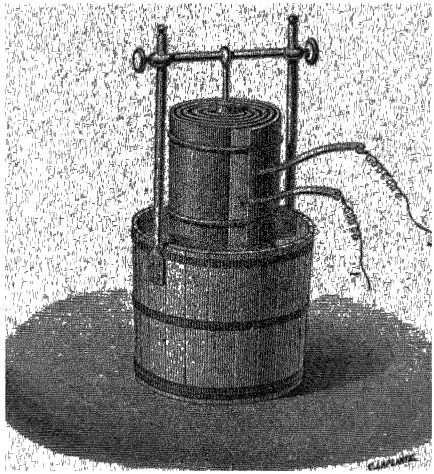


Fig 98.—Hare's Deflagrator.

157. Polarization of Plates. All the forms of battery which we have thus far described, are liable to a rapid decrease of power, owing to causes which are partly chemical and partly electrical.

The chemical action which takes place in each cell consists primarily in the formation of sulphate of zinc, at the expense of the zinc plate, the sulphuric acid, and the oxygen of the water with which the acid is diluted, the hydrogen of the water being thus liberated. As this action proceeds, the liquid becomes continually less capable of acting powerfully on the zinc. Again, a portion of the zinc which has been dissolved becomes deposited on the copper plate, thus tending to make the two plates alike, and so to destroy the current, which essentially depends on the difference between them.

But the most important cause is to be found in what is called the *polarization* of the copper plate, that is to say, in the deposition of a film of hydrogen on the surface of the plate. This film not only interposes resistance by its defect of conductivity, but also brings to bear an electromotive force in the direction opposed to that of the current.

These obstacles to the maintenance of a constant current were first overcome by Daniell.

158. Daniell's Battery.—In the cell devised by Daniell, there is a

porous partition of unglazed earthenware, separating the two liquids, which are in contact one with the zinc, and the other with the copper plate. These two liquids are not precisely alike, that which is in contact with the copper being not simply dilute sulphuric acid like the other, but containing also as much sulphate of copper as it will take up. For the purpose of keeping it saturated, crystals of sulphate of copper are suspended in it near its surface by means of a wire basket of copper. The effect of this arrangement is, that the hydrogen is intercepted before it can arrive at the copper plate, and the deposit which takes place on the copper plate is a deposit of copper, the hydrogen taking the place of this copper in the saturated solution.

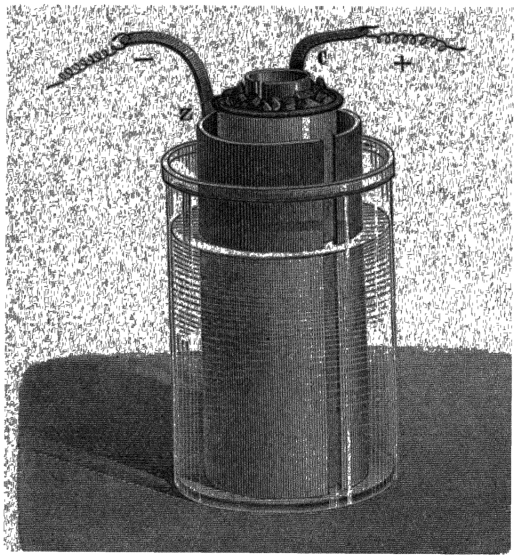


Fig. 99.—Daniell's Cell.

The current given by a battery of these cells remains nearly constant for some hours.

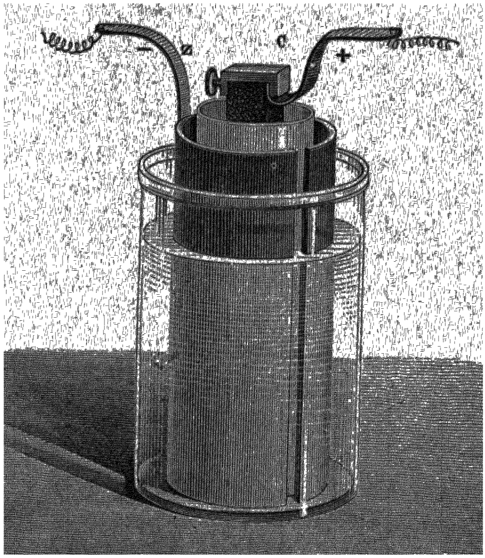
In the figure, the copper plate C is represented as a cleft cylinder occupying the interior, with the crystals of sulphate of copper piled up round it. The entire cylinder surrounding these is the porous partition, outside of which is the cleft cylinder of zinc Z, the whole being contained in a vessel of glass.

It is more usual in this country to dispense with the glass vessel, and interchange the places of the zinc and copper in the figure, the copper plate being a cylindrical vessel of copper containing the saturated solution. In this is immersed the porous vessel containing the other fluid with the zinc plate immersed in it. The cells thus constructed are usually arranged in square compartments in a wooden box.

159. Bunsen's Battery.—One of the best-known forms of battery is that which was invented by Bunsen in 1843, being substantially

identical with one previously invented by Grove, except that carbon is substituted for platinum.

The usual construction of its cells is very clearly represented in Fig. 100, and the mode of connecting them in Fig. 101. The cleft cylinder is the zinc plate, which is immersed in dilute sulphuric acid. Within this is the porous cylinder, similar to Daniell's, containing *strong nitric acid,* in which is immersed a rectangular prism, of a very dense kind of charcoal, obtained from the interior of the retorts at gas works, being deposited there in the manufacture of gas.



In this cell the hydrogen is intercepted on its way to the carbon plate by the nitric acid, with which it forms nitrous acid.

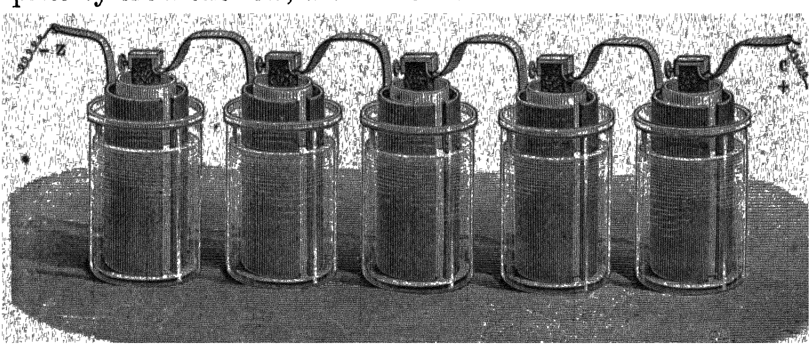


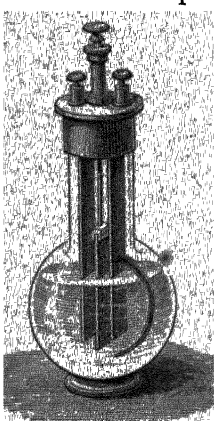
Fig. 101—Bunsen's Battery.

Grove's battery possesses some advantages over Bunsen's, but its first cost is much greater.

160. Amalgamated Zinc.—When the poles of a battery are insulated from one another, there ought to be no chemical action in the cells.

Any action which then goes on is wasteful, and is an indication that unproductive consumption of zinc goes on when the current is passing, in addition to the consumption which is necessary for producing the current. This wasteful action, which is called *local action*, goes on largely when the zinc plates are of ordinary commercial zinc, but not when they are of perfectly pure zinc. In this respect amalgamated zinc behaves like pure zinc, and it is accordingly almost universally employed. The amalgamation, which must be often renewed in the case of a battery in constant use, is performed by first cleaning the zinc plates with dilute acid, and then rubbing them with mercury.

161. Bichromate. Fuller. Leclanché.—The most convenient cells for most class experiments are the bichromate of potash **bottle-cells,** one of which is represented in Fig. 102. The liquid is a solution of bichromate of potash, with a little sulphuric acid added. In this liquid two flat plates of carbon are suspended, an[illegible] between them is a flat plate of zinc, which [illegible] be slid up and down by means of a rod [illegible]jecting through the top of the cell. It is slid up when not in use, and is then just clear of the liquid. By pushing it down the cell is brought into full action, and as soon as the experiment for which it is required is concluded the zinc should again be raised out of the liquid. The cell is not suited for long-continued work, but it gives powerful effects when only used for a few minutes at a time. It has the conveniences of great portability and of freedom from noxious fumes.



Bichromate ..le-cell.

For long-continued use the two-fluid bichromate battery known as Fuller's is very effective. One plate is of carbon immersed in the solution above described. The other plate is a block of zinc cast on to the end of a thin copper rod, at the top of which is the binding screw. Mercury is poured in till the zinc is completely covered, and the cell is filled up with dilute acid. The copper rod is usually coated with gutta percha to protect it from the acid. If not so coated it must be amalgamated.

The favourite battery for electric bells is that of Leclanché. One of its plates is a zinc rod immersed in a solution of sal-ammoniac.

The other is a plate of carbon, tightly packed in a porous cell with a mixture of peroxide of manganese and gas-carbon, moistened with water, or with the same liquid as that in the outer vessel. It is suitable for sending brief currents at any time during many months.

CHAPTER XIV.

GALVANOMETER.

162. Œrsted's Experiment.—The discovery by the Danish philosopher Œrsted, in 1819, that a magnetized needle could be deflected by an electric current, was justly regarded with intense interest by the scientific world, as affording the first indication of a definite relation existing between magnetism and electricity.

Œrsted's experiment can be repeated by means of the apparatus represented in Fig. 103. Two insulated metallic wires are placed in the magnetic meridian, one of them above, and the other below a magnetized needle. If a current be sent through one of these wires, the needle will be deflected, and if the current be strong, the deflection will amount nearly to a right angle. The direction of the deflection will be reversed if the current be passed through the lower instead of the upper wire. It will also be reversed by reversing the direction of the current. In the figure, the current is supposed to be passing above the needle from south to north. In this case the north end of the needle moves to the west, and the south end to the east. On making the current pass in various directions, either horizontally, vertically, or obliquely, near one pole of the needle, it will be found that deviation is always produced except when the plane containing the pole and current is perpendicular to the length of the needle.

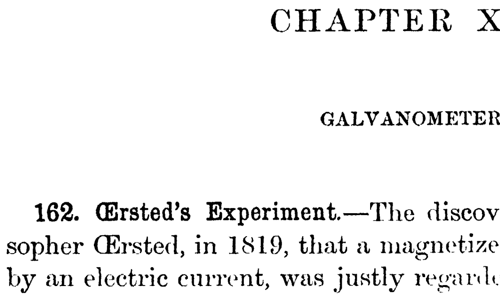


Fig 103.—Œrsted's Experiment.

163. Ampère's Rule.—The direction in which either pole of a needle is deflected by a current, whatever their relative positions may be, is given by the following rule, which was first laid down by Ampère.

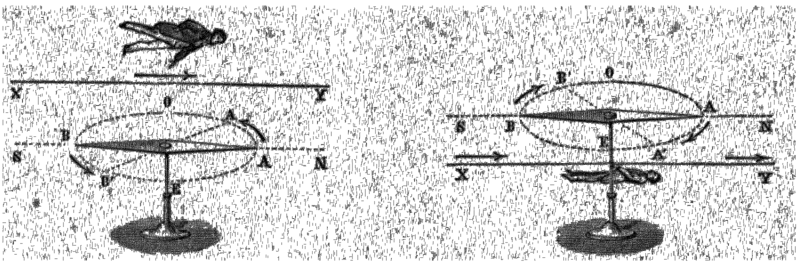


Fig. 104 Ampère's Rule **Fig. 105.**

Imagine an observer to be so placed that the current passes through him, *entering at his feet* and leaving at his head (or that he is swimming with the current and looking at the needle), then the deflection of a *north-seeking pole* will be *to his left.* The deflection of a south-seeking pole will be in the opposite direction. The two figures 104, 105 illustrate the application of this rule to the two cases just considered. A is the austral or north-seeking pole of the needle, and B the boreal or south-seeking pole.[1]

164. Lines of Magnetic Force due to Current.—The relation between currents and magnetic forces may be more precisely expressed by saying that a current flowing through a straight wire produces circular lines of force, having the wire for their common axis. A pole of a magnet placed anywhere in the neighbourhood of the wire, experiences a force tending to urge it in a circular path round the wire, and the direction of motion round the wire is opposite for opposite poles. Fig. 106 represents three of the lines of force for a north-seeking pole, due to a current flowing through a straight wire from the end marked + to the end marked −. The lines of force are circles (shown in perspective as ellipses), having their centre

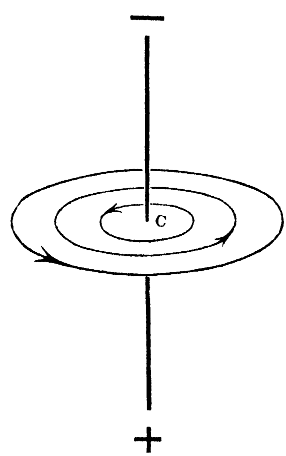


Fig 106.—Lines of Force due to Current

[1] A corkscrew advancing in the direction of the current turns the same way as the north-seeking pole.

at a point C in the wire, and having their plane perpendicular to the length of the wire. The arrows indicate the direction in which a north-seeking pole will be urged.[1] This direction is from right to left round the wire as seen from the wire itself by a person with his feet towards + and his head towards −, according to Ampère's rule. The figure may be turned upside down, or into any other position, and will still remain true.

165. Reaction of Magnet on Current.—While the wire, in virtue of the current flowing up through it, urges an austral pole from A towards A′ (Fig. 107), it is itself urged in the opposite direction C C′. If an observer be in imagination identified with the wire, the current being supposed, as in Ampère's rule, to enter at his feet, and come out at his head, the force which he will experience from a north-seeking pole directly in front of him will be a force to his right. It will be noted that the magnetic influence which thus urges him to the right would urge a north-seeking pole from his front to his back. *A conductor conveying a current is not urged along lines of magnetic force, but in a direction which is at right angles to them, and at the same time at right angles to its own length.*[2]

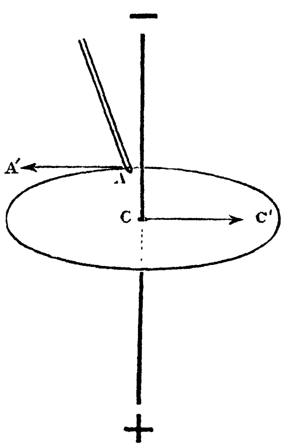


Fig. 107.—Reaction on Current.

166. Numerical Estimate of Currents.—The numerical measure of a current denotes the quantity of electricity which flows across a section of it in unit time. It is sometimes called *strength* of current, sometimes, especially by French writers, *intensity* of current, sometimes simply *current* or *amount* of current. If a thin and a thick wire are joined end to end, it has the same value for them both; just as the same quantity of water flows through the broad as through the contracted parts of the bed of a stream. Hence the name *intensity* is obviously inappropriate, for, with the same total quantity of

[1] The *direction of a line of magnetic force* is by general agreement taken to mean the direction in which a *north-seeking* pole would be urged.

[2] The following brief rules, the second of which will be established hereafter, are collected here for convenience of reference:—

In field from front to back, current from foot to head is urged to right.

In field from front to back, motion to right produces current from head to foot.

electricity flowing through both, the current is, properly speaking, more *intense* in the thin than in the thick wire

Currents may be measured experimentally by various tests, which are found to agree precisely. The most convenient of these for general purposes is the deflection of a magnetized needle. The force which a given pole experiences in a given position with respect to a wire conveying a current, is simply proportional to the current. Hence the name *strength* of current admits of being interpreted in a sense corresponding to that in which we speak of the strength of a pole. Instruments for measuring currents by means of the deflections which they produce in a magnetized needle are called *galvanometers.*

167. Multiplier.—The idea of carrying a wire several times round a needle in a vertical plane is due to Schweiger. The form of apparatus designed by him, called *Schweiger's multiplier*, is represented in Fig. 108. The name *multiplier* is derived from the fact that, if the current is not sensibly diminished by increasing the number of convolutions of wire through which it has to pass, the force exerted on the needle is n times as great with n convolutions as with only 1, since each convolution exerts its own force on the needle independent of the rest. Cases, however, frequently occur in which the increased *resistance* introduced by increasing the number of convolutions outweighs the advantage of multiplicati[illegible] that a short thick wire with few convolutions gives a more powerful effect than a long thin wire with many. This is especially the case with thermo-electric currents.

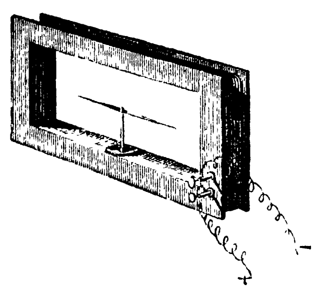


Fig 108.—Schweiger's Multiplier

The difference between the rectangular and the circular form is merely a matter of detail. Whichever form be adopted, all parts of the coil contribute to make the needle deviate in the same direction. For instance, in Fig. 109, if the current proceeds in the direction indicated by the arrows, the application of Ampère's rule to any one of the four sides of the rectangle shows that the austral pole a will be urged towards the front of the figure. When the coil is circular, and the needle so small that

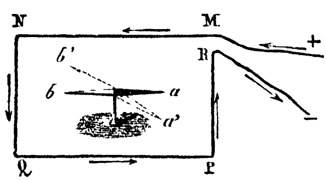


Fig 109.

each pole is nearly in the centre, equal lengths of the current, in whatever parts of the circle they may be situated, exert equal forces upon the needle, and all alike urge the poles in directions perpendicular to the plane of the coil.

168. Sine Galvanometer.—The sine galvanometer, which was invented by Pouillet, is represented in Fig. 110. The current which is to be measured traverses a copper wire, wrapped round with silk for insulation, which is carried either once or several times round a vertical circle; and this circle can be turned into any position in azimuth, the amount of turning being indicated on a horizontal circle. In the centre of the vertical circle, a declination needle is mounted, surrounded by a horizontal circle for indicating its position, this circle being rigidly attached to the vertical circle. Suppose that, before the current is allowed to pass, both the needle and the vertical circle are in the magnetic meridian, and that the needle consequently points at zero on its horizontal circle. On the current passing, the needle will move away. The vertical circle must then be turned until it overtakes the needle; that is, until the needle again points at zero. This implies turning the circles through an angle α equal to that by which the needle finally deviates from the magnetic meridian. In this position the terrestrial couple tending to bring back the needle to the meridian is proportional to $\sin \alpha$ (§ 116). The forces exerted upon the two poles by the current are perpendicular to the plane of the vertical circle, and are simply proportional to the current. Hence, in comparing different observations made with the same instrument the amounts of current are proportional to the sines of the deviations

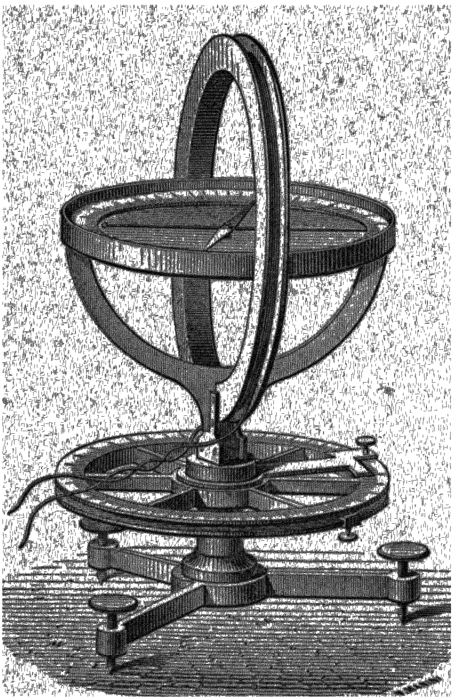


Fig. 110.—Sine Galvanometer.

169. Tangent Galvanometer.—The tangent galvanometer, which is simpler in its construction and use, and is much more frequently employed, consists of a declination needle mounted in the centre of a vertical circle whose plane always coincides with the magnetic meridian, the length of the needle being small in comparison with the radius of the circle.

Let o (Fig. 111) be the centre of suspension, $a\,b$ the initial position of the needle, and $a'\,b'$ its deflected position. The force F exerted on either pole by the current is sensibly the same at a' as at a, on account of the smallness of the needle, and it acts in the direction $l\,k$, while the horizontal force of the earth upon the pole acts along $a'\,m$; and these two forces give a resultant along $o\,a'$. Hence, taking the triangle $o\,l\,a'$ as the triangle of forces, the force exerted by the current is to the horizontal force exerted by the earth as $l\,a'$ to $o\,l$, or as $\tan\,\alpha$ to unity; that is, the current is proportional to the tangent of the deflection.

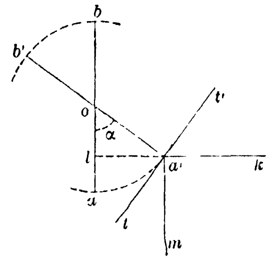


Fig 111.—Principle of Tangent Galvanometer

In order to permit the deviation of the short needle to be accurately read, a long pointer is attached to it, usually at right angles, the two ends of which move along a fixed horizontal circle.

When the needle is deflected, it is no longer in the plane of the coil, and this circumstance complicates the relation between current and deflection. Helmholtz has overcome this difficulty by placing the needle midway between two equal and parallel coils, whose distance apart is equal to the radius of either, the two being connected in series so that the same current flows through both. The lines of magnetic force in the intervening region can be shown to be very nearly straight.[1]

170. Differential Galvanometer.—The coil of a galvanometer sometimes consists of two distinct wires, having the same number of convolutions, and connected with separate binding-screws. This arrangement allows of currents from two distinct sources being sent at the same time round the coil either in the same or in opposite directions. In the latter case, the resultant effect upon the needle will be that due to the difference of the two currents; and if they

[1] A drawing of the lines will be found in Maxwell's *Electricity and Magnetism*, vol. ii. fig. xix.

are not exactly equal, the direction of the deflection will indicate which of them is the greater. An instrument thus arranged is called a *differential galvanometer.*

171. Astatic Needle.—The sensibility of the galvanometer is greatly increased by employing what is called an *astatic* needle. It consists of a combination of two magnetized needles *with their poles turned opposite ways.* The two needles are rigidly attached at different heights to a vertical stem, and the system is usually suspended by a silk fibre, which gives greater freedom of motion than support upon a point. On account of the opposition of the poles, the directive action of the earth on the system is very feeble. If the magnetic moments of the two needles were exactly equal, and their axes exactly parallel, the resultant moment would be zero, and the system would remain indifferently in all azimuths.

One of the needles $a\,b$ (Fig. 112) is nearly in the centre of the coil C D E F through which the current passes. The other $a'\,b'$ is just above the coil. When a current traverses the coil in the direction of the arrows, the action of all parts of the current upon the lower needle tends to urge the austral pole a towards the back of the figure, and the boreal pole b to the front. The upper needle $a'\,b'$ is affected principally by the current in the upper part CD of the coil, which urges the austral pole a' to the front of the figure, and the boreal pole b' to the back. Both needles are thus urged to rotate in the same direction by the current, and as the opposing action of the earth is greatly enfeebled by the combination, a much larger deflection is obtained than would be given by one of the needles if employed alone.

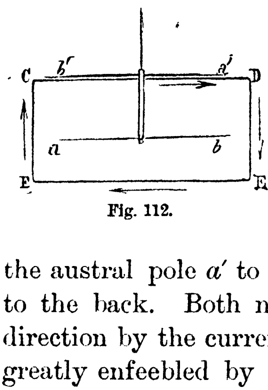


Fig. 112.

If the two needles had rigorously equal moments, the system would be said to be *perfectly astatic.* The smallest current in the coil would then suffice to set the needles at right angles to the meridian, and no measure would be obtained of the amount of current.

Fig. 113 represents an astatic galvanometer, as usually constructed. The coil is wound upon a wooden frame, which supports the divided circle in whose centre the upper needle is suspended. The ends of the coil are connected with two binding-screws, for making connection with the wires which convey the current to be measured.

The needles are usually two sewing-needles, and the upper one often carries a light pointer. The suspending fibre is attached at its upper end to a hook, which can be raised or lowered, and when the instrument is not in use this is lowered till the upper needle rests upon the plate beneath it, so as to relieve the fibre from strain. In using the instrument care must be taken to adjust the three levelling-screws so that the needle swings free.

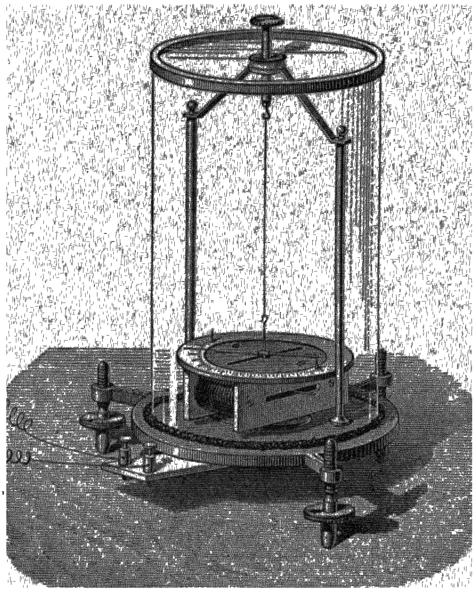


Fig. —Astatic Galvanometer.

172. Thomson's Mirror Galvanometer.—A more sensitive instrument is the mirror galvanometer of Sir W Thomson. Its needle, which is very short, is rigidly attached to a small light concave mirror, and suspended in the centre of a vertical coil of very small diameter by a silk fibre. A divided scale is

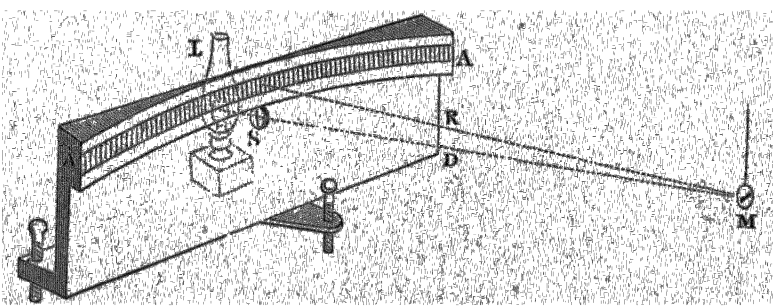


Fig. 114.—Mirror and Scale.

placed in a horizontal position in front of the mirror, at the distance of about a yard, and the image of an illuminated slit, which is thrown by the mirror upon this scale, serves as the index. The arrangement of the mirror and scale, which is the same as in the case of the quadrant electrometer described in a previous chapter, is exhibited in

Fig. 114. M is the mirror of silvered glass, slightly concave, with a small piece of magnetized watch-spring attached to its back, the two together weighing only a grain and a half, and suspended by a few fibres of unspun silk. AA is a divided scale forming an arc of a horizontal circle about the mirror as centre. Immediately below the centre of this scale is a circular opening S with a fine wire stretched vertically at the back of it. A paraffin lamp L is placed directly behind this opening, so as to shine through it upon the mirror, which is at such a distance as to throw upon the screen a bright image of the opening with a sharply-defined dark image of the wire in its centre. The image of the wire is employed as the index in taking the readings.

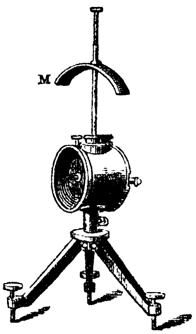


Fig. 115.—Thomson's Mirror Galvanometer

In order to obviate the necessity of keeping the needle in the meridian, with the lamp east or west of it, and to admit of other positions, a controlling magnet M (Fig. 115) is provided, which can be raised or lowered, and can also be turned round. When it is low down it overpowers the earth's magnetism, and compels the needle to take any position that may be required.

For maximum sensitiveness, the magnetic field around the needle should be made as weak as possible. For this purpose the needle should be placed in or not far from the meridian, and the magnet, after being turned into such a position as directly to oppose the earth's action on the needle, should be lowered till its force is a very little less than that of the earth. The operator in making the adjustment watches the vibrations of the needle, as indicated by the movements of the image on the scale, and knows that the force on the needle is diminishing when he sees the vibrations becoming slower.

For use at sea the galvanometer is modified by fastening the supporting fibre of silk at both ends, so as to keep it tight, with the needle and mirror attached at its centre, care being taken to make the direction of the fibre pass through the common centre of gravity of the needle and mirror, in order that the rolling of the ship may not tend to produce rotation. In this form it is called the *marine galvanometer.*

173. Direct-reading Galvanometers. Ammeters.—In many galvanometers the connection between strength of current and magnitude of deflection cannot be expressed by a simple law.

It is becoming customary in instruments for commercial use to place the graduations not at equal distances, but at such distances that the reading shall be simply proportional to the current. The instruments are then called *direct-reading* galvanometers.

This mode of graduation is especially common in instruments intended for measuring very strong currents, such as those produced by dynamos. These instruments have a controlling magnet, whose field is so strong as completely to overpower that of the earth. Their readings usually give the current in "amperes," in which case the instrument is called an *ammeter* (short for ampere-meter).

174. General Law for Magnetic Force due to a Current.—In every case, the magnetic force at a given point due to a current, can be computed by dividing the current into elementary portions, each sensibly straight, and compounding by the parallelogram of forces the effects due to these separate elements. The force due to each element is normal to the plane drawn through the element and the given point, and is proportional to $C\frac{l}{r^2}\sin\theta$, where C denotes the strength of the current, l the length of the element, r the distance between the element and the given point, and θ the angle between the joining line and the element. The force at the centre of a single circular current of radius a is therefore $C\frac{2\pi a}{a^2}=C\frac{2\pi}{a}$, and the force at the centre of a circular galvanometer-coil of n convolutions, if all can be regarded as in one plane and of the same radius a, is $C\frac{2\pi n}{a}$.

175. Instantaneous Current; Ballistic Galvanometer.—When the duration of a current is small in comparison with the time of vibration of the needle, and the total deflection small, the velocity which the current gives the needle is jointly proportional to the duration of the current and its average strength. It is, therefore, simply proportional to the quantity of electricity which passes. This velocity is equal to that acquired in the return movement to zero: and this latter obviously follows the same law as the motion of a simple pendulum, for in both cases the effective force is proportional to the sine of the displacement. In the case of the pendulum, the square of the velocity acquired in the whole descent is proportional to the vertical height descended, and this vertical height multiplied by the diameter of the circle in which the pendulum moves, is equal to the square of the chord; hence, the velocity acquired is propor-

tional rigorously to the chord, and approximately to the arc of descent if small. The same rule must hold for the needle; that is to say, the velocity acquired must be proportional to the extreme displacement. *The quantity of electricity transmitted through the galvanometer coil by an instantaneous discharge is* therefore *proportional to the distance to which the needle swings.*

Galvanometers specially intended for this use are called *ballistic galvanometers.*

176. The Galvanometer a True Measurer of Current.—This reasoning assumes the principle that the force exerted by a current on a needle is a true measure of the strength of the current (defined as the quantity of electricity conveyed per unit of time); and conversely, the observed fact that, when known quantities of electricity are discharged through a galvanometer, the swings produced are proportional to these quantities, establishes the principle. The experiment has frequently been made by discharging a condenser (§ 73) which has been charged by a galvanic battery; and Faraday obtained a similar result with Leyden-jars which had been charged by a powerful frictional machine, the jars being discharged through a wet thread or string leading to the galvanometer. He found that the swing was independent of the length and thickness of the thread or string, as well as of the number of jars employed, and was proportional to the number of turns that had been given to the electrical machine in charging the jars.

The proportionality of force to current might have been inferred *à priori* from the consideration that, if we have two parallel wires close together, conveying equal currents, the resultant force on a pole will be the sum of the forces due to each, and will therefore be double of the force due to one alone. The force will not be altered by allowing the wires to touch each other all along their length; and in this position they form a single conductor conveying a double current.

177. Needle Deflected by Motion of a Charged Body.—The question has been raised whether the carrying of electricity by the motion of a charged body produces effects similar to those of a current flowing through a conductor, and in particular, whether it is capable of deflecting a magnetized needle. The matter was put to the test by Professor Rowland, in an experiment performed at the laboratory of the Berlin University.[1]

[1] See *Phil. Mag.* September, 1876, pp. 211-216.

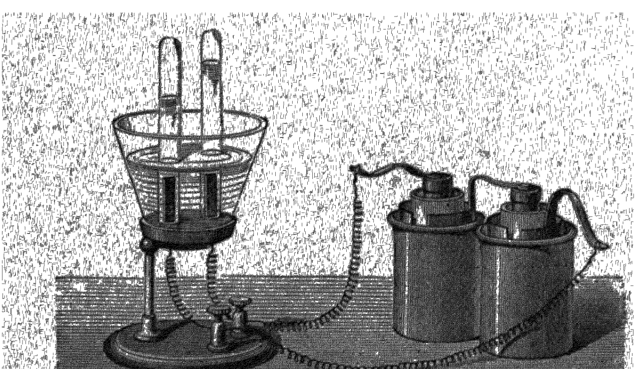
The carrier of the electricity was a rapidly revolving horizontal disc of ebonite, gilt on both sides, and maintained in a high state of electrification by means of a fixed discharging point connected with one of the coatings of a battery of Leyden-jars. The needle to be deflected was suspended over it near its circumference, the length of the needle being perpendicular to the radius of the disc, so that the motion of the electricity beneath the needle was parallel to its length. Between the needle and the revolving disc, a larger fixed disc of glass, gilt on one side and connected with the earth, was interposed; and there was a similar disc on the lower side The needle was one of an astatic pair, the other needle being at a much greater height; and both were inclosed in a brass case, to protect them from electrostatic influences. The deflection was observed by means of a mirror attached to the stem of the needles, and a telescope for viewing in the mirror the reflected image of a scale. The disc, which was 8⅓ inches in diameter, revolved at the rate of about 60 turns per second, and the deflections observed amounted to from 5 to 7½ divisions of the scale, the deflection being to the one side or the other, according as the charge of the disc was positive or negative. The observations extended over several weeks, and conclusively proved, subject to small errors of observation and reduction, that the magnetic effect of carrying a charge of electricity is the same as that of the flow of the same quantity of electricity in the same time through a conductor.

CHAPTER XV.

ELECTRO-CHEMISTRY.

178. Electrolysis.—When a current is passed through a compound liquid, decomposition is frequently observed, two of the component substances being separated, one at the place where the current enters and the other at the place where it leaves the liquid. This decomposition is called *electrolysis*, and the substance decomposed or *electrolysed* is called the *electrolyte*. The action only occurs in the case of liquids, and these must be conductors.

The process may be illustrated by the decomposition of water as represented in Fig. 116. The apparatus consists of a vessel con-



taining water to which a little sulphuric acid has been added, and in which two strips of platinum are immersed, connected respectively with the two poles of a battery. When the connections are completed, bubbles make their appearance at the surfaces of the two

strips and rapidly rise to the surface. If two tubes filled with the liquid are inverted over the two strips, the gases will bubble up through the liquid into the upper part of these tubes and the level of the liquid will gradually fall as shown in the figure. It will be found that the volume of the hydrogen is about double that of the oxygen.

The two strips of platinum are called the poles or *electrodes* of the decomposing cell; the one in connection with the positive pole of the battery is called the *anode* (literally the *way up*) and the one in connection with the negative pole the *cathode* or *kathode* (*way down*). The direction of the current through the liquid is from the anode to the kathode. The gas which is given off at the anode (in the present case oxygen) is called the *anion* (*that which goes up*) and that which is given off at the kathode the *kathion* or *cation* (*that which goes down*). The anion is often called the *electro-negative* element, because it moves as if attracted by the positive and repelled by the negative pole. For a similar reason the kathion is called the *electro-positive* element.

In many cases, the separation effected by the direct action of the current is followed by secondary actions due to chemical affinities.

Thus, in the decomposition of acidulated water above described, the first effect, according to modern theory, is a breaking up of the sulphuric acid ($S\,O_3$, $H_2\,O$) into hydrogen and sulphion (2 H and $S\,O_4$), the latter being a substance which has never been obtained by itself. The hydrogen travels to the negative pole and there escapes. The sulphion goes to the positive pole, but instead of escaping enters into combination with the hydrogen of the liquid, forming again the primitive compound ($S\,O_3$, $H_2\,O$) and leaving the oxygen of the liquid to escape.

179. Transport of Elements.—It is a remarkable fact that the separated elements never make their appearance except at the electrodes. Nothing is seen of them, nor is any action exhibited, at intermediate points. The appearance is as if the gases could vanish from one extremity and appear at the other without passing through the intermediate space. The only possible explanation of this phenomenon seems to be what is known as Grotthus'

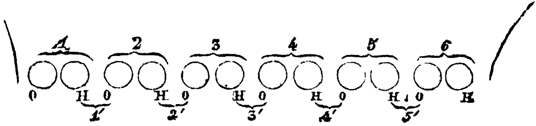


Fig. 117.—Grotthus' Hypothesis.

hypothesis, that all the particles of the water in the course of the current undergo continual decomposition and recomposition. Thus if Fig. 117 represent a line of particles traversed by the current from left to right, there will be a continual stream of hydrogen particles along this line from left to right, and a stream of oxygen[1] particles from right to left. The hydrogen of molecule 1 will combine with the oxygen of molecule 2 to form a new molecule 1′; the hydrogen of molecule 2 will combine with the oxygen of molecule 3 to form a new molecule 2′, and so on. The oxygen of molecule 1 is given off at the left-hand extremity, which we suppose to be the point of contact with one of the strips of platinum, and the hydrogen of molecule 6 at the other strip. The molecules 1′, 2′, 3′ . . . are then in their turn decomposed to form a new set. In actual cases, the number of molecules, instead of being only six as represented in the figure, is of course many millions.

180. Discovery of Potassium.—When a compound formed by the union of a metal with a non-metallic substance is submitted to electrolysis, the metal always comes to the negative pole. It was in this way that several of the metals were first obtained from their oxides by Sir Humphry Davy. Potassium, for example, was obtained by placing a piece of potash on a platinum disc connected with the negative pole of a battery of 250 cells, and then applying a platinum wire connected with the positive pole to its upper surface. The potash, which had been allowed to contract a little moisture from the atmosphere, in order to give it sufficient conducting power, soon began to fuse at the points of contact of the electrodes. A violent effervescence occurred at the upper or positive electrode; while at the lower surface small globules appeared resembling quicksilver, some of which instantly burst into flame, while others merely became tarnished and afterwards coated over with a white film.

181. Electrolysis of Salts.—When a salt of any of the less inflammable metals is submitted to electrolysis, a continual deposition of the metal is observed on the negative electrode; while, at the positive electrode, oxygen is disengaged, and acid set free. These effects occur, for example, if platinum electrodes are plunged in a solution of sulphate of copper. If copper is employed as the positive electrode, the oxygen will combine with it instead

[1] Or sulphion particles according to modern theories. The explanation as it stands in the text represents the views held by Faraday and his contemporaries.

of being given off, and the oxide thus formed will be dissolved by the acid.

182. Voltameters.—The quickness with which a given electrolyte is decomposed is simply proportional to the strength of the current; and any apparatus designed for measuring the strength of a current on this principle is called a *voltameter*. The apparatus in Fig. 116 is one of its forms. The commonest form consists of a cell containing a solution of sulphate of copper, with two copper plates for electrodes. As the current passes, the anode is gradually dissolved away, and an equal quantity of copper is deposited on the kathode. The quantity of electricity that has passed through the cell can accordingly be determined by weighing the plates. Copper, however, has the drawback that it is to some extent acted on by the solution when no current is passing. When the greatest possible accuracy is required, two plates of silver immersed in a solution of nitrate of silver are preferred.

183. Quantitative Laws of Electrolysis.—The following table gives, in the column headed "chemical equivalent," the quantities of the several elements deposited by one and the same quantity of electricity. It will be seen from the table that the same element in different states of combination may have different equivalents. The general law for such cases is that the equivalent is equal to the atomic weight (which is definite for each element) divided by the "valency." The "valency" expresses the number of atoms of hydrogen which are replaced by one atom of the substance.

	Atomic weight.	Valency	Chemical equivalent
Electro-positive.			
Hydrogen,	1	1	1
Potassium,	39·1	1	39·1
Sodium,	23·	1	23·
Gold,	196·6	3	65·5
Silver,	108·	1	108·
Copper (Cupric),	63·	2	31·5
„ (Cuprous),	63·	1	63·
Mercury (Mercuric),	200·	2	100·
„ (Mercurous),	200·	1	200·
Tin (Stannic),	118·	4	29·5
„ (Stannous),	118·	2	59·
Iron (Ferric),	56·	3	18·7
„ (Ferrous),	56·	2	28·
Nickel,	59·	2	29·5
Zinc,	65·	2	32·5
Lead,	207·	2	103·5
Aluminium,	27·	3	9·

	Atomic weight.	Valency.	Chemical equivalent.
Electro-negative.			
Oxygen,	16·	2	8·
Chlorine,	35·5	1	35·5
Iodine,	127·	1	127·
Bromine,	80·	1	80·
Nitrogen,	14·	3	4·3

The quantity of a substance that is deposited by the passage of a unit of electricity is called the *electro-chemical equivalent* of the substance. The electro-chemical equivalent of silver, according to the latest and best determinations, is ·01118 of a gramme for one C.G.S. unit of electricity, or ·001118 of a gramme for one "coulomb." The electro-chemical equivalents of the other substances mentioned in the above table can be calculated from this by simple proportion.

The above table holds good also for battery cells, provided that there is no wasteful "local action." The quantity of zinc dissolved in each cell of a battery is chemically equivalent to the quantity of any metal that is deposited in an electrolytic cell by the same current.

184. **Chemical Relations of Electro-motive Force.**—The energy of the current produced by a battery is equal to the potential energy of chemical affinity which runs down in its production; and this latter is measured by the heat of combination due to the chemical action which goes on in the battery. If there are decomposing cells in circuit, they contribute negative heat of combination, and there can be no current unless the total heat of combination for the whole circuit be positive.

The energy of the current for a given time is equivalent to the total heat of combination due to the action which takes place in this time; and if this energy be divided by the quantity of electricity conveyed by the current in the time, the quotient is called the *electro-motive force* of the circuit.

The quantity of electricity conveyed bears a definite relation of equivalence to the zinc dissolved in any one cell; and hence the electro-motive force of a cell is proportional to the heat of combination due to the action which takes place during the consumption of a given quantity of zinc. This heat is about twice as great for a Grove's cell as for a Daniell, and hence the electro-motive force of the former is about double that of the latter.

185. **Secondary Batteries.**—We have stated in § 157 that in some

forms of battery a reverse electromotive force is produced by the deposition of gas on the surface of the plates, and that this action is called *polarization*. A similar effect occurs in the voltameter of Fig. 116; and if the oxygen and hydrogen, as they are given off, are collected in separate tubes, each platinum plate being in contact both with one of the gases and with the liquid, the voltameter, after being disconnected from the source of the original current, can be used as a battery cell giving a current in the opposite direction. This is the principle of *Grove's Gas Battery*, in which the tubes may be filled with oxygen and hydrogen either by electrolysis or in any other manner, and any number of the cells may be connected in series. During the flow of the current which this battery gives, the two gases of each cell gradually unite through the medium of the acidulated water between them.

Any cell which is first subjected to electrolysis by a current from an external source, and then gives of itself a current in the opposite direction, is called a *secondary cell*, and a combination of such cells is a *secondary battery*. Recent improvements in their construction have called public attention to them as an important means of storing up energy, to be used when and where it is wanted. Hence they have received the name of *storage batteries* or *accumulators*.

186. Faure's Accumulator.—Faure's Accumulator consists of two leaden plates of large surface, covered with minium (red oxide of lead), rolled up together with flannel between them, and immersed in dilute sulphuric acid. The primary current, which is usually supplied by a dynamo machine, changes a portion of the minium on the anode into peroxide of lead, and reduces a portion of the minium on the cathode to metallic lead in a spongy condition. It is probably to the presence of these two substances with dilute sulphuric acid between them, that the secondary current is due. A battery of these cells freshly charged by a dynamo machine will give a powerful current for some hours; and an interval of a few hours or even of a few days between charging and using does not involve very much loss.[1]

Planté had previously constructed secondary cells consisting in like manner of two lead plates spirally coiled in dilute sulphuric acid, and had greatly improved their action by coating them with peroxide, which he did, not by mechanical means, but by sending

[1] The favourite accumulator at present (1893) is that of Crompton and Howell. The plates are of a porous kind of lead, and are not in the form of rolls but of a number of parallel plates. No minium is used.

through the cell a succession of currents in opposite directions, with intervals of rest between.

Planté also constructed a "rheostatic machine" containing some hundreds of these cells each connected to a separate condenser, and with an arrangement by which the connections of the cells can be instantaneously altered. During the charging process, poles of the same name in the secondary cells are all connected together, one set being connected with the positive pole of the charging battery, and the other set with the negative pole, the charging battery being usually composed of two large Bunsen cells. A small movement suffices to alter the connections so as to arrange the secondary cells in series, and a battery is thus obtained whose electromotive force is hundreds of times greater than that of the charging battery. The process of charging is instantaneous, and a quick succession of discharges of the secondary battery can be obtained by turning a handle, the effects being somewhat similar to the discharges of a Ruhmkorff coil (§ 275).

187. Applications of Accumulators.—Accumulators serve two important purposes. They can be used either like a fly-wheel, to smooth down irregularities in the supply of energy, or, like a mill-reservoir, to store up a large quantity of energy which is running to waste, and give it out later.

For example, a gas-engine usually gives alternately a quick stroke and a slow one; and when it is employed to drive a dynamo for supplying incandescent lamps, a corresponding flickering is seen in the lamps. This evil can be effectually cured by connecting the two terminals of the dynamo with the two terminals of a storage battery. The battery, after it has been charged up to a certain point, absorbs energy during each quick stroke, and gives it out again during the slow stroke, thus keeping the electromotive force of the main circuit nearly constant.

On the other hand, storage cells can be charged during the day by employing the surplus power of a factory engine to drive a dynamo, and can be used at night to supply electricity for lighting the factory. Or they can be charged by a dynamo driven by a fixed engine, and can then be placed in a tram-car to supply motive power for its journey.

188. Electro-metallurgy.—Electrolysis is largely used in connection with the reduction of metals from their ores, especially in the case of copper. Electrolytically deposited copper is remarkable for its

purity, and is almost universally employed when very high conductivity is required.

Electrolysis is also used still more extensively for the two kindred purposes of electro-plating and electrotype. In the former the electrolytic deposit is intended as a permanent covering, and should adhere perfectly so as to form one mass with the body which it covers. In the latter, the adhesion is temporary, and must not be too close, the object being merely to obtain an exact copy of the original form.

189. Electro-gilding and Electro-plating.—The deposition of a coating of gold or silver on the surface of a less precious metal is merely an example of the electrolysis of a salt, as described in § 181. The metal in solution is always deposited on the negative electrode, hence we have merely to make the negative electrode consist of the article which we wish to coat. The only points to be decided practically relate to the means of making the deposit solid and firmly

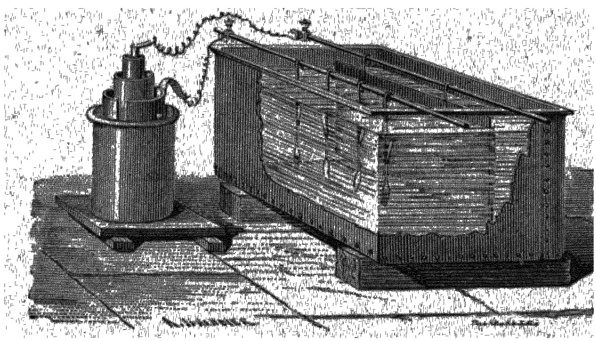


Fig. 118.—Apparatus for Electro-gilding

adherent. These ends have been completely attained by the methods patented about 1840 by Elkington in England and Ruolz in France.

The solutions are always alkaline, and usually consist of the cyanide or chloride of the metal, dissolved in an alkaline cyanide.

To prepare the *gold bath,* 50 grammes of fine gold are dissolved in aqua regia; and the solution is evaporated till it has the consistence of syrup. Water is then added, together with 50 grammes of cyanide of potassium, and the mixture is boiled. The quantities named give about 50 litres of solution.

The negative electrode consists of the article to be gilded. The positive electrode is a plate of fine gold, which constitutes a soluble

electrode, and serves to keep the solution at a constant strength. In order that the gilding may be well done, the bath must be maintained, during the operation, at a temperature of from 60° to 70° Centigrade.

Fig. 118 represents a form of apparatus which is very frequently employed. The poles of the battery are connected with two metallic rods resting on the top of the cistern which contains the bath. The articles to be gilded are hung from the negative rod. From the positive rod is hung a plate of gold, whose size should be proportional to the total surface of the articles which form the negative electrode.

The *silver bath* is a solution containing 2 parts of cyanide of silver, 10 of cyanide of potassium, and 250 of water. The operation of plating is the same as that of gilding, except that the apparatus is usually on a larger scale, and that the temperature may be lower.

In both cases the surfaces to be coated must be thoroughly cleansed from grease. For this purpose they are subjected to the processes of pickling and dipping, which we cannot stay to describe.

Other bodies, as well as metals, can be coated, if their surfaces are first covered with some conducting material. Baskets, fruits, leaves, &c., have thus been gilded or silvered.

Similar processes are employed for depositing other metals, of which copper is the most frequent example.

190. Electrotype.—Electrotyping consists in obtaining copper casts or facsimiles of medals, engraved plates, &c., by means of electrolytic deposition. The first successful attempts in this direction were made about 1839 by Jacobi at St. Petersburg and Spencer in England. The art is now very extensively practised.

If a fac-simile of a medal is required, a cast is first taken of it, either in fusible alloy, plaster of Paris, or gutta percha softened by heating to 100° C., this last material being the most frequently employed. The fusible alloy is a conductor; the other materials are not, and their surfaces are therefore rendered conducting by rubbing them over with plumbago. The mould thus prepared is made to serve as the negative electrode in a bath of sulphate of copper, a copper plate being used as the positive electrode. When the current passes, copper is deposited on the surface of the mould, forming a thin sheet, which, when detached, is a fac-simile of one side of the original medal. A similar process can be applied to the other side, and thus a complete copy can be obtained.

In operations of this kind, the bath itself is often made to serve as the battery. Fig. 119 represents such an arrangement.

In the interior of a vessel containing a saturated solution of sulphate of copper, a second vessel is supported, consisting either of porous earthenware or of a glass cylinder closed below by a membrane. In this second vessel is placed acidulated water, with a cylinder of zinc suspended in it. The mould is placed in the outer vessel under the bottom of the porous cylinder, and is connected with

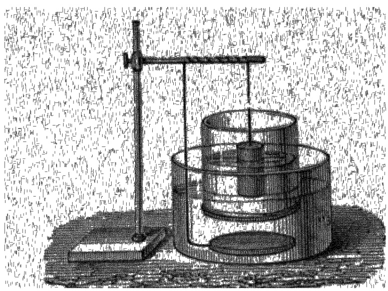


Fig 119 —Bath and Battery in one

the zinc by a stout wire which completes the circuit. The arrangement is evidently equivalent to a Daniell's cell. The current passes through the liquids from the zinc to the mould, electrolysing the solution of sulphate of copper; and as the metal travels with the current, it is deposited on the surface of the mould. The strength of the solution is kept up by suspending in it crystals of sulphate of copper contained in a vessel pierced with holes.

191. Applications of Electrotype.—One of the commonest applications of electrotype is to the production of copies of wood engravings. The original blocks as they leave the hand of the engraver, could not yield a large number of impressions without being materially injured by wear. When many impressions are required, they are not taken directly from the wood, but from an electrotype taken in copper from a gutta-percha mould The process of deposition is continued only for twenty-four hours, and the plate of copper thus obtained is very thin. It is strengthened by filling up its back with melted type-metal Such plates will afford about 80,000 impressions, and it is from them that nearly all the illustrations in popular works are printed. Postage stamps, which must be exactly alike in order to prevent counterfeits, are also printed from electrotypes, and, on account of the great number of impressions required, the electrotypes

themselves need frequent renewal; but the operations necessary for this purpose do not sensibly injure the original.

Copperplate engravings and even daguerreotypes can be very accurately reproduced in copper. No preparation of the surface is necessary, as the thin film of oxide which is present is quite sufficient to prevent the deposit from adhering too closely.

Gasaliers are usually of cast-iron coated with copper by electrolysis. The copper is not, however, deposited on the surface of the iron, as the contact of the two metals would greatly promote the oxidation of the iron, if any of it were accidentally exposed to the air. The iron is first painted over with red-lead, which, when dry, is covered with a very thin layer of plumbago to render it conducting; and it is on this that the copper is deposited.

CHAPTER XVI.

OHM'S LAW.

192. Brief Statement.—The strength of the current which traverses a circuit depends partly on the electromotive force of the source of electricity, and partly on the resistance of the circuit. For equal resistances, it is proportional to the whole electromotive force tending to maintain the current, and for equal electromotive forces it is inversely as the whole resistance in the circuit. Hence, when proper units are chosen for expressing the current C, the resistance R, and the electromotive force E, we have

$$C = \frac{E}{R},$$

or *the current is equal to the electromotive force divided by the resistance.* This is Ohm's law, so called from its discoverer.

193. Explanation of the term Electromotive Force.[1]—We may define electromotive force as *the quotient of the energy of a current by the quantity of electricity which it conveys.* This definition implies that electromotive force is a quantity of the same nature as difference of potential; for when electricity passes from one conductor to another, the work done in the passage is equal to the quantity of electricity multiplied by the difference of potentials of the two conductors.

When a steady current is flowing through a galvanic circuit, there must be a gradual fall of potential in every uniform conductor which forms part of the circuit; since, in such a conductor, the direction of a current must necessarily be from higher to lower potential. These gradual falls are exactly compensated by the abrupt rises (diminished by the abrupt falls, if any) which occur at the various places of contact of dissimilar substances.

[1] The abbreviation *e.m.f.* is commonly used for *electromotive force.*

If we imagine a large and deep trough of water of annular form, divided into compartments by transverse partitions; and suppose that circulation in one direction is maintained by pumping water over the partitions, we have a rough representation of the distribution of potential in the cells of a battery; the rise of level in passing across a partition being analogous to the rise of potential in traversing a surface of junction.

The electromotive force of a galvanic battery may be defined as the *algebraic sum of the abrupt differences of potential which occur at the junctions of dissimilar substances.* In a battery consisting of a number of similar cells arranged in series, it is of course proportional to the number of cells.

194. Explanation of the term Resistance.—When the current of a circuit is taken through the coil of a galvanometer, it is found that, by introducing different lengths of connecting wire, very different amounts of deflection can be obtained. The longer the wire which connects either pole of the battery with the galvanometer, the smaller is the deflection; and a small deflection indicates a feeble current. The current is in like manner weakened by introducing a fine instead of a stout wire, if their length and material be the same, or by introducing an iron wire instead of a copper wire of the same dimensions. These differences in the properties of the different wires are expressed by saying that they have different resistances.

It is found that, to produce no change in the deflection, the length of the wire must vary directly as its cross-section; that is to say, if $l, l', l'' \ldots$ be the lengths of different wires employed, and $s, s', s'' \ldots$ their sectional areas, their resistances will be equal, if

$$\frac{l}{s}=\frac{l'}{s'}=\frac{l''}{s''} \ldots$$

This is on the supposition that the wires are all of precisely the same material. Every substance has its own specific resistance, the reciprocal of which is its electrical conductivity and is precisely analogous to thermal conductivity. Denoting specific resistances by $r, r', r'', \ldots$ the condition of equal resistances, when the materials are different, is

$$\frac{rl}{s}=\frac{r'l'}{s'}=\frac{r''l''}{s''} \ldots$$

and the resistance of any wire is expressed by the formula $\frac{rl}{s}$, l denoting its length, s its sectional area, and r the specific resistance of its material.

To express, in terms of the equivalent length of one wire, the resistance of a circuit composed of several, we can employ the relation

$$\frac{rl}{s}=\frac{r'l'}{s'};\ \text{whence } l=\frac{s}{s'}\,\frac{r'}{r}\,l'.$$

l denoting the length of one kind of wire equivalent to the length l' of the other. The length l is called the reduced length of the wire whose actual length is l'.

195. Fuller Statement.—The following more complete statement of Ohm's law will now be intelligible.

When a current is flowing steadily through a conductor which contains no source of electromotive force, it flows from sections of higher to sections of lower potential; and the *difference of potential between two sections is equal to the intervening resistance multiplied by the current.*

The resistance may vary with the temperature and other physical conditions of the conductor, but is independent of the strength of current, so that at a given temperature it varies in the exact ratio of the difference of potential of the two bounding sections. This proportionality has been tested with extreme rigour by experiments devised for the purpose by Maxwell.

In travelling round a complete circuit in the direction of the current, the *total fall of potential* computed by the above rule (in other words the *current multiplied by the whole resistance*) must be *equal to the whole electromotive force.*

196. Rheostat. Resistance Coils.—Wheatstone's *rheostat* was one of the earliest instruments contrived for the comparison of resistances. It consists (Fig. 120) of two cylinders, one of brass, and the other of non-conducting material, so arranged that a copper wire can be wound off the one on to the other by turning a handle. The surface of the non-conducting cylinder B has a screw-thread cut in it, for its whole length, in which the wire lies, so that its successive convolutions are well insulated from each other. Two binding-screws are provided for introducing the rheostat

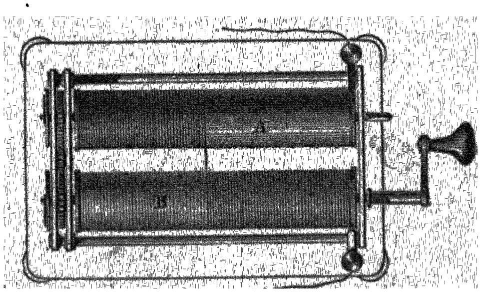


Fig 120.—Rheostat

into a circuit; and the resistance which is thus introduced depends on the length of wire which is wrapped upon the non-conducting cylinder, for the brass cylinder A has so large a section that its resistance may be neglected. The amount of resistance can thus be varied as gradually as we please by winding on and off. The handle can be shifted from one cylinder to the other. The figure shows it in the position for winding wire off A on to B. The number of convolutions of wire on B can be read off on a graduated bar provided for the purpose, and parts of a revolution are indicated on a circle at one end.

Fig. 121 represents a very direct mode of measuring resistances by the rheostat. The current traverses a galvanometer B, a rheostat R, and the conductor m, whose resistance is to be measured, the whole of the wire of the rheostat being wound on the brass cylinder. The deflection of the galvanometer having been observed, the conductor m is taken out of circuit, the two wires at a and b are directly connected, and as much of the rheostat wire is brought into circuit as suffices to reduce the deflection to its former amount.

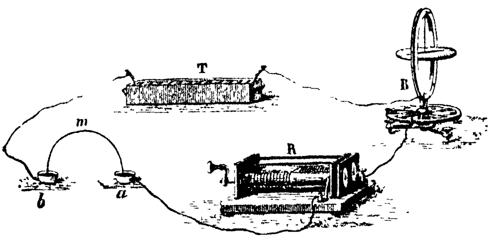


Fig. 121.—Measurement of Resistance.

Measurements of resistance are now usually made by comparison

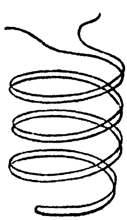


Fig. 122.—Resistance Coil.

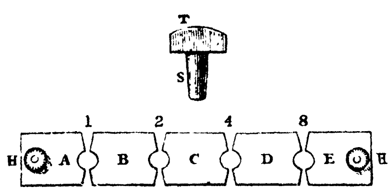


Fig. 123.—Mode of Connecting Resistance Coils

with standard coils of wire which at a certain specified temperature have known resistances. The most usual material for the wire is German silver, this being a metal whose resistance is comparatively little affected by temperature. The coil is always wound double, as in Fig. 122, because this arrangement prevents it from influencing

or being influenced by magnets and currents in the neighbourhood. In fact, the influences exerted or received by the two strands of the coil cancel one another.

The coils are usually fixed in a box in such a manner that, by combining separate coils, resistances of 1, 2, 3 or any integer number of *ohms* up to 5000 or 10,000 can be obtained. The mode of combining them will be understood by reference to Fig. 123.

A, B, C, D, E are stout plates of brass, which can be connected together by the insertion of plugs in the hollows between them. The plugs (one of which is shown in the figure) are of brass, with ebonite handles, and exactly fit the hollows, so as to give contact over a large surface. The ends of one coil are attached to A and B, the ends of the next to B and C, and so on, the resistances of the coils being marked in figures above them. When the hollow between two plates is open, the current can only pass from one to the other by going through the intervening coil; but when the plug is inserted, the resistance between the plates is inappreciable.

Hence the resistance between the two binding-screws H, H, attached to the extreme plates, will be the sum of the unplugged resistances.

197. Specific Resistances and Conductivities.—Numerous experimenters have compared the specific resistances of the different metals. Though the results thus obtained exhibit some diversity, they all agree in making silver, gold, and copper the three best conductors. Slight impurities, especially in the case of copper, have a very great effect in diminishing conductivity, or, in other words, in increasing resistance. Resistance is also increased, in the case of metals, by increase of temperature; but the opposite rule holds for insulators, such as gutta-percha and india-rubber.

The order of the metals as regards their conductivity for heat is nearly the same as for electricity. The effects of impurity and of change of temperature are also, upon the whole, alike in the two cases.

The following are E. Becquerel's determinations of specific electrical resistance at the temperature 15° C., the resistance of silver at 0° C. being denoted by 100:—

SPECIFIC RESISTANCES AT 15° C.

Silver,	107	Palladium,	715
Copper,	112	Iron,	825
Gold,	155	Lead,	1213
Cadmium,	407	Platinum,	1243
Zinc,	414	Mercury,	5550
Tin,	734		

On comparing these results with the thermal conductivities obtained by Wiedemann and Franz, it will be observed that the order is precisely the same as far as the comparison extends, and that the numerical values are nearly in inverse proportion, showing that electrical and thermal *conductivities* are nearly in direct proportion.

198. Resistance of Liquids.—In measuring the resistance of a liquid which is decomposed by a current, care must be taken to avoid complications arising from the reverse e.m.f. called out by the decomposition. The liquid may be contained in a cylindrical tube of glass, closed at both ends by metal plates, and having two small holes in its side near the ends, through which wires (which must both be of one metal) pass into the interior of the liquid, in a direction perpendicular to the length of the tube.

Let a measured current pass steadily through the whole column of liquid, the metallic plates at the ends serving as terminals; and let the difference of potential of the two wires be measured. This may be regarded as the difference of potential of the two cross sections in which the wires lie; and the resistance of the column of liquid between these cross sections will be equal to the difference of their potentials [redacted] by the strength of the current. Arrangements are sometimes made for including a variable and known resistance, such as a box of resistance coils, in the same circuit. When there is the same difference of potential between the ends of this known resistance as between the two wires above mentioned, the resistance of the liquid column between the cross sections is equal to the known resistance, and no measurement of current is necessary.

The resistance even of the best conducting liquids, except mercury, is enormously greater than that of metals. For instance, in round numbers, the resistance of dilute sulphuric acid is a million times, and that of solution of sulphate of copper ten million times greater than that of pure silver. The resistance of pure water is very much greater than either of these.

In the cells of a galvanic battery, the current has to traverse liquid conductors, and the resistance of these is sometimes a large part of the whole resistance in circuit. It is diminished by bringing the plates nearer together, and by increasing their size, since the former change involves diminution of length, and the latter increase of sectional area in the liquid conductor to be traversed. This is the only advantage of large plates over small ones, the electromotive force being the same for both. The advantage of the double coppers

in Wollaston's battery (§ 155) is similarly explained, the resistance with this arrangement being about half what it would be with copper on only one side of the zinc, at the same distance.

199. Choice of Galvanometer.—When stout wire is employed for a galvanometer coil the resistance is small, but it is not practicable to multiply convolutions to any great extent. Short coils of thick wire are accordingly employed in connection with thermopiles, the resistance in the pile itself being so small that the resistance of the galvanometer may be regarded as the total resistance in circuit.

When, on the other hand, the resistance in the other parts of the circuit is very great, the resistance of the galvanometer coil becomes comparatively immaterial, so that, within moderate limits, the force which deflects the needle is nearly proportional to the number of convolutions, and a coil composed of a great length of wire will give the maximum effect.

In both cases, for a given length and diameter of wire, the sensibility increases with the conductivity of the metal composing the wire. Copper is the metal universally employed, and its purity is of immense importance for purposes of delicacy, as impurities often increase its resistance by 50 or even 100 per cent.

200. Resistances in Series and in Parallel.—When two or more wires are connected *in series*, so that whatever flows through one must flow through all, *the resistance of the whole is the sum of the resistances of the wires composing it.*

On the other hand, when two or more wires are arranged *in parallel circuit*, so as to constitute so many independent channels of communication between the same two points, the joint resistance is evidently less than the resistance of any one of the wires. A circuit is said to be *divided* when such an arrangement occurs in any part of it, and the current is also said to be divided. Thus, in Fig. 124 the current from the positive source s to the negative source s' is divided between the two wires acb, adb, which connect the small mercury cups a, b.

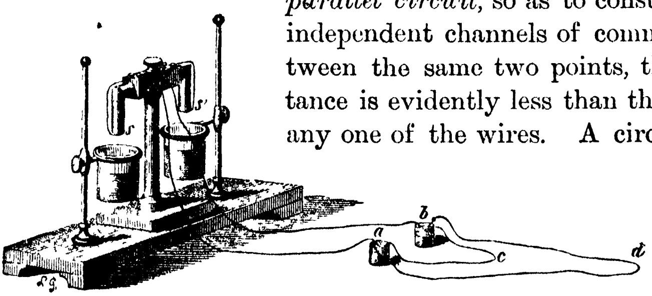


Fig. 124.—Divided Circuit.

Let E denote the difference of potential between the two points

a, b, which are thus joined, and r_1, r_2, &c., the resistances of the separate paths; then the currents through the separate paths will be $\frac{E}{r_1}$, $\frac{E}{r_2}$, &c. The total current between the two points is the sum of these, or $E\left(\frac{1}{r_1}+\frac{1}{r_2}+\&c.\right)$, and this must be equal to $\frac{E}{R}$, where R denotes the resistance of a single wire equivalent to the system. Hence we have

$$\frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}+\&c.;$$

or, *the reciprocal of the joint resistance is the sum of the reciprocals of the separate resistances.*

201. Currents in Parallel Branches. Shunts.—If two points, A, B, are connected by any number of parallel branches, in which the currents are i_1, i_2, &c., and the resistances r_1, r_2, &c., we have

$$i_1 r_1 = i_2 r_2 = \&c.;$$

each of these equal quantities being the difference of potential between A and B. Hence the currents through parallel branches are inversely as the resistances.

If there are 3 branches, the current in the first branch will be to the whole current as $\frac{1}{r_1}$ to $\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$. If there are only 2 branches, the ratio will be $\frac{1}{r_1}$ to $\frac{1}{r_1}+\frac{1}{r_2}$, or r_2 to r_2+r_1.

One of the two branches of a divided circuit is often called a *shunt*, and a portion of the whole current is said to be shunted through it. Thus, a delicate galvanometer is often accompanied by an adjustable shunt, through which the larger part of the whole current can be diverted, in order to avoid injury to the galvanometer. The shunt usually contains three resistances, any one of which can be brought into use at pleasure by inserting a brass plug in the proper hole. They are usually $\frac{1}{9}$, $\frac{1}{99}$, and $\frac{1}{999}$ of the resistance of the galvanometer. Putting $r_2=\frac{1}{999}r_1$, we shall have $\frac{i_1}{i_1+i_2}$ $=\frac{\frac{1}{999}}{1+\frac{1}{999}}=\frac{1}{1000}$. Thus the galvanometer will only get $\frac{1}{1000}$ of the whole current. With the plug in one of the other holes the galvanometer would get $\frac{1}{10}$ or $\frac{1}{100}$ of the whole current.

202. Grouping of Cells in Battery.—Suppose that we have a number n of precisely similar cells, each having electromotive force e and resistance r, and that we connect them in a series, as in Figs. 90, 101, with a conductor of resistance R joining their poles. The whole electromotive force in the circuit will then be $n\,e$, and the

whole resistance will be $nr+R$; hence the strength of current will be

$$C=\frac{ne}{nr+R}.$$

This formula shows that, if the external resistance R is much greater than the resistance in the battery nr, any change in the number of cells will produce a nearly proportional change in the current; but that when the external resistance is much less than that of one cell, as is the case when the poles are connected by a short thick wire, a change in the number of cells affects numerator and denominator almost alike, and produces no sensible change in the current. It is impossible, by connecting any number of similar cells *in a series*, to obtain a current exceeding $\frac{e}{r}$, which is precisely the current which one of the cells would give alone if its plates were well connected by a short thick wire.

It is possible, however, by a different arrangement of the cells, to obtain a current about n times as strong as this, namely, by connecting all the zinc plates to one end of a conductor, and all the

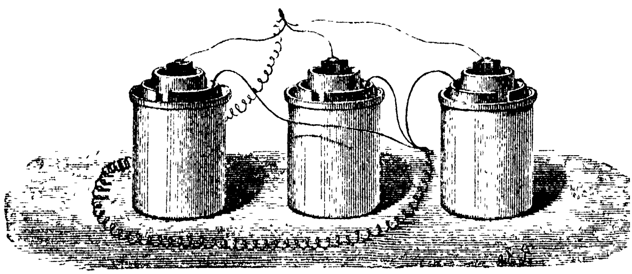


Fig 125 —Cells with similar Plates connected

carbons or coppers to the other end, as in Fig. 125. In the arrangement of three cells here figured, the current which passes through the spiral connecting wire is the sum of the currents which the three cells would give separately. The arrangement is equivalent to a single cell with plates three times as large superficially, and at the same distance apart. The electromotive force with n cells so arranged is simply e, but the resistance is only $\frac{r}{n}+R$, so that the current is

$$C=\frac{e}{\frac{r}{n}+R}=\frac{ne}{r+nR}.$$

This system of arrangement may be called *arranging the cells as*

one element, or arranging them *in parallel*. It has sometimes been called the *arrangement for quantity*, the arrangement in a series being called the *arrangement for intensity*.

If in Fig. 125 we substitute for each of the three cells a *series* consisting of four cells, the electromotive force in circuit will be $4e$, and the resistance in circuit will be $\frac{4r}{3}+R$, for each series has a resistance of $4r$, and three parallel series connected at the ends are equivalent to a single series, of the same electromotive force as one of the component series, and of one-third the resistance. The current will therefore be

$$C=\frac{4e}{\frac{4r}{3}+R}=\frac{12e}{4r+3R}=\frac{e}{\frac{r}{3}+\frac{R}{4}}.$$

The question often arises, What is the best manner of grouping a given number of cells in order to give the strongest possible current through a given external conductor? The answer is, they should be so grouped that the internal and external resistance should be as nearly as possible equal; for example, if we have 12 cells as above, and the resistance R in the given conductor is $\frac{4}{3}$ of the resistance of one of these cells, the arrangement just described is the best.[1]

203. Distribution of Potential in a Voltaic Circuit.—When the electrodes of a battery are not connected, their difference of potential, supposing them to be of the same metal, is a measure of the electromotive force of the battery. On joining them by a connecting wire, their difference of potential will be diminished, and will (§ 195) be the same fraction of the whole electromotive force that the resistance in the connecting wire is of the whole resistance.

In a battery of four cells, like that represented in Fig. 90, when the extreme plates are connected by a wire whose resistance is double that of the battery, the fall of potential in the connecting wire will be two-thirds, and the fall of potential in the battery will be one-third, of the whole electromotive force. To avoid fractions,

[1] Instead of 3 and 4, put x for the number of series, and y for the number of cells in a series. Then the current will be $\frac{e}{\frac{r}{x}+\frac{R}{y}}$, and will vary inversely as $\frac{r}{x}+\frac{R}{y}$. Now the product of $\frac{r}{x}$ and $\frac{R}{y}$ is given, being the quotient of rR by the whole number of cells; and when the product of two variables is given, their sum is least when they are equal, and increases as they are made more and more unequal. As x and y must be integers, exact equality cannot generally be obtained.

let the electromotive force of each cell be denoted by 3. Then the total electromotive force will be 12, the fall of potential in the connecting wire will be 8, in the battery 4, and in each cell 1.

The potential, as indicated by an electrometer connected by a copper wire with different parts of the circuit in succession, will have the under-mentioned values, according as the circuit is open or closed. The quantity x may have any value whatever, either positive or negative, but will have the same value for the whole circuit. It will be zero if the negative terminal of the battery is to earth. If any one point of the circuit is to earth, the potential of this point will be zero, and the value of x will thus be determined,

Open (Break being at positive terminal)		Closed	
Connecting wire,	x	Connecting wire,	$x+8$ to x
1st Zinc,	x	1st Zinc,	x
Acid,	$x+3$	Acid,	$x+3$ to $x+2$
1st Copper,	$x+3$	1st Copper,	$x+2$
2d Zinc,	$x+3$	2d Zinc,	$x+2$
Acid,	$x+6$	Acid,	$x+5$ to $x+4$
2d Copper,	$x+6$	2d Copper,	$x+4$
3d Zinc,	$x+6$	3d Zinc,	$x+4$
Acid,	$x+9$	Acid,	$x+7$ to $x+6$
3d Copper,	$x+9$	3d Copper,	$x+6$
4th Zinc,	$x+9$	4th Zinc,	$x+6$
Acid,	$x+12$	Acid,	$x+9$ to $x+8$
4th Copper,	$x+12$	4th Copper,	$x+8$

The distribution of potential in the closed circuit is graphically represented by the crooked line A 3 2 5 4 7 6 9 C (Fig. 126); resistances being represented by horizontal, and potentials by vertical distances. A C represents the total resistance in circuit; A B being the resistance of the battery, and B C that of the connecting wire. A D represents the total electromotive force. The points C and A are to be regarded as identical; in other words, the diagram ought to be bent round a cylinder so as to make one of these points fall upon the other.

204. Measurement of Resistance of Battery.—The resistance of a battery may be measured in various ways, of which we shall begin with the simplest.

Let the poles of the battery be directly connected with a galvanometer whose resistance is either very small or accurately known, and let the deflection be noted. Then let a wire of known resistance be introduced into the circuit, and the deflection again noted. The two

currents thus measured will be inversely as the resistances, since the electromotive force is the same in both cases. Let the resistance of the galvanometer coil and connections be denoted by G, that of the wire introduced in the second case by W, and that of the battery by x. Then if the amounts of current be denoted by C_1, C_2, we have $\frac{C_1}{C_2} = \frac{x+G+W}{x+G}$; whence x can be determined.

This mode of determination is not very accurate; in the first place, because the electromotive force of a battery is not a constant

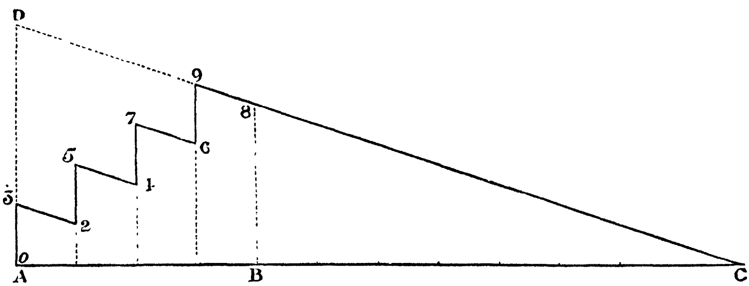


Fig. 126.—Curve of Potential for Closed Circuit.

quantity but usually diminishes as the current increases; and secondly, because the measurement of a current by a galvanometer is not a very exact operation. Some better methods will be described later.

205. Measurement of Electromotive Force.—The most direct mode of comparing the electromotive forces of cells of different kinds, would be to observe how many cells of the one kind arranged in series must be opposed to a given number of the other kind, in order that the resultant electromotive force may be nil as indicated by the absence of deflection in a galvanometer forming part of the circuit. For example, if two Daniell's cells and one Grove's cell be connected with each other and with a galvanometer, in such a manner that the current due to the Daniell is in one direction, and that due to the Grove is in the opposite direction, the current actually produced will be in the direction of the greater electromotive force. It will thus be shown whether the electromotive force of a Grove's cell is more or less than double that of a Daniell's. This method has not been much used.

Another method of comparison consists in first connecting the two cells to be compared, so that their electromotive forces tend the same

way, and then again connecting them so that they tend opposite ways, the resultant current being observed in both cases with the same galvanometer. The resistance in circuit is the same in both cases, being the resistance of the galvanometer plus the sum of the resistances of the cells; hence the currents will be simply as the electromotive forces, that is to say, as E_1+E_2 to E_1-E_2, if E_1 and E_2 denote the electromotive forces of the cells. Hence the ratio of E_1 to E_2 is easily computed.

This method is liable to the objection that increase of current gives increase of polarization (§ 157) and consequent diminution of electromotive force; besides the objection that the measurement depends upon the reduction of the indications of a galvanometer to proportional measure. The first objection can be obviated by introducing large resistance into the circuit so as to render the currents feeble.

Another and very effective method is to employ a galvanometer whose coil is so long and fine that its resistance is hundreds of times greater than that of the cell to be tested. The current given by a cell will then be practically independent of the resistance of the cell, and will be proportional simply to its e.m.f. Thomson's "potential galvanometer" is intended for this purpose, and gives the e.m.f. in absolute measure.

206. Determination by Electrometer.—The statical electromotive force of a battery or cell (that is, its electromotive force when no current is passing, the poles being disconnected) can be directly observed by connecting its poles to opposite quadrants of Thomson's electrometer; for the instrument will then show the difference of potential between them, and this difference of potential is the electromotive force.

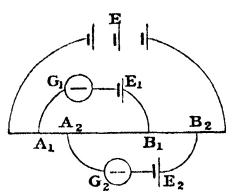


Fig. 127 —Latimer Clark's Method

207. Latimer Clark's Potentiometer.—Another mode of statically comparing electromotive forces is illustrated by Fig. 127. A battery of greater electromotive force than either of those which are to be compared is employed to send a current round a circuit containing a long uniform wire. By Ohm's law, the difference of potential between two points in this wire is proportional to the intervening length. In the method now to be explained we find two points in it whose difference of potential is equal to the statical difference of

potential between the poles of one battery; and also two points whose difference of potential is that of the other battery. Then the ratio of the electromotive forces will be the ratio of the resistances between these points, which is known. E_1 and E_2 in the figure are the two batteries which are to be compared, and E is the third battery, more powerful than either, which, alone of the three, gives a current when the adjustments are complete. G_1 is a galvanometer which shows what current is passing through the battery E_1, and this current is to be reduced to zero by properly choosing the points A_1 B_1. In like manner the current through E_2 and G_2 is to be reduced to zero by proper choice of the points A_2 B_2. Then the electromotive force of E_1 is to that of E_2 as the resistance A_1 B_1 is to the resistance A_2 B_2, and these are simply as their lengths.

The points A_1 A_2 may coincide with each other and with one end of the uniform wire.

In the *Potentiometer* designed by Latimer Clark for carrying out this principle, the uniform wire is coiled in a helical groove on the surface of a non-conducting cylinder, and is made to travel past a contact-maker by turning a handle.

By this method Mr. Clark has found that the statical electromotive forces of a cell of Grove, Bunsen, Daniell, and Wollaston are approximately as 100, 98, 56, and 46. The last of these, being a one-fluid battery, is liable to fall off 50 per cent or more when in action, owing to the deposition of hydrogen on the copper plate.

Mr. Clark has designed a standard cell which is largely employed for comparisons of e.m.f. by statical methods such as the above. "It is formed by employing pure mercury as the negative element, the mercury being covered by a paste made by boiling mercurous sulphate in a thoroughly saturated solution of zinc sulphate, the positive element consisting of pure distilled zinc resting on the paste." It must not be used for producing a current; but its statical electromotive force is very constant and permanent.

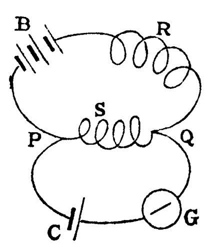


Fig. 128.

208. Bosscha's Method.—The following method gives the electromotive force of a battery through which a current is passing; and also gives the resistance of the battery.

B (Fig. 128) is the battery, which sends a current through the circuit P S Q R, containing two variable resistances, R and S. The

resistance S is to be varied until the difference of potential between its ends P and Q is equal to the statical difference of potential between the poles of a standard cell C, as shown by the absence of a current through the galvanometer G.

R is then to be increased by any convenient amount r, and S is to be increased by an amount s such that the galvanometer again indicates no current.

Then if e denote the statical electromotive force of the standard cell, and E the electromotive force of the battery B during the experiment, the strength of the current through S in the first observation is $\frac{e}{S}$, and is also $\frac{E}{B+S+R}$. We may therefore equate these two expressions, and by alternation we have

$$\frac{E}{e}=\frac{B+S+R}{S}. \tag{1}$$

Similarly, from the second experiment, we have

$$\frac{E}{e}=\frac{B+S+s+R+r}{S+s}. \tag{2}$$

By taking the differences of numerators and denominators we deduce

$$\frac{E}{e}=\frac{s+r}{s}, \tag{3}$$

an equation which determines the value of E, since e, s, and r are known.

Again by equating the right-hand members of (1) and (3) and subtracting unity, we obtain $\frac{B+R}{S}=\frac{r}{s}$, whence $B=\frac{Sr}{s}-R$; thus the resistance B of the battery is determined. It is advantageous to make R zero.

The fundamental idea which underlies these last two methods—the idea of measuring the resistance of a battery by finding two points in a circuit such that no current flows through the battery when its terminals are connected with them—was first suggested and carried out by Poggendorff.

209. Wheatstone's Bridge.—The method usually employed for measuring the resistances of wires was first contrived by Christie, but became generally known through a famous paper of Wheatstone's in the *Philosophical Transactions* (in which due credit is given to Christie); and the special instrument employed in it is called Wheatstone's Bridge. It depends on the principle that in two portions of a conductor traversed by the same current the falls of potential are proportional to the resistances.

The poles P, N of a battery (Fig. 129) are connected by two independent channels of communication A C B, A D J E B. The former is a uniform wire: the latter consists of the wire D, whose resistance is to be determined, and of a standard resistance-coil E.

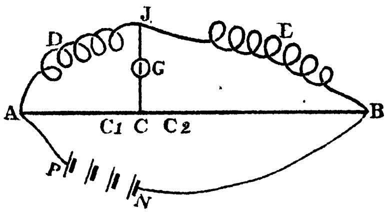


Fig 129.—Wheatstone's Bridge.

The observation has for its immediate object to find what point in the uniform wire A B has the same potential as the junction J of the other two. When this point C is found, and connected with J through a galvanometer G, no current will pass across, and the needle of the galvanometer will not move. If a point C_1 on the positive side of C were connected with J, a current would run from C_1 to J, and if a point C_2 on the negative side were connected, the current would be from J to C_2. The deflection diminishes as the right point C is approached, and becomes reversed in passing it. When it is found we know that the resistances in A C and C B have the same ratio as those of D and E, each of those ratios being in fact equal to the fall of potential between A and J C divided by the fall between J C and B. As the resistance of E is known, and the resistances of A C, C B are as their lengths, which are indicated on a divided scale, the resistance of D can be computed by simple proportion.

In Wheatstone's original arrangement, the resistances of the two portions A C, C B were equal, and the resistances of the other two portions A D J, J E B were made equal by the help of a rheostat.

210. Conjugate Branches.—Wheatstone's bridge may be otherwise described as consisting of six branches connecting four points, two and two, in every possible way, the four points being A, B, C, J in Fig. 129. A battery is inserted in the branch which connects two of these points, A and B, and a galvanometer is inserted in the branch which connects the other two, C and J. These two branches may be called *opposite*, and in like manner A C is opposite to B J, and B C to A J. The condition of no current going through the galvanometer is expressed in § 209 as a proportion. Multiplying extremes and means, and writing the names of the branches for the resistances in them, the condition is

$$AC \,.\, BJ = BC \,.\, AJ,$$

where each member of the equation is the product of the resistances in opposite branches. When this condition is fulfilled, the remaining pair of opposite branches A B and C J are conjugate, that is to say, a battery in one produces no current in the other. The symmetry of the relations shows that the battery may change places with the galvanometer.

211. Loop Test.—The following method of finding the position of a fault in a telegraph wire is an application of the principle of Wheatstone's bridge. We suppose the fault to consist in loss of insulation at some point of the wire, so that the resistance between this point and the ground is much less than it ought to be, though it may still be as great as that of some miles of wire.

The fault is known to be between two given stations. At one of these stations let the end of the faulty wire be joined to the end

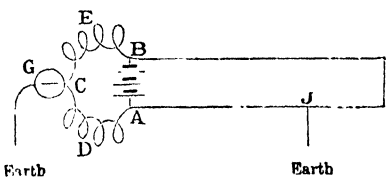


Fig. 130.—Loop Test.

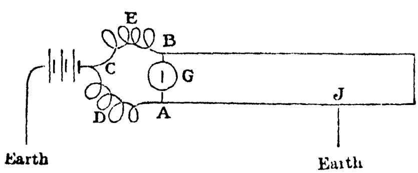


Fig. 131.—Loop Test

of another wire; and at the other station let the ends A, B of the same two wires be put in connection with the two poles of a battery (Fig. 130). Also let A and B be connected by a circuit containing two variable resistances D, E, and let an intermediate point C be connected, through a galvanometer G, with the ground. Let the resistances D, E be made such that no current goes through the galvanometer. Then we know that the point C has the same potential as the earth, and in these circumstances the faulty point J of the wire will also be at the potential of the earth; for if there were a current flowing through the fault to the earth the battery would be steadily giving off electricity of one sign while having no outlet for electricity of the opposite sign, and this cannot be. The points J and C are therefore at the same potential, like the points J and C in Fig. 129; and a comparison of the two figures shows that the same reasoning applies to both. The loop AJB formed by the two telegraph wires is therefore divided by the point J in the ratio of the two known resistances D and E. That is, we have $\frac{\text{resistance of A J}}{\text{resistance of B J}} = \frac{D}{E}$. This determines the position of the point J.

The positions of the battery and galvanometer may be interchanged, as in Fig. 131, and the equation above obtained will still apply; for when no current flows through the galvanometer in this new arrangement, the two paths C D A J and C E B J, which lead from C to J, must be divided proportionally at A and B.

212. Usual Forms of Bridge.—One of the commonest forms of Wheatstone's bridge is that known as the *metre bridge*. Its construction will be understood from Fig. 132.

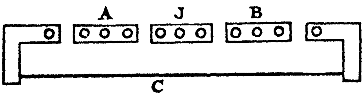


Fig. 132.—Metre Bridge.

A uniform wire of platinum, a metre long, has its ends attached to two stout strips of copper. These can be connected by other similar strips so as to form a closed circuit; but gaps can be left at four places when desired. The small circles represent binding-screws which can be used either for attaching the ends of wires or for holding short strips of copper to fill the gaps. A sliding piece not shown in the figure strides across the wire, and carries a binding-screw attached to a key, by depressing which the binding-screw is put in connection with a point of the wire. The position of this point is read off on a metre scale divided to millimetres, along which the sliding piece travels.

The wires from the battery are attached to A and B, and the wires from the galvanometer to J and the sliding piece, which we will suppose makes contact at C. In the ordinary use of the bridge the two outer gaps are filled up by their copper strips, and the two resistances to be compared are inserted in the two inner gaps. When the sliding piece has been adjusted so as to give a balance, the resistances in the two inner gaps are proportional to the lengths of the two portions into which the point C divides the uniform wire; the resistances of the copper strips being negligible unless C is very close to one end. The two lengths are read off on the metre scale.

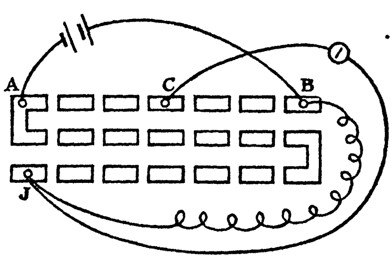


Fig 133.

Fig. 133 shows the commonest arrangement of a box of resistance coils intended to be used as a Wheatstone's bridge.

The coils, which are out of sight in the interior of the box, and

their brass connecting pieces, which are visible at the top, are arranged in three rows, so connected at the ends as to form a single series of which B and J are the terminals. The actual forms of the brass connecting pieces were described and figured in § 196. We now indicate them merely by rectangles with gaps between. There are single binding-screws at A and C, and double binding-screws at B and J.

The wire whose resistance is required is connected to B and J. As regards the battery and galvanometer, one must be connected to A and B, and the other to C and J. Their positions may be interchanged.

The relations of the parts are best understood by regarding A, B, C, J as four points connected by six branches. The branches A B and C J are to be conjugate. This requires that the product of A C and B J shall equal the product of B C and A J. In practice, the resistance A C is either made equal to B C or 10 or 100 times B C. The required resistance B J will then be either equal to A J or one-tenth or one-hundredth of A J. A J can be adjusted to be any integer number of ohms, as described in § 196.

213. Foster's Method.—The following method, due to Professor Carey Foster, is much used for measuring the small differences between standard coils.

The two coils whose difference is required are inserted in the two *outer* gaps of the metre bridge, and two other resistances nearly equal to one another are inserted in the two inner gaps. The position of C for balance is observed. Call it C_1.

The two coils to be compared are then interchanged, and the new position of C is found. Call it C_2.

In the notation of § 210, we have

$$\frac{AJ}{JB}=\frac{AC}{CB}; \text{ whence } \frac{AJ}{AJ+JB}=\frac{AC}{AC+CB}.$$

But A J and J B are unchanged, and the changes in A C and C B do not affect their sum. Hence A C is unchanged in magnitude; that is, the difference of the two coils which have been inserted in the gap in A C is equal to the resistance of the portion C_1 C_2 of the metre wire.

214. Network of Conductors. Kirchhoff's Laws.—In investigating the steady currents in a network of conductors when the resistances and electromotive forces are given, the two following obvious principles, which are known as Kirchhoff's laws, will give the necessary equations.

I. *At any point of meeting of different branches, the algebraic sum of the currents is zero, opposite signs being attributed to inward and outward currents.*

II. *In travelling round any portion that forms a closed circuit, the algebraic sum of the electromotive forces is equal to the algebraic sum of the products of current by resistance.*

The first law merely expresses that there is no accumulation (positive or negative) going on at the point.

The second law expresses that the total increase of potential in travelling round the closed circuit is zero. A positive e.m.f. is a sudden rise. A positive product of current by resistance is a gradual fall.

215.—The following is an example (see Fig. 134):—

A coil of resistance R connects the terminals of a battery of resistance R_1 and electromotive force E_1, and also connects the terminals of a second battery R_2, E_2, like terminals being connected to the same end of the coil. Required the currents in the system.

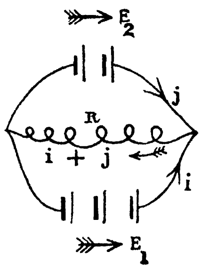


Fig. 134.

Let i denote the current through the first battery, reckoned positive when it has the same direction as E_1; and j the current through the second battery, reckoned positive when it has the same direction as E_2. Then the first law shows that the current through the coil will be $i+j$, reckoned positive when it has the direction due to E_1 and E_2. The second law, applied to the circuit formed of the first battery and the coil, gives

$$E_1 = iR_1 + (i+j)R,$$

and for the circuit formed of second battery and coil,

$$E_2 = jR_2 + (i+j)R.$$

These two equations suffice to give i and j. A third equation might be written down for the circuit formed of the two batteries; but it would be an equation derivable from the other two.

Solving the equations, we find

$$i = \frac{(E_1 - E_2)R + E_1R_2}{RR_1 + RR_2 + R_1R_2}, \quad j = \frac{(E_2 - E_1)R + E_2R_1}{RR_1 + RR_2 + R_1R_2}, \quad i + j = \frac{E_1R_2 + E_2R_1}{RR_1 + RR_2 + R_1R_2}.$$

Supposing E_1 to be greater than E_2 the expression for j shows that the current through the second battery will be reversed if E_2 is less than $E_1R/(R + R_1)$.

216. Conjugate Branches when there are Several Batteries.—When

there are batteries in more branches than one, the current in any branch will be the algebraical sum of the currents due to the several batteries considered separately. Hence when there is equality between the two products of opposite resistances, as in last section, the current in either of the two remaining branches will be independent of the electromotive force of the battery in the other; and these two branches are still said to be conjugate. In estimating the resistance of any branch which contains a battery, the resistance of the battery must of course be included.

Thus far we have not discussed the effect of change of resistance in one of two conjugate branches. The introduction of additional resistance into any branch can affect the current in the rest only by altering the difference of potentials between the ends of this branch; and the same remark applies to the introduction of a source of electromotive force into any branch. Two changes, one of resistance, and the other of electromotive force, in a branch, will have the same effect on the rest of the circuit, if they have the same effect on the difference of potentials of the ends of this branch. Hence if the current in one of two branches be independent of the electromotive force in the other, it must also be independent of the resistance in the other.

As this reasoning may appear doubtful to some of our readers, we subjoin a formal investigation leading to the same result.

217. Investigation of Condition of Conjugateness.—Let A, B, C, J (Fig. 129 or Fig. 135), be four points connected, two and two, by six branches. Let the resistance in the branch connecting A and B be denoted by A B or B A, and the electromotive force in it (positive if tending from A to B) by ab. Let the current in this branch (positive if from A to B) be denoted by γ, and the currents in the branches BC, CA, by α, β. Then the current in the branch A J (positive if from A to J) will be $\beta-\gamma$;

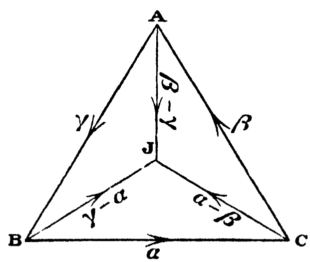


Fig 135.—Theory of Conjugate Branches.

for this current together with γ carries off from A the supply brought by . Similarly, the currents in B J, C J, will be $\gamma-\alpha$, $\alpha-\beta$. Then, since the sum of the falls of potential in travelling round a circuit must equal the sum of the rises, we have, for the circuit J B C, the equation

$$\alpha . \mathrm{BC}+(\alpha-\beta)\,\mathrm{CJ}+(\alpha-\gamma)\,\mathrm{JB}=bc+cj+jb.$$

Similarly, for the circuits J C A, J A B, we have

$$\beta . \mathrm{CA}+(\beta-\gamma)\,\mathrm{AJ}+(\beta-\alpha)\,\mathrm{JC}=ca+aj+jc$$
$$\gamma . \mathrm{AB}+(\gamma-\alpha)\,\mathrm{BJ}+(\gamma-\beta)\,\mathrm{JA}=ab+bj+ja.$$

These three equations are sufficient to determine the three currents α, β, γ, in terms of the electromotive forces and resistances. Multiplying the equations in order, by 1, l, m (l and m being multipliers to be afterwards determined), and adding; the coefficients of α, β, γ will be

$$\mathrm{BC}+(1-l)\,\mathrm{CJ}+(1-m)\,\mathrm{BJ},$$
$$l . \mathrm{CA}+(l-m)\,\mathrm{AJ}+(l-1)\,\mathrm{CJ},$$
$$m . \mathrm{AB}+(m-1)\,\mathrm{BJ}+(m-l)\,\mathrm{AJ},$$

and the second member of the equation will be

$$bc+l . ca+m . ab+(l-m)\,aj+(m-1)\,bj+(1-l)\,cj.$$

To find the value of α, we must equate the coefficients of β and γ to zero, and then divide the second member by the coefficient of α. The electromotive force aj appears only in the term $(l-m)\,aj$, and the resistance AJ only in the terms $(l-m)\,\mathrm{AJ}$ and $(m-l)\,\mathrm{AJ}$. Hence the equality of l to m is the condition alike of the disappearance of aj and of AJ. Putting $l=m$, and equating the coefficients of β and γ to zero, we have

$$l=\frac{\mathrm{CJ}}{\mathrm{CA}+\mathrm{CJ}}=\frac{\mathrm{BJ}}{\mathrm{AB}+\mathrm{BJ}},$$

whence

$$\mathrm{CJ} . \mathrm{AB}=\mathrm{BJ} . \mathrm{CA},$$

which is, accordingly, the condition of conjugateness. That is to say, *if the product of one pair of opposite resistances be equal to the product of another pair, the remaining pair of branches will be so related that the current in each is independent of the electromotive force and resistance in the other.*

218. Thomson's Method of Measuring the Resistance of a Galvanometer.—The resistance of a galvanometer can be measured without the use of another galvanometer, by the following method due to Sir Wm. Thomson.

In a system of six branches joining four points, let a battery and a contact key respectively be in one pair of opposite branches. Then, if the products of the resistances in opposite branches be equal for the four remaining pairs, we know by § 210 that no current will pass through the branch containing the key, and hence making or breaking contact with the key will be nugatory; hence the galvano-

meter will not have its deflection altered by making or breaking contact with the key. The experiment is to be conducted by altering the resistance in one of the branches until the key has no effect on the galvanometer. The resistance of the galvanometer is then calculated from the equality of the products of opposite pairs.

This method was suggested by the following.

219. Mance's Method of Finding the Resistance of a Battery.— In this method a galvanometer and a contact key are in a pair of opposite branches, and the battery is in one of the four remaining branches, while the other three contain known resistances. The observation is made by varying one of these resistances till the galvanometer is not affected by the key. The branches containing the key and the galvanometer are then conjugate (and the resistance of the battery can be calculated), if the putting down of the key does not alter the electromotive force of the battery. This condition is seldom fulfilled.

In this method, as well as in that described in the preceding section, the galvanometer does not stand at zero, but shows a steady deflection, which is unaltered by opening or closing the branch containing the key.

CHAPTER XVII.

RELATIONS BETWEEN ELECTRICITY AND HEAT.

220. Heating of Wires.—The heating of a wire by the passage of a current may conveniently be exhibited by the aid of the apparatus represented in Fig. 136. Two uprights mounted on a stand are furnished, at different heights, with pairs of insulated binding-screws $a\,a'$, $b\,b'$, $c\,c'$, having wires stretched between them. A current can thus be sent through any one of the wires, by connecting the terminals of a battery with the binding-screws at its extremities. When this is done with a battery of suitable power, the wire is first seen to droop in consequence of expansion, then to redden, and finally to melt, becoming inflamed if the metal is sufficiently combustible.

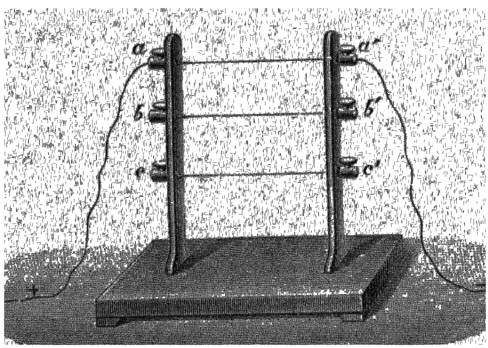


Fig 136 —Stand for Heating Wires.

If a file is attached to one of the terminals of a battery, and the other terminal is drawn along the file, a rapid succession of sparks will be obtained; and if the battery be sufficiently powerful, globules of incandescent metal will be scattered about with brilliant effect.

221. Joule's Law.—The energy of a current is equal to the product of the quantity of electricity that passes and the electro-motive force that drives it. As the numerical measure of a current is the quantity of electricity which passes in unit time, it follows that the energy of a current C lasting for a time t, is E Ct, E denoting the electro-motive

force. But again, by Ohm's law, E is equal to the product of the current C and the whole resistance R. The expression for the energy therefore becomes

$$C^2 R t, \qquad (1)$$

and this energy is all transformed into heat in the circuit, subject to a small correction for the Peltier and Thomson effects which will be described in a later section (§ 228). It has accordingly been found, first by Joule, and afterwards by Lenz, Becquerel, and others, that the formula C^2Rt represents the quantity of heat generated by a current under ordinary circumstances. The experiments have usually been conducted by passing a current through a spiral of wire immersed in water or alcohol, and observing the elevation of temperature of the liquid.

This law of Joule's, like that of Ohm, may be applied to any part of a circuit, as well as to the circuit considered as a whole; that is to say, if the circuit consists of parts whose resistances are $r_1, r_2, \ldots,$ the quantities of heat generated in them are respectively $C^2 r_1 t$, $C^2 r_2 t, \ldots,$ and are therefore proportional to the resistances $r_1, r_2 \ldots$ of the respective parts, since C and t are necessarily the same for all.

222. Relation of Heat in Circuit to Chemical Action in Battery.—The energy of a current, and consequently the heat developed in the circuit, is the exact equivalent of the potential energy of chemical affinity which runs down in the cells of the battery. This fact, first verified approximately by Joule, has been more accurately confirmed by the experiments of Favre, who introduced into the muffle of his mercurial calorimeter, described and figured in Part II., a small voltaic cell with its poles connected by a fine wire. He found that the consumption of 33 grammes of zinc in the cell corresponded to a generation of heat amounting to 18,796 gramme-degrees. But the chemical action in the cell is complex. The 33 grammes of zinc unite with 8 grammes of oxygen, and in so doing generate 42,451 gramme-degrees. The combination of these 41 grammes of oxide of zinc with 40 grammes of sulphuric acid, produces 10,456 gramme-degrees, making in all 52,907. But an equivalent of water undergoes decomposition, and this *absorbs* 34,463, which must be subtracted from the above sum, leaving 18,444 gramme-degrees as the balance of heat generated in the whole complex action. The heat actually observed in the experiment agrees almost precisely with this calculated amount. (Compare § 184.)

223. Distribution of Heat in Circuit.—These experiments also served to verify the application of Joule's law to each part of the circuit considered separately. By introducing the cell into the muffle whilst a spiral of fine wire connecting the poles was outside, and then introducing the spiral while the cell was outside, Favre was able to measure separately the heat generated in the cell and in the spiral, and these were found to be proportional to their resistances.

If wires of different diameter or of different electrical conductivity form parts of the same circuit, so as to be traversed by the same current, the bad conductors will become more heated than the good, and the fine wires more than the coarse. All parts of the length of a uniform wire will be uniformly heated. The specific resistance of platinum is ten times greater than that of copper; hence ten times as much heat will be generated in a platinum as in a copper wire by a given current, if the diameters of the two wires be the same.

The *elevation of temperature* is greater in a fine than in a coarse wire, not only because of its greater resistance, which leads to the development of a greater quantity of heat in it, but also on account of its smaller capacity for heat, and its smaller surface. When the current is passed for so short a time that the heat emitted may be neglected, the elevation of temperature varies directly as the resistance per unit length, and inversely as the capacity per unit length. The resistance varies inversely and the capacity directly as the section of the wire, and hence the elevation of temperature is inversely as the square of the section, or as the fourth power of the diameter.

On the other hand, if the current be continued till the permanent temperature is attained, capacity ceases to have any influence, and the heat emitted in unit time must be equal to the heat received. If x denote the elevation of temperature, the heat emitted is approximately $2\ \pi\ r\ B\ x$ by Newton's law, r being the radius of the wire, and B a constant. The heat received is $\frac{A}{\pi r^2}$, A being another constant. By equating these two expressions, we find that $r^3 x$ is equal to a constant, and hence x varies inversely as r^3, that is, the elevation of temperature is inversely as the cube of the diameter.

To obtain the most rapid production of heat in the circuit considered as a whole, we must reduce the resistance to a minimum; for the heat produced in unit time is EC, which, by Ohm's law, is the same as $\frac{E^2}{R}$, and therefore varies inversely as R the total resistance.

224. Mechanical Work done by Current.—Favre's experiments also furnished a confirmation of the fact, that when a current is called upon to perform mechanical work, the amount of heat generated in the circuit is diminished by the equivalent of this work. He inclosed a battery of five cells in the muffle of one calorimeter, and an electro-magnet in another calorimeter; the connections between the coil of the electro-magnet and the poles of the battery being made by short thick wires whose resistance could be neglected. The electro-magnet attracted an armature, and thus raised a weight by means of external pulleys.

It was found that when the armature was fixed, so that no mechanical work could be performed, the heat developed was the precise equivalent of the chemical action which took place in the battery; but when the electro-magnet was allowed to raise the weight, the amount of heat indicated by the calorimeters was sensibly less. The difference was measured, and compared with the work done in raising the weight. The comparison indicated 444 kilogrammetres of work for each kilogramme-degree of heat that disappeared, a result which agrees sufficiently well with the established value of Joule's equivalent (425 kilogrammetres).

225. Thermo-electric Currents.—Electric currents can be produced by applying heat or cold to one of the junctions in a circuit composed of two different metals. This was first shown by Seebeck of Berlin in 1821. It may be illustrated by employing a rectangular frame (Fig. 137), having three sides formed of a copper plate, and the fourth of a cylinder of bismuth. It must be placed in the magnetic meridian, with a magnetized needle in its interior. On heating one of the junctions with a spirit-lamp, the needle will be deflected in such a direction as to indicate the existence of a current, which in the copper portion of the circuit, flows from the hot to the cold junction, and in the bismuth portion from the cold to the hot. If cold instead of heat be applied to one junction, the direction of the current will still be from the warmer junction through the copper to the colder junction, and from this through the

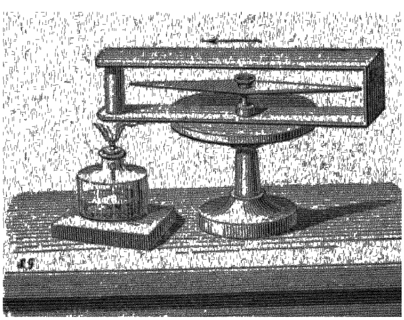


Fig 137.—Thermo-electric Current.

bismuth to the warmer junction. Antimony, if employed instead of copper, gives a still more powerful effect.

226. Though a circuit composed of bismuth and antimony is specially susceptible of thermo-electric excitation, the property is possessed, in a more or less marked degree, by every circuit composed of two metals, and even by circuits composed of the same metal in different states. If, for example, a knot or a helix (as in Fig. 138), be formed in a piece of platinum wire, and heat applied at one side of it, a current will be indicated by a delicate galvanometer. In metals which are usually heterogeneous in their structure, such as bismuth, it is not uncommon to find currents produced by heating parts which appear quite uniform. If the ends of two copper wires be bent into hooks, and one of them be heated; on placing them in contact, a current will be produced due to the presence of a thin film of oxide on the heated wire. With two platinum wires, no such effect is obtained.

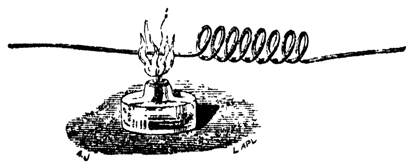


Fig. 138.—Current with one Metal.

227. **Thermo-electric Order.**—According to Becquerel's experiments, the metals may be ranged in the following order, as regards the direction of the current produced by heating a junction of any two of them:—*Bismuth, platinum, lead, tin, copper, silver, zinc, iron, antimony;* that is to say, if a junction of any two of these metals be heated, the direction of the current at the junction in question will be from that which stands first in the list to the other. His experiments have also established the important fact that the current obtained by heating all the junctions B, C, D, E, F, of a chain of dissimilar metals to one common temperature, is the same as that obtained by uniting the two extreme bars A B, F G, directly to each other, and heating their junction to the same temperature.[1]

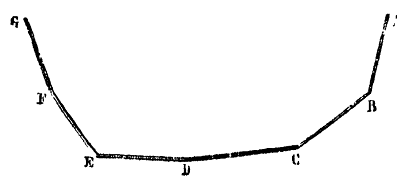


Fig. 139.

[1] The more accurate statement is, that the *electro-motive force* is the same in the two cases. The *current* will be sensibly the same if the resistance in B C D E F is insignificant in comparison with the rest of the circuit. In order that there may be a current, the circuit must of course be completed, and not left open as in Fig. 139. In the case of an open circuit, the result of the heating will simply be to produce difference of potential between

228. Source of Thermo-electric Force.—The source of the electro-motive force in a thermo-electric arrangement is to be found in two effects called, from the names of their discoverers, the Peltier effect and the Thomson effect. They may both be described as *thermal effects of a current which are reversed by reversing the direction of the current.*

Peltier found that if two wires of different metals are joined at one end so as to form a single conductor, the junction becomes hotter when a current is sent through the conductor in one direction than when it is sent in the opposite direction.

Thomson found that a wire of one metal, if it is not at a uniform temperature, but is hotter at one end than at the other, is more heated by a current in one direction than by a current in the opposite direction.

In both cases, the quantity of heat generated in the conductor in unit time by a current C is represented by the expression

$$C^2 R \pm C S,$$

R denoting the resistance of the conductor, S a quantity which we shall suppose to be essentially positive, and the positive or negative sign being taken according to the direction of the current.

The term $\pm$ C S represents an amount of energy which is converted from the electrical to the thermal form by a current in one direction, and from the thermal to the electrical form by a current in the other direction. When the conversion is from the thermal to the electrical form, the current is aided by an electro-motive force S, and when the conversion is from the electrical to the thermal form, the current is opposed by an electro-motive force S. S is the measure of the thermo-electric force, and the direction of this force is always that direction for which the sign of the term C S is negative.

These statements are true both of a thermo-electric circuit as a whole, and of any portion of it considered separately. If the portion considered be an indefinitely short portion containing a junction, the term $C^2 R$ will vanish, and the heating effect will be $\pm$ C S. A junction would therefore be cooled by sending through it a current in the proper direction, if we could prevent the conduction of heat to it from neighbouring parts.

229. Thermo-electric Diagram.—The statement of the quantita-

tive laws of thermo-electricity is greatly facilitated by the use of what is called a thermo-electric diagram.

In a thermo-electric diagram each metal has its own line. Let A A and B B (Fig. 140) be the lines of two metals, which we will call A and B. Horizontal distances measured from the vertical line A p q B represent absolute temperatures. Let p P and its equal q Q represent any absolute temperature. Then the area of the rectangle P p q Q represents the thermo-electric force at a junction of the two metals at this temperature. If P be above Q, as in the figure, the force tends from Q to P. The temperature corresponding to the point of intersection N is the *neutral point* of the two metals. At temperatures below N the thermo-electric force at a junction is from the metal B to the metal A, but at temperatures above N it is from A to B, being in both cases directed upwards in the diagram.

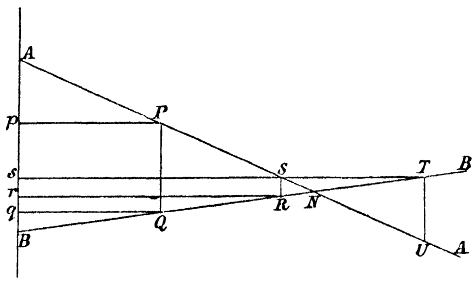


Fig 140.

Again if pP and sS represent the temperatures of two points of the metal A, the thermo-electric force in the intervening portion is represented by the area Pp sS, which is bounded on three sides by straight lines, and on the fourth side by the line of the metal, which is not in all cases straight. The direction of this force is from the lower point S to the higher P, so that if the line of the metal slopes like A A in the figure, the force tends from the hot to the cold end. In the portion of the metal B represented by Q R, the force tends from Q to R, that is from the cold to the hot end. In every case the thermo-electric force in each portion of a circuit is directed up-hill in the diagram; and conversely a current generates more heat by running downhill than by running uphill.

230. Thermo-electric Pair.—If the circuit consist simply of a wire of the metal A, joined at both ends to a wire of B, one junction being at the temperature pP or qQ, and the other at the higher temperature sS or rR, both being below the neutral point, the thermo-electric forces in Q R, R S, and S P, all tend in one direction round the circuit, namely, the direction Q R S P Q. Their joint force is represented by the area qQ R S Pp. The force at the

junction P Q is from Q to P, and opposes the other three. It is represented by the area qQ Pp, which must accordingly be subtracted. The remainder is the area P Q R S, which accordingly represents the resultant thermo-electric force of the circuit.

231. Properties of the Neutral Point.—Supposing the lower of the two temperatures to be fixed, and to be below the neutral point N, the area P Q R S will be a maximum when S and R coincide with N, being then identical with the triangle P Q N. If the higher

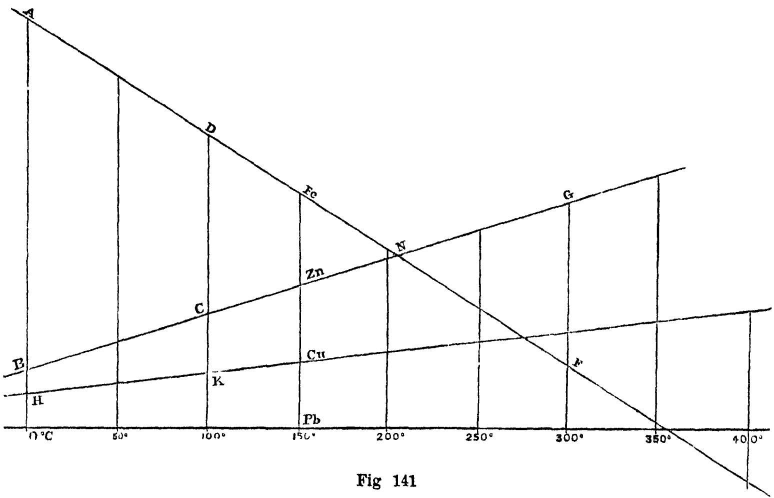


Fig 141

temperature be above the neutral point, and be represented by sT, the area which represents the thermo-electric force will be the difference of the two triangles P Q N and N T U. If the lines of the two metals are straight, these two triangles will be equal when one junction is as much above the neutral point as the other is below it, and the thermo-electric force will then be *nil*. At higher temperatures of the hot junction the triangle N T U is larger than the triangle P Q N, and the thermo-electric force is accordingly reversed.

232. General Formula.—Denoting absolute temperature by x, and heights above any fixed horizontal line by y, the thermo-electric force in any part of the circuit is represented by the sum of the elementary strips $x\,d\,y$ taken along the corresponding portion of the diagram, and the resultant thermo-electric force round a circuit is the sum of these strips taken round the whole boundary which represents the circuit. This sum $\Sigma\, x\, d\, y$ is the area enclosed by the boundary, and is positive when the boundary is traced in the

opposite direction to watch hands. The direction of the resultant force will be that which makes this area positive. If one part of the boundary crosses another, we must mark arrows along the boundary in directions corresponding to one direction round the circuit; then areas encircled by arrows against watch hands are to be reckoned positive, and those encircled by arrows with watch hands negative.

233. Application to several Metals.—Fig. 141 is a thermo-electric diagram containing portions of the lines of four metals: Iron (Fe), Zinc (Zn), Copper (Cu), and Lead (Pb).

The Thomson effect is nil in lead, and the lead line is accordingly horizontal. The lines for zinc and copper slope in the direction of ordinary writing; in these the Thomson effect is said to be positive. The line for iron slopes the contrary way, and the Thomson effect in this metal is said to be negative. In a metal for which the Thomson effect is positive, the thermo-electric force due to inequality of temperature tends from the cold to the hot portions of the metal, and conversely the metal would be more heated by a current from hot to cold, than by a current from cold to hot. In a metal in which the Thomson effect is negative the thermo-electric force is from hot to cold, and more heat would be produced by a current from cold to hot than by one from hot to cold.

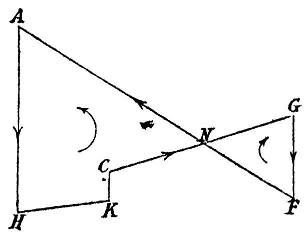


Fig. 142.

The line of lead, being horizontal, is usually employed as the axis of x, and temperatures are laid off along it. In the figure the area A B C D represents the thermo-electric force of an iron-zinc couple, A H K D that of an iron-copper couple, B H K C that of a zinc-copper couple, and so on, the junctions in each case being at 0° and 100° C.

For a circuit composed of three wires of iron, zinc, and copper respectively, with junctions at the following temperatures:

Iron-zinc 300°, Zinc-copper 100°; Copper-iron 0°,

we must trace the boundary A H K C G F A, marking our course with arrows, as shown in Fig. 142. We shall thus obtain the area A H K C N encircled by arrows against watch hands, and the area N G F encircled by arrows with watch hands. The resultant

thermo-electric force is represented by their difference, and is in the direction A H K C G F A.

Numerous experiments, of which the most important are those of Professor Tait, have shown that the lines of most metals are straight within the limits of temperature to which the experiments extended. The line of iron is straight up to a temperature just below redness, but exhibits some remarkable bends before a white heat is reached.

For a further discussion of the laws of thermo-electricity, see the Note at the end of this chapter.

234. Nomenclature.—It is convenient to call the ordinates in the diagram "*thermo-electric height.*" Thus A B is the difference of the heights of iron and zinc at 0°, or more definitely, it is the height of iron above zinc at 0°.

The difference of the ordinates at two points of one of the lines, divided by the difference of the abscissas, may be called the "*tangent of the slope.*" Its sign is the same as that of the Thomson effect.

The student is likely in the course of his reading to meet with the phrases "electric convection of heat" and "specific heat of electricity." We will therefore explain these somewhat misleading terms.

In a uniform linear conductor along which a current is flowing, there is, in addition to the frictional heating, which is proportional to the square of the current, a warming or cooling effect proportional (at given temperature) to the steepness of the thermometric gradient at the point which is warmed or cooled, changing sign with the gradient, and vanishing at points of maximum or minimum temperature, where the gradient vanishes. This is the Thomson effect. Let us compare it with what happens when a stream of liquid flows through a pipe surrounded at alternate points in its length with hot and cold jackets, the average temperature of the liquid being the same as the average temperature of the pipe. It will carry heat from the hotter to the colder portions, thus cooling the hottest parts, warming the coldest parts, and at the same time carrying forward the points of maximum and minimum temperature. If, at each point of the pipe (supposed straight and horizontal), we erect an ordinate to represent its temperature, and call the curve of which they are the ordinates "the temperature curve;" the effect of the flow of liquid on this curve will be twofold: (1) It will carry the

temperature curve forward; (2) It will make the temperature curve flatter.

In the Thomson effect, an electric current carries the temperature curve forward in copper, and backward in iron; but in neither case does it make the temperature curve flatter. Erroneous statements on this point[1] will be found in some text-books. They have arisen from taking the phrase "electric convection" too literally. In speaking of electric convection, it is customary to say that the "specific heat of electricity" is positive in a metal in which the temperature curve is carried forward, and is negative in a metal in which the temperature curve is carried backward.

235. Thermo-electric Pile.—If a thermo-electric chain be composed

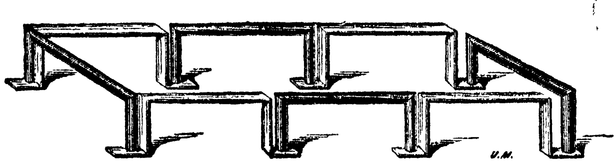


Fig. 143 —Pouillet's Thermo-pile

of two metals occurring alternately (as in Fig. 143), no effect will be obtained by equally heating *two consecutive* junctions; for the current which would be generated by heating the one is in the opposite direction to that due to the heating of the other. If we number the junctions in order, we shall obtain a current in one direction by heating any junction which bears an odd number, and in the opposite direction by heating any one that bears an even number. The thermo-electric pile, or *thermo-pile,* whose use has been described in Part II. in connection with experiments on radiant heat, is an arrangement of this kind, in which all the odd junctions are presented together at one end, and all the even junctions at the other, the two metals composing the pile being antimony and bismuth. The electro-motive force obtained with a given difference of temperature between the ends of the pile is proportional to the number of junctions, except in so far as accidental differences may exist between different junctions.

Of late years some thermo-piles have been constructed which have sufficient electro-motive force for the deposition of metals from solutions, and they have to some extent been employed commer-

[1] See a discussion in *Nature*, vol. 34, pp. 75, 120, 143.

cially for this purpose. The number of pairs of metals is usually about 70, the materials being in one instance iron and type-metal, in another iron and galæna. These piles have the form of a hollow cylinder with junctions facing alternately inwards and outwards The inner junctions are exposed to the flame of a Bunsen gas-burner, while the outer junctions are kept cool by the contact of the air.

236. Observations of Temperature by Thermo-electric Junctions.—Thermo-electric currents may be employed either for comparing small differences of temperature (which is the function of the thermopile), or for testing equality of temperature. As an example of the latter application, suppose a circuit to be formed of two long wires, one of iron and the other of copper, connected at both ends, and covered with gutta-percha or some other insulator except at the two junctions. Let one junction be lowered to the bottom of a boring, or any other inaccessible place whose temperature we wish to ascertain, and let the other junction be immersed in a vessel of water containing a thermometer. If one of the wires be carried round a galvanometer, the direction in which the needle is deflected will indicate whether the upper or lower junction is the warmer, and if we alter the temperature of the water in the vessel till the deflection is reduced to zero, we know that the two junctions are at the same temperature, which we can read off by the thermometer immersed in the water.

237. Pyro-electricity.—Another relation between heat and electricity may here be mentioned, though it belongs rather to electrostatics than to current electricity.

When a crystal of tourmaline is heated or cooled, observation shows that, while the crystal is gaining heat, one end of it has a charge of positive and the other of negative electricity; and while it is losing heat these charges are reversed. This phenomenon is called pyro-electricity, and it is always associated with a departure from symmetry in crystalline form, which enables us to distinguish one end of the crystal from the other.

238. Effect of Light on Electrical Resistance.—Mr. Willoughby Smith has discovered that the electrical resistance of crystalline selenium is diminished by the action of light. A strong instantaneous effect is observed at the moment when light first falls upon the substance, and the effect gradually increases for some time if the exposure to light is continued Professor W. G. Adams found that exposure to the light of an ordinary wax taper at a distance

of 20 centims. diminished the resistance of a plate of selenium by about one-eighth part of the whole.

Selenium is a very bad conductor, its resistance being more than a thousand million times that of iron.

The most striking effects are obtained by constructing a so-called "selenium cell" on the following plan. A strip of mica or some other substance of high insulating power is notched at both edges, and a copper wire is wound round it leaving alternate notches vacant. Its ends are secured, one of them being attached to a binding-screw. A second wire is then wound in the intervening notches and similarly secured. It must not touch the first, but must be everywhere very near it. The face of the plate is then thinly covered with selenium, which must be melted on and allowed to cool slowly so as to assume the crystalline form. The selenium affords the only medium of electrical communication between the two wires, and if the two binding-screws are connected with a battery, a high-resistance mirror-galvanometer being also introduced into the circuit, the exposure of the face of the cell to various degrees of light will give strongly marked effects on the galvanometer. If a disc of cardboard be cut away in sectors and rapidly rotated between the face of the cell and the sun or any strong light, in such a manner that the cell is alternately in light and shadow as the sectors pass, the fluctuations of current thus produced can be detected by means of a telephone, which gives a very audible hum. This is a severe test of the quickness of the action, for a thermo-pile gives no sound under the same circumstances. This combination of a selenium cell with a telephone is called a *photophone*, and with a modified form of it articulate sounds have been transmitted to a distance by light. The invention is due to Professor Graham Bell, the inventor of the telephone, and the form of cell above described was introduced by Mr. Shelford Bidwell.

NOTE.

Mathematical Investigation relating to Thermo-electricity.

Let $\pi_1\ (a\,b)$ denote the *e.m.f.* tending from the metal A to the metal B at their junction at temperature t_1. It will follow from this definition that $\pi_1\ (b\,a) = -\pi_1\ (a\,b)$. The experimental fact that, in a closed circuit of three metals

A, B, C at uniform temperature, there is no resultant thermo-electric force, proves that

$$\pi_1(a\,b)+\pi_1(b\,c)+\pi_1(c\,a)=0, \quad \text{or} \quad \pi_1(a\,c)=\pi_1(a\,b)+\pi_1(b\,c). \tag{1}$$

Again, the experimental fact that the resultant *e.m.f.* of a closed circuit is independent of the temperatures of all parts except the junctions, shows that the resultant *e.m.f.* due to difference of temperature in a single metal depends on the temperatures of its ends only, and is independent of its length. The same conclusion might be drawn from the fact that no current is produced in a closed circuit of one homogeneous metal by any distribution of temperature in its various portions. Hence the *e.m.f.* in a short portion of nearly uniform temperature varying from t at one end to $t+dt$ at the other, may be expressed as $\sigma\,dt$, where σ is a function of t having different values for different metals. The coefficient σ (called the Thomson coefficient) is regarded as positive when the *e.m.f.* tends from the colder to the warmer end. We shall denote its value in the metal A by σ_a, its value in B by σ_b, and so on.

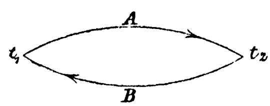


Fig 144.

Consider a closed circuit (Fig. 144), consisting of a wire of one metal A joined at both ends to a wire of another metal B, one junction being at the temperature t_1, and the other at t_2. The resultant *e.m.f.*, regarded as positive when it is in the direction of the arrows, is

$$\pi_1(b\,a)+\int_{t_1}^{t_2}\sigma_a\,dt+\pi_2(a\,b)+\int_{t_2}^{t_1}\sigma_b\,dt, \tag{2}$$

or

$$\pi_2(a\,b)-\pi_1(a\,b)+\int_{t_1}^{t_2}(\sigma_a-\sigma_b)\,dt. \tag{3}$$

In any reversible thermo-dynamic engine, if we divide each portion of heat taken in or given out by the absolute temperature at which the exchange occurs, and reckon heat given out opposite in sign to heat taken in, the sum of the quotients is zero. Applying this principle to the present case, we have

$$\frac{\pi_2(a\,b)}{t_2}-\frac{\pi_1(a\,b)}{t_1}+\int_{t_1}^{t_2}\frac{(\sigma_a-\sigma_b)}{t}\,dt=0,$$

or, writing y as an abbreviation for $\dfrac{\pi(a\,b)}{t}$,

$$y_2-y_1+\int_{t_1}^{t_2}\frac{\sigma_a-\sigma_b}{t}\,dt=0, \tag{4}$$

By supposing a small change in t_2, while t_1 remains constant, we deduce from (4)

$$\frac{dy}{dt}+\frac{\sigma_a-\sigma_b}{t}=0, \tag{5}$$

which gives by integration

$$\int_{t_1}^{t_2}(\sigma_a-\sigma_b)\,dt=-\int_{y_1}^{y_2}t\,dy=t_1y_1-t_2y_2+\int_{t_1}^{t_2}y\,dt.$$

But t_1y_1 is $\pi_1(a\,b)$ and t_2y_2 is $\pi_2(a\,b)$. Hence expression (3) for the resultant thermo-electric force in the circuit reduces to

$$\int_{t_1}^{t_2}y\,dt, \tag{6}$$

which denotes an area such as A B C D or B H K C in Fig. 141, if y denote the length intercepted between the lines of the two metals on the ordinate corresponding to the abscissa t.

Writing $y\ (a\,b)$ instead of y for greater explicitness, we have, by dividing equation (1) by t_1,

$$y\,(a\,b) + y\,(b\,c) = y\,(a\,c), \tag{7}$$

a property which enables us to combine the lines of more than two metals in the same diagram.

The quantity y in the above investigation is the difference of the ordinates of the two metals A and B in a thermo-electric diagram. The line of some one standard metal is arbitrary both in form and position, and its choice will determine the rest. Adopting as the standard a metal (lead) for which σ is zero at all temperatures, and assigning to it a line which is straight and horizontal, we find from (5), by identifying the standard with A, that for any other metal (identified with B)

$$\frac{\sigma}{t} = \frac{d\,y}{d\,t}. \tag{8}$$

Hence the line of a metal slopes one way or the other according to the sign of σ. The *e.m.f.* due to difference of temperature in a small portion of one metal is $\sigma\,d\,t$, which by (8) is equal to $t\,dy$. It always tends in the direction in which y increases.

The *e.m.f.* at a junction with the standard metal, reckoned positive when it tends from the standard to the other metal, is $t\,y$, and is represented by the area of the rectangle whose sides are the two co-ordinates.

If y_a and y_b are the ordinates of the lines of two metals A and B, $y_b - y_a$ will be the quantity denoted by y in (4), (5), and (6), and by $y\,(a\,b)$ in (7).

If we travel completely round any thermo-electric circuit consisting of any number of metals, and trace the corresponding course on the thermo-electric diagram, this latter course will consist partly of vertical movements corresponding to the junctions, and partly of slanting or horizontal movements along the lines of the metals. In each of these two kinds of movement the *e.m.f.* (reckoned positive if it tends in the direction in which we are travelling) is the sum of the elementary areas $t\,d\,y$. The resultant *e.m.f.* is therefore $\Sigma\,t\,d\,y$ taken all round the course, and this is identical with the area which the course encloses.

"The specific heat of electricity" is a name which has been given to the Thomson coefficient σ for the reason explained in § 234, but it is not desirable that it should be retained, the analogy to convection of heat being very imperfect and misleading. The other coefficient π (more commonly printed Π) is called the Peltier coefficient.

CHAPTER XVIII.

ELECTRO-DYNAMICS.

[This chapter, with the exception of the last three sections, is of little importance, and should be omitted at first reading.]

239. Meaning of Electro-dynamics.—A wire through which a current is passing is found to be capable of producing movements in other wires also conveying currents. The theory of these movements, or more generally, of *the mechanical actions of currents upon one another*, constitutes a distinct branch of electrical science, and is called *electro-dynamics*. It stands in very close relation to electro-magnetism; and if the laws of either of the two sciences are given, those of the other may be deduced as consequences.

The science of electro-dynamics was founded by Ampère. Figs. 145, 146 represent an arrangement which he devised for rendering a conductor movable without interruption of the current conveyed by it.

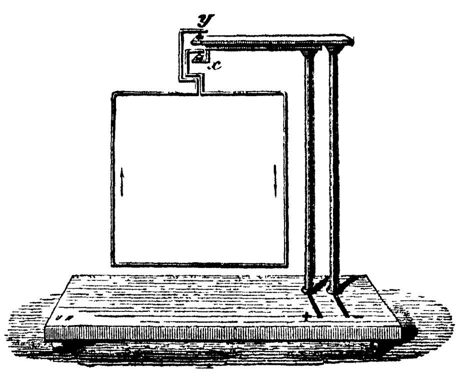


Fig. 145.—Ampère's Stand.

A wire is bent into the form of a nearly complete rectangle, and its two ends terminate in points, one above the other, so arranged that a vertical through the centre of gravity passes through them both. Accordingly, if either or both of these points be supported, the wire can turn freely about this vertical as axis. The points dip into two small metallic cups xy containing mercury, and the weight is usually borne by the upper point alone, which touches the bottom of its cup. The cups are attached to two horizontal arms of metal, supported on metallic pillars, which can be connected with the two terminals of a battery. The wire thus forms part of the circuit, the current being down one side of the rectangle and up the other. Instead of the rectangular the circular form may be employed, as in Fig. 147.

If a magnet be placed beneath, as in Fig. 146, the wire frame will set its plane perpendicular to the length of the magnet, the relative position assumed being

the same as if the wire frame were fixed, and the magnet freely suspended, if we neglect the disturbing effect of the earth's magnetism.

240. Mutual Forces between Conductors conveying Currents.—The following elementary laws, regarding the mutual forces exerted between conductors through which currents are passing, were established by Ampère. For

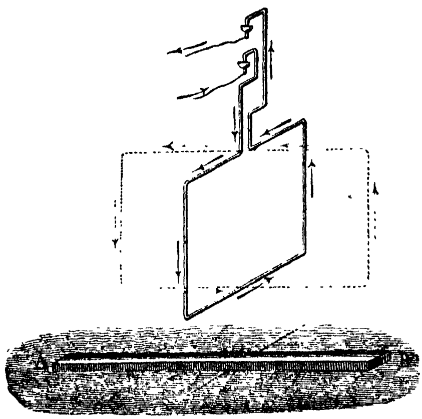


Fig. 146.—Action of Magnet on Movable Circuit.

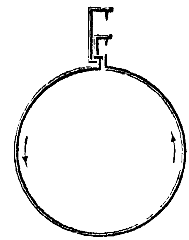


Fig. 147.

brevity of expression, it is usual to speak, in this sense, of the *mutual forces between currents*, or of the *mutual mechanical action of currents*.

I. *Parallel currents, if in the same direction, attract, and if in the opposite direction, repel each other.*

The apparatus employed for demonstrating this twofold proposition, consists of two metallic pillars *t*, *v* (Fig. 148), which are respectively connected at their

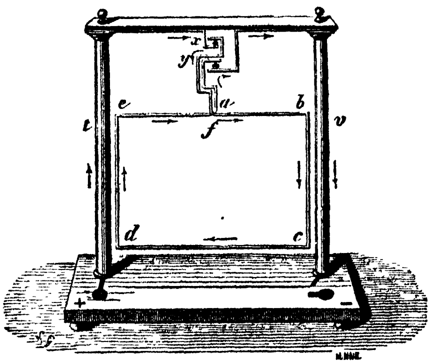


Fig. 148.—Attraction of Parallel Circuits.

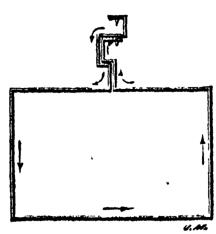


Fig. 149.—Apparatus for Repulsion.

upper ends with the two cups of mercury *x*, *y*. The rectangular conductor *a b c d e* is suspended with its terminal points in these cups so as to complete the circuit between the pillars. When the current is passed, this movable conductor

always places itself so that its plane coincides with that of the two pillars, and so that currents in the same direction in the pillars and in the wire are next each other, as shown in the figure.

For establishing repulsion, a slightly different form of wire is employed, which is represented in Fig. 149. When this is hung from the cups, in the position which the figure indicates, the currents in the pillars are in opposite directions to those in the neighbouring portions of the movable conductor, and the latter accordingly turns away until it is stopped by collision with the wires above.

II. *Currents whose directions are inclined to each other at any angle, attract each other if they both flow towards the vertex of the angle,*[1] *or if they both flow from it, and repel each other if one of them flows towards the angle, and the other from it.*

A consequence of this law is that two currents, as A B, D C (Fig.150), crossing one another near O in different planes, tend to set themselves parallel, and so that their directions shall be the same. For there is attraction between the portions A O and D O, and also between the portions O B and O C; whereas there is repulsion between A O and O C, and between O B and O D. Accordingly, if the movable conductor of Fig. 148 or 149 be traversed by a current, and another wire carrying a current be placed horizontally at any angle underneath its lower side, the movable conductor will turn on its point of suspension till it becomes parallel to the wire below it; and in the position of stable equilibrium

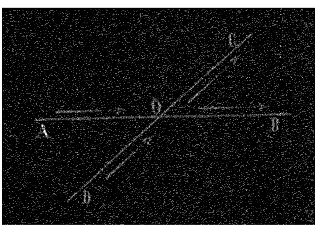


Fig. 150.—Tendency to set Parallel.

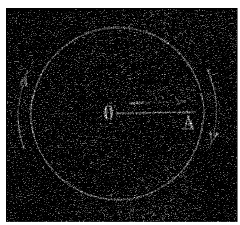


Fig 151.—Continuous Rotation of Radial Current.

the current in its lower side will have the same direction as that in the influencing wire.

241. Continuous Rotation produced by a Circular Current.—Suppose we have a current flowing round a circle (Fig. 151), and also a current flowing along O A, which is approximately a radius of this circle. First let the current in O A be from the centre towards the circumference, as indicated in the figure. Then, by law II., O A is attracted on one side and repelled on the other, both forces combining to make O A sweep round the circle in the opposite direction to that in which the circular current is flowing. If the current in O A were from circumference to centre, the tendency would be for O A to sweep round the circle in the same direction as the circular current.

The reasoning still holds if O A is in a plane parallel to that of the circular

[1] If the currents are not in the same plane, we must substitute *their common perpendicular* for *the vertex of the angle*, in the enunciation of this law.

current, O being a point on the axis of the circle and the length of O A being not greater than the radius.

A circular current may also produce continuous rotation in a conductor parallel to the axis of the circle, and movable round that axis. Fig. 152 represents an arrangement for obtaining this effect.

A coil of wire through which a current can be sent, is wound round the copper basin E F, its extremities being connected with the binding-screws *m*, *o*. From

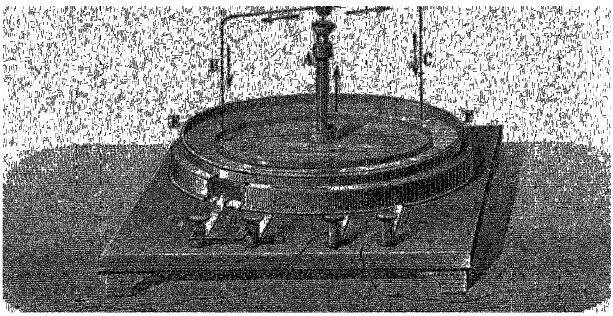


Fig. 152.—Apparatus for Continuous Rotation.

the centre of the basin rises the little metallic pillar A, terminating above in a cup containing mercury. This pillar is connected with the binding-screw *n*. The basin, which is connected with the binding-screw *p*, contains water mixed with a little acid to improve its conducting power, and a movable conductor B C rests, by a point, on the bottom of the cup of mercury, while its lowest portion, which consists of a light hoop, dips in the acidulated water. By connecting *m* and *n* a single circuit is obtained, of which *o* and *p* are the terminals, so that if *o* is connected with the positive and *p* with the negative pole of a battery, the current entering at *o* first traverses the wire coil, then ascends the pillar A, returns down the sides B, C to the floating ring and liquid, and so escapes to *p*. As soon as these connections have been completed, the movable conductor commences continuous rotation in the direction opposite to that of the current in the coil.

If, instead of connecting *m* and *n*, we connect *n* and *o*, and lead the positive wire from the battery to *p* and the negative wire to *o*, the course of the current will be from *p* to the acid, thence up the sides B, C, and inwards along the top of the movable conductor to the mercury cup, then down the pillar to *n*, thence to *o*, and through the coil from *o* to *m* in the same direction as in the former experiment; but the rotation of the movable conductor will now occur in the opposite direction to that before observed, and therefore in the same direction as the current.

242. Action of an Indefinite[1] Rectilinear Current upon a Finite Current movable around one Extremity.—A finite current movable about one extremity may also be caused to rotate continuously about this extremity by the action of an indefinite rectilinear current. This is clearly indicated by Fig. 153.

[1] The word *indefinite*, in this application, simply means *of great length in comparison with the distance and length of the movable current.*

In the right-hand diagram, the current O A flowing outwards from the centre of motion O, and acted on by the indefinite current M N, is first attracted into the position O A′. In this new position it is repelled by n N, and attracted by M n. It is thus brought successively into the positions O A″, O A‴, O A$^{\text{IV}}$. In this last-

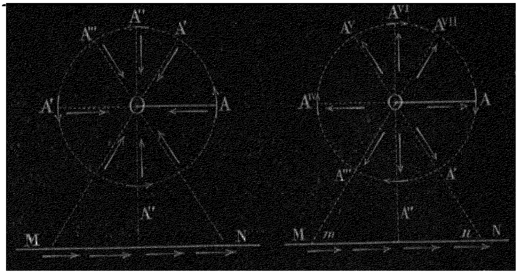


Fig 153 —Rotation of Radial Current

mentioned position, the two currents being parallel and opposite, there is repulsion; and after passing it, there is again repulsion on one side and attraction on the other, till it is carried round to its first position O A. It is thus kept in continual rotation. If the movable current flows inwards to the centre of motion O, as in the left-hand diagram, while the direction of the indefinite current is the same as before, the direction of rotation will be reversed.

243. Action of an Indefinite Rectilinear Current on a Finite Current Perpendicular to it.—Let M N, in the upper half of Fig. 154, be an indefinite rectilinear current, and A D a portion of another current either in the same or in any other plane. In the latter case let DC be the common perpendicular. Then, if the currents have the directions represented by the arrows, an element at p will attract an element at m with a force which we may represent by a line mf'; and an element at p' equal to that at p and situated at the same distance from C on the other side, will repel the element at m with an equal force, represented by mf. Constructing the parallelogram of forces, the resultant force of these two elements upon m is represented by the diagonal m F, which is parallel to M N and in the opposite direction to the indefinite current. As this reasoning applies to all the elements of both currents, it follows that the current A B will experience a force tending to give it a motion of translation parallel to M N. This motion will be opposite to the direction of the indefinite current when the direction of the finite current is towards the common perpendicular D C, as in the upper diagram, and will be in the same direction

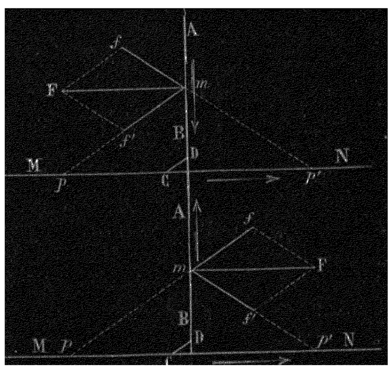


Fig 154.—Translation Parallel to Indefinite Current

as the indefinite current when the direction of the finite current is from the common perpendicular, as in the lower diagram.

244. Action upon a Rectangular Current movable about an Axis Perpendicular to an Indefinite Current.—It follows from the preceding section that if a finite current A B (Fig. 155), perpendicular to an indefinite

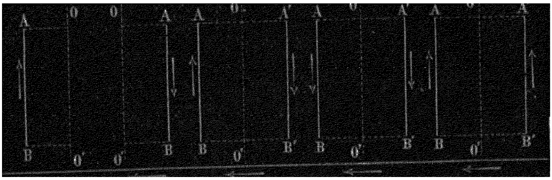


Fig. 155.—Position assumed by Perpendicular Current.

current, is movable round an axis O O′ parallel to itself, the plane A B O O′ will place itself parallel to the indefinite current, and A B will place itself in advance or in rear of the axis according as the current in A B is from or towards the indefinite current.

If a pair of parallel and opposite currents B A, A′ B′, rigidly connected together, and movable round the axis O O′ lying between them, are submitted to the action of the indefinite current, the forces upon them will conspire to place the system in the position indicated in the figure. If the two currents A B, A′ B′ are both in the same direction, their tendencies to revolve round the axis O O′ will counteract each other.

The action upon the near side of the rectangle (Fig. 156) contributes to produce the same effect, since this side tends to set itself parallel to the influencing current, and so that the directions of the two shall be the same.

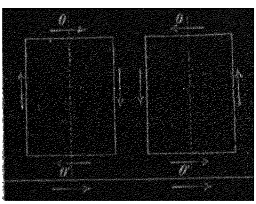


Fig. 156.—Position assumed by Rectangular Current.

The action upon the further side of the rectangle tends to produce an opposite effect; but, in consequence of the greater distance, this action is feebler than that upon the near side. The system accordingly tends to take the position of stable equilibrium represented in the right-hand half of the figure. The diagram on the left hand represents a position of unstable equilibrium.

What is here proved for a rectangular current, is true for any closed plane circuit movable round an axis of symmetry perpendicular to an indefinite rectilinear current; that is to say, any such circuit tends to place itself so that the current in the near side of it is in the same direction as the indefinite current.

These results can be verified experimentally by the aid of the apparatus represented in Fig. 157. C C, D D are two cups (shown in section) surrounding the metallic pillar A B at its upper and lower ends, and containing a conducting liquid. The lower cup is insulated from the pillar, and connected with the binding-screw *g*. The liquid in the upper cup C C is connected with the upper end of the pillar by the bent arm *d m*. *o* K is a light horizontal rod supported

on a point at B, and carrying a counterpoise K at one end, while the other carries a wire $m\,n\,o\,p$, whose two ends $n\,m$ and $o\,p$ descend vertically into the two cups, the middle portion of the wire being wrapped tightly round the rod. The binding-screw f is connected with the lower end of the pillar. If a current enters at f and leaves at g, its direction in the long vertical wire $o\,p$ will be descending; and it will be ascending if the connections are reversed. By sending a current at the same time through a long horizontal wire in the neighbourhood of the system, movements will be obtained in accordance with the foregoing conclusions.

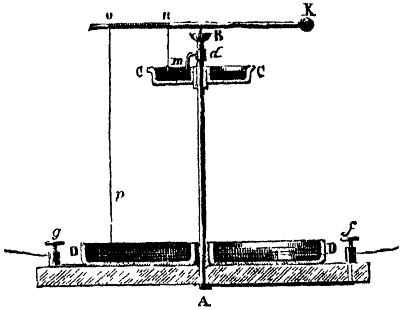


Fig. 157.—Position assumed by Vertical Current

245. Sinuous Currents.—A sinuous current exhibits the same action as a rectilinear current, provided that they nowhere deviate far from each other. This principle can be exemplified by bringing near to a movable conductor (Fig. 158) another conductor consisting of a wire doubled back upon itself, having one of its portions straight, and the other sinuous, but very near the first. A current sent through this double wire traverses the straight and the sinuous

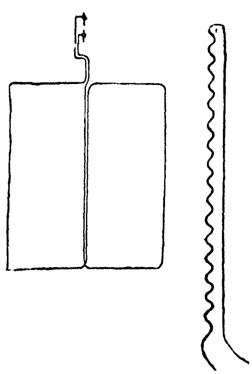
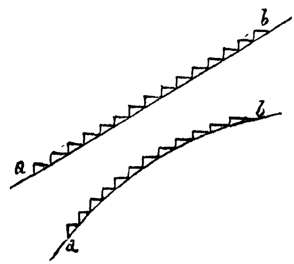



Fig 158 Sinuous Currents Fig 159.

portions in opposite directions, and it will be found that their joint effect upon the movable conductor is inappreciable.

This principle holds not only for rectilinear currents but for currents of any form, and is very extensively employed in the analytical investigations of electrodynamics. In computing the action exercised by or upon a conductor of any form, it is generally convenient to substitute for the conductor itself an imaginary conductor, nearly coincident with it, and consisting of a succession of short straight portions at right angles to one another (Fig. 159).

246. Mutual Action of Two Elements of Currents.—Ampère based his analytical investigations on the assumption that the action exercised by an

element (*i.e.* a very short portion) of one current upon an element of another, consists of a single force directed along the joining line. This assumption conducted him to a formula for the amount of this force, which has been found to give true results in every case capable of being tested by experiment. Nevertheless, it is by no means certain that either Ampère's formula or his fundamental assumption is true. Other assumptions have been made, leading to other formulæ in contradiction to that of Ampère, which also give true results in every case capable of being experimentally tested.[1] The fact is that experiments can only be performed with complete circuits, and the contradictions which subsist between the different assumptions, in the case of the several parts of a circuit, vanish when the circuit is considered as a whole. All the formulæ, however, agree in making the mutual force or forces between two elements vary inversely as the square of their distance, and directly as the products of the currents which pass through them. Professor Clerk Maxwell[2] discards all assumptions as to mutual actions between elements at a distance, and employs the principle that a circuit conveying a current always tends to move in such a manner as to increase the number of magnetic force-tubes (§§ 53, 109) which pass through it. The work done in any displacement is measured by the number of tubes thus added; but tubes which cross the circuit in the opposite direction to those due to the current in the circuit are to be regarded as negative.

We have seen (§ 164, 165) that the lines of magnetic force due to a current are circles surrounding it; and also that, when a line of magnetic force cuts a current, the latter experiences a force tending to move it at right angles to the plane of itself and the line of force. In the case of two parallel currents, each is cut at right angles by the lines of magnetic force due to the other; the direction of the force experienced by either current is therefore directly to or from the other current; and the criterion of § 165 will be found to indicate attraction when the directions of the currents are the same, and repulsion when they are opposite.

In Fig. 152 the lines of magnetic force cut OA in a direction perpendicular to the plane of the diagram, OA accordingly experiences a force perpendicular to its own length in the plane of the diagram; and the same remarks apply to AB in Fig. 155. All the mutual mechanical actions of currents are in fact thus explicable.

247. Action of the Earth on Currents.—In virtue of terrestrial magnetism, movable circuits, when left to themselves, take up definite positions having well-marked relations to the lines of terrestrial magnetic force. For example, in the apparatus of Fig. 157, the vertical wire *op* will place itself to the west or east (magnetic) of the pillar AB, according as the current in *op* is ascending or descending. This effect is due to the horizontal component of terrestrial magnetism.

In the apparatus of Fig. 152, if the current be sent only through the movable portion, continuous rotation will be produced, which will be with or against the hands of a watch according as the current in the top wires is inwards or outwards. This effect is due to the vertical component of the earth's magnetism, acting on the currents in the horizontal wires. Vertical lines of magnetic force falling on

[1] Maxwell, § 526, second edition.

[2] Maxwell "On Faraday's Lines of Force." *Camb. Trans.* 1858, p. 50.

a horizontal current give the latter a tendency to move perpendicular to its own length in a horizontal plane.

248. Solenoids.—If we suspend from Ampère's stand (Fig. 145) a plane circuit, whether rectangular or circular, it will place itself perpendicular to the magnetic meridian, in such a manner that the current in its lower side is from east to west; or, in other words, so that the ascending current is in its western and the descending current in its eastern side; this effect being due to the action of the horizontal component of terrestrial magnetism upon the ascending and descending parts of the current. If, then, we have a number of such circuits, rigidly connected together at right angles to a common axis, and with their currents all circulating the same way, their common axis will tend to place itself in the magnetic meridian, like the axis of a magnet. Such a system was called by Ampère a solenoid (σωλην, a tube), and was realized by him in the following manner.

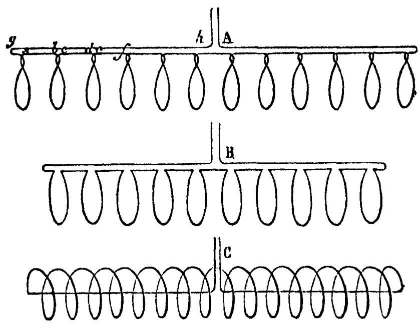


Fig 160—Solenoids

Imagine a wire bent into such a shape as to consist of a number of rings united to each other by straight portions. It will differ from a theoretical solenoid only by having currents in these straight portions; but if the two ends of the wire be carried back till they nearly meet in the middle of the length, as shown at A and B (Fig. 160), the currents in these returning portions, being opposite to those in the other straight portions, will destroy their effect, and the resultant electro-dynamic action of the system will be simply due to the currents in the rings. The same effect is more conveniently obtained by substituting for the rings and intermediate straight portions, a helix, which, by the principles of sinuous currents, is equivalent to them. Each spire of the helix represents a circle perpendicular to the axis, together with a straight portion parallel to the axis and equal to the distance between two spires. The effect of all the straight portions is exactly destroyed by the wires which return from the ends of the helix and meet in the middle. This arrangement, which is represented at

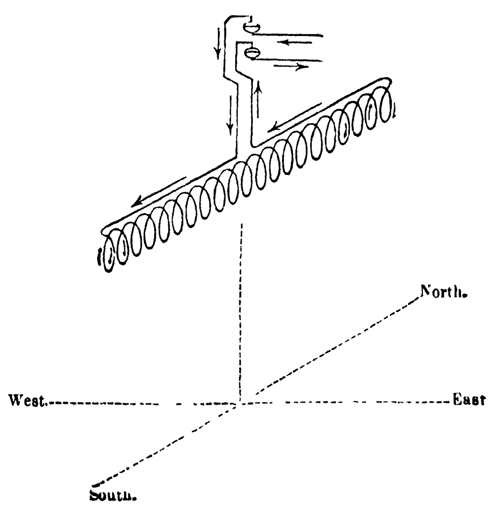


Fig 161.—Orientation of Solenoid.

C, is that which is universally adopted, the returning wires being sometimes in the axis, and sometimes on the outside of the helix.

If a solenoid, thus constructed, be suspended on an Ampère's stand, as in Fig. 161, and a current sent through it, it will immediately place its axis parallel to a declination needle. It may accordingly be said to have poles. In Fig. 162, A represents the austral or north-seeking, B the boreal or south-seeking pole of the solenoid; that is to say, the direction of the current is against or with the hands of a watch according as the austral or boreal pole is presented to the observer. The same difference is illustrated by Fig. 161.

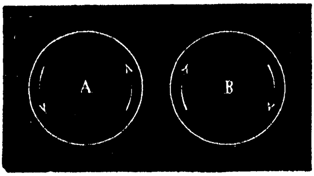


Fig. 162.—Poles of Solenoid.

249. Dip of Solenoid.—If a solenoid could be balanced so as to be perfectly free to move about its centre of gravity, it would place its axis parallel to the dipping-needle. The experiment would be scarcely practicable with a solenoid properly so called, on account of its weight; but it can be performed with a single plane circuit, such as that shown in Fig. 163. If such a circuit is nicely balanced

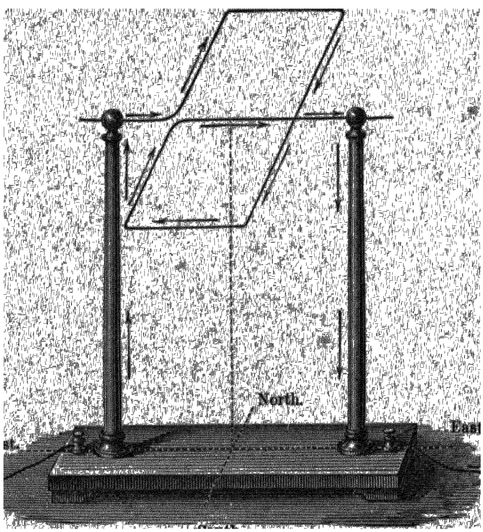


Fig. 163.—Dip of Element of Solenoid.

about an axis through its centre of gravity, and placed so that it can turn freely in the plane of the magnetic meridian, the passing of a current through it will cause it to set its plane perpendicular to the direction of a dipping-needle. This effect is due to the action of terrestrial magnetism on the upper and lower sides of the rectangle. The plane of the rectangle is represented in the figure as coinciding with the direction of dip. In this position the action of terrestrial magnetism urges the upper side backwards and the lower side forwards, and stable equilibrium will be attained when the rectangle has turned through 90°.

250. Mutual Actions of Solenoids.—Solenoids behave like magnets not only as regards the forces which they experience from terrestrial magnetism, but also as regards the actions which they exert upon one another. The similar poles of two solenoids repel, and the unlike poles attract each other, as we may easily prove by suspending one solenoid from an Ampère's stand and bringing another near it.

The reason of these attractions and repulsions is illustrated by Fig. 165. If two austral poles are placed opposite each other, as in the upper part of the figure, the currents are circulating round them in opposite directions, and, by the

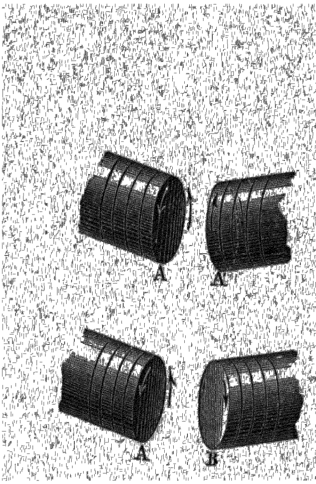


Fig 164 —Mutual Action of Solenoids.

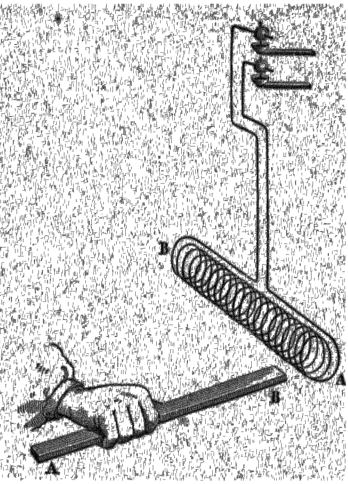


Fig 165 —Action of Magnet on Solenoid.

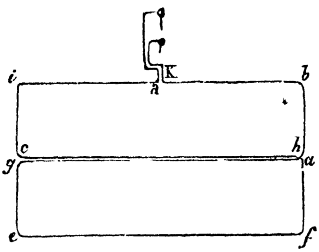
laws of parallel currents, should therefore repel each other, whereas if two dissimilar poles be placed face to face, the currents which circulate round them are in the same direction, and attraction should therefore ensue.

Lastly, if one pole of an ordinary magnet be brought near one pole of a suspended solenoid, as in Fig. 165, repulsion or attraction will be exhibited according as the poles in question are similar or dissimilar. In the position represented in the figure, this action is mainly due to the action of the boreal pole of the magnet upon the descending currents in the near side of the solenoid. This action consists in a force to the left hand, nearly parallel to the axis of the solenoid, which tends to make the solenoid rotate about its supports, and thus to bring the end A of the solenoid into contact with the end B of the magnet.

It may be shown, by the aid of Ampère's formula for the mutual force between two elements, that the mutual action of two solenoids is equivalent to four forces, directed along lines joining the poles of the solenoids, and varying inversely as the squares of the distances between the poles; the forces between similar poles being repulsive, and the other two attractive. The analogy between solenoids and magnets is thus complete.

251. Astatic Circuits.—When it is desired to eliminate the influence of terrestrial magnetism in electro-dynamic experiments, an astatic circuit may be

employed as the movable conductor. Two such circuits are represented in the accompanying figures (Figs. 166, 167). In each of them the current in one half of the circuit circulates with, and in the other against the hands of a watch, thus producing equal and opposite tendencies to orientation, which destroy one another.



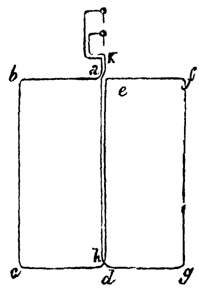
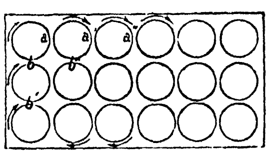


Fig. 166. Astatic Circuits. Fig. 167.

252. Ampere's Theory of Magnetism.—In accordance with the preceding facts, Ampère propounded the hypothesis that what is called magnetism consists in the existence of electric currents circulating round the particles of magnetic bodies. In iron or steel, when unmagnetized, according to this theory, the currents around different particles have different directions; but when it is magnetized, the directions of all are the same. Fig. 168 represents an ideal section of a



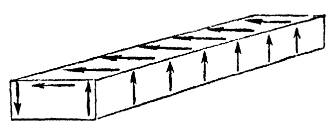


Fig. 168. Amperian Currents in Magnets. Fig. 169.

magnetized bar at right angles to the direction of its magnetization. On the neighbouring faces of any two particles, the currents are in opposite directions, hence, by the laws of sinuous currents, there is a mutual destruction of force through the whole interior, and the resultant effect is the same as if there were currents circulating round the exterior of the magnet, as represented in Fig. 169.

Magnetization by influence depends, according to this theory, on the tendency of currents to set themselves parallel and in similar directions; and if the substance magnetized possesses coercive force, the direction thus impressed on its currents persists after the influence is removed. In soft iron, on the contrary, they resume their former irregularity.

Ampère's theory of magnetism is in complete accordance with all known facts. But it admits of question whether it is simpler to deduce the laws of magnetism and electro-magnetism from those of electro-dynamics; or to adopt the reverse order, and deduce the laws of electro-dynamics from those of electro-magnetism.

253. Rotations of Magnets.—The following experiment is due to Ampère.

A magnet, loaded with platinum at its lower end, floats upright in mercury contained in a glass vessel (Fig. 170). A cavity is hollowed out in the top of the magnet. This contains mercury, in which a point dips. On connecting one of

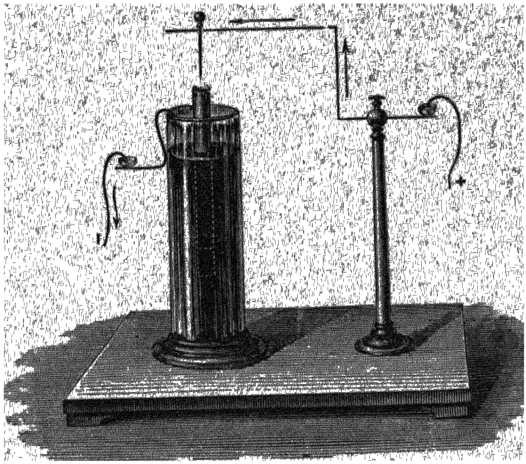


Fig. 170.—Rotation of Magnet

the terminals of a battery with this point, and the other with the outer edge of the mercury in the vessel, the magnet is seen to rotate on its axis. If the north-seeking pole is uppermost, and the positive pole of the battery is connected with the point, the direction of rotation is N.E.S.W.

The Amperian explanation of this phenomenon is, that it is due to the action

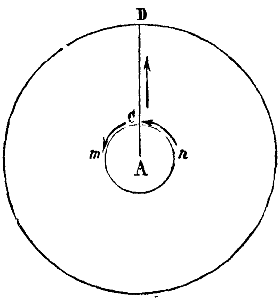


Fig 171.—Explanation of Rotations.

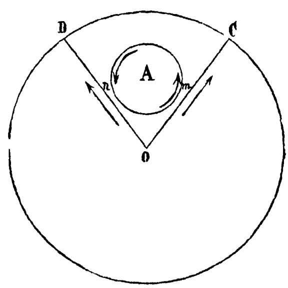


Fig 172.—Explanation of Rotations.

between the outward-flowing current in the mercury and the Amperian currents which circulate round the magnet. The latter, as represented by the arrows n C, C m in Fig. 171, are opposite to watch-hands. The outward-flowing current in CD attracts the current in C m, since they are both directed away from the angular point C, and repels the current in n C. Hence the magnet is made to rotate in the direction m C n, opposite to that of the Amperian current.

The experiment is sometimes varied by making the point dip in the mercury in

the vessel, the magnet being allowed to float freely near it, and a metallic ring being immersed at the outer edge of the mercury, to which the current flows out in all directions from the point. As soon as the circuit is completed, the magnet begins to revolve round the point. The rotation will be in the same direction as in the other form of the experiment; that is to say, if the current flows outwards from the point to the edge of the vessel, the direction of rotation will be opposite to that of the Amperian currents in the magnet. This is easily explained by the laws of parallel currents, for the current in O C (Fig. 172) attracts the Amperian current at m, and the current in O D repels the current at n. The magnet will therefore move from O D to O C, and will revolve round O in the direction N.E.S.W.

254. Magnetization by Currents.—Ampère's theory of magnetism leads naturally to the conclusion that a bar of iron or steel may be magnetized by means of a current. Arago was the first to establish this fact, but without a clear apprehension of the conditions necessary for success, or of the criterion for determining which will be the austral, and which the boreal pole. Ampère conceived the idea of introducing the needle to be magnetized into the axis of a solenoid, and the result confirmed his prediction that the poles of the needle would be turned the same way as those of the magnetizing helix. This is what must happen if the currents in the helix force the Amperian currents in the bar into parallelism with themselves, so that all rotate the same way.

It is to be remarked that, in this process of magnetization, the portions of the currents parallel to the axis of the helix produce no effect. The wire through which the current is to be sent may be wound like thread upon a reel, returning alternately from end to end, and all the convolutions will contribute to magnetize the bar the same way, although it is evident that the helices are in this case alternately right-handed and left-handed. The north-seeking and south-seeking poles may be in all cases distinguished by the rule that if a corkscrew be turned in the direction in which the current circulates it will travel towards the north-seeking pole; or it may be remembered by the rule, that if I identify my own body in imagination with a portion of the wire, and suppose the current to enter at my feet, while my face is towards the needle, the north-seeking pole will be to my left. In 1 and 2 (Fig. 173), a will be the austral (or north-seeking), and b the boreal pole of the inclosed needle, when the current in the helix has the direction indicated by the arrows.

If the direction of winding is changed, in the manner represented

in 3, so that, as seen from one end, the direction in which the current circulates is in one part with and in another against the hands of a watch, consequent points (§ 126) will be formed at the points

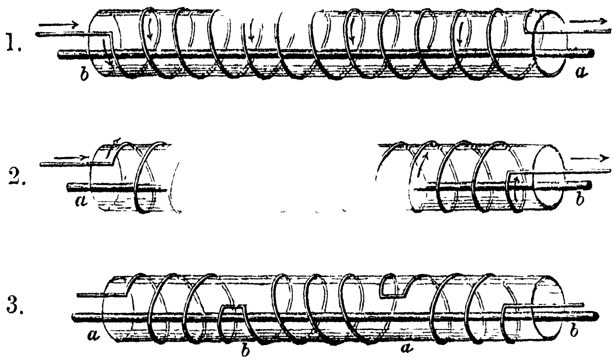


Fig. 173.

1 Right-handed Helix. 2. Left-handed Helix. 3 Arrangement for Consequent Points.

of change. Thus, if the current enters at the left-hand end of the coil, the points $a\,a$ will be austral, and the points $b\,b$ boreal poles.

255. Electro-magnets.—A cylindrical bar of iron can be powerfully magnetized by wrapping round it a coil of insulated wire and sending a current through this coil. Stout copper wire is generally employed for this purpose. Such an arrangement is called an *electro-magnet.*

The bar has often the horse-shoe form, as in Fig. 174, and in this case the central part is usually left bare. The direction of winding on the ends must be such that, if the bar were straightened out, the current would circulate in the same direction round every part. This is clearly shown in the figure. Electro-magnets have been constructed capable of sustaining a load of many tons.

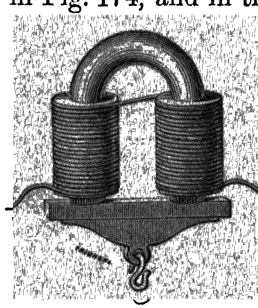


Fig 174.—Horse-shoe Electro-magnet.

Besides the enormous power that can be given them, electro-magnets have the advantage of being readily made or unmade instantaneously, by completing or interrupting the circuit to which the coil belongs. This principle has received very numerous and varied applications, some of which will be mentioned in later chapters.

256. Residual Magnetism.—When the current round an electro-magnet is interrupted, the destruction of the magnetization is not complete. The small remaining magnetization is called *remanent* or *residual* magnetism. It is frequently sufficiently powerful to retain the armatures in contact with the magnet, and thus necessitates the employment of *opposing springs* if instantaneous separation is desired. The mere act of separation suffices to destroy the greater part of the residual magnetism.

Fig. 175 represents an electro-magnet E E′, furnished with an

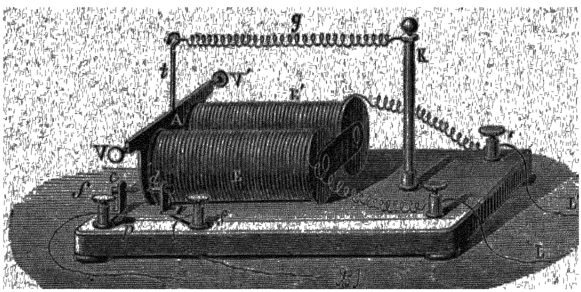


Fig. 175.—Electro-magnet with Opposing Spring.

opposing spring *g*. The armature A, with its lever *t*, turns about the axis V V′. The opposing spring *g* has one end fixed at K, and the other attached to the end of the lever *t*. It therefore tends to remove the armature from the magnet. *c* and *d* are two points whose distance can be regulated, which serve to limit the movements of the armature.

CHAPTER XIX.

INDUCTION OF CURRENTS, OTHERWISE CALLED MAGNETO-ELECTRIC INDUCTION

257. Induced Currents.—Induced currents may be described as currents produced in conductors by the influence of neighbouring currents or magnets. Their discovery by Faraday in 1831 constitutes an epoch in the history of electrical science. We shall first describe some modes of producing them; and then state their general laws.

258. Currents induced by Commencement and Cessation of Currents.—Let two coils be wound upon the same frame B, one of them, called

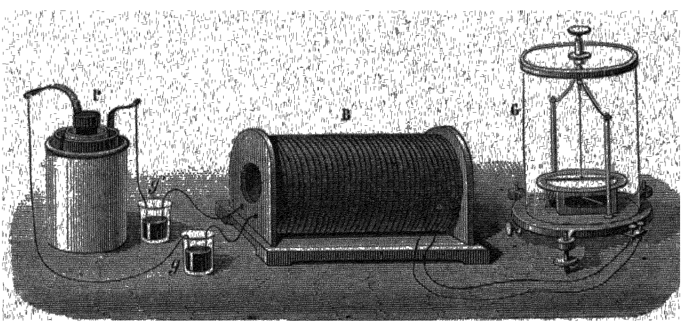
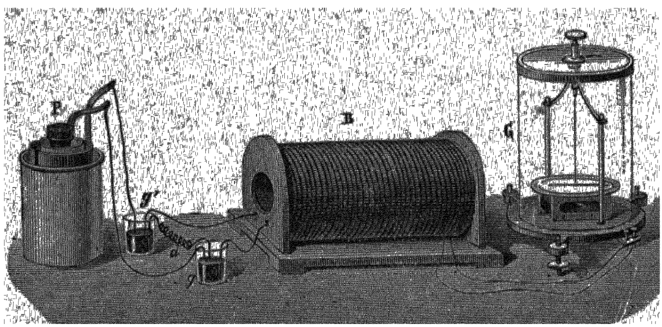


Fig. 176.—Current induced by Commencement or Cessation.

the secondary coil, having its ends connected with the binding-screws of the galvanometer G, while the ends of the other, which is called the primary coil, dip in two cups of mercury *gg′* connected with the two plates of the voltaic element P. As long as the current is passing steadily in the primary coil, the needle of the galvanometer remains undeflected; but if the current be stopped, by lifting a wire out of one of the mercury cups, the needle is immediately deflected, indicating the existence of a current in the same direction as that which

was previously circulating in the primary coil. This effect is very transitory. The needle appears to receive a sudden impulse which immediately passes away. If the current be then re-established, there is a deviation to the other side, indicating a current in the opposite direction to that in the primary coil; and this deviation, like that which occurred before, is merely the effect of an instantaneous impulse, the needle making a few oscillations from side to side, and then remaining steadily at zero. This experiment, which is substantially the same as that by which Faraday first made the discovery, establishes the following proposition:—*When a current begins to flow, it induces an inverse current in a neighbouring conductor; when it ceases, it induces a direct current; and both the currents thus induced are merely instantaneous.*

259. Currents induced by Variations of Strength of Primary Current.—Employing the same apparatus, let us, while the primary current

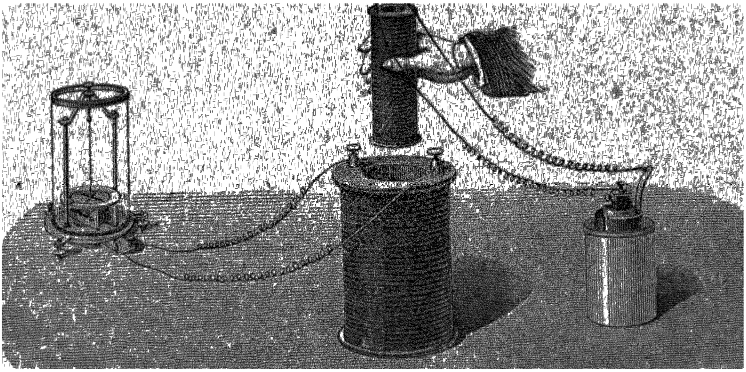


[illegible] induced by Chan[illegible]gth.

is passing, connect the two mercury cups by the wire d (Fig. 177), thus dividing the circuit (§ 200), and causing a great diminution of the current in the primary coil. At the instant of making this connection, the needle of the galvanometer is affected, moving in the same direction as if the primary current were stopped; and on lifting the connecting wire out of one of the cups, so as to produce a sudden increase in the current in the primary coil, the needle moves in the opposite direction. *When a current receives a sudden increase, this produces an inverse current in a neighbouring conductor; and when it is suddenly decreased, a direct current is induced.*

260. Currents induced by Variations of Distance.—Currents may also be induced by change of distance between the primary and secondary conductors. Let the secondary coil, for example, be hollow,

as in Fig. 178, and let the primary coil, with the current passing in it, be suddenly introduced into its interior. The galvanometer will indicate the production of an inverse current in the secondary coil.



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When the needle has come to rest, let the primary coil be withdrawn, and a direct current will be indicated by the galvanometer. These currents differ from those previously mentioned in being less sudden. They last as long as the relative motion of the two coils continues. *When a conductor conveying a current approaches or is approached by a neighbouring conductor, an inverse current is induced in the latter; and when one of these conductors moves away from the other, a direct current is induced.*

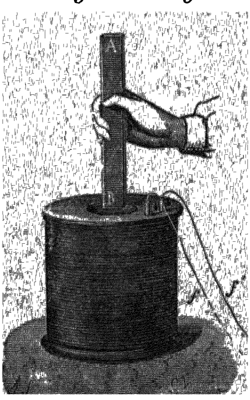


Fig. 179.—Current induced by Motion of Magnet

261. Induction by Magnet.—Let a hollow coil be connected with a galvanometer, and a magnet held over it, as in Fig. 179. As long as the magnet remains stationary, no current is indicated; but when one pole of the magnet is thrust into the interior of the coil, the needle is deflected by an impulse which lasts only as long as the motion of the magnet. If the magnet is allowed to remain at rest in this position, the needle, as soon as it has time to recover from its oscillations, stands at zero; but on withdrawing the magnet, another current will be indicated in the opposite direction to the former.

Currents may also be induced, with even more striking effect, by moving one pole of a magnet towards or from one end of a soft-iron bar previously placed in the interior of the coil (Fig. 180). These currents are due to the magnetism produced and destroyed in the

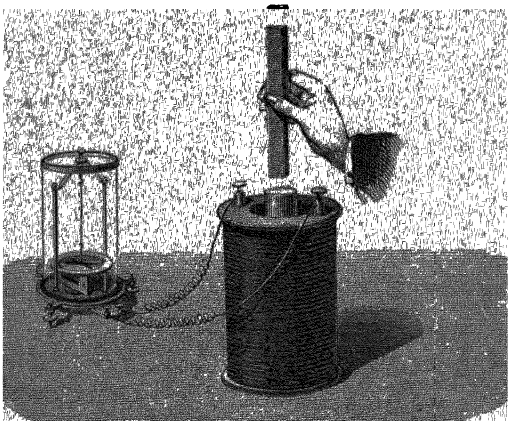


Fig. 180.—Current induced by Magnetization of Soft Iron.

soft iron. *When the intensity of magnetization of a piece of iron or steel undergoes changes, currents are induced in neighbouring conductors.* The directions of these currents can be inferred from the preceding rules by supposing a solenoid to be substituted for the magnet.

262. Lenz's Law.—*The currents induced by the relative movement either of two circuits or of a circuit and a magnet are always in such directions as to produce mechanical forces tending to oppose the movement.* For example, when two parallel wires, through one of which a current is passing, are made to approach, an opposite current is induced in the other; and opposite currents by their mutual repulsion resist approach. This general law as to the direction of induced currents was first distinctly enunciated by Lenz, a Russian philosopher.

263. Arago's Rotations.—Faraday successfully applied his discovery of magneto-electric induction to account for a phenomenon first observed by Arago in 1824, and subsequently investigated by Babbage and Sir John Herschel. A horizontal disc of copper *b b* (Fig. 181), placed in the interior of a box, is set in rapid rotation by turning a handle. Just over the copper disc, but above the thin glass plate which forms the top of the box, a magnetized needle *a a*

is balanced horizontally. When the disc is made to rotate, the needle is observed to deviate from the meridian in the direction of the rotation. When the speed of rotation exceeds a certain limit, the needle is not only deflected, but carried round in continuous rotation in the same direction as the disc.

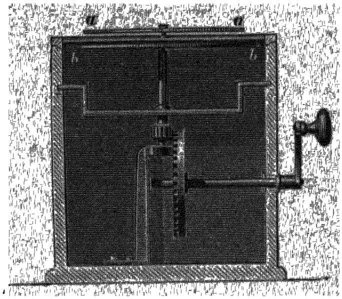


Fig. 181.—Arago's Rotations

The explanation is to be found in the currents which are induced in the disc by its motion in the vicinity of the magnetized needle. The forces between these currents and the needle are (by Lenz's law) such as to urge the disc backwards; and, from the universal relation which subsists between action and reaction, they must be such as to urge the needle forwards, hence the motion. The direction of the induced current through the centre of the disc at any instant is along that diameter which is directly under the needle, the circuit being completed through the lateral portions of the disc; and it is evident that a current thus flowing parallel to the needle underneath it tends to produce deflection. If the continuity of the disc is interrupted by radial slits, the observed effect is considerably weakened inasmuch as the return circuit is broken. Faraday succeeded in directly demonstrating the existence of currents in a disc rotating near a fixed magnet, by exploring its surface with the amalgamated ends of two wires connected with a galvanometer.

The experiment performed by Arago may be reversed by setting the magnet in rotation, and observing the effect produced on the disc. The latter, if delicately suspended, will be found to rotate in the same direction as the magnet. This experiment was first performed by Babbage and Herschel. Its explanation is identical with that just given. In both cases the induced rotation must be slower than that of the body turned by hand, as the existence of the induced currents depends upon the motion of the one body relative to the other.

When an iron disc is used instead of a copper one, magnetism is induced in the portions which pass under the poles of the magnet; and as this requires a sensible time for its disappearance, there is always attraction between the poles of the needle and the portions

of the disc which have just moved past. The needle is thus drawn forwards by magnetic attraction, and the observed effect is similar to that obtained with the copper disc, though the cause[1] is altogether different.

264. Copper Dampers.—Precisely similar to the above is the explanation of the utility of a copper disc in checking the vibrations of a magnetized needle under which it is fixed. As the needle swings to either side, its motion induces currents in the copper which urge the needle in the opposite direction to that in which it is moving. When it rests for an instant at the extremity of its swing, the currents cease; and as soon as it begins to return, the currents again resist its motion. A copper plate thus used is called a *damper*, and the vibrations thus resisted and destroyed are said to be *damped*. The name is applied to any other means for gradually destroying vibrations.

The resistance which induced currents oppose to the motion producing them is well illustrated by Faraday's experiment of the *copper cube*. A cube of copper is suspended by a thread, and set spinning by twisting the thread and then allowing it to untwist. If, while spinning, it is held between the poles of a powerful magnet, like that represented in Fig. 88, it is instantly brought almost to rest. If the poles are brought very near together, so as to heighten the intensity of the field, and a thin sheet of copper is inserted between them and moved rapidly in its own plane, the operator feels its motion resisted by some invisible influence. The sensation has been compared to that of cutting cheese. Foucault's apparatus for the heating of a copper disc by rotating it between the poles of a magnet (see Part II.), is another illustration of the same principle. In all cases where induced currents are generated, they yield their full equivalent of heat, in accordance with the principles of § 221.

The advantage of employing copper in experiments of this kind arises from its superior conductivity, to which the induced currents are proportional.

265. Direction of Induced Currents specified by reference to Lines of Magnetic Force.—We have already mentioned, in connection with the mutual forces between magnets and currents (§ 165), that a

[1] That is to say, the *main cause;* for there must be induced currents in the iron as well as in the copper, though inferior in strength, on account of the inferior conductivity of the former metal.

wire conveying a current experiences force perpendicular to its length, and at the same time perpendicular to the lines of magnetic force, when placed in a magnetic field. We have seen that, if the current is from foot to head, and the lines of force (for an austral pole) run from front to back, the force experienced by the wire is a force to the right. Motion of the wire to the right will diminish this force by diminishing the current, motion to the left will increase it by increasing the current. Let the direction of the lines of magnetic force be called *from front to back;* then the motion of a conductor *to the right* generates a current in it *from head to foot,* and motion in the opposite direction generates an opposite current. We shall have frequent occasion to recur to this criterion of direction, which applies to every case of induced currents.

As the generation of currents by induction depends not on absolute but on relative motion, namely the relative motion of the conductor and the lines of magnetic force, the criterion of direction will take the following form when the conductor is supposed to be stationary, and the lines of force to move. Let the direction of the lines of magnetic force be called *from front to back,* then *if the lines of force move so as to cut through the conductor from right to left, a current will be induced in the conductor from head to foot.*

If the conductor forms part of a closed circuit, we shall have a continuous current flowing through it as long as the motion lasts. If the circuit is open, there will merely be an incipient current, which, if its direction be from head to foot, will reduce the end of the conductor which we are regarding as its foot to a higher electrical potential than the other, and this difference of potential will be maintained as long as the motion lasts.

266. Quantitative Statements.—In order to state the quantitative laws of induced currents in the simplest and most general manner, we must employ the conception of tubes of force as explained in § 52 (but they will now be tubes not of electrical but of magnetic force), and we must suppose them to be arranged in the equable manner described in § 53. That is to say, we must suppose the whole field cut up into tubes of force in such a manner that, if a cross section (an equipotential surface as regards magnetic potential) be made in any part of the field, the number of tubes per unit of sectional area is equal to the intensity in that part of the field. It is more usual to speak of *number of lines of force* than of *number*

of tubes, the convention being that each tube contains one line; but the counting of tubes rather than lines has the advantage of naturally allowing fractional parts to be reckoned, and not suggesting the idea of discontinuity.

The tubes of force due to a magnet are to be regarded as rigidly attached to the magnet, and carried with it in all its movements, whether of translation or rotation.[1] They undergo no change of size or form unless the magnet itself undergoes changes in its magnetization.

These conceptions being premised, the quantitative laws of induced currents can be stated with great simplicity and complete generality.

1. When a conductor is moved in a magnetic field, the ELECTROMOTIVE FORCE generated by the motion is equal to the number of tubes which the conductor cuts through per unit time.

2. If the conductor forms part of a closed circuit, the CURRENT generated in the circuit is the quotient of the number of tubes cut through per unit time, by the resistance of the circuit; and lastly,

3. The whole QUANTITY of electricity conveyed by the current is the quotient of the number of tubes cut through, by the resistance of the circuit. The quantity of electricity conveyed by a current of brief duration is measured by observing the swing of a galvanometer needle (§ 175). It is proportional to the greatest deviation of the needle from zero, provided that this deviation is small, and that the duration of the current is less than that of the swing. When experiments on induced currents are made under these conditions, it is found that the deviation of the needle depends only on the initial and final positions of the body which is moved, being independent both of the path taken and of the velocity.

The dependence of the quantity of electricity induced upon the number of tubes cut through, was discovered by Faraday, who established it experimentally by moving a loop of wire in various ways in the vicinity of a magnet. The three foregoing laws were all, in fact, substantially established by the series of researches in which these experiments occur.[2]

[1] It may be noted that when a cylindrical magnet is rotated on its axis, it induces no current in a neighbouring wire, inasmuch as its lines of force cut such a wire once positively and once negatively.

[2] *Researches*, vol. iii. series xxviii.

In counting the tubes cut through, it is necessary to attend to the direction of the current due to the cutting of each tube. Those tubes which are so cut as to give currents in one direction round the circuit (when tested by the criterion of § 265) must have one sign given them, and those which give currents in the opposite direction must be reckoned as of the opposite sign. It is in every case the algebraic sum that is to be taken; and if a tube is cut once positively and once negatively, it may be left out of the reckoning.

267. Deduction of the Laws from Maxwell's Rule.—The laws in the preceding section are deducible by the principle of energy from Maxwell's rule (§ 246), which asserts that the mechanical work requisite to produce a given displacement of a circuit in a magnetic field is equal to the strength of the current multiplied by the number of tubes of force cut through. Call this number n and the current C. Then, if there is no other electro-motive force in the circuit beside that which is due to the motion, the work $\mathrm{C}\,n$ must be equal to the energy of the current, which is $\mathrm{E\,C}\,t$, t denoting the time. Hence we have

$$\mathrm{E}=\frac{n}{t}$$

that is, the electro-motive force is equal to the number of tubes cut through per unit time.

If the circuit contains a battery of electro-motive force E_0, the energy supplied by this battery is $\mathrm{E}_0\,\mathrm{C}\,t$; hence we have

$$\mathrm{E\,C}\,t=\mathrm{E}_0\,\mathrm{C}\,t+\mathrm{C}\,n,$$

Or $$\mathrm{E}=\mathrm{E}_0+\frac{n}{t},$$

that is, the electro-motive force $\mathrm{E}-\mathrm{E}_0$ produced by the motion is equal to the number of tubes cut through per unit time; and these are to be counted as positive when the direction of the motion is opposed to the forces of the field, since we have supposed positive work to be supplied. The induced current is therefore *with* or *against* the original current according as the motion is *against* or *with* the mutual forces between the current and the field.

268. Tubes of Force for Resultant and Components.—The principle of superposition can be applied to tubes of force. For if we resolve the whole force into any two components, these three forces will be represented by the three sides of a triangle as in Fig. 182, where B C represents the resultant, and B A, A C the two components.

By producing each side, and drawing a parallel to it through the opposite angle, we obtain longitudinal sections of three tubes, whose cross sections are proportional to the perpendiculars on the sides of the triangle A B C from the opposite angles. These perpendiculars are inversely as the sides (since side × perpendicular = double area of triangle). Hence the cross sections of the three tubes are inversely as the forces, and the number of tubes per unit area of cross section will be directly as the forces.

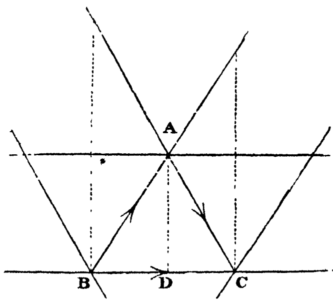


Fig 182.—Superposition of Tubes.

In moving parallel to B C, no resultant tubes are cut, and as many component tubes are cut from the right to the left as from the left to the right of a person travelling with the component forces. In moving perpendicular to B C, through a distance equal to A D the perpendicular from A on B C, the fraction $\frac{BD}{BC}$ of one component tube is cut, and the fraction $\frac{DC}{BC}$ of another, the sum of these fractions being unity. Hence, in every motion, *the number of component tubes cut* (reckoned algebraically) *is equal to the number of resultant tubes.*

In dealing with terrestrial magnetism, it is often convenient to consider separately the tubes of vertical and of horizontal force; for example, effects depending only on horizontal force can be determined by considering the horizontal tubes alone.

269. Uniform Field.—In a field of uniform intensity the lines of force cannot be curved, for if they were, the equipotential surfaces, which cut them at right angles, would not be equidistant, but would be nearer together on the side next the centres of curvature than on the opposite side, and the force would be greatest where these surfaces were nearest (§ 49). The lines of force in a field of uniform intensity are therefore straight; and, since the tubes of force must have a constant cross section (§ 52), these tubes must be cylinders or prisms, and the lines of force parallel. A field of uniform intensity of force is therefore also uniform as regards direction of force.

The electro-motive force generated by the motion of a straight wire of length L in a field of uniform intensity I, with a velocity of translation V, being equal to the number of tubes cut through in unit time, will be L V I, if the length of the wire and the direction of

motion are perpendicular to each other and to the lines of force. For any other position of the wire, and for any other direction of motion, the number of tubes cut through will evidently be less. If the length of the wire is parallel to the lines of force, no tubes will be cut through, whatever be the direction of motion; and if the direction of motion be parallel to the lines of force, no tubes will be cut through, whatever be the position of the wire. In these two cases, then, there is no generation of electro-motive force tending to produce a current along the wire.

Terrestrial magnetism furnishes us with an example of a uniform field, so long as we confine our attention to a space of moderate dimensions, such as the interior of a room.

270. Movement of Lines of Force with Change of Magnetization.—As long as a piece of iron or steel remains unchanged in its magnetization, its tubes of force are to be conceived of as a rigid system rigidly connected with it. When the intensity of magnetization is increased, new tubes are added and the old ones are crushed together. The new tubes are to be regarded as coming into existence at the magnetic axis of the magnet, and pushing the old ones further away from the axis. When the intensity of magnetization falls off, a reverse motion occurs, and the axis absorbs those tubes which lie next it.

Similar remarks apply to changes of strength in a current. The lines of magnetic force due to a current in a straight wire are circles, and the tubes of force are rings, having the wire for their common axis. When the current receives an increase of strength, the new rings must all be conceived of as starting from the wire, and pushing out the old rings before them, and on the diminution or cessation of the current a reverse movement occurs.

When a current suddenly commences in a wire, or a piece of soft iron is suddenly magnetized, the effect upon a neighbouring conductor is the same (so far as this source of magnetic force is concerned) as if the conductor were suddenly moved up from a great distance into its actual position. The experimental results described in §§ 258–261 are thus only particular cases of the general principles of §§ 265, 266.

271. Unit of Resistance.—Units of *length*, *mass*, and *time*, having been selected, unit *force* is defined as that which, acting on unit mass for unit time, generates unit velocity.

A magnetic pole of unit strength, or a unit *pole*, is defined as that

which attracts or repels an equal pole at unit distance with unit force.

Unit *intensity of field* is defined as the intensity at a place where a unit pole experiences unit force.

A unit *current*, or a current of unit strength, is one which, for each unit of its length, affects a unit pole at unit distance with unit force. In passing through a circular coil of unit radius and length l, the force which it exerts on a unit pole at the centre is l.

Unit *electro-motive force* is the electro-motive force existing in a circuit in which unit current does unit work in each unit of time; and unit *resistance* is the resistance of a circuit in which unit electro-motive force would produce unit current.

The course of the above investigation shows that the units of length, mass, and time are sufficient to determine all the other units mentioned. It can be further shown[1] that the unit of resistance is independent of the unit of mass, and depends only on the units of length and time, being directly as the unit of length, and inversely as the unit of time—a property which is also characteristic of the unit of velocity. Hence a resistance, like a velocity, can be adequately expressed in *metres per second*. The unit of resistance now commonly employed is the *ohm*, which is defined as *ten million metres per second*. It is the resistance of a column of pure mercury at 0° C. 1·063 metre long and a millimetre in diameter, or of about 50 metres of pure copper wire a millimetre in diameter.

272. Induction by Means of Terrestrial Magnetism.—If a wire ring, or any other form of closed circuit, receives a movement of translation in a uniform field, no current is generated, because the same number of force-tubes are cut negatively as positively. Whatever currents are generated by the motion of a closed circuit in the terrestrial magnetic field, must therefore be due solely to rotational movements. Suppose the circuit to consist of a single circle of wire, and let it be initially placed so that its plane is perpendicular to the dipping-needle, and therefore perpendicular to the lines of magnetic force. In this position, the number of force-tubes which it incloses is equal to the product of the inclosed area by the total intensity of terrestrial magnetic force, that is to $\pi r^2 \mathrm{I}$, I denoting this intensity, and r the radius of the circle. Now let the ring rotate through 180° about any diameter, so that it comes back into its original place, but facing the opposite way. During this semi-revolution, each half of

[1] See Appendix at the end of this Part.

the ring has cut through all the tubes which passed through the ring, and though in one sense the two halves have been cutting the tubes in opposite directions, the application of the criterion of § 265 shows that the resulting currents are in the same direction round the circuit. The number of tubes cut through is therefore to be reckoned as $2\pi r^2 I$, and the quotient of this by the time occupied in a semi-revolution is the average electro-motive force (§ 266). If the rotation be uniform, the actual electro-motive force is greatest in the middle of the semi-revolution, and is zero at its commencement and termination. During the other half-revolution the circumstances are precisely the same, except that the two halves of the ring have changed places. If we compare the currents in two positions of the ring which differ by 180°, we see that the current round the ring has the same direction in space, but opposite directions as regards the ring itself.

If, instead of a single ring of wire, we have a circular coil consisting of any number of convolutions, with its two ends united, the same principles apply. If there are n convolutions, the electro-motive force will be n times greater than with one, but as the resistance is also n times greater the strength of current is the same.

In the apparatus called *Delezenne's Circle*, a coil of wire revolves about a diameter, but the two ends of the coil, instead of being directly united, are so connected with the two ends of the axis of rotation that the circuit is completed through a galvanometer. On rotating the coil rapidly by means of a handle provided for the purpose, a current is indicated by the galvanometer, and this current is found to be strongest (for a given rate of rotation) when the axis is perpendicular to the dipping-needle. If the axis is inclined at an angle θ to the dipping-needle, the current is proportional to $\sin \theta$; and if the axis is parallel to the dipping-needle there is no current at all. For a given position of the axis, the current varies directly as the speed of rotation. When the time of a revolution is only a small fraction of the time in which the needle would oscillate, the variations of electro-motive force, and consequently of current, which take place during a revolution, have not time to manifest themselves, and the deflection of the needle is that due to the average current. It is necessary, however, that a commutator be employed to prevent the reversal of the current at each half-revolution. The proportionality of the current to $\sin \theta$ is easily inferred from the principles of the foregoing sections; for if the plane of a circle, instead of being per-

pendicular to the lines of force, is inclined to them at an angle θ, the number of force-tubes which it incloses will be not $\pi r^2 I$, but $\pi r^2 I \sin\theta$.

273. British Association Experiment.—The first experiments for constructing a coil whose resistance should be a known number of metres per second, were conducted by a committee of the British Association in 1862. A circular coil of wire, with its ends joined, was made to revolve rapidly, at a measured rate, about a vertical axis; and the current induced was measured by the deflection of a magnetized needle suspended, within a glass case, in the centre of the coil. The part of the earth's magnetic force which comes into play in this arrangement, is only the horizontal component, and it is worthy of remark that variations in the horizontal intensity do not alter the deflection of the needle, since they affect to the same extent the amount of the induced current, and the terrestrial couple on the needle tending to resist deflection.

All the other elements involved were determined by observation, and hence the value of R in metres per second was calculated. By comparing the resistances of other coils with that of the coil used in this experiment (a comparison which can be made with great accuracy by means of Wheatstone's bridge), their values in metres per second were at once determined; and it was easy to construct a resistance-coil of ten million metres per second, or any other desired amount of resistance. As the resistances of metals are increased by heat, a standard coil can only be correct at one particular temperature.

274. Induction of a Current on Itself: Extra Current.—If two portions of the same wire are side by side, the sudden commencement or cessation of a current in one, induces a current in the other, just as if they were portions of two unconnected circuits. An action of this kind occurs whenever a current commences or ceases in a coil, each convolution exercising an inductive influence on the rest. This action is called the *induction of a current upon itself*, and the current due to it is called an *extra current.*

The extra current on the commencement of the primary current is inverse, and merely acts as a hindrance to commencement; but the extra current on the stoppage of the primary current is direct, and is often a strongly-marked phenomenon. Hence it is that, with batteries of ordinary power, a spark is obtained on breaking, but not on making connection. The spark is particularly brilliant when a coil of many convolutions is included in the circuit, and especially if

this coil incloses a core of soft iron. If an observer holds in his hands two metallic handles permanently connected with the two ends of such a coil, and if the circuit of the battery is alternately made and broken, he will receive a shock from the extra current at each interruption. If the interruptions succeed each other rapidly, the physiological effect may become very intense. Many of the machines employed for medical purposes are constructed on this plan.

Special contrivances are provided for producing a rapid succession of interruptions at regular intervals. They are called *rheotomes* or *contact-breakers.* Sometimes they consist of toothed wheels turned by hand,—sometimes of vibrating armatures moved automatically.

275. Ruhmkorff's Induction-coil.—Induced currents capable of producing very striking effects are furnished by the apparatus first successfully constructed by Ruhmkorff, and hence known as Ruhmkorff's coil.

It contains two coils of wire, one of them forming part of the circuit of a battery, and called the primary coil; while in the other, called the secondary coil, the induced currents are generated. In the axis of the coils is a bundle of stout straight wires of soft iron, with a disc of the same material at each end, to which the wires are united. Around this core is wound the primary coil, consisting of a copper wire about two millimetres in diameter. The ends of this wire are shown at f and f'. The secondary coil consists of much finer wire (about a quarter of a millimetre in diameter) and of much greater length. In large instruments the primary coil may have a length of 80 metres, and the secondary a length of 150 kilometres (94 miles). Special precautions must be taken to insulate the different convolutions of the secondary coil from one another, and from the primary coil. The two ends of the secondary wire are at the binding-screws A, B, which are supported on glass pillars. It is obvious that if currents are alternately passed and stopped in the primary coil, there will be an alternate generation of currents (or at all events of electro-motive forces) in opposite directions in the secondary coil. The action of the core is similar to that of the soft-iron bar in Fig. 180, and its inductive effect is always in the same direction as that of the primary coil, for the primary coil may itself be regarded as a temporary magnet with its poles turned the same way as those of the core.

The wooden stand which forms the base of the instrument is

hollow, and contains a condenser of the kind described in § 73, its two coatings being connected with the primary coil at the two sides of the break. It mitigates the spark which occurs at the break, and the two opposite electricities, after rushing into its two coatings, rebound, so as to produce a reverse current in the primary, thus doubling the inductive effect of a mere stoppage. When the primary current is supplied by an alternating dynamo (without any breaking of contact) the condenser is not required.

The successive makes and breaks are effected automatically in

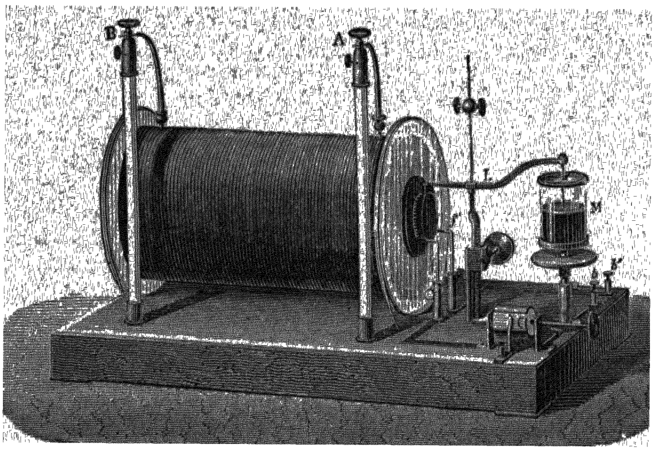


Fig. 183.—Ruhmkorff's Coil.

various ways. In small instruments the arrangement adopted is usually the same as that of the vibrating alarum described in § 300, but for large instruments Foucault's contact-breaker is preferred. It is represented in its place in Fig. 183.

The wires from the battery are attached at *b* and *b′*. The current entering for example at *b*, passes to the commutator C, and thence, through a brass bar let into the table, to the end *f* of the primary coil. Having traversed this coil, it comes out at *f′*, and is conducted to a vertical pillar, carrying at its upper end a spring, to which the transverse lever L is attached. One end of the lever carries a point which just dips in the mercury of the vessel M, the bottom of which is metallic, and is in communication with *b′*. The other end of the lever carries a small armature of soft iron just above the end of the core.

When the current passes, the core becomes magnetized and attracts this armature, thus lifting the point at the other end of the lever out of the mercury and breaking circuit. The core being thus

demagnetized, the elasticity of the spring releases the armature, and the point again dips in the mercury, and completes the circuit. A thin layer of absolute alcohol is usually poured on the surface of the mercury, and serves, by its eminent non-conducting power, to make the interruptions and renewals of the current more sudden.

The *commutator* C serves to stop the current from passing or to make it pass in either direction, at pleasure. It is represented in end view and bird's-eye view in the two parts of Fig. 184. There is a cylinder of insulating material, turning by means of metallic axle-ends on metallic supports connected with the two ends of the primary coil. One axle-end is permanently connected by means of the screw *g* with the brass plate C on the surface of the cylinder, and the other axle-end is in like manner connected with the plate C′ diametrically opposite to C. The contact-springs *f f′* are in permanent connection with the two binding-screws A A′ which receive the wires from the battery. When *f* presses against C, and *f′* against C′, as shown in the figure, A is connected with one end of the primary coil and A′ with the other, and when the commutator is turned (by its milled head) through 180°, these connections will be reversed. If it is turned through 90°, the connections will be interrupted, as the contact springs will bear against insulating portions of the cylinder. The milled head is, of course, insulated from the axle-ends so as to protect the operator.

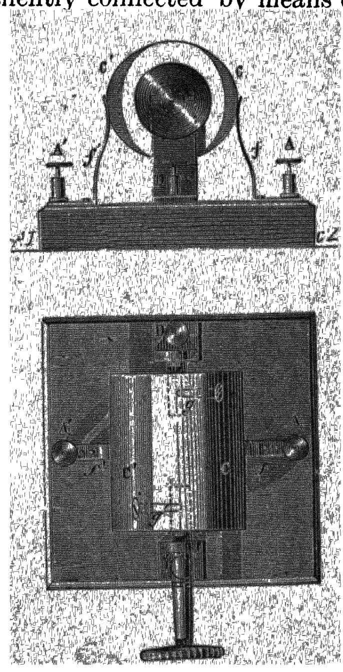


Fig 184.—Commutator

276. Spark from Induction-coil.—When the ends of the secondary coil are connected, currents traverse it alternately in opposite directions, as the primary circuit is made and broken. These opposite currents convey equal quantities of electricity, and if they are employed for decomposing water in a voltameter, the same proportions of oxygen and hydrogen are collected at both electrodes. If, however, the ends are disconnected, so that only disruptive discharge can

occur between them, the inverse current, on account of its lower electro-motive force, is unable to overcome the intervening resistance, and only the direct current passes (that is, the current produced by breaking the primary circuit). The sparks are usually from 1 inch to about 18 inches long, according to the size and power of the apparatus, and exhibit effects comparable to those obtained by electrical machines. A Leyden battery may be charged, glass pierced, or combustible bodies inflamed.

The great electro-motive force of the induced current, which enables it to produce these striking effects, depends on the great number of convolutions of the secondary coil, and on the suddenness of the interruptions of the primary current. The average electro-motive force is the product of the number of convolutions by the number of tubes of force which cut through them, divided by the time occupied (§ 263).

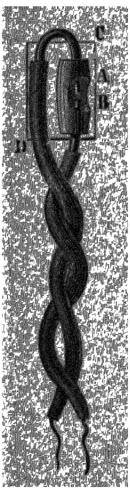
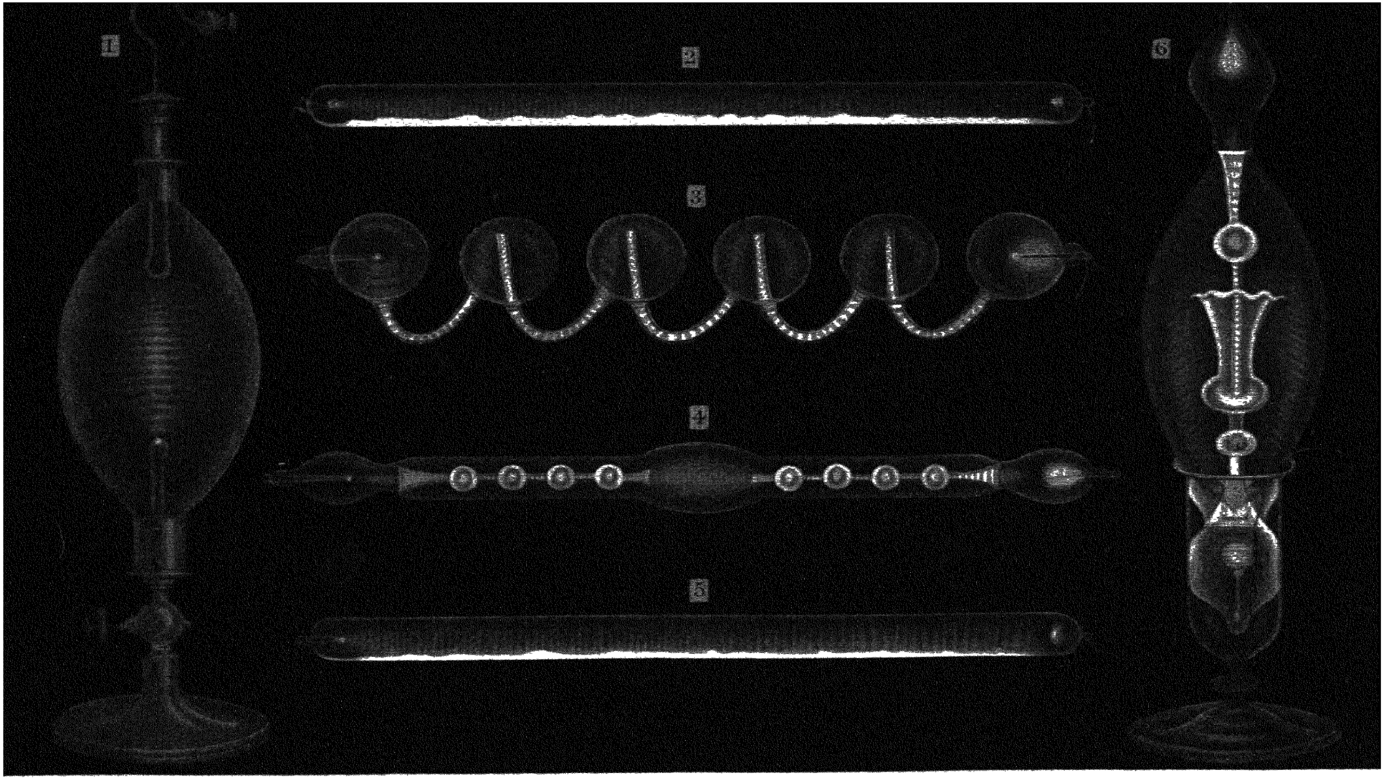


Fig 185.
Statham's Fuse

The discharges from a Ruhmkorff's coil become more violent and detonating if the two electrodes of the secondary coil are connected respectively with the two coatings of a Leyden jar, but the length of the spark is very much diminished.

Induction-coils are often used for firing mines, by means of Statham's fuse, which is represented in the annexed figure (Fig. 185). Two copper wires covered with gutta-percha have their ends separated by a space of a few millimetres, and inclosed in a little cylinder of gutta-percha containing sulphuret of copper. This, again, is inclosed in a cartridge, C D, which is filled up with gunpowder. The two wires are connected with the two ends of the secondary coil, and when the instrument is set in action, sparks pass between the ends A, B, heating the sulphuret of copper to redness, and exploding the powder.

277. Discharge in Rarefied Gases.—When the ends of the secondary coil are connected with the electrodes of the electric egg (Fig. 186), which has first been exhausted as completely as possible by the air-pump, a luminous sheaf, of purple colour, is seen extending from the positive ball to within a little distance of the negative ball. The latter is surrounded by a bluish glow. The blue and purple lights are separated by a small interval of darkness. If other gases are used instead of air, the tints change, but there is always a decided



ELECTRIC DISCHARGE IN RAREFIED GASES.

difference of tint between the positive and negative extremities. By the aid of the commutator it is easy to reverse the current, and thus produce at pleasure an interchange of the appearances presented by the two terminals.

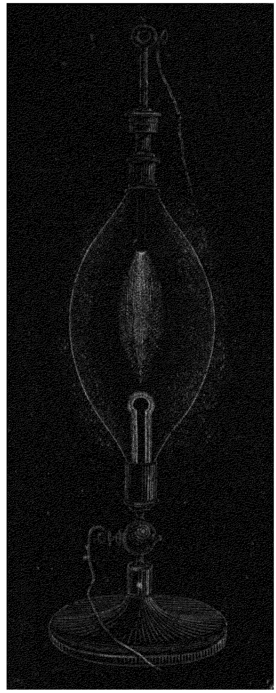


Fig. 186.—Electric Egg.

If, before exhausting, we introduce into the egg a little alcohol, turpentine, or other volatile liquid, the light presents a series of bright bands alternating with dark spaces. Plate II. fig. 1 represents these stratifications as seen in vapour of alcohol.

The phenomenon of stratification is seen to more advantage in long tubes than in the electric egg; and the presence of alcoholic or other vapour may be dispensed with if the exhaustion be carried sufficiently far, as in the tubes constructed by Geissler of Bonn, which contain various gases very highly rarefied, and have platinum wires sealed into their extremities to serve as electrodes. Four such tubes are represented in Plate II. Certain substances, such as uranium glass, and solution of sulphate of quinine, become luminous in the presence of the electric light, and are called *fluorescent.* Such substances are often introduced into Geissler's tubes, for the sake of the brilliant effects which they produce.

If a Geissler's tube is fastened to the front of a disc which can be rapidly rotated, the instantaneousness of the discharges, combined with the persistence of visual impressions, cause it to present the appearance of the rays of a star. The star appears stationary if an exact number of discharges occur in the time of one revolution. A slight increase or diminution in the speed of rotation will then make the star appear to revolve forwards or backwards.

The discharge in a Geissler's tube is capable of deflecting a magnetic needle, and can itself be deflected to one side of the tube by the action of a magnet.

278. Experiments of Gassiot and De La Rue.—By means of a battery of some thousands of cells, discharge in rarefied gases can be obtained

without the use of an induction-coil, and with the advantage of greater steadiness. This was done by Mr. Gassiot, and on a larger scale by Dr. De La Rue. The stratifications were still observed, and appeared absolutely fixed in position to the naked eye. When examined by a revolving mirror they were found to exhibit the appearance of a rapid succession of discharges.

Dr. De La Rue's battery consisted of 11,000 small chloride of silver cells; and when all the cells were used, a steady stream of fire passed between the terminals as soon as they were brought within about half an inch of each other, in air at atmospheric pressure.

279. Spectra.—The spectrum of an electric discharge usually consists of the spectrum of the gas in which it occurs, superposed on the spectrum of the metallic terminals, a small portion of the latter being vaporized. The source of electricity usually employed is the Ruhmkorff coil. As the discharge is rendered more violent, whether by increasing the strength of the primary current, or by connecting the terminals of the secondary with a Leyden jar, the number of lines in the spectrum is usually increased. The spectra of gases are usually observed by means of Geissler's tubes; which, for this purpose, are sometimes constructed of such a shape as to admit of directing the spectroscope along the axis of the tube, so as to obtain greater concentration of light.

280. Change of Resistance with Diminution of Pressure.—The resistance of a gas to electric discharge diminishes as the pressure diminishes. A coil which can only give a half-inch spark in air at atmospheric pressure may give a discharge between terminals more than a foot apart in highly rarefied air. But this diminution does not go on indefinitely. At a certain very small pressure (different for different gases) the resistance is a minimum, and at the lowest pressures attainable by modern methods the resistance is many times greater than at atmospheric pressure. This fact was for a long time disputed, but the earlier experiments of Gassiot and Alvergniat have been confirmed by the later ones of De La Rue and Crookes, and the result is now conclusively established.

De La Rue employed a vacuum tube connected with a bulb containing caustic potash which had absorbed the last traces of carbonic acid gas, after the tube, originally filled with this gas, had been exhausted by a Sprengel pump. No discharge passed until heat was applied to the bulb, so as to drive off some of the gas, and thus diminish the perfection of the vacuum. The discharge ceased as the

potash cooled down, and could be renewed any number of times at pleasure.

Crookes, who has given increased command of vacua by his improvements in the Sprengel pump, has succeeded in measuring the pressures at which the resistances of air and some other gases are a minimum, and has arrived at the conclusion that across an absolute vacuum discharge would be impossible. The explanation stated roughly seems to be that the particles of gas act as carriers of the discharge. When there are too many carriers they hamper one another, and when there are no carriers at all nothing is carried.

281. Crookes' "Radiant Matter."—Crookes, by carrying exhaustion to a higher point than had before been reached, has obtained a new set of phenomena. An influence (to which he gives the name of *radiant matter*) emanates from the negative terminal, and proceeds only in straight lines. Glass and precious stones exposed to this influence become strongly phosphorescent, and a delicately-balanced glass wheel with blades like a paddle-wheel is set in rotation by the successive impacts of the "radiant matter" upon its blades. The influence is only exerted on surfaces which are directly visible from the negative terminal, and is stopped by glass, so that a piece of glass interposed casts a shadow. By employing as the negative terminal a platinum saucer shaped like a concave mirror the influence can be made to converge on an object placed at its centre of curvature, and platinum foil can thus be heated to incandescence. No special effect is exhibited by the positive terminal. These phenomena are at their best when the vacuum is such as to give minimum resistance.

282. Tesla's Experiments with High Frequencies.—Tesla has shown that the character of the discharge is greatly altered by raising the frequency of alternation in the primary to a high pitch. This he has done sometimes by employing a specially constructed alternating dynamo giving many thousand reversals per second, but more usually and with more strongly marked effects by employing the oscillatory discharge of a jar or battery of jars, the oscillations in this case ranging from hundreds of thousands to some millions per second. It is necessary so to regulate the dimensions of the primary and secondary conductors that the electric oscillations in the one correspond in natural period with those of the other, so that the effects are exalted by means of resonance, as in Hertz's experiments.

A discharge, in the form of powerful brushes and luminous streams, can thus be obtained, covering an area of several square feet, and this not only in exhausted spaces but in ordinary air.

To produce discharge in a Geissler tube by such means, it is sufficient to connect one terminal of the tube with a terminal of the coil, and to bring within a moderate distance of the other terminal of the tube a large conductor, such as the human body.

When the frequency is sufficiently high, the effects are mainly due to the electrostatic induction of the alternating charges at the terminals of the secondary; and as this action can take place through glass, there is no necessity for wires sealed through the glass. It is only necessary to have two small pieces of metal inside the tube, one at each end, and to have two conductors of considerable size serving as terminals of the secondary coil and brought near the ends of the tube.

Either by means of large luminous brushes, or of Crookes' phosphorescence, it is hoped that these excessively rapid alternations will furnish, when fully worked out, a more economical system of electric lighting than any at present in use. Tesla expresses the conviction that, "to whatever kind of motion light may be due, it is produced by tremendous electrostatic stresses vibrating with extreme rapidity."[1]

It is noteworthy that the currents which produce the brilliant discharges above mentioned can pass through the human body without producing injury or inconvenience. The Geissler tube can conveniently be excited by placing one hand on a terminal of the coil and the other on a terminal of the tube.

[1] For an account of Tesla's experiments see No. 97 of *Journal of Inst. of Elec. Eng.*, issued April, 1892. (E. & F. N. Spon.)

CHAPTER XX.

MAGNETO AND DYNAMO MACHINES.

283. Magneto-electric Machines.—Faraday's discovery of the induction of currents by magnets, was speedily utilized in the construction of magneto-electric machines, which, without a battery, and with no other stimulus than that afforded by the presence of a permanent magnet, enable the operator, by the expenditure of mechanical work, to obtain powerful electrical effects. The first machine of this kind was constructed in 1833 by Pixii, and had a magnet revolving close to a double coil, in which a current was thus generated. The construction was improved by Saxton, and afterwards by Clarke, who made the magnet fixed, and caused the coil, which is much lighter, to rotate in front of it.

284. Clarke's Machine.—In this machine there is a compound horse-shoe magnet fixed to a vertical support. Close in front of the magnet, near its poles, are two connected coils t, t', each containing a soft-iron core. The two cores are united by a plate of copper on the side next the magnet, and by a plate of soft iron on the remote side. The direction of winding in the two coils is the same as for an ordinary horse-shoe electro-magnet. The coils are mounted on an axis f, which passes through the support of the steel magnet, and carries a pinion. By means of an endless chain passing over this pinion, and over a large wheel to which a handle is attached, the pinion, and with it the coils, can be made to revolve rapidly. The ends of the wire which forms the two coils are connected respectively with the two metallic pieces E, E′ (Fig. 188), which are mounted on the axis, but insulated from it and from each other.

Let us now examine the formation of the currents. The two iron cores, with their connecting iron plate, may be regarded as a temporary horse-shoe magnet, whose poles are always of opposite name

to those of the steel magnet which are respectively nearest to them. The intensity of magnetization is greatest when the soft-iron magnet is horizontal, vanishes when it is vertical, and in passing through the vertical position undergoes reversal. If we call one direction of magnetization positive and the opposite direction negative, the

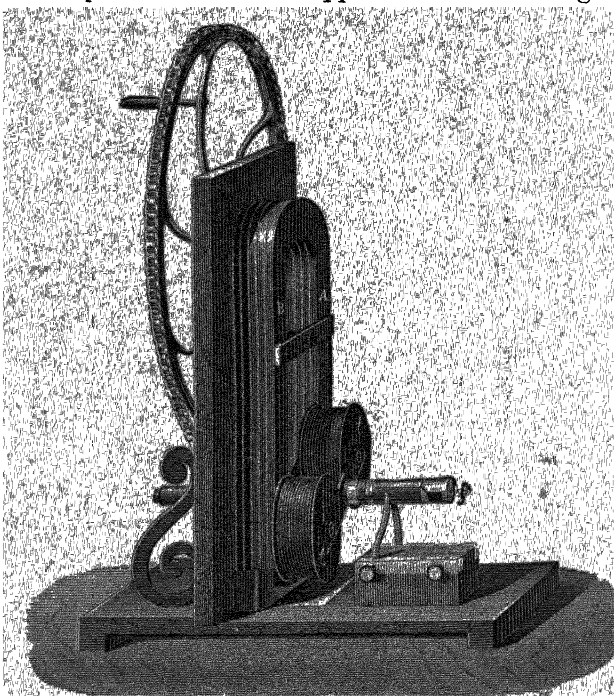


Fig. 187.—Clarke's Machine.

strongest positive magnetization corresponds to one of the two horizontal positions, and the strongest negative to the other, the two positions differing by 180°. While the magnet, then, is revolving from one horizontal position to the other, its magnetization is changing from the strongest positive to the strongest negative, and this change produces a current in one definite direction in the surrounding coil. During the next half-revolution the magnetization is again gradually reversed, and an opposite current is generated in the coil. If we examine the direction of the currents due to the cutting across of the lines of force of the permanent magnet by the convolutions of the coil, we shall find that they concur with those due to the action of the cores. The current in the coils circulates

in one direction as long as the electro-magnet is moving from one horizontal position to the other, and changes its direction at the instant when the cores come opposite the poles of the steel magnet.

By the aid of the commutator represented in Fig. 188, the currents may be made to pass always in the same direction through an external circuit. *r* and *r'* are two contact-springs bearing against the two metal pieces E, E′, which are the terminals of the coil. At the instant when the current in the coil is reversed, these springs are in contact with intermediate insulating pieces which separate the metallic pieces E, E′. When the current in the coil is in one direction (say from E to E′), *r* is in contact with E, and *r'* with E′. When the current in the coil is in the opposite direction (E′ to E), *r* is in contact with E′, and *r'* with E; thus in each case *r* is the positive and *r'* the negative spring, and the current will be from *r* to *r'* in an external connecting wire. O O, O′ O′, are metallic pieces insulated from each other, and connected with the springs *r r'* respectively. Binding-screws are provided for attaching wires through which the current is to be passed.

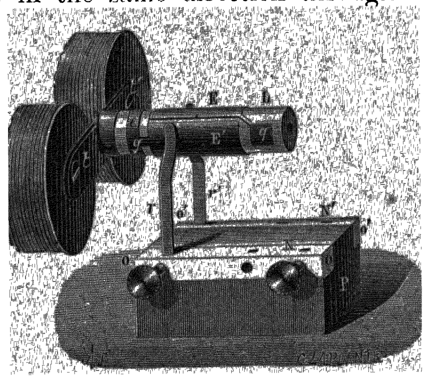


Fig. 188 —Commutator of Clarke's Machine

With this machine water can be decomposed, wire heated to redness, or soft iron magnetized; but these effects are usually on a small scale on account of the small dimensions of the machine

For giving shocks, two wires furnished with metallic handles are attached to the binding-screws, and a third spring is employed which puts the terminals E E′ in direct connection with each other twice in each revolution, by making contact with two plates *q*. When these contacts cease, the current is greatly diminished by having to pass through the body of the person holding the handles, and the extra-current thus induced gives the shock To obtain the strongest effect, the hands should be moistened with acidulated water before grasping the handles.

285. Magneto-electric Machines for Lighthouses.—Very powerful effects can be obtained from magneto-electric machines of large size driven rapidly. Such machines were first suggested by Professor

Nollet of Brussels; and they have been constructed by Holmes of London and the Compagnie l'Alliance of Paris. It is by means of these machines that the electric light is maintained in several lighthouses; they have also been employed to some extent in electro-metallurgy. Fig. 189 represents the pattern adopted by the French company.

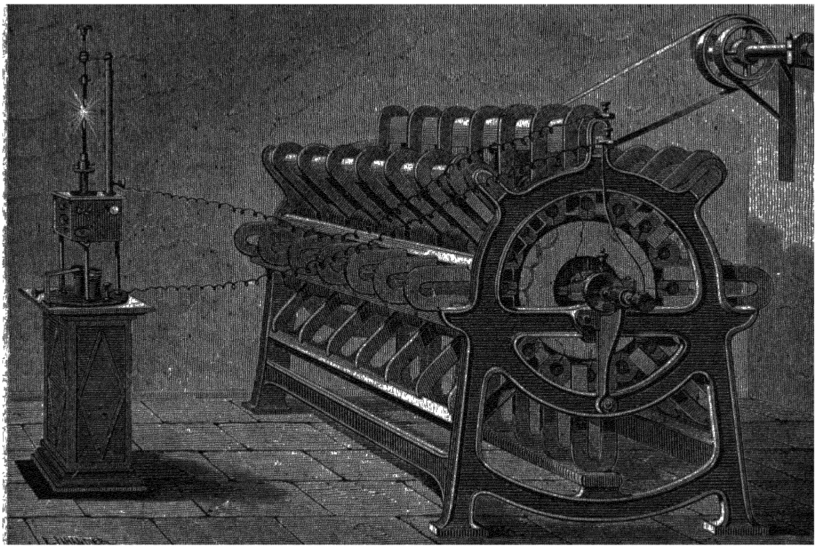


Fig. 189.—Lighthouse Machine.

It has eight rows of compound horse-shoe magnets fixed symmetrically round a cast-iron frame. They are so arranged that opposite poles always succeed each other, both in each row and in each circular set. There are seven of these circular sets, with of course six intervening spaces. Six bronze wheels, mounted on one central axle, revolve in these intervals, the axle being driven by steam-power transmitted by a pulley and belt. The speed of rotation is usually about 350 revolutions of the axle per minute. Each of the six bronze wheels carries at its circumference sixteen coils, corresponding to the number of poles in each circular set. The core of each coil is a cleft tube of soft iron, this form having been found peculiarly favourable to rapid demagnetization.

Each core has its magnetism reversed sixteen times in each revolution, by the influence of the sixteen successive pairs of poles between which it passes, and the same number of currents in alternately

opposite directions are generated in the coils. The coils can be connected in different ways, according as great electromotive force or small resistance is required. The positive ends are connected with the axle of the machine, which thus serves as the positive electrode, and a concentric cylinder, well insulated from it, is employed as the negative electrode.

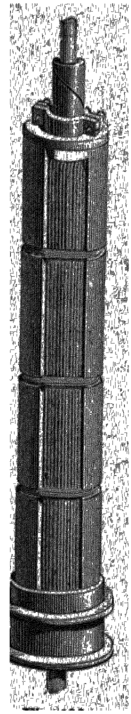


Fig 190
Siemens' Armature

When the machine is employed for the production of the electric light, the currents may be transmitted to the carbon points in alternate directions, as they are produced. For electro-metallurgical purposes they are brought into one constant direction by a commutator, as in Clarke's machine above described. The driving-power required for lighthouse purposes is about three horse-power.

Machines of this class are seldom constructed now, as the same power is obtained with only a fifth of the weight by employing electro-magnets instead of permanent magnets.

286. Siemens' Armature.—An important improvement in Clarke's machine was introduced by Siemens of Berlin in 1854. It consists in the adoption of a peculiar form of electro-magnet, which is represented in Fig. 190. The iron portion is a cylinder with a very deep and wide groove cut along a pair of opposite sides, and continued round the ends. The coil is wound in this

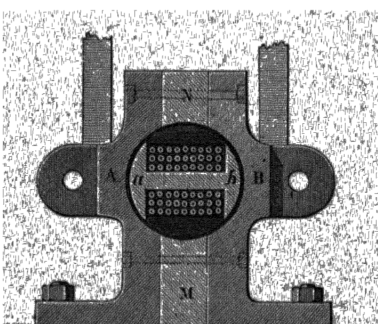


Fig. 191.—Section of Siemens' Armature.

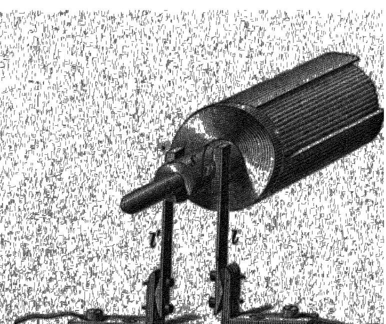


Fig 192.—Commutator

groove like thread upon a shuttle. Regarded as an electro-magnet, the poles are not the ends of the cylinder, but are the two cylindrical

faces which have not been cut away. In Fig. 191, *a b* is a section of the armature with the coil wound upon it. A B M N is a socket within which the armature revolves, the portions A B being of iron, and M N of brass.

The advantage of Siemens' armature is that, on account of the small space required for its rotation, it can be kept in a region of very intense magnetic force by the use of comparatively small magnets. Its form is also eminently favourable to rapid rotation. It is placed between the opposite poles of a row of horse-shoe magnets which bestride it along the whole of its length, as shown at the top of Fig. 193, and is rotated by means of a driving-band passing over the pulley shown at the lower end of Fig. 190.

The polarity of the electro-magnet is reversed at each half-revolution as in Clarke's arrangement, and the alternately opposite currents generated are reduced to a common direction by a commutator nearly identical with Clarke's, and represented in Figs. 190, 192. Siemens' machines are much more powerful than Clarke's when of the same size.

287. **Accumulation by Successive Action: Wilde's Machine.**—By employing the current from a Siemens' machine to magnetize soft iron, we can obtain an electro-magnet of much greater power than the steel magnets from whose induction the current was derived. By causing a second coil to rotate between the poles of this electro-magnet, we can obtain a current of much greater power than the primary current. This is the principle of Wilde's machine, which is represented in Fig. 193. It consists of two Siemens' machines, one above the other. The upper machine derives its inductive action from a row of steel magnets M, whose poles rest on the soft-iron masses *m*, *n*, forming the sides of the socket within which a Siemens' armature *r* rotates. The currents generated in the coil, after being reduced to a uniform direction by a commutator, flow to the binding-screws *p*, *q*. These are the terminals of the coil of the large electro-magnet A B, through which accordingly the current circulates. The core of this electro-magnet consists of two large plates of iron, connected above by another iron plate, which supports the primary machine. Its lower extremities rest, like those of the primary magnets, on two iron masses T, T, separated by a mass of brass *i*; and a second Siemens' armature F, of large size, revolving within this system, furnishes the currents which are utilized externally.

Wilde's principle can be carried further. The current of the second armature can be employed to animate a second electro-magnet of greater power than the first, with a third Siemens' armature revolving between its poles. This was actually done by Wilde. By

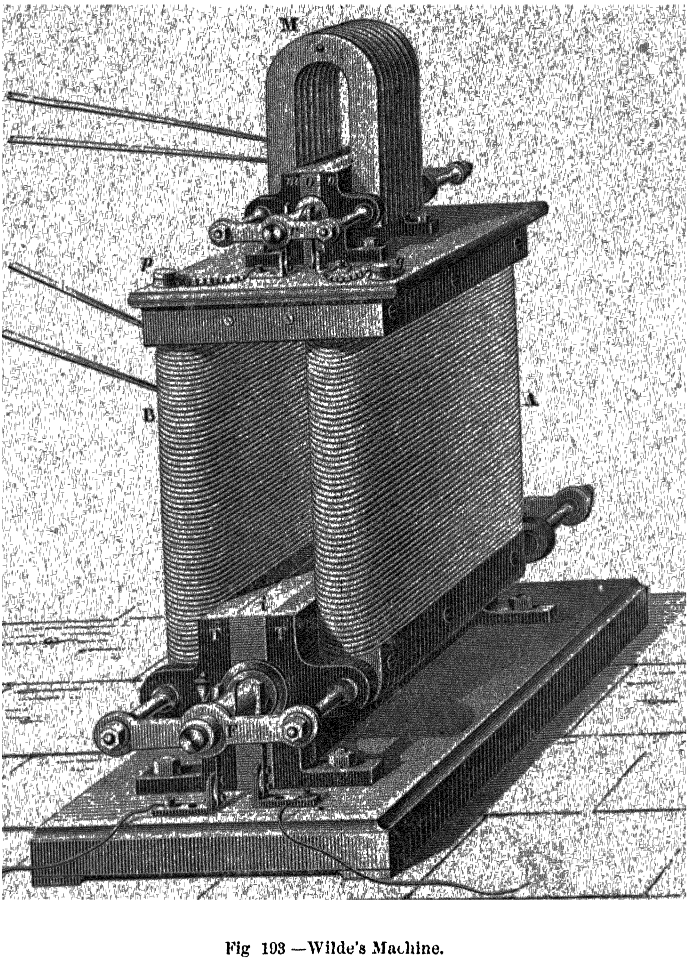


Fig 193 —Wilde's Machine.

means of the current from this triple machine, driven by 15 horse-power, a bar of platinum 2 feet long and a quarter of an inch in diameter was quickly melted. This system of accumulation could probably be carried several steps further if desired.

288. Accumulation by Mutual Action; Dynamo Machines.—Siemens and Wheatstone nearly simultaneously proposed the construction

of a magneto-electric machine in which the induced current, or a portion of it, is made to circulate round the soft-iron magnet which produced it. Iron has usually some traces of permanent magnetism, especially if it has once been strongly magnetized. This magnetism serves to induce very feeble currents in a revolving armature. These currents are sent round the iron magnet, thus increasing its magnetization. This again produces a proportionate increase in the induced currents; and thus, by a succession of mutual actions, very intense magnetization and very powerful currents are

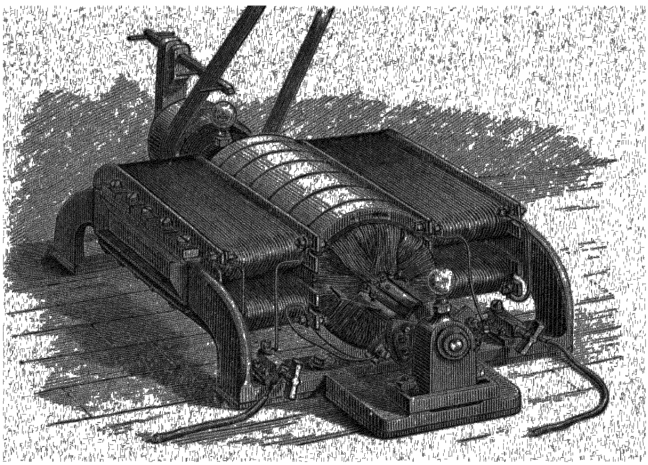


Fig 194.—Siemens' Dynamo.

speedily obtained. Machines constructed on this principle are called *dynamos*.

289. Siemens' Dynamo.—One of the earliest patterns of dynamo constructed by Messrs. Siemens for commercial purposes is shown in Figs. 194, 195.

The armature consists of a hollow iron cylinder, on the outside of which the wire is wound lengthwise in from ten to twenty successive sections, each section being wound in the same manner as the whole coil of the original Siemens' armature (Fig. 190), and being like it connected at its ends to two opposite segments of a commutator, as well as to one end each of the two adjacent sections; so that the whole wire forms one continuous coil, connected at a number of equidistant points with the successive segments of a commutator. The commutator is on the same plan as that of

Clarke (Fig. 188), but instead of only two segments has a considerable number. Two contact springs, or flexible bundles of wire, called brushes, rub upon the commutator at two points fixed in space, as it revolves, one of them receiving from the armature positive and the other negative electricity. The armature is rapidly rotated between two sets of curved iron bars, one set above the armature, as shown in Fig. 194, and the other, precisely similar set below. These bars are prolonged at both ends through the four fixed coils shown in Figs. 194, 195, so that the upper bars, for example, form the core of the two upper fixed coils. The currents in these two coils are in

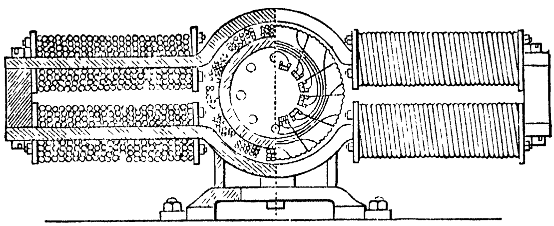


Fig 195 —Section of Siemens' Machine.

opposite directions, so that the bar has a "consequent point" in the middle of its length. If these middle points are north poles, then the middle points of the lower bars are south poles, and each section of the coil by revolving between these two poles has currents generated in it which are reversed twice in each revolution, namely, in the two positions (differing by 180°) in which the maximum number of tubes of force pass through it. The two segments of the commutator which are in direct connection with the section in question make contact with the collecting brushes midway between these two reversals; and the current given off is due partly to this section and partly to the neighbouring sections on each side of it.

The fixed magnets are called the *field-magnets*, because their function is to produce a strong magnetic field for the armature to move in. Their coils are of stout wire, and the connections are such that the whole current generated in the armature passes through them; the armature coils, the field-magnet coils, and the external circuit being joined in series.

290. Gramme's Machine.—Another well-known type of magneto-electric machine is that invented by M. Gramme. Let C D E F (Fig. 196) be a ring of soft iron, wrapped round with insulated

copper wire, and revolving in its own plane between the poles P, P′ of a fixed magnet. The ring will, at any given instant, consist virtually of two semicircular magnets, F C D, F E D, having a pair of similar poles at F, and the other pair at D, these being the points directly opposite the poles of the fixed magnet. Since the poles of the ring remain fixed in space, the electric effect in the copper wire is the same as if the wire coil alone rotated, its core remaining stationary. The effect of this rotation would be, that in the portion C F E of the coil there would be electromotive force tending to produce a current in one direction,—say the direction C F E; while in the other half, C D E, there would be electromotive force tending to produce a current in the opposite direction—that is the direction C D E. The effects in the two halves are opposite as regards the current which they tend to produce in the coil as a whole; but they are the same as regards the electromotive force between the opposite points C and E; and if the two ends of an external conductor be maintained in rubbing contact with the coil at these two points, a permanent current will flow through it in virtue of this electromotive force.

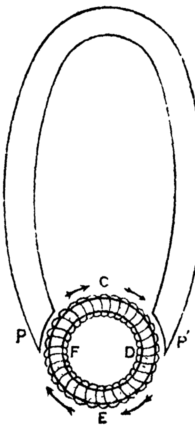


Fig. 196—Magnet and Ring.

The above reasoning may be put in the following form. Nearly all the tubes of force which run from one pole to the other of the permanent magnet are concentrated in the substance of the iron ring, one half traversing the upper and the other the lower half-ring. Each convolution of the coil, in ascending from its lowest position E by way of F to its highest position C, cuts each of these tubes once, and all in the same direction, namely, from below to above. In descending on the other side by way of D to E, the same tubes are cut, each once, in the opposite direction, namely, from above to below. Hence the movement in E F C generates electromotive force in one direction through the wire composing the coil, and the movement in C D E generates electromotive force in the opposite direction; both parts of the motion conspiring to produce difference of potential between the convolution at C and that at E.

The details of the armature of Gramme's machine are shown in Fig. 197, in which different parts are represented in different stages of construction.

The ring or core consists of a bundle of iron wires, shown in section at A. The copper wire, covered as usual with an insulating material, is divided into a number of separate coils, as B B. The two ends of each coil are respectively connected to two thick pieces of copper (one of which is marked R R in the figure), which are the segments of the commutator, their number being equal to the number of separate coils. In passing the two points most remote from the poles, these coppers rub against two brushes, connected respectively with two binding-screws, one forming the positive and the other the negative electrode of the machine. As each brush makes

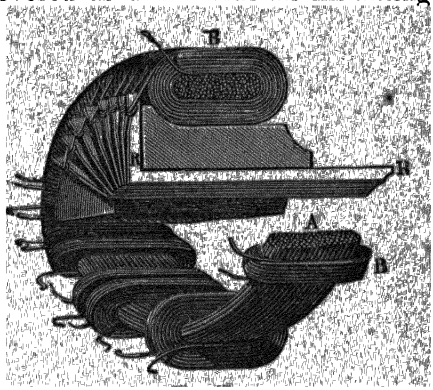


Fig 197.

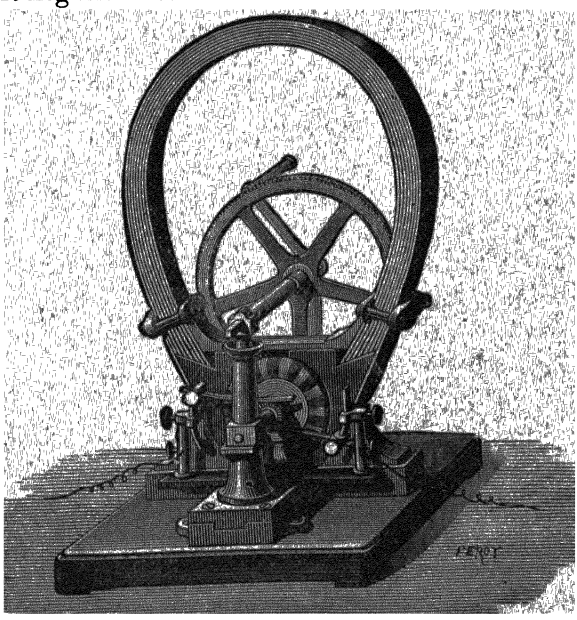


Fig 198.—Gramme's Magneto-electric Machine, for hand power.

contact with two or more coppers at the same time, the current is never interrupted, and undergoes but small fluctuations of strength,—a remark which also applies to Siemens' machine.

In consequence of the great steadiness of the current thus obtained, such machines can be used instead of galvanic batteries for nearly all purposes. The machine as constructed for hand use is shown in Fig. 198. Fig. 199 represents a larger pattern, intended to be

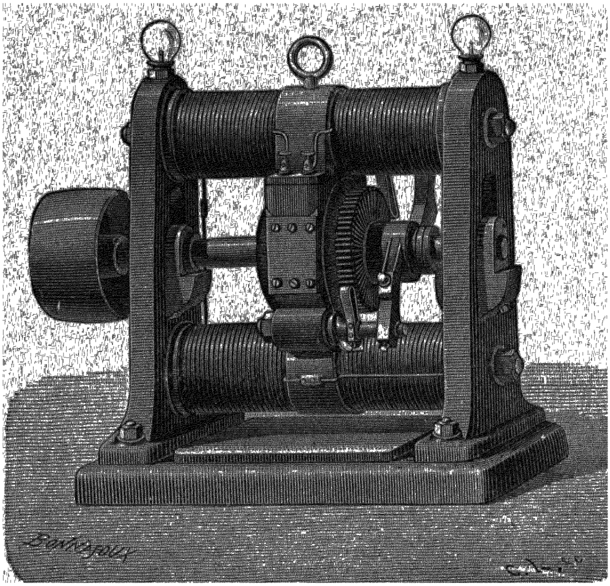


Fig. 199.—Gramme's Dynamo-electro Machine, for steam-power.

driven by steam power. The armature revolves between the poles of four very powerful electro-magnets actuated by the current which the machine produces. The two upper magnets may be regarded as forming one magnet, with a consequent point in the centre, directly above the armature; and the two lower magnets give, in like manner, a consequent point, opposite in name to the former, directly below the armature.

The first machine with a ring like that of Gramme was a small laboratory model constructed by Pacinotti at Pisa in 1860, and described by him in an Italian publication in 1864, but it did not become generally known, nor was any large machine of the kind made, till the reinvention of the ring by Gramme, who was the first to introduce the now usual arrangement of collecting currents by flexible bundles of wire rubbing on a commutator of many segments.

291.—The machines of Siemens and Gramme above described are

among the earliest types of commercial dynamos. Later patterns contain many improvements. One of the most important consists in making the cores, pole-pieces, and yokes (*i.e.* pieces connecting the cores) of the field-magnets more massive. This diminishes what is called the magnetic resistance, and thus makes the field stronger.

292. Classification of Dynamos.—Dynamos which give an external current constantly in one direction are called *direct-current* machines.

Two different arrangements are in use for supplying their field-magnets with current; thus we have the distinction of *series dynamos* and *shunt dynamos.*

In the *series dynamo*, the whole current from the armature passes through the field-magnet coils, which must be composed of stout wire, so as to have low resistance and not consume much of the energy of the current.

In the *shunt dynamo*, the current from the armature divides into two parts, the larger part going through the external circuit, and the smaller through the field-magnet coils, which must in this case consist of a great length of fine wire, so as to have much greater resistance than the external circuit.

In many modern installations, a combination of the two systems is adopted, the field-magnets being wound with two coils, one in parallel and the other in series with the external circuit. This is called *compound* winding. It is specially useful for keeping the e.m.f. of the machine nearly constant in spite of changes of resistance in the external circuit.

All these belong to the class of direct-current dynamos as distinguished from *alternate-current* dynamos. These latter give currents alternately in one direction and the opposite, the reversals succeeding each other usually some hundreds or thousands of times in a second. For this purpose no commutator is required, inasmuch as the current in the armature itself is alternating in all machines; but the two collecting springs rub without interruption on the surfaces of two revolving cylinders, to which the ends of the armature-coil or of its several sections are connected. Each cylinder gives off positive and negative electricity alternately, and when the one is giving off positive the other is giving off negative. This is the favourite plan for lighthouses, because the alternating currents make the two carbons of the electric lamp burn away equally, and thus facilitate the keeping of the light in the focus of the optical apparatus. The field-magnets of an alternate-current machine, if

they are electro-magnets, must be excited by a current distinct from that of the machine itself; as alternating currents will not serve for this purpose.

293.—In most of the alternate-current dynamos now constructed, the field-magnets are arranged in two circles facing each other with a narrow space between. In this intervening space the armature rotates. It consists of a series of flat coils equal in number and about equal in area to the ends of the field-magnets in either circle.

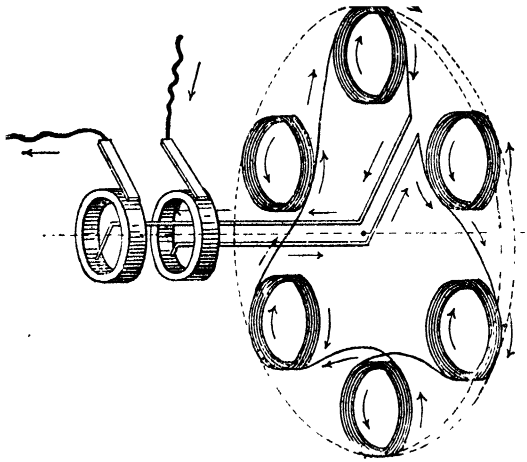


Fig. 200.—Armature of Alternator.

These coils have usually no iron core, but are wound upon an insulating material which holds them in their places. The field-magnets are so arranged that unlike poles face each other, and also unlike poles succeed each other. The arrangement of the armature-coils and collecting rings is shown in Fig. 200[1] By the passage of one of the armature coils between a pair of poles two successive currents are produced, one by the passage of its forward and the other of its hinder half, these currents having opposite directions. While the forward half is passing between one pair of poles, the hinder half is passing between the next pair of poles, and the two currents due to these crossings are in the same direction. This action goes on in all the coils simultaneously, and if they are connected in series in the manner shown in the figure their electromotive forces will be com-

[1] This figure is borrowed with permission from Professor S. P. Thompson's *Dynamo-Electric Machinery,* to which we would refer our readers for further information.

bined.[1] The number of reversals of current in one revolution of the armature is equal to the number of pairs of poles between which it revolves. A dynamo of this kind is called *multipolar*.

The field-magnets are electro-magnets supplied with a constant current by a *separate exciter*, that is by a relatively small direct-current dynamo employed for the purpose.

294. Conditions of Self-excitation in a Dynamo.—It might be inferred from § 288 that the initial traces of magnetism in the field-magnets will be gradually augmented even when the armature is made to revolve slowly. Such an inference would however be erroneous. A dynamo does not work at all unless the speed of revolution exceeds a certain critical value, which is definite when the resistance in circuit is given. The reason may be stated as follows.

Let C stand for the current in the armature. We shall assume that either the whole of C or a definite fraction of it flows through the coils of the field-magnets. Let M denote the intensity of magnetization of these magnets, and n the number of revolutions of the armature per minute. Then in comparing the results obtained with the same machine at different speeds, C will be jointly proportional to n and the intensity of the field in which the armature revolves, which latter may be taken as proportional to M, that is, we may assume

$$kC = nM, \text{ or } n = k\frac{C}{M},$$

k denoting a constant multiplier.

Now the value of $\frac{C}{M}$ increases with the magnetization except when the magnetization is feeble, in which case it is nearly constant Let this constant value of $\frac{C}{M}$ for weak magnetization be denoted by k'. Then the least possible value of n will be kk'. This is the critical speed. At lower speeds the field-magnets will not be excited, and at higher speeds they will be excited more and more as the speed is increased, since M increases with $\frac{C}{M}$.

295. An empirical formula which gives a fair approximation to the truth in ordinary cases can be obtained by assuming what

[1] In any two consecutive coils the currents are opposite ways round, because they are moving across fields in which the lines have opposite directions. Hence the direction of winding must be reversed from each coil to the next, as shown in the figure.

mathematicians call a "homographic relation" to exist between C and M. The most general expression of such a relation is,

$$CM + aC + bM + c = 0, \qquad (1)$$

a b c denoting constants. In the present case c is zero, since C and M vanish together, the formula is thus reduced to

$$CM + aC + bM = 0. \qquad (2)$$

When C is very great the third term may be neglected, and we have therefore $M + a = 0$. But M in this case is the magnetization at saturation, which we will denote by m, hence the constant a denotes $-m$ and the formula may be written

$$CM - Cm + bM = 0. \qquad (3)$$

When C has the value b, we can divide out by it and we obtain

$$2M - m = 0, \text{ or } M = \tfrac{1}{2}m.$$

The constant b therefore denotes that value of C which produces half-saturation.

Dividing by Mm, we may write equation (3) in the form

$$\frac{C}{M} = \frac{C + b}{m}, \qquad (4)$$

which shows that the least value of $\frac{C}{M}$ is $\frac{b}{m}$. Hence, by the preceding section, the critical speed is

$$n = k\frac{b}{m}. \qquad (5)$$

At double the critical speed the value of $\frac{C}{M}$ will (by the preceding section) be doubled. It will therefore be $\frac{2b}{m}$, which when substituted in equation (4) gives $C = b$, showing that at double the critical speed the field-magnets will be half-saturated. This result is probably not far from the truth. Equation (3), when written in the form

$$M = \frac{mC}{C + b},$$

is known as the Lamont-Frölich formula.

CHAPTER XXI.

ELECTRIC TELEGRAPHS.

296. Electric Telegraph: History.—The discovery that electricity could be transmitted instantaneously to great distances, at once suggested the idea of employing it for signalling. Bishop Watson, already referred to in § 38, performed several experiments of this kind in the neighbourhood of London, the most remarkable being the transmission of the discharge of a Leyden-jar through 10,600 feet of wire suspended between wooden poles at Shooter's Hill. This was in 1747. A plan for an alphabetical telegraph to be worked by electricity is minutely described in the *Scots Magazine* for 1753, but appears to have been never experimentally realized. Lesage, in 1774, erected at Geneva a telegraph line, consisting of twenty-four wires connected with the same number of pith-ball electroscopes each representing a letter. Reusser, in Germany, proposed, in the same year, to replace the electroscopes by spangled panes exhibiting the letters themselves. The difficulty of managing frictional electricity was, however, sufficient to prevent these and other schemes founded on its employment from yielding any useful results. Volta's discoveries, by supplying electricity of a kind more easily retained on the conducting wires, afforded much greater facilities for transmitting signals to a distance.

Several suggestions were made for receiving-apparatus to exhibit the effects of the currents transmitted from a voltaic battery. Sömmering of Munich in 1811 proposed a telegraph, in which the signals were given by the decomposition of water in thirty-five vessels, each connected with a separate telegraph wire. Ampère, in 1820, proposed to utilize Œrsted's discovery, by employing twenty-four needles, to be deflected by currents sent through the same

number of wires; and Baron Schilling exhibited in Russia, in 1832, a telegraphic model in which the signals appear to have been given by the deflections of a single needle.[1]

Weber and Gauss carried out this plan in 1833, by leading two wires from the observatory of Göttingen to the Physical Cabinet, a distance of about 9000 feet. The signals consisted in small deflections of a bar-magnet, suspended horizontally with a mirror attached, on the plan since adopted in Thomson's mirror galvanometer.

At their request the subject was earnestly taken up by Professor Steinheil of Munich, whose inventions contributed more perhaps than those of any other single individual to render electric telegraphs commercially practicable. He was the first to ascertain that earth-connections might be made to supersede the use of a return wire. He also invented a convenient telegraphic alphabet, in which, as in most of the codes since employed, the different letters of the alphabet are represented by different combinations of two elementary signals. Two needles were employed, one or the other of which was deflected according as a positive or a negative current was sent, the deflections being always to the same side. Sometimes the needles were merely observed by eye, sometimes they were made to strike two bells, and sometimes to produce dots, by means of capillary tubes charged with ink, on an advancing strip of paper, thus leaving a permanent record on the strip in the shape of two rows of dots. His currents were magneto-electric, like those of Weber and Gauss.

The attraction of an electro-magnet on a movable armature furnishes another means of signalling. This was the foundation of

[1] The contributions of the late Sir Francis Ronalds to the art of telegraphy must not be altogether overlooked. According to an able notice in *Nature*, Nov. 23, 1871, "Sir Francis, before 1823, sent intelligible messages through more than eight miles of wire insulated and suspended in the air. His elementary signal was the divergence of the pith-balls of a Canton's electrometer produced by the communication of a statical charge to the wire. He used synchronous rotation of lettered dials at each end of the line, and charged the wire at the sending end whenever the letter to be indicated passed an opening provided in a cover; the electrometer at the far end then diverged, and thus informed the receiver of the message which letter was designated by the sender. The dials never stopped, and any slight want of synchronism was corrected by moving the cover. Hughes' printing instrument is the fully-developed form of this rudimentary instrument. A gas pistol was used to draw attention, just as now a bell is rung. The primary idea of reverse currents is to be found where Sir Francis suggests that the wire when charged with positive electricity should discharge not to earth but into a battery negatively charged. Equally interesting is the discussion on what we now call lateral induction, then known as compensation. The author clearly saw that in the underground wires which he suggests as substitutes for aerial lines, this induction would be or might be a cause of retardation."

Morse's telegraphic system, and was employed by Wheatstone for ringing a bell to call attention before transmitting a message.

About the year 1837 electric telegraphs were first established as commercial speculations, in three different countries. Steinheil's system was carried out at Munich, Morse's in America, and Wheatstone and Cooke's in England. The first telegraphs ever constructed for commercial use were laid down by Wheatstone and Cooke, on the London and Birmingham and Great Western Railways. The wires, which were buried in the earth, were five in number, each acting on a separate needle; but the expensiveness of this plan soon led to its being given up. The single-needle and double-needle telegraphs of the same inventors have been much more extensively used, the former requiring only one wire, and the latter two.

Wheatstone made several subsequent contributions to the art of telegraphy, some of which we shall have occasion to mention in later sections.

297. Needle Telegraph. Dial Telegraph.—In the needle telegraphs of Wheatstone and Cooke, which were the commonest instruments in this country before the taking over of the telegraphs by the Post Office, the deflections were produced in the same way as those of a galvanometer. By combining deflections to left and right an alphabet was constructed which, as devised by Wheatstone and Cooke, was framed with a view to being easily remembered. This has been superseded by the Morse alphabet, which is constructed on the principle of assigning the briefest signs to the letters of most frequent occurrence, and has now become international.

Dial telegraphs, in which the indications were given by a pointer directed towards any one of the letters of the alphabet arranged in a circle, were very common in counting-houses till they were superseded by the telephone. They were on the "step by step" plan. Every current that was sent advanced the pointer one step, and on the reversal (or in some instruments on the cessation) of the current the pointer advanced another step. The number of such steps was regulated by the advance of the handle of the sending instrument, which also travelled round a circle formed of the letters of the alphabet. When all was right the pointer of the receiving instrument was at the same letter as the handle of the sending instrument and in case of disarrangement, both could be brought to the intermediate space between Z and A. Descriptions of two such instruments will be found in previous editions of this work.

298. Batteries. Wires.—All the public telegraphs in this country have now for many years been worked by voltaic currents, the magneto-electric system, which was tried on some lines, having been found to involve a needless expenditure of labour. The modified Daniell's which was described in our earlier editions has been largely replaced by Fuller's bichromate battery, which we have described in § 161.

The wires for land telegraphs are commonly of what is called galvanized iron, that is, iron coated with zinc, supported on posts by means of glass or porcelain insulators, so contrived that some part of the porcelain surface is sheltered from rain, and insulates the wires from the posts even in wet weather. Wires thus suspended are called *air-lines*.

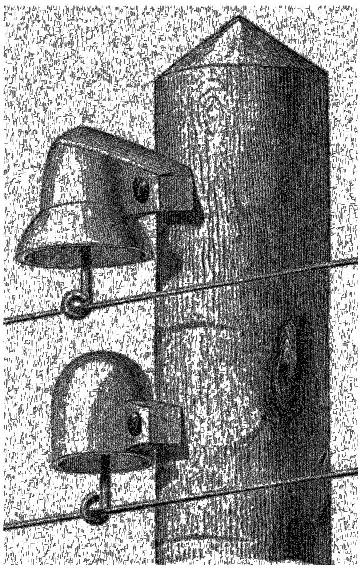


Fig. 201.—Insulators.

Underground wires are, however, sometimes employed. They are insulated by a coating of gutta percha, and are usually laid in pipes, an arrangement which admits of their being repaired or renewed, without opening the ground except at the drawing-in boxes. There is less leakage of electricity from subterranean than from air lines, but their cost is greater, and they are less suited for rapid signalling, on account of the retardation caused by the inductive action between the wire and the conducting earth, which is similar to that between the two coatings of a Leyden-jar.

299. Earth-Connections.—The early inventors of electric telegraphs supposed that a current could not be sent from one station to another without a return wire to complete the circuit. Steinheil, while conducting experiments on a railway, with the view of ascertaining whether the rails could be employed as lines of telegraph, made the discovery that the earth would serve instead of a return wire, and with the advantage of diminished resistance, the earth, in fact, behaving like a return wire of infinitely great cross-section, and therefore of no resistance. The earth-connection is usually made by

connecting one terminal of the sending battery and one terminal of the receiving instrument to large plates of metal buried in the soil.

We are not, however, to suppose that the current really returns from the receiving to the transmitting station through the earth. The duty actually performed by the earth consists in draining off the opposite electricities which would otherwise accumulate in the terminals. It keeps the two terminals at the same potential, and as long as this condition is fulfilled, the current will have the same strength as if the terminals were in actual contact.

300. Alarum.—In connection with all telegraphs an alarum is used for calling attention. There are several different kinds. Fig. 202 represents one of the simplest. It contains an electro-magnet *e*, with an armature *f* fixed to the end of an elastic plate. When no current is passing through the coil, the armature is held back by the elasticity of this plate, so as to press against a contact-spring *g* connected with the binding-screw *m*. The terminals of the coil are at the binding-screws *p*, *p'*, the former of which is in connection with the armature, and the latter with the earth. As long as the armature presses against the spring *g*, there is communication between the two binding-screws *m* and *p'* through the coil, but the passing of a current produces attraction of the armature, which draws it away from *g* and interrupts the current. The electro-magnet is thus demagnetized, and the armature springs back against *g*, so as to allow a fresh current to pass. The armature is thus kept in continual vibration; and a hammer K, which it carries above, produces repeated strokes on a bell T.

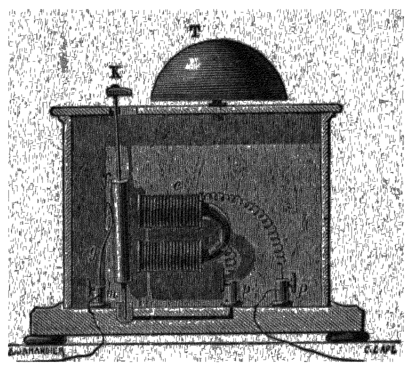


Fig 202—Vibrating Alarum.

301. Morse's Telegraph.—The basis of nearly all the telegraphs now used in all countries is Morse's system, which was first tried in America about 1837.

His receiving instrument, or *indicator*, in its primitive simplicity, consists (Fig. 203) of an electro-magnet, a lever movable about an axis, carrying a soft-iron armature at one end, and a pencil at the

other, and a strip of paper which is drawn past the pencil by a pair of rollers.

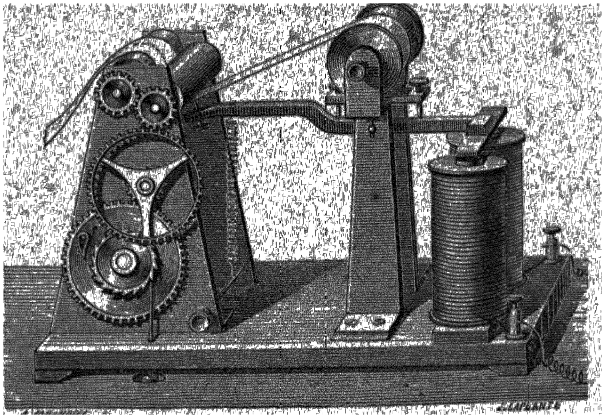
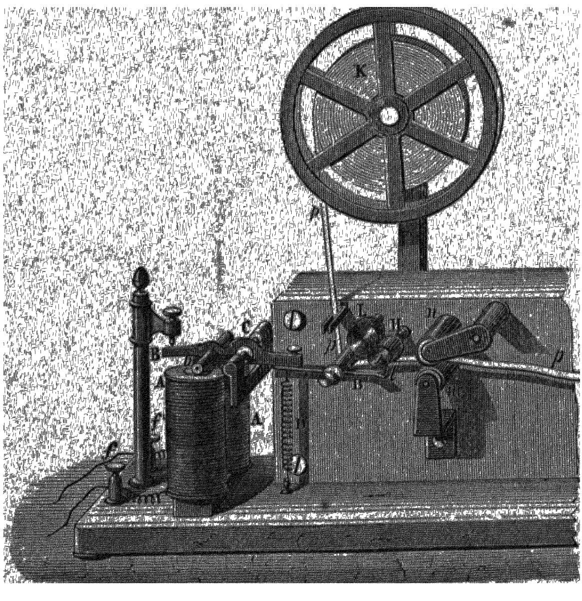


Fig 203.—Morse's Telegraph

As the pencil soon became blunt, and was uncertain in its marking a point, which scratched the paper, was substituted This has now



to a great extent been superseded by an ink-writer, which requires the exertion of less force, and at the same time leaves a more visible trace. Fig. 204 represents Morse's indicator as modified by Digney.

A train of clock-work, not shown in the figure, drives one of a pair of rollers *n m*, which draw forward a strip of paper *p p* forming part of a long roll K. The same train turns the printing-cylinder H, the surface of which is kept constantly charged with a thick greasy ink by rolling-contact with the ink-pad L. The armature B B′ of the electro-magnet A is mounted on an axis at C, and carries a style at its extremity just beneath the printing-cylinder. When a current passes, the armature is attracted, and the style presses the paper against the printing-cylinder, causing a line to be printed on it, the length of which depends on the duration of the current, as the paper continues to advance without interruption. The lines actually employed are of two lengths, one being made as short as possible (-) and called a *dot*, the other being about three times as long (—) and called a *dash*. The opposing spring D restores the armature to its original position the moment the current ceases.

Morse's key (Fig. 205) is simply a brass lever, mounted on a hinge at A, and pressed up by the spring *f*. When the operator puts down the key, by pressing on the button K with his finger, the projections *c d* are brought into contact, and a current passes from the battery-wire P to the line-wire L. When the key is up, the projections *a b* are in contact, and currents arriving by the line-wire pass by the wire R to the indicator or the relay. By keeping the key down for a longer or shorter time, a dash or a dot is produced at the station to which the signal is sent. The dash and dot are combined in different ways to indicate the different letters, as shown in the following scheme, which is now generally adopted both in Europe and America:—

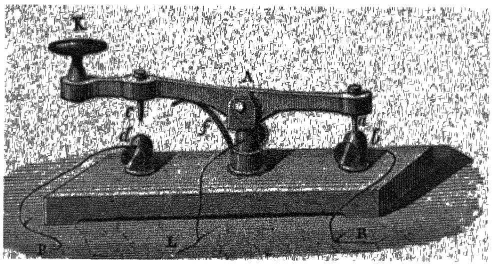
MORSE'S ALPHABET.

A ·—	J ·———	T —	1 ·————
Ä ·—·—	K —·—	U ·· ·	2 ··———
B —···	L ·—··	Ü ··——	3 ···——
C —·—·	M ——	V ···—	4 ····—
D —··	N —·	W ·——	5 ·····
E ·	O ———	X —··—	6 —····
É ··—··	O ———·	Y —·——	7 ——···
F ··—·	P ·——·	Z ——··	8 ———··
G ——·	Q ——·—	Ch ————	9 ————·
H ····	R ·—·		0 —————
I ··	S ···	Understood ···—·	

A space about equal to the length of a dash is left between two letters, and a space of about twice this length between two words.

In needle-telegraphs, the dot is represented by a deflection to the left, and the dash by a deflection to the right.

302. Relay.—Fig. 206 represents Morse's indicator in connection with what is called a *relay;* that is to say, an apparatus which, on receiving a feeble current from a distance, sends on a much stronger current from a battery on the spot. The key B being up, a current arriving by the line-wire passes through the key from c to a, thence through another wire to the coil of the electro-magnet belonging to the relay, and through this coil to earth. The electro-magnet of the relay attracts an



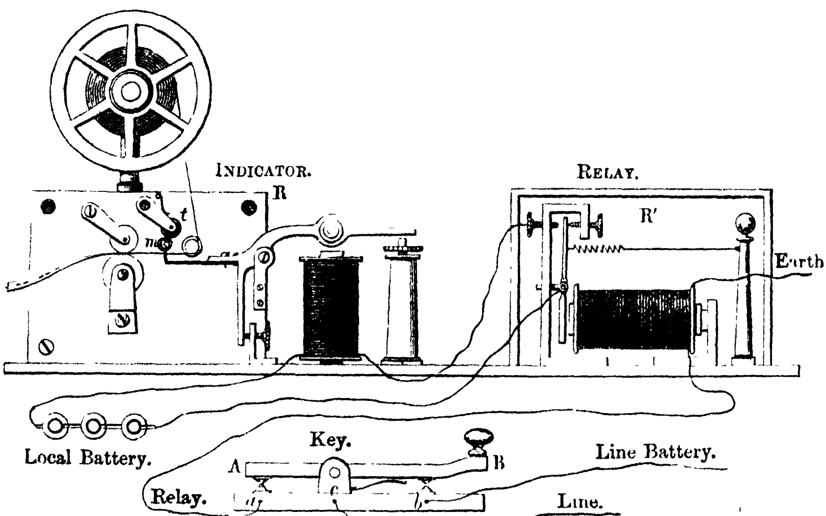


Fig. 206.—Morse's Apparatus, with Relay

armature, the contact of which with the magnet completes the circuit of the local battery, in which circuit the coil belonging to the indicator is included. The armature of the indicator is thus compelled to follow the movements of the armature of the relay.

Relays are used when the currents which arrive are too much

enfeebled to give clear indications by direct action. They are also frequently introduced at intermediate points in long lines which could not otherwise be worked through from end to end. The analogy of this use to change of horses on a long journey is the origin of the name. Relays are also frequently used in connection with alarums when these are large and powerful.

303. Electro-chemical Telegraph.—Suppose a metallic cylinder, permanently connected with the earth, to be revolving, carrying with it on its surface a strip of paper freshly impregnated with cyanide of potassium. Also suppose a very light steel point permanently connected with the line-wire, and resting in contact with the paper. Every time that a current arrives by the line-wire, chemical action will take place at the point of contact, and the paper at this point will be discoloured by the formation of prussian blue. This is the principle of Bain's electro-chemical telegraph, which leaves a record in the shape of dots and dashes of prussian blue. The apparatus for sending signals is the same as in Morse's system. The paper must not be too wet, or the record will be blurred; neither must it be too dry, for then no record will be obtained.

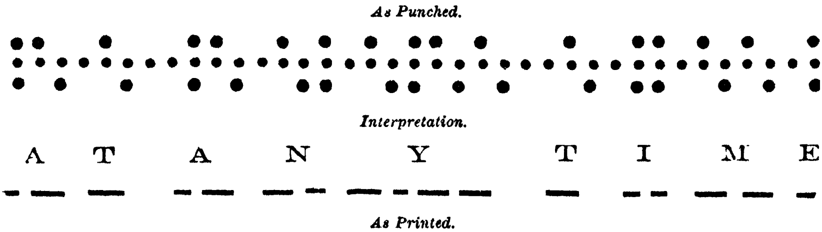
For a full account of Hughes' printing telegraph, which has done excellent service, and of some other telegraphs, which are interesting as ingenious curiosities, we must refer to previous editions.

304. Wheatstone's Automatic System.—A very effective contrivance for increasing the speed of signalling is Wheatstone's automatic apparatus, which is very extensively adopted by the authorities of the postal telegraphs. The first step towards sending a message by this system consists in punching the message in a ribbon of stiff paper. The punching is done by a special instrument, the operator having merely to put down three keys, one of which represents *dot*, another *dash*, and the third *blank*. The holes punched are in three rows. Those in the middle row are equi-distant, and are intended to perform the office of the teeth of a rack in guiding the paper uniformly forwards. Those in the two outside rows contain the message, a dot being represented by a pair of holes exactly opposite each other (:) one in each row, and a dash by two holes ranged obliquely (˙.).

The punched strips are then put through the transmitting instrument, and, by regulating the movements of two pins, cause the transmission of the currents necessary for printing the message at

the receiving station. From 100 to 150 words are thus transmitted per minute and automatically printed.[1]

The following is a specimen of three consecutive words of a telegraphic message, as it appears on the punched strip at the sending station, and on the printed strip at the receiving station:



The practical limit to speed, in lines of considerable length, arises not so much from the difficulty of making quicker movements, as from the blending together of successive signals in travelling a great distance, especially if part of the distance be under ground or under water. This evil is partly remedied by making each signal consist, not of a single current, but of two; thus a dot will be produced by an instantaneous current, *immediately* succeeded by another of opposite sign; a dash by an equally short current followed *at a longer interval* by an opposite one. In this way, though a greater number of currents are required for each word, a greater number of words can be distinctly signalled in a given time; and, by sending three properly adjusted currents for each signal, a still greater speed of distinct transmission is possible. The transmitting instrument of Wheatstone's automatic system does in fact send three currents for each dot or dash.

305. Duplex Working.—Of late years methods of sending two messages in opposite directions along the same wire at the same time have been very extensively introduced. This mode of operating is called *duplex* telegraphy. It requires that the current sent from either station shall not affect the receiving instrument at that station.

One method of attaining this end depends upon the principle that, if the coil of an electro-magnet be tapped at any point in its length,

[1] We learn from Mr. Preece's Presidential Address to the Institution of Electrical Engineers, Jan. 1893, that the speed has been gradually increased till it now sometimes reaches 600 words per minute.

a current sent into it at this point will traverse the two portions of the coil in opposite directions; whereas a current sent in at either end of the coil traverses the whole of it in one direction. Suppose this coil to be the coil of the electro-magnet which actuates the receiving instrument at any station. The currents from the transmitter at this station are sent into the coil at a point near its middle, and, circulating in opposite directions in its two portions, annul each other's effect upon the core, the arrangements being such that one of these portions forms the inner and the other the outer portion of the whole coil, and that, with equal currents, their effects in magnetizing the core are equal. On the other hand, currents arriving from another station enter at one end of the coil, and circulate round its whole length in one direction.

In order to obtain an exact balance, the two currents which circulate in opposite directions must be equal, and as they are both produced by the same electromotive force, the resistances in the two partial circuits must be equal. One of these includes the line-wire. The other goes to earth, and on its way thither is made to pass through a resistance which must be adjusted to exact equality with the line-wire. This mode of duplex sending is called the *Differential* method, from a remote resemblance to the differential galvanometer.

Another mode of duplex working is known as the *Bridge* method, because it depends upon the principle of Wheatstone's Bridge.

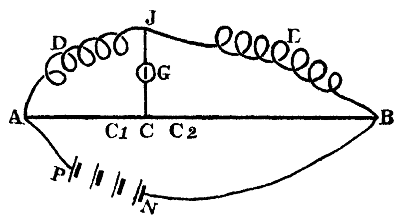


Fig 207.—Wheatstone's Bridge.

In Fig. 207 (which we here reproduce from § 209) it is allowable to suppose the three conductors which meet at B to be disconnected from each other, and separately put to earth; for this is merely equivalent to keeping the point B at the potential of the earth, and will not affect the difference of potential of any two points.

Let P N be the battery at any station, its pole N being connected to earth by the wire N B, G the receiving instrument at the same station, D, A C, C B three resistances at this station, the point B of C B being to earth, and E the line-wire leading to a distant station, where it finally reaches earth at B. Then by the principle of the bridge, if the four resistances D, E, A C, C B are proportional, the

current of the battery will not affect the receiving instrument. On the other hand, the battery at the distant station will affect it, for this battery is in the branch J B, and will always produce a difference of potential between the points J and C, whatever be the relation of the resistances.

When both stations are sending to each other simultaneously, the actual current at any point will be the algebraic sum of the currents due to the two batteries.

It is practically necessary, in most cases, to connect the point C with a condenser, whose capacity should bear the same ratio to that of the line-wire E as the resistance in C B to the resistance in E.

306. Quadruplex Telegraphy.—Duplex telegraphy is converted into quadruplex by the following additional device, which is called *Diplex* working:—

At each station the current passes through two receiving instruments, one of which is only affected by strong currents, while the other is only affected by currents of a particular sign—say positive. A weak positive current will affect the second instrument and not the first, while a strong negative current will affect the first and not the second. There are, accordingly, two sending instruments employed at each station, one sending weak currents of a particular sign, and the other sending strong currents of the opposite sign. This arrangement, combined with one of the duplex arrangements above described, permits the sending of two currents each way at the same time, so that four currents are simultaneously passing through the same wire, producing signals in four separate instruments.

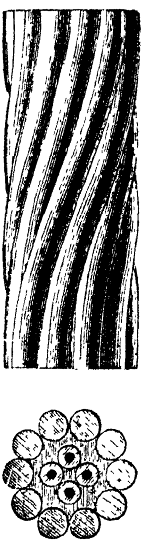


Fig 208.
Submarine Cable.

307. Submarine Telegraphs.—The first submarine telegraph cable was laid between Dover and Calais in 1850; but, being insufficiently protected against the friction of the rocks, it only lasted a few hours. The two Atlantic cables which were laid in 1866 appear to be still in perfect order.

Submarine cables are now usually constructed by embedding a certain number of straight copper wires in gutta percha (Fig. 208), which insulates them from each other; this is surrounded with tarred hemp, and several strands of iron wire are wound outside of all. The copper wires in the interior

are the conductors for the transmission of the signals; the gutta percha is for insulation; the hemp and iron are for protection.

The Atlantic cables contain a central conductor, consisting of seven copper wires, twisted together and covered with three layers of gutta percha, forming altogether a cylinder $\frac{5}{8}$ of an inch in diameter. This is covered with a layer consisting of five strands of hemp, served with a composition consisting of 5 parts of Stockholm tar, 5 of pitch, 1 of linseed-oil, and 1 of bees'-wax. Lastly, the whole is covered by 18 strands of charcoal iron, each strand consisting of seven wires $\frac{7}{10}$ of a millimetre in diameter. On leaving the machine which put on the wire covering the cable was passed through a cauldron containing a mixture of pitch, tar, and linseed-oil. The difficulty of obtaining sufficiently good insulation has thus been completely surmounted.

A second difficulty attaching to submarine telegraphy depends upon the inductive action of the surrounding water, or of the iron sheath. This action, which is found quite sensible in subterranean lines of no great length, becomes of immense importance in long submarine cables. The cable forms one enormous condenser, the central conductor representing the inner coating, and the sea-water, or iron sheath, the outer coating of a Leyden-jar. In the Atlantic cables, the retardation of the signals due to this cause is so considerable that it would be barely possible to obtain a speed one-fifth of that usually attained on land-lines, if the same modes of sending and receiving signals were employed. The electrical capacity of the cable is in fact so enormous, that a long time is required to give it a full charge from a battery, or to discharge it again. The signals accordingly lose all their sharpness, and run into one another, unless special precautions are taken. After sending a current from one pole of the battery, the cable must be discharged, either by putting it to earth, or, still better, by connecting it for an instant with the other pole of the battery. The residual effects of the first current are thus quickly destroyed, and the line is left free for a second signal.

As the first effect received through such a cable is very slight, a very sensitive receiving instrument is necessary for quick working. Thomson's mirror galvanometer (§ 172) is the instrument which was first employed, the signals being read off by an attendant who watched the movements of the spot of light, dots and dashes being represented by deflections in opposite directions. The *siphon-*

recorder of the same inventor is now used, which writes the signals with ink discharged from a very light glass siphon, the siphon being moved by a very light coil of fine copper wire, suspended by a silk fibre between the poles of a very powerful permanent magnet.[1] The coil turns in one direction or the other according as the current transmitted is positive or negative, thus producing opposite sinuosities in the ink record which is traced upon an advancing strip of paper. The regular flow of the ink is assisted by electrical attraction, on the principle of the bucket or watering-pot described in § 44; but with this difference, that it is not the ink but the paper that is electrified. An electrical machine of peculiar and novel construction, bearing some resemblance to the replenisher of § 84, is employed for this purpose.

308. Telephone.—The articulating telephone invented by Professor

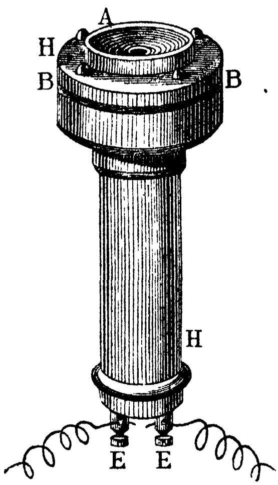


Fig. 209.—Telephone.

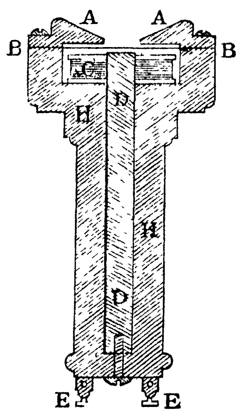


Fig. 210.—Section of Telephone.

Graham Bell is represented in Figs. 209, 210. D D is a steel magnet, C a coil of very fine silk-covered copper wire, surrounding the magnet close to one end, and having its terminals in permanent connection with the two binding-screws E E. B B is a thin disc of soft iron (usually one of the ferrotype plates prepared for photographers), tightly clamped, in its circumferential portion, between

[1] D'Arsonval's galvanometer is an adaptation of the principle of this instrument. The current to be measured passes through a small light coil suspended between the poles of a steel magnet.

the two parts of the wooden case H H, which are held together by screws, while its central portion is left free and nearly touches the end of the magnet. A A is the mouth-piece, through which the speaker directs his voice upon the iron disc.

Two telephones must be employed, one for transmitting, and the other for receiving, one binding-screw of each being connected with the line-wire, and the other with the earth or with a return-wire, so that their coils form parts of one and the same circuit, and every current generated in the one traverses the other. The mouth-piece A of the receiving telephone is held to the ear of the listener, and he is able to hear the words which are spoken into the transmitting telephone. There is a great falling off in loudness, and a decided nasal twang is imparted, but so much of the original character is preserved that familiar voices can be recognized. Conversations have thus been carried on through 60 or 70 miles of submarine telegraph cable, and through as much as 200 miles of wire suspended in the air on poles.

These results, which came upon the scientific world as a most startling surprise, are explained as follows. The voice of the speaker produces changes of pressure in the air in front of the iron disc, and thus causes the disc alternately to advance and recede, its movements keeping time with the sonorous vibrations, and the amplitudes of its movements being approximately proportional to those of the particles of air which convey the sound. Now a piece of soft iron, when brought near a magnet, exercises a *quasi* attraction upon the lines of force, causing them to be more closely aggregated in its own neighbourhood, and more widely separated in the other parts of the field. Hence when the disc approaches the magnet, it causes the lines of force to move in towards the axis of the disc, and when it recedes it causes them to open out again.

The lines of force thus cut the convolutions of the coil in opposite directions, according as the disc is approaching or receding, and give rise to alternate currents. These currents, passing through the coil of the receiving telephone, strengthen or weaken, according to their direction, the magnetism of its steel core, and increase or diminish the attraction of the latter for the iron disc. The disc is accordingly set in vibration, and imitates on a diminished scale the movements of the disc of the transmitter. Thus the original sonorous vibrations, having first been converted into undulating currents

of electricity, are reproduced as sonorous vibrations. The currents are excessively feeble, probably millions of times feebler than ordinary telegraphic currents; but on the other hand the ear is extremely sensitive to movements however small which recur periodically. Lord Rayleigh has made experiments from which it appears that the note of a whistle is audible at a distance at which the amplitude of the vibrating particles of air is less than a millionth of a millimetre.

When the telephone is employed for conversing through one of a number of telegraphic wires suspended on the same poles, it is found that messages sent by ordinary telegraphic instruments along the other wires are audible in the telephone as a succession of loud taps, so loud in fact as seriously to interfere with the telephonic conversation. This is an illustration of the principle, that the starting or stopping of a current in one wire gives rise to an induced current in a neighbouring wire; but the induced currents in this case, though so loudly audible in the telephone, have never been detected by any other receiving instrument. The telephone appears likely to supplant the galvanometer as a means of detecting feeble currents.

309. Microphone.—Fig. 211 represents one of the best forms of the microphone of Professor Hughes, the inventor of the well-known printing telegraph.

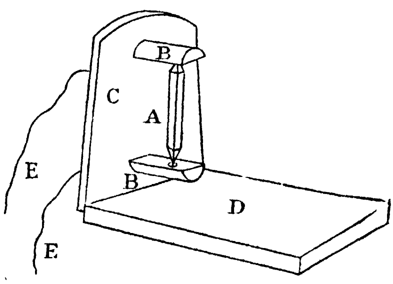


Fig. 211.—Microphone.

A is a stick of carbon about an inch long, sharpened at both ends, which rest in cavities in the two horizontal supports B B, also of carbon. The upper end of A is free to rattle about in the cavity which contains it, but not to fall away. The two wires E E are in connection respectively with the two supports B B, and are used for putting the instrument into circuit with a receiving telephone at another station. A battery, usually consisting of two or three very small cells, is also introduced into the circuit. The back C in which the supports B B are fixed, and the base D, are of wood, and, besides insulating the carbons, serve to convey to them the sonorous vibrations of the air or of surrounding bodies. These vibrations produce alternate increase and diminution of pressure at the points

of contact of the carbons with one another, and as increase of pressure gives closer contact and consequently diminished resistance, the current in the circuit undergoes corresponding changes of strength. These changes act upon the receiving telephone, and cause it to emit sounds which are often much louder than the originals. The microphone in fact acts as a relay, turning on and off the current of the battery, like the Morse relay described in § 302.

The action is improved by employing carbon which has been "metallized" by heating it white hot, and then plunging it in mercury.

The back C should be attached to the base D by a pivot which permits it to be inclined to one side. The best results for speech are usually obtained with an inclination of some 20 or 30 degrees from the vertical. When this inclination is too small there is an increase of noise in the receiving telephone, but a loss of distinctness. A microphone of the above kind transmits spoken sounds with as much distinctness as a telephone, and with much greater loudness. It has also a surprising power of transmitting very faint sounds produced by rubbing or striking the base or back with light bodies. Sounds of this kind which are quite inaudible at the place where they are produced, are easily heard by a person with his ear to the receiving telephone.

310. Telephonic Transmitters.—Though Bell's original telephone is still used as a receiving instrument, it has been almost entirely superseded as a transmitter by various forms of the microphone on the following plan.

There is a funnel for receiving the voice and converging the waves of sound upon a thin iron diaphragm, as in Bell's telephone. This diaphragm, by its vibrations to and fro, increases and diminishes the pressure at one or more points of contact in a local circuit containing a small battery and the primary coil of a miniature Ruhmkorff. The secondary coil of the Ruhmkorff is connected to the line-wire which leads to the receiving instrument.

311. Hughes' Induction Balance.—Bell's telephone, as above stated, is an extremely sensitive indicator of the currents induced in a wire by the commencement or cessation of currents in a neighbouring wire. Professor Hughes has taken advantage of this property to construct a very sensitive instrument for the instantaneous testing of metals.

A current from two or three cells is sent through two small

primary coils at a considerable distance apart. Near to them are placed two secondary coils, in circuit with a telephone, and so arranged that the induced currents in them are opposite. When the induced currents are exactly equal they destroy one another; and the adjustments are first made so as to obtain this result, that is, so as to obtain no sound in the telephone when the primary circuit is momentarily made and broken. The balance is then ready for making comparisons. Within each secondary coil is a box for containing the specimens to be compared. If precisely similar specimens are placed in the two boxes, no effect is obtained; but the slightest difference suffices to disturb the balance of the two currents and give a sound. A counterfeit coin can thus be easily distinguished from a genuine one; and even two genuine coins of the same kind will disturb the balance if one is a little more worn than the other. The best conducting metals give the most powerful effects, and a piece of wire gives a very much stronger effect when its ends touch so as to form a closed circuit than when they are apart. The effect of iron is exceptional, depending partly on currents induced in it and partly on its magnetic properties. Some successful attempts have been made, by Professor Roberts-Austen of the Mint, to apply the instrument to the testing of alloys.

312. Electrically-controlled Clocks.—Various schemes have been proposed for utilizing electricity in connection with the driving and government of clocks. In some of them, electricity is employed either to wind up the driving-weight, or to fulfil the office of a driving-weight by its own action, a pendulum being employed as the regulator, as in ordinary clocks. In others, electricity both drives and regulates the clock (or even a considerable number of clocks), by means of currents which keep time with the movements of a standard clock, electricity having thus to do the work both of driving and regulating the dependent clocks.

But the system which has given the best practical results is that introduced by Mr. R. L. Jones, in which the dependent clocks are complete clocks, able to go of themselves, and keep moderately good time, without the aid of electricity. The duty devolving on the electric currents is merely to supply the small amount of accelerating or retarding action necessary to prevent the dependent clocks from gaining or losing on the standard clock by whose movements the currents are timed.

The arrangements for attaining this end are shown in the annexed

figures 212, 213, which represent the pendulums of the controlling and controlled clocks respectively. These pendulums are supposed to be almost precisely of the same length, so that they would nearly synchronize if disconnected

The controlling pendulum, in its movement to either side, comes in contact with one or the other of two weak springs S S′, which are connected with the poles of a battery P N, having one of its middle plates connected with the earth, so as to keep its poles at potentials differing from that of the earth in opposite directions. In the position represented in the figure, a current is being sent from the positive pole P into the wire W. When the pendulum swings over to the other side, a negative current will be sent.

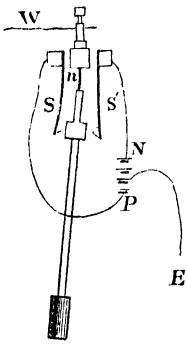


Fig 212.—Controlling Pendulum.

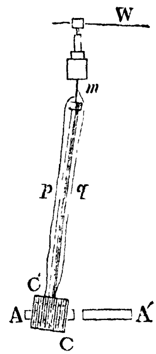


Fig 213.—Controlled Pendulum

The bob C C of the controlled pendulum (Fig. 213) is a hollow cylinder of soft iron encircled by a coil, whose ends are connected through two suspending springs at *m* with the wire W and the earth respectively. The consequence of this arrangement is that, whenever a current arrives by the wire W, the bob becomes an electro-magnet.

Two steel magnets A A′ are fixed, with their poles turned opposite ways, in such a position that the hollow bob of the pendulum always encircles one or both of them. Suppose, in the figure, that the poles A A′ which are turned outwards, are the two austral poles, so that the two boreal poles are facing each other. Then matters are to be so arranged that, in the position represented, the pendulum being near the left extremity of its swing, the right-hand end of the coil is a boreal pole, and magnetic force urges the pendulum to the left. When the pendulum is near the right extremity of its swing, the current is in the opposite direction, and consequently the boreal pole of the coil is its left-hand end. The pendulum will thus experience magnetic force urging it to the right. If the pendulum tends to gain upon the standard, its return from the extremities of its swing is thus opposed for a longer time than its outward movement is aided; and if it tends to lose, the assistance to its motion lasts longer than the opposition. Its tendency to deviate from the

standard clock either way is thus checked, and the correcting action is greater as the deviation from coincidence is greater. The controlling power thus obtained is so great, that even if the electrical connections are interrupted during several consecutive beats, the accumulated errors will be completely wiped off after the connections are restored.

CHAPTER XXII.

ELECTRIC LIGHT.

313. Arc Light.—When two pointed pieces of a conducting kind of carbon, such as that which is deposited in the retorts at gasworks, are connected with the poles of a powerful battery, as in Fig. 214,

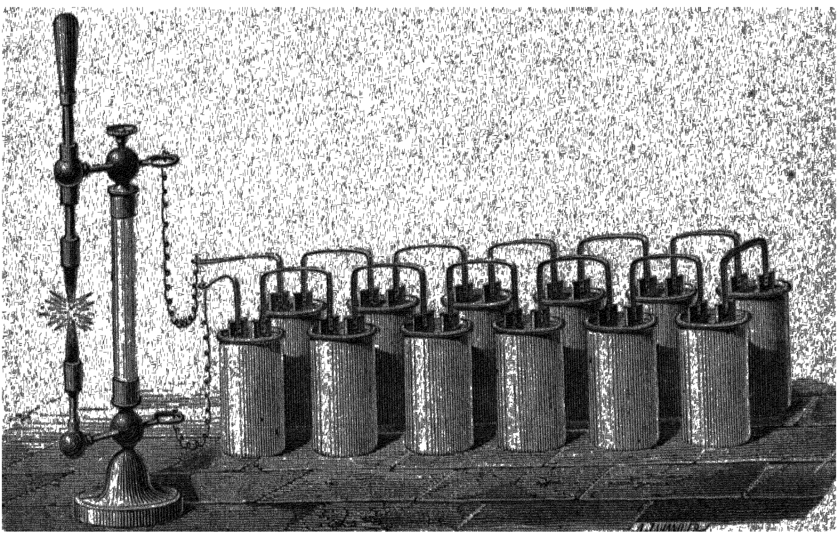


Fig. 214.—Electric Light.

a brilliant light is obtained by bringing them together so as to allow discharge to take place between them. This discharge, when once obtained, will not be interrupted by separating the points to some distance,—greater in proportion to the electromotive force of the battery; and the interval will be occupied by a luminous arch (known as the *voltaic arc*) of intense brightness and excessively high temperature. This brilliant experiment was first performed by Sir Humphry

Davy, at the commencement of the present century, with a battery of 3000 cells. The light appears to be mainly due to the incandescence of particles of carbon which traverse the space between the points.

This transport of particles can be rendered visible to a large

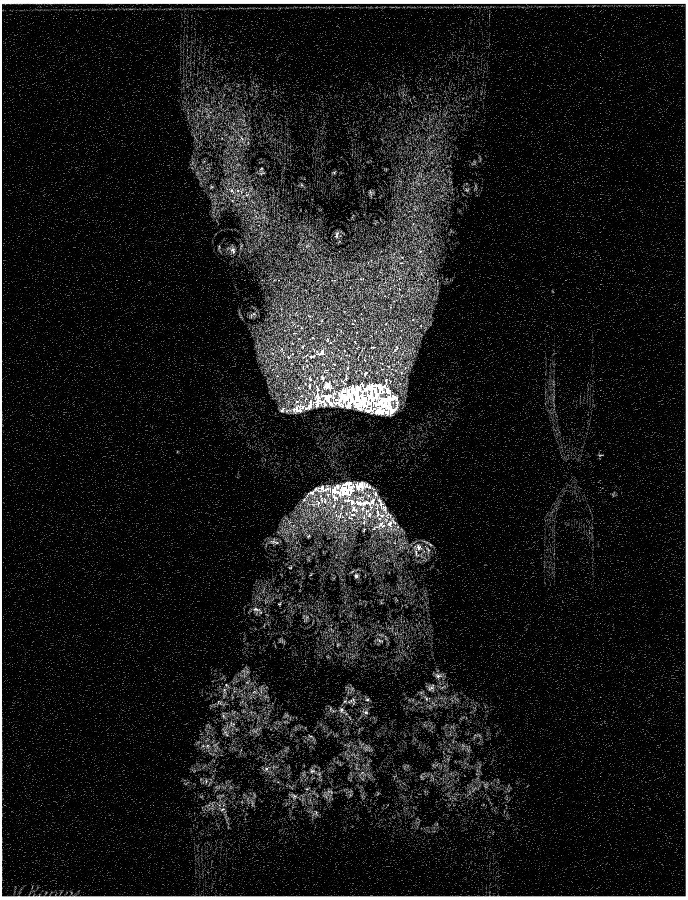


Fig. 215.—Image of the Carbon Points.

number of spectators by throwing an image of the heated points on a screen with the aid of a lens. Fig. 215 represents the image thus obtained, the natural size of the carbons being indicated by the sketch at the right hand. On watching the image for some time, incandescent particles will be observed traversing the length of the arc, some-

times in one direction and sometimes in the other, the prevailing direction being, however, that of the positive current. This circumstance, which appears to be connected with the higher temperature of the positive terminal, explains the difference between the forms assumed by the two carbons. The point of the positive carbon becomes concave, while the negative carbon remains pointed and wears away less rapidly. This difference is more precisely marked when the experiment is performed *in vacuo;* a kind of cone then grows up on the negative carbon, while a conical cavity is formed in the positive carbon. These phenomena are less clearly exhibited in air, on account of the combustion occasioned by the presence of oxygen.

The voltaic arc exceeds in temperature as well as in brightness all other artificial sources of heat. Despretz succeeded by its means in fusing and even volatilizing many substances which had previously proved refractory. Carbon itself was softened and bent, welded, and apparently reduced to vapour, which was condensed, in the form of black crystalline powder, on the walls of the containing vessel.

The voltaic arc must be regarded as an instance of conduction rather than of disruptive discharge, the air being rendered a conductor by the high temperature to which it is raised. Hence it is that, although discharge does not commence between the points till they have been brought close together, it is not interrupted by subsequently removing them to a considerable distance.

The voltaic arc is acted on by a magnet, according to the same laws as any other current. M. Quet, by employing a very powerful electro-magnet, with its poles at equal distances on opposite sides of the line joining the points, repelled the arc laterally to such an extent that it resembled a blowpipe flame (Fig. 216).

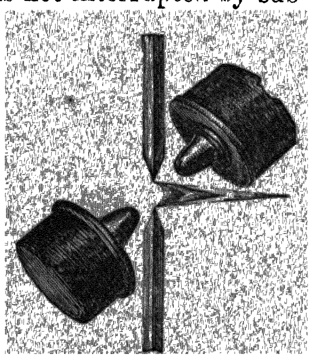


Fig. 216.—Action of Magnet on Voltaic Arc.

314. Character of the Light.—The light of the voltaic arc has a dazzling brilliancy, and attempts were long ago made to utilize it. The failures of these attempts were due not so much to its greater costliness in comparison with ordinary sources of illumination, as to the difficulty of using it effectively. Its brilliancy is painfully

and even dangerously intense, being liable to injure the eyes and produce headaches. Its small size detracts from its illuminating power—*it dazzles rather than illuminates*—and it cannot be produced on a sufficiently small scale for ordinary purposes of convenience. There is no mean between the absence of light and a light of overpowering intensity. It is much used for lighthouses, for the lighting of open spaces, and for throwing images on a screen in lecture-illustrations.

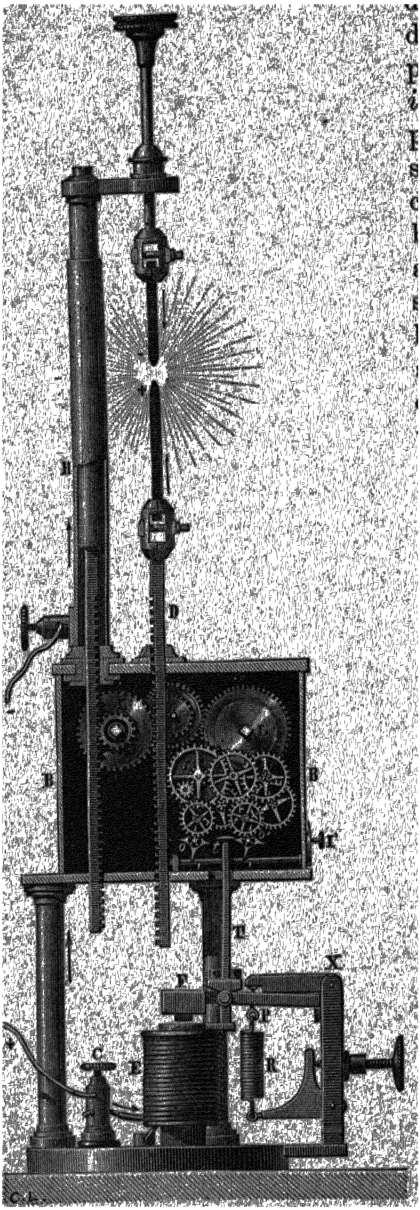


Fig. 217.—Foucault's Regulator.

As the carbons undergo waste by combustion, it is necessary to employ some means for keeping them at a nearly constant distance, so as to give a steady light. Several different regulators have been employed for this purpose, all of them depending on the principle that the strength of the current diminishes, as the distance, and consequently the resistance, increases. We will briefly describe Foucault's.

315. Foucault's Regulator.—It contains two systems of wheelwork, one for bringing the carbons nearer together, and the other for moving them further apart. Fig. 217 represents the apparatus, with the omission of a few intermediate wheels. L′ is a barrel driven by a spring inclosed within it, and driving several intermediate wheels which transmit its motion to the fly *o*.

L is the second barrel, driven by a stronger spring, and driving in like manner the fly o'. The racks which carry the carbons work with toothed wheels attached to the barrel L′, the wheel for the positive carbon having double the diameter of the other, because experience has shown that it is consumed twice as rapidly. The current enters at the binding-screw C, traverses the coil of the electro-magnet E, and passes through the wheel-work to the rack D, which carries the positive carbon. From the positive carbon it passes through the voltaic arc to the negative carbon, and thence, through the support H, to the binding-screw connected with the negative pole of the battery.

When the armature F descends towards the magnet, the other arm of the lever F P is raised, and this movement is resisted by the spiral spring R, which, however, is not attached to the lever in question, but to the end of another lever pressing on its upper side and movable about the point X. The lower side of this lever is curved, so that its point of contact with the first lever changes, giving the spring greater or less leverage according to the strength of the current. In virtue of this arrangement, which is due to Robert Houdin, the armature, instead of being placed in one or the other of two positions, as in some other regulators, has its position continuously varied according to the strength of the current. The anchor T *t* is rigidly connected with the lever F P, and follows its oscillations. If the current becomes too weak, the head *t* moves to the right, stops the fly o' and releases *o*, which accordingly revolves, and the carbons are moved forward. If the current becomes too strong, *o* is stopped, o' is released, and the carbons are drawn back. When the anchor T *t* is exactly vertical, both flies are arrested, and the carbons remain stationary. The curvature of the lever on which the spring acts being very slight, the oscillations of the armature and anchor are small, and very slight changes in the strength of the current and brilliancy of the light are immediately corrected.

316. Jablochkoff Candle.—The modern revival of interest in the electric light dates from the Paris Exhibition of 1878; when some of the streets of Paris were for the first time lighted by electric lamps constructed on a plan devised by M. Jablochkoff Instead of placing the two carbons end to end, and providing mechanism for keeping them at the proper distance, he dispensed with mechanism, and placed them side by side, with an insulating substance between

them, which is gradually consumed. A A (Fig. 218) are the two carbons, separated by a stick of plaster of Paris B. The heat produced by the electric current fuses the plaster of Paris between the points of the carbons, and the fused portion acts as a conductor of high resistance, becoming brightly incandescent. To light the lamp, a piece of carbon, held by an insulator, is laid across the two carbon points until the light appears, and is then removed. The lower ends of the carbons are inserted in copper or brass tubes C C, separated from each other by asbestos; and these tubes are connected by binding-screws with the two wires which convey the current.

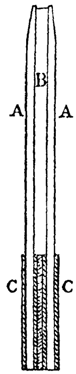


Fig. 218. Jablochkoff Candle.

When the current employed flows always in the same direction, the positive carbon is made twice as large in section as the negative, because it is consumed about twice as fast. When the current is alternating, which is the preferable arrangement, they are made equal.

The light was surrounded by a globe of opal glass, which served to diffuse its intensity and prevent dazzling.

317. Incandescent or Glow Lamps.—Another form of electric light, invented about the same time by Swan in England and Edison in America, is more suitable for the lighting of rooms and other purposes not requiring excessive brilliancy, but it gives much less light for the same expenditure of power, as compared with arc lamps. A filament of carbon prepared from bamboo, paper, or some other fibrous material, and about as thick as sewing thread, is inclosed within a vacuous glass vessel, its two ends being attached to wires which pass through the base of the lamp and serve as electrodes. A current of proper strength heats the carbon filament to whiteness, causing it to emit a soft and brilliant light, and the carbon is not consumed, as there is no oxygen to produce combustion. The vacuum must be the most perfect that can be obtained with the best Sprengel pump. An early form of these lamps is represented in Fig. 219. The thin black line in the interior represents the carbon filament, which is highly elastic and takes two turns at its upper end.

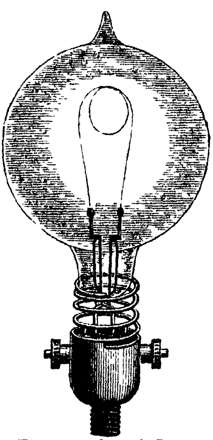


Fig. 219.—Swan's Incandescent Lamp.

CHAPTER XXIII.

ELECTROMOTORS.

318. Electromagnetic Engines.—Electromagnetic engines, more briefly styled *electromotors*, are machines driven by currents, their action depending on mechanical forces called out either between magnets and magnets, or between magnets and currents, or between currents and currents. Since their first construction in 1834 by Jacobi of St. Petersburg, who propelled a boat on the Neva by means of one, they have remained in the position of scientific toys till the recent electrical revival; but they are now becoming important in connection with the electrical transmission of driving power to a distance. A waterfall or a fixed steam-engine can be employed to generate a current of electricity by means of a dynamo or other magneto-electric machine, and this current can be made to propel a carriage or drive machinery at a distance by means of an electromotor, which receives the current and reconverts it into mechanical effect. There is always some waste of energy in this double process of conversion; but the waste is less than by any other mode of transmitting power, if the distance be considerable.

The attempts of early inventors to bring such machines, with galvanic batteries supplying their currents, into competition with steam-engines, necessarily resulted in failure, on the score of expense. A current can be more cheaply produced by a steam-engine driving a dynamo machine than by a galvanic battery; and less work would be obtainable from an electromotor driven by this dynamo than from the steam-engine direct.

319.—The most successful electromotors hitherto employed have been continuous-current dynamo machines, such as that of Siemens (Fig. 194), or of Gramme (Fig. 198), used not as dynamo machines but in the converse manner. To explain how the motion is pro-

duced, we may begin with a somewhat simpler case—that of the original Siemens' armature depicted in Figs. 190, 191. If a current from without be sent through the coil of this armature so as to make it an electro-magnet with a and b (Fig. 191) as poles, there will be a dead point in the position represented in the figure, where the fixed pole A of the field magnet is repelling a and the other fixed pole B is repelling b; but if the armature be slightly displaced from this position these two repulsions will concur in rotating the armature through about 180°. By the action of the commutator (Fig. 192) the current is interrupted for an instant at each of the two dead points (which are 180° asunder), and is then again supplied in such a direction that the pole which has just passed a pole of the field magnet is repelled by it, the repulsion lasting till the next interruption of the current. This armature is actually employed in the manner here described for some kinds of light work such as the driving of sewing-machines; but the occurrence of the dead points is a disadvantage. This difficulty could be surmounted by employing two such armatures in positions differing by 90° from each other; but still greater steadiness is obtained by employing the arrangement of Fig. 194, which resembles a combination of a number of such armatures with their poles ranged at equal distances round the circumference of the circle described.

Again in the case of the Gramme ring (Fig. 196) it is clear that if a current were sent into the coil at C from an external source and drawn off at E, this current would divide itself into two parts, one going through CDE and the other through CFE. If we suppose C to be the top and E the bottom, the current both at D and at F will circulate in the direction of watch-hands; hence both halves of the coil combine to give one pole at C and the other at E. The attractions and repulsions between these poles and the poles P P′ of the field magnet will constantly tend to turn the ring in one direction; for instance, if C is similar to P, C will be urged to the right and E to the left.

The rotations in all these cases might also have been deduced from the tendency of wires conveying a current to move across tubes of magnetic force. The two explanations in fact are fundamentally identical.

320. Quantitative Relations.—The following investigation shows the relations which exist between the work employed in driving the generator and the work given out by the motor.

Let E denote the e.m.f. of the *generator* (that is of the dynamo employed to generate the electricity), and e the reverse e.m.f. of the *motor* (which is itself a dynamo worked backwards). The whole e.m.f. in the circuit is $E-e$, and if R denote the whole resistance of the circuit, the expression for the current will be

$$C=\frac{E-e}{R}.$$

In each unit of time the quantity CE of mechanical energy is converted into electrical energy in the generator, and the quantity Ce of electrical is converted into mechanical energy in the motor.

The efficiency, as measured by the ratio of the mechanical energy given out to that put in, is accordingly $\frac{e}{E}$, and the work wasted is $C(E-e)$, which, on reference to the value above obtained for C, will be seen to be equal to C^2R, the well-known expression for the heat generated.

The expression for the efficiency shows that for economical working, the reverse e.m.f. should be a large fraction of the direct.

On the other hand, to obtain the greatest amount of work from the motor *in a given time*, when E and R are given, the product $e(E-e)$ must be made a maximum, that is, E must be divided into two parts whose product is the greatest possible; hence the two parts must be equal, e will be $\frac{1}{2}E$, and the efficiency will be $\frac{1}{2}$.

321. Economy in Transmission.—When large quantities of electrical energy are to be transmitted to great distances, whether for driving motors or for supplying electric lamps, it is important that the work spent in heating the intervening conductor should be as small as possible. The expression for this work is C^2r, where r denotes the resistance of the conductor. This latter factor can be diminished by increasing the size of the conductor, but a stout rod of copper is very expensive when the distance is several miles. The other factor C^2 can be diminished without change in the amount of energy transmitted, if we at the same time increase the electromotive force e, so as to keep the product Ce unchanged in value. If electricity is ever to be transmitted with commercial success over such distances as 50 or 100 miles, it must be by the employment of excessively high electromotive forces.

The electromotive force of a circuit supplied by a direct-current dynamo is usually called the *tension* (or the *pressure*) *of the current.*

In the case of an alternating current, the actual e.m.f. varies from

instant to instant according to a periodic law, and its *tension* or *pressure* is understood to mean the e.m.f. of a direct current which, flowing through the same resistance, would have the same energy per unit of time. It is the square root of the mean square of the actual e.m.f.

The objections to high-tension currents are the increased tendency to leakage, and the more dangerous character of the shock which will be received by a person inadvertently touching the conductor. From 500 to 600 volts is the highest tension which the Board of Trade has hitherto allowed for direct currents in circuits liable to be touched by the public.

322. Transformation of Currents.—In order to obtain economy in transmission combined with safety to the user, means are employed for transforming currents into others of higher or lower tension without much loss of energy.

In the case of direct currents, two methods are available. The first consists in employing the original current to charge storage cells arranged in series of a certain length, and rearranging these cells (usually in a shorter series) to produce the new current.

The second consists in employing the original current to supply an electromotor which drives by mechanical means a second dynamo, whose armature, if the tension is to be lowered, will have thicker wire than that of the original dynamo. The simplest mode of making the connection is to mount the armature of the motor and the armature of the second dynamo on the same axle. They may even be wound upon the same core and turn in the same magnetic field.

For alternate currents an apparatus specially called a *transformer* is employed, consisting of a primary and a secondary coil, both of copper wire, surrounding the same iron core. It has very various forms, one form resembling a Ruhmkorff coil with the iron wires of the core prolonged at both ends and opened out widely. Another form may be described as a composite iron ring, with the two coils wound one upon each half of it.

The weights of the two coils should be nearly equal, one of them being of thicker wire, and the other having a larger number of turns. The tensions of the two currents will be nearly in the ratio of the number of turns, and the currents themselves will be nearly in the inverse ratio. A transformer can thus be used either to increase the tension and diminish the current or to increase the current and diminish the tension.

It is now a common thing for electricity, like gas, to be supplied to a number of houses through "mains" proceeding from a central station; the "mains" consisting, in this case, of stout copper wires well insulated and laid in pairs, one wire for the direct and the other for the return current. In connection with such systems of electrical supply, it is usual to employ one of the above modes of transformation, the transformer being interposed between the mains and the houses.

In the great electric installation which has its head-quarters at Deptford, two of the dynamos at Deptford give alternate currents at a tension of 2400 volts, which undergo three transformations before reaching the consumer. They are first raised to a tension of 10,000 volts in the circuit connecting Deptford with the London centres. At these centres they are lowered again to 2400 volts in the street mains, and thirdly, there is at each consumer's house a transformer by which they are lowered to 100 volts.

323. Electric Welding.—Transformation of currents has been applied with great success by Professor Elihu Thomson to the welding of metals. The current of a powerful alternating dynamo is transformed into a current of very much lower tension and very much greater quantity.

The pieces which are to be welded together are held in massive metallic clamps which can be made to approach each other by turning a screw; and the clamps are connected with the terminals of the secondary coil. The high initial resistance of the joint compared with that of the continuous metal causes great concentration of heat at the place where heat is wanted; and the high temperature immediately after union is a further cause of high resistance and consequent concentration of heat. The weld is accordingly effected without much heating of other parts.

324. Series and Shunt Motors.—When a direct-current dynamo is giving a current, this current calls out an opposing mechanical force —in other words a force tending to make the armature rotate backwards. Hence if the field magnets retain their polarity unchanged, a reverse current supplied from without will drive the armature forwards. Whenever a dynamo is driven by a current, the current due to the rotation of its armature is opposite to the driving current.

When the brushes of a direct-current dynamo are in position, it is mechanically convenient that the armature should turn in such a

direction that the friction pulls and does not push them. It being understood that the armature is always to turn in this one direction, the following rules will be found to hold for the two kinds of machine.

In the case of a shunt dynamo, an external current in either direction drives the dynamo forwards. This appears from a glance at the three sketches, Fig. 220, in all of which *a* represents the

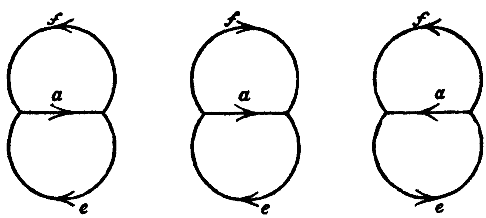


Fig. 220.

current in the armature, *f* that in the field magnets, and *e* the external current, their directions being indicated by the arrow-heads.

In the first sketch the current *a* is produced by the dynamo, and divides into *e* and *f*. In the second and third the current *e* in the external circuit divides into *a* and *f*. If the driving current *e* is in the original direction, the second sketch shows that the polarity of the field magnets is reversed, while the current in the armature is in its original direction, and therefore the armature will be driven forward. If the driving current is reversed, the third sketch shows that the field magnets retain their original polarity, but that the current through the armature is reversed, and hence the armature will still be driven forward.

In the case of a series dynamo, an external current in either direction drives the armature backwards; for the polarity of the field and the current in the armature either retain their original directions or are both reversed. The brushes must therefore be attached in a manner adapted to backward rotation.

When the field of a dynamo *is produced by a separate exciter*, or by permanent steel magnets, *the driving current must be reverse to make the armature rotate forwards.*

325. Rotating Field.—An alternating current can be used to drive a series dynamo, since, as we have seen above, the direction in which it is driven is independent of the direction of the current.

It has also been found that, under suitable conditions, one alter-

nating dynamo can drive another, and make it do mechanical work.

But a more promising application of alternating currents to motors is furnished by the discovery, made independently by Ferraris and Tesla, that two such currents, differing by a quarter-period in phase, can be made to give a rotating magnetic field, which will produce rotation of an armature (or of a magnet) placed in it.

In fact, if we denote by X the magnetic force due to the first coil at its centre, and by Y that due to the second, we shall have approximately—

$$X = a \cos \omega t \quad , \quad Y = a \sin \omega t \quad ,$$

ω being a quantity such that when t is put equal to the periodic time, ωt becomes 2π, and a being the maximum value of either component.

If the coils have a common centre and have their axes at right angles, the resultant of X and Y (which act along their axes), is evidently a, and if θ denote its inclination to X we have $\tan \theta = Y/X = \tan \omega t$, $\theta = \omega t$, showing that the direction of the resultant revolves with uniform angular velocity ω.

A bar magnet, or a bar of soft iron, mounted in the field thus produced, will tend to place its length in the direction of the resultant force, and will, under proper conditions, rotate with the velocity of the field.

CHAPTER XXIV.

FURTHER DISCUSSION OF ELECTROMAGNETIC THEORY.

326. Shell.—An important conception in connection with the relations between magnetism and electricity is that of a *magnetic shell.* By this name is denoted a thin lamina, either plane or curved, magnetized in directions everywhere normal to its surfaces. It is supposed to be so thin that the areas of its two faces may be regarded as equal; and if $d\,\mathrm{S}$ denote the area of either face of one of its elements, taken small enough to be regarded as plane, the quotient obtained by dividing the magnetic moment of the element by $d\,\mathrm{S}$ is called the *strength of the shell.* This strength is supposed to be the same for all elements of the shell unless the contrary is stated.

It is convenient to suppose the thickness of the shell to be uniform and the magnetization to be solenoidal. Then, denoting the thickness by l, the magnetization by I, and the strength of the shell by Φ, the volume of an element is $l\,d\,\mathrm{S}$, and its moment $\mathrm{I}\,l\,d\,\mathrm{S}$; hence $\Phi = \mathrm{I}\,l$. The *strength* Φ of the shell is therefore the *product of its thickness by the intensity of its magnetization;* or by its *surface-density* of free magnetism.

327.—We shall now investigate the potential of an element of a magnetic shell.

First consider an indefinitely thin small magnet, having an amount m of free magnetism at one end A, and an amount $-m$ at the other end O. The potential at a distant point P will be

$$m/\mathrm{AP} - m/\mathrm{OP} = m(\mathrm{OP} - \mathrm{AP})/\mathrm{OP}\,.\,\mathrm{AP}.$$

But, if θ denote the angle A O P, we have $\mathrm{OP} - \mathrm{AP} = \mathrm{OA}\cos\theta$. Hence the potential is $m\,.\,\mathrm{OA}\cos\theta/\mathrm{OP}\,.\,\mathrm{AP}$.

But $m\,.\,\mathrm{OA}$ is the moment of the small magnet, and $\mathrm{OP}\,.\,\mathrm{AP}$ is the square of its distance from P, say r^2.

If we have any number of small magnets close together, with their axes parallel, θ will be the same for them all, and their joint potential at P will be their joint moment multiplied by cos θ, and divided by r^2. Applying this result to an element of a magnetic shell, the moment of the element is $\Phi\, d\,S$, and the potential due to it at P is $\Phi\, d\,S \cos\theta / r^2$.

Imagine a small cone having $d\,S$ as its base, and P as its vertex, θ is the angle between r and the normal to the element, hence $d\,S \cos\theta$ is the cross section of the cone, at distance r from vertex, and $d\,S \cos\theta / r^2$ is the solid angle of the cone. Denoting this solid angle by $d\,\omega$, the potential of $d\,S$ at P is $\Phi\, d\,\omega$, and the potential of the whole shell at P is $\Phi\,\omega$, ω denoting the solid angle which the edge of the shell subtends at P.

The potential due to an element will be positive or negative according as the positive or the negative face of the element is turned towards P; and if a line from P cuts the shell at two points on the same side of P, elements at these points taken so as to subtend the same angle at P, will cancel each other's effects. The elementary solid angles that remain after this cancelling will constitute a solid angle bounded by the edge of the shell.

Since a closed curve drawn on the surface of a sphere of unit radius divides the surface into two parts, whose sum is the whole surface 4π, the edge of a shell subtends at any point P two solid angles, whose arithmetical sum is 4π. It is easy in each case to select the right one by attending to the elements which compose the solid angle.

328. Change $= 4\,\pi\,\Phi$ in crossing Shell.—If the shell is closed, its potential at external points is zero, and its potential at points in the inclosed hollow is $4\,\pi\,\Phi$.

If the shell is plane, its potential vanishes at external points in its plane, and has opposite signs in the two regions separated by this plane. At points close to the shell the potential is $2\,\pi\,\Phi$ on the positive side, and $-2\,\pi\,\Phi$ on the negative side.

For a shell of any form, if we take two points separated from each other by only the thickness of the shell, it will readily be seen that the angles ω subtended at these two points make up $4\,\pi$ when considered without regard to sign. Attending to sign, one of them is positive and the other negative; hence their algebraic difference is $4\,\pi$, and the difference of their potentials is $4\,\pi\,\Phi$, which is accordingly the work that would be done by the shell on a posi-

tive unit pole, travelling round from the point on the positive face to the opposite point on the negative face. This result we shall find to be of great importance.

329. Shell equivalent to Current.—The importance of magnetic shells in electromagnetic theory arises from the fact that the effect of a shell at external points is in most respects the same as the effect of a current in a circuit coinciding with the edge of the shell.

The first step towards the establishment of this identity is furnished by the experimental fact, that the action of a current, flowing round a circle or other plane figure, is the same at distant points as the effect of a short magnet, placed within the figure, and having its magnetic axis perpendicular to the plane of the figure.

The next step is theoretical. Let ABCD be any closed curve, and AC any line joining two of its points. The effect of a current of given strength flowing round ABCD will be the same as the combined effect of two currents of the same strength flowing round ACD and CAB, the direction of flow being in each case indicated by the order of the letters. For the currents in AC and CA being equal and opposite will annul each other's effects. By successive applications of this principle, it can be shown that currents of equal strength round the meshes of any network are together equivalent to a single current of the same strength round the edge of the net.

330.—By dividing the area of a plane circuit into small equal square meshes, we can show that the moment of the equivalent magnet is proportional to the area, and can be represented by $k\,\mathrm{C\,S}$, S denoting the area of the circuit, C the strength of the current in it, and k a constant depending only on the units employed. Hence a plane shell bounded by the circuit and of strength $\Phi = k\mathrm{C}$ will be equivalent to the current.

To determine the value of k, we shall take the simplest case—that of the force produced by a circular current at its centre. This force is proportional directly to C and to the length $2\pi r$ of the circuit, and inversely to r^2. Hence its value is $k'\mathrm{C}\,2\pi r / r^2$ or $k' 2\pi\mathrm{C} / r$, k' being a constant depending on the units employed. The electro-magnetic system of units makes $k' = 1$; hence in this system the force at the centre is $2\pi\mathrm{C} / r$.

We have to compare this with the force at the same point due to a shell of strength $k\,\mathrm{C}$, having the circle for its edge; we know that

all shells which fulfil these conditions have equal potentials, and therefore exert the same forces. We may conveniently regard the shell as hemispherical. The solid angle subtended at the centre is 2π; hence the potential at the centre is $2\pi k C$. To compute the solid angle subtended at a point on the axis at a small distance x from the centre, we must add or subtract the angle subtended by an equatorial zone of breadth x and area $2\pi r x$; this angle is $2\pi x / r$. The change of potential is therefore $2\pi x k C / r$. This is the work done by the force in moving over the distance x; hence the force is $2\pi k C / r$. This is to be identical with the force $2\pi C / r$ due to the current. Therefore the constant k is unity.

A shell of strength C of any form can be divided into elements each of which may be regarded as plane. Each element is equivalent to a current of strength C round it; and the aggregate of these currents reduces to a single current of strength C round the edge of the shell. We have accordingly the proposition that *a magnetic shell is equivalent to a current round its edge, the strength of the shell being equal to the strength of the current as expressed in electro-magnetic measure.*

331. Line-integral for Closed Path.—A closed curve which cuts a shell once is said to be *singly linked* with the curve which forms the edge of the shell. In travelling along such a curve from the positive face round to the negative face of a shell of strength C, the work done, or the fall of potential, is $4\pi C$, and in completing the curve by cutting through the shell the potential rises again to its original value. When the equivalent current is substituted for the shell, the work in describing the whole of the closed curve is $4\pi C$, for the force at all points of the path is the same as before, with the exception of the indefinitely small part representing the thickness of the shell, and the work in this is negligible.

A closed curve which cuts a shell n times in the same direction (so that in travelling along it in one direction we cut through the shell n times from the negative to the positive face) is said to be *linked n times* with the curve which forms the edge of the shell, and the work in it due to the current is $4\pi n C$.

For a closed path not linked with the circuit, the work due to the current will be the same as that due to a shell which is not cut by the path, and will therefore be zero.

The magnetic potential due to a shell at a given point has a definite value; but the magnetic potential due to the equivalent

current at the same point has an unlimited number of values differing each from the next by $4\pi C$.

It is to be noted that though we can speak definitely of *the* current equivalent to a given shell, we cannot speak definitely of *the* shell equivalent to a given current, for all shells which have the same edge and strength and agree in sign are equivalent to one another.

It is to be understood throughout the above discussion that the work spoken of is the work done on a unit pole of positive sign.

This work is called the *line-integral of magnetic* force along the path in question. We have thus the proposition that the line-integral of magnetic force due to a current C, taken along a closed curve, is zero if the closed curve is not linked with the circuit in which the current flows, and is $4n\pi C$ if the linking is n-fold.

We may remark that the linking of two curves is mutual. Arrange two strings so that the second makes n turns round a straight portion of the first, and then straighten the second. It will be seen that the first now makes n turns round the second.

o **332. Force within Circular Helix.**—When the circuit has the form of a uniform circular helix (like a single layer of wire wound on a Gramme ring), the line integral taken along any closed curve which runs once through its interior will from above be $4\pi n C$, if C denote the current, and n the number of turns. Let this curve be a circle of radius r coaxal with the helix. Its length will be $2\pi r$, which divided into the work $4\pi n C$ gives $2nC/r$ as the average working force. If the turns are close together, the force will be uniform, and will have this value at all points on the circle. It is to be noted that the radius of the convolutions does not enter the expressions at all; the wire carrying the current may therefore be wound in any number of layers, and the above expressions for the force and for the line-integral of force remain true. When the value of C is stated in amperes, the product nC is called the number of *ampere-turns*. This number, it will be observed, has a constant ratio to the line-integral.

o **333. Force within Straight Helix.**—When the circuit has the form of a uniform straight[1] helix, if a pole moves from the centre of one convolution to the centre of the next, the work done is the same as if the pole were stationary and the helix moved an equal distance (which we will call one step) in the opposite direction. In this

[1] Strictly, a helix whose axis is straight.

latter displacement each convolution moves into the position previously occupied by the next, and the total motion and total work are the same as if the extreme convolution moved in succession to the positions occupied by all the other convolutions, and one step further.

This reasoning applies not only to a straight helix of any length, but also to a circular helix, whether forming a portion of a circle or a complete circle.

In the case of the straight helix, if its length is very great in comparison with its diameter, the work done on a unit pole near the middle in moving from the centre of one convolution to the centre of the next will be the same as if there were only one convolution, and the particle moved along its axis from a great distance on one side to a great distance on the other. Since this path could be converted into a closed path embracing the helix, by an external movement in which no work is done, the work in it is $4\pi C$. Let n turns of the helix occupy a length l measured along the axis of the helix, then l/n is the distance between two convolutions, and in moving through this distance the work done is $4\pi C$; hence the force is $4\pi C n/l$.

*334. **Flux of Force.**—In a foot-note to § 52 we have pointed out an analogy between lines of force and lines of flow. It will be convenient now to expound this analogy more fully.

In the "*steady flow*" of a liquid (that is, when the state of things does not alter, but the velocity remains always the same at the same point), if we consider an imaginary tube formed by lines of flow, the quantity of liquid contained between two given sections of it remains always the same, hence the quantity which enters the included space at one of these sections must be equal to the quantity which leaves it at the other. This is true not only when the sections are perpendicular to the flow, but when they are oblique. The quantity that crosses any one section of the tube, whether perpendicular or oblique, plane or curved, must be equal to the quantity that crosses any other section in the same time. The volume of liquid that crosses a section of the tube in unit time is called the flux through the tube; and the volume that crosses any surface in unit time is called the flux through or across the surface. If the lines of flow are normal to the surface this flux is obviously equal to the surface multiplied by the velocity of the liquid. If they are inclined to the normal at an angle ϕ, the flux will be the

product of the surface, the velocity, and $\cos \phi$, or will be equal to the surface multiplied by the normal component of velocity. Denoting an element of any surface by $d\,S$, and the component velocity normal to this element by N, the flux across the surface will be the sum of such terms as $N\,d\,S$ taken over the whole surface. This sum $\Sigma\,N\,d\,S$ is the general expression for a flux. In the steady flow of heat, the flux of heat across a surface is $\Sigma\,N\,d\,S$, where $N\,d\,S$ denotes the quantity of heat that crosses $d\,S$ in unit time, and N is consequently the product of the thermal conductivity by the gradient of temperature along the normal.

In the steady flow of electric currents, N will denote the electric conductivity multiplied by the gradient of electric potential along the normal to $d\,S$.

In like manner, if N denote the normal component of the intensity of force at a point, $N\,d\,S$ is called the flux of force across or over the element $d\,S$, and the total value of $\Sigma\,N\,d\,S$ taken over any surface is called the *flux of force* over the surface.

The intensity of force at a point is analogous to the velocity of a particle of the liquid, and the lines of force are analogous to the lines of flow of the liquid. Lines of force are always supposed to be drawn at such a distance apart that the number of lines per unit area of perpendicular section is equal to the intensity of the force; whence it will follow that the number of lines which cross any section is equal to the value of $\Sigma\,N\,d\,S$ taken over the section.

Accordingly, the three expressions,

"flux of force across a surface,"

"integral of normal force over a surface,"

"number of lines of force that cross a surface,"

denote substantially the same thing, namely, $\Sigma\,N\,d\,S$.

We shall have frequent occasion to speak of the flux of (magnetic) induction B over a surface, that is to say $\Sigma\,N\,d\,S$, where N denotes the component of B normal to $d\,S$.

335. Analogy of μ to Thermal Conductivity.—We shall now show that the flux of magnetic induction B from one medium to another of different permeability follows the same laws as the steady flux of heat from one substance to another of different conductivity.

We shall use accented letters for the second medium, and unaccented for the first; shall treat the surface of junction as plane; and shall regard H, I, and B as positive when they tend from

the first medium to the second. We shall employ the subscripts n and t to denote normal and tangential components.

Since B is $H + 4\pi I$, we have, by resolving along the normal,

$$\left.\begin{aligned} B_n &= H_n + 4\pi I_n \\ B'_n &= H'_n + 4\pi I'_n \end{aligned}\right\} \qquad (1)$$

But the surface density at the junction is $I_n - I'_n$, and the repulsion due to this on either side is $2\pi(I_n - I'_n)$. Since this repulsion changes from the negative to the positive direction in passing from the first medium to the second, we have

$$H'_n - H_n = 4\pi(I_n - I'_n); \text{ hence } B_n = B'_n. \qquad (2)$$

Since the surface layer exerts no tangential force, we have

$$H_t = H'_t. \qquad (3)$$

If the media are isotropic and free from permanent magnetism, the directions of H, I, and B will coincide, making with the normal a common angle θ in the first medium, and θ' in the second. Then our last two results may be written,

$$B\cos\theta = B'\cos\theta' \qquad H\sin\theta = H'\sin\theta' \qquad (4)$$

$$\text{whence } \frac{\tan\theta}{\tan\theta'} = \frac{B}{H} \cdot \frac{H'}{B'} = \frac{\mu}{\mu'} \qquad (5)$$

$$\frac{B\sin\theta}{B'\sin\theta'} = \frac{\cos\theta'\sin\theta}{\cos\theta\sin\theta'} = \frac{\tan\theta}{\tan\theta'} = \frac{\mu}{\mu'}. \qquad (6)$$

that is, the tangential components B_t and B'_t of the flux of induction are directly as the permeabilities μ and μ', whereas the normal components B_n and B'_n are equal. The fluxes B and B′ themselves are as the secants of their inclinations to the normal.

336.—In the steady flow of heat across the surface of junction of two conductors, the normal components of flow must be equal, since each of them is the quantity of heat which in unit time crosses unit area of the junction.

The tangential flow in either medium is the product of the conductivity and the tangential gradient of temperature. This tangential gradient is the same for the two media, since the temperatures at the junction are common to both. Hence the tangential components of flow are directly as the conductivities. The tangents of the inclinations of the flows to the normal are the quotients of the tangential by the normal flows, and are therefore directly as the conductivities. Also, since the normal components are equal, the flows themselves are as the secants of their inclinations to the normal.

All these results are identical with those just obtained in the analogous problem of magnetic induction, the permeability μ taking the place of thermal conductivity.

Another analogy (from which the name *permeability* is derived) is that of a liquid forced through porous material with such small velocity that its inertia does not influence the direction of its motion. At the junction of two such materials the normal flow will be the same in both, and the tangential flows will be directly as the permeabilities. In working out the analogy pressure will take the place of temperature.

337. Analogy to Ohm's Law.—The analogy between magnetic induction and flow of heat is thus completely established; and it is often convenient to speak of the total "flux of induction" across a surface when we wish to denote that which is analogous to the total flow of heat (per unit time) across a surface, not necessarily a surface of junction, but any surface that we choose to imagine. This total flux is the sum of such terms as $N\,dS$, dS denoting an element of the surface, and N the component of B normal to the element. Its numerical value is often called the "number of lines of induction" that pass through the surface. A more exact designation is "number of unit tubes of induction."

In one point the analogy fails. Lines of induction are closed curves, and if we travel along one of these curves in the direction of the induction we come round to the point from which we started, whereas a line of flow of heat must have two distinct ends at different temperatures.

The analogy with flow of electricity is closer. All the reasoning of § 336 on thermal currents is equally applicable to electrical currents, and the analogy can now be extended to closed circuits.

The electrical resistance of a uniform circuit is $\frac{1}{k}\frac{l}{S}$, l denoting the length of the circuit, S its sectional area, and k the specific conductivity, or the reciprocal of the specific resistance. The analogue of this is $\frac{1}{\mu}\frac{l}{S}$, which is called the "magnetic resistance" of the circuit. For a non-uniform circuit it is $\Sigma\frac{dl}{\mu S}$.

The current C in a circuit has for its analogue the total flux BS.

Again, since the electromotive force E which forces the current C through the resistance R is CR, the magnetic analogue of E is BS multiplied by $\frac{1}{\mu}\frac{l}{S}$, that is $\frac{B}{\mu}l$, that is H l.

If the circuit is not uniform, but the values of k and S vary from point to point, the e.m.f. between two consecutive sections at distance dl will be $C\frac{dl}{kS}$, and the magnetic analogue of this is $BS\frac{dl}{\mu S}$, that is $\frac{B}{\mu}dl$, or $H\,dl$. The analogue of the total e.m.f. in the circuit is therefore $\Sigma H\,dl$, or the line-integral of H taken round the circuit, which has accordingly received the name of *magneto-motive force.* The "sections" here considered must be surfaces to which B and H are everywhere normal. If the distance dl between two consecutive sections is sensibly different at different points of the sections, we may confine our attention to a single narrow tube of induction, and the above reasoning shows that the analogue of e.m.f. is the line-integral of H along any such tube.

If the magnetizing force is due to a current C in a wire of n convolutions linked with the magnetic circuit (for instance n turns of wire wound upon an iron ring), the value of this line-integral is $4\pi n C$ (§ 331).

If there are interruptions in the iron of the circuit, so that the circuit consists partly of iron, partly of air, and partly of copper, gutta-percha, or other non-magnetic material, the case is comparable with that of an electric circuit consisting partly of copper and partly of materials whose conductivity is only about one-thousandth of that of copper, the whole being immersed in a medium of the same small conductivity. There would be leakage through this surrounding medium; and in like manner in a dynamo there is leakage of magnetic induction through the air and other bodies which surround the armature.

In one respect the analogy with Ohm's law fails. The magnetic "resistance" of the circuit is not independent of the magnetomotive force, since it involves μ, which varies with H. On this account some writers prefer the name *magnetic reluctance.*

338. Work Dissipated in Reversals of Magnetization.—The work done by the magnetizing force H in a unit cube of iron when the magnetization I is increased by dI can be computed by multiplying the small quantity of magnetism removed from the negative to the positive end by H, and by the distance unity. It is therefore $H\,dI$. If the iron were perfectly "soft" in the magnetic sense, the total work $\Sigma H\,dI$ in a complete cycle would be zero; but owing to retentiveness, now usually called *hysteresis,* a portion of the work given is dissipated in an internal operation resembling friction, and

produces its equivalent of heat. The value of the integral $\Sigma \mathrm{H}\, d\mathrm{I}$ for a complete cycle, after the steady condition of recurrence has set in, is accordingly the amount of work wasted per unit volume of the iron in each complete cycle in mere magnetizing and demagnetizing.

It is desirable to use the softest possible iron for the armature-cores of dynamos, not only because it can be the most strongly magnetized, but also because it has the least hysteresis, and therefore involves the smallest wasteful generation of heat in magnetizing and demagnetizing.

Fig. 221 exhibits in graphic form the changes in B produced by a cyclic change of H in a very soft iron ring. H is represented by horizontal and B by vertical distance, the scale for H being 1000 times as large as that for B. The numbers marked are the C. G. S. values of the two variables, H ranging from $+7$ to -7, and B from $+11{,}000$ to $-11{,}000$. The area of the curve is the value of $\Sigma \mathrm{H}\, d\mathrm{B}$ in going once round. But B is $\mathrm{H} + 4\pi \mathrm{I}$, hence the area is $\Sigma \mathrm{H}\, d\mathrm{H} + 4\pi \Sigma \mathrm{H}\, d\mathrm{I}$. The first term $\Sigma \mathrm{H}\, d\mathrm{H}$, being the total increase of $\frac{1}{2}\mathrm{H}^2$, vanishes for a complete cycle. Hence the area is $4\pi \Sigma \mathrm{H}\, d\mathrm{I}$ and is proportional to the energy wasted in overcoming hysteresis.

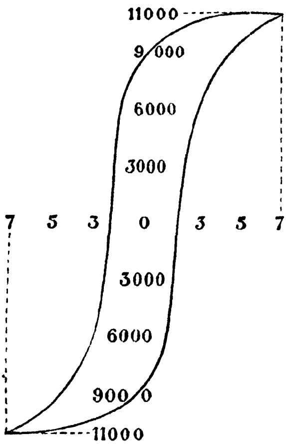


Fig. 221.—Hysteresis Curve.

339. Foucault Currents.—Another and quite distinct source of heating of iron by rapid reversals of magnetization is the production of electric currents in it by the movement of the lines of force across it. These are known by the name of *Foucault currents*, from an experiment of Foucault's in which a copper disc is heated by rapid rotation between the poles of a magnet. To guard against them in armature-cores, the iron, while continuous in the direction of magnetization, has interruptions at short intervals in the direction of the e.m.f. induced by its movement in the field. It is composed either of thin plates or of wires, and these are separated by a small thickness of some insulating substance, such as mica, paper, or varnish.

340. Potential Energy of Shell and Magnet.—We have seen that the potential at a point P, due to a magnetic shell of strength Φ, is

$\omega\Phi$, ω denoting the solid angle which the shell subtends at P. As regards sign, it is convenient to define magnetic potential as the work done by the magnetic forces in going *from* P *to* infinite distance. The potential of the shell will then be positive if the face which it turns towards P (or, when it turns both faces towards P, the face which preponderates by the amount ω,) is positive.

Let a quantity m of magnetism be collected at P, then the work which would be done in repelling m to infinity, which is called the *mutual potential energy* of m and the shell, is $m\omega\Phi$ Now the intensity of the force due to m at distance r is $\frac{m}{r^2}$; and the surface of a sphere of radius r described round P as centre is $4\pi r^2$. The product of these two, that is $4\pi m$, is the flux of force from m through this spherical boundary, and is the same at all distances. $4\pi m$ is, in fact, the whole flux of force from m, or the whole number of lines of force emitted by m; or the surface-integral of the force due to m taken over any closed surface which surrounds m. The flux from m within a solid angle ω is, therefore, $m\omega$, being the same fraction of the whole flux that ω is of the whole solid angle 4π. Thus the expression $m\omega\Phi$, for the potential energy of m due to the shell, is equal to the strength of the shell multiplied by the flux of force from m through the shell, the flux being regarded as positive when it is incident on the positive face.

If we have any number of quantities of magnetism m_1, m_2, &c., at various points not in the substance of the shell, the total flux due to them over any surface is the algebraical sum of their separate fluxes; also their total potential energy relative to the shell is the sum of their separate energies. Therefore the mutual potential energy of a shell and a magnet is equal to the strength of the shell, multiplied by the total flux of force due to the magnet which is incident on the positive face of the shell.

341. Work in Displacement.—The work done by the mutual forces in any change of the relative position of the shell and magnet is the loss of potential energy; it is, therefore, equal to the strength of the shell multiplied by the diminution in the flux incident on its positive face, or by the increment of the flux incident on its negative face. The lines of force due to the shell itself proceed from its positive face through the air round to its negative face; they are accordingly incident on its negative face. Thus the work done is positive when the additional lines,

due to the magnet, are in the same direction as the lines due to the shell itself. Since forces tend to do positive work, the system tends to move in such a way as to produce increased flux in the direction of the flux due to the shell.

342. Coefficient of Mutual Induction.—These conclusions are applicable to a current flowing along the edge of the shell. The work done by the mutual mechanical forces of a current and a magnet in any slow movement is equal to the strength of the current multiplied by the increase in the number of lines of force (in the same direction as those due to the current) which are girdled by the current. If the movement is quick it will produce a temporary variation of the strength of the current, but the proposition will still remain true for each instant.

The magnet may be a second shell, and for this we may substitute a second current. This leads to the result that the mutual potential energy of two currents is the product of three factors, namely, the strength of the first, the strength of the second, and the number of lines of force due to unit current in one circuit which are embraced by the other circuit.

Since the potential energy cannot have two unequal values, the number of lines due to unit current in one circuit which go through the other circuit is the same whichever of the two we regard as carrying the unit current. This number is called the *coefficient of mutual induction* of the two circuits.

343. Electro-motive Force due to Motion.—Denoting by N the number of lines of force from a shell or magnet which pass through a given circuit in the direction regarded as positive, the work done *against* the mutual forces of the current and magnet in a small movement of the magnet is $-\mathrm{C}\,d\mathrm{N}$, C denoting the current in the circuit. The quantity of electricity that passes in the positive direction is $\mathrm{C}\,dt$, dt denoting the time occupied in the small motion. The electromotive force produced in the circuit by the motion can be calculated by dividing the work by this quantity, and is, therefore, $-d\mathrm{N}/dt$. Thus we are led to the result that the e.m.f. induced in a circuit by the motion of a magnet is equal to the rate of decrease of the flux of induction through the circuit.

This rule continues to hold when the flux of induction is due to a current, either in a second circuit or in the first circuit itself, and whether the change of flux is due to motion of one of the circuits, or to change of strength of current. *Change in the total amount*

of flux of induction through a circuit, from whatever cause it arises, calls out in the circuit an e.m.f. equal to the rate of decrease of the flux.

As regards sign, either direction having been selected as the positive direction for C, if we suppose a corkscrew, surrounded by the current, to be turned in this direction, the direction in which the corkscrew advances is to be regarded as the positive direction for flux, since this is the direction of the flux due to a positive current in the circuit.

344. **Self-induction.**—Let L denote the *coefficient of self-induction* of a circuit, that is to say, the flux of induction through it due to unit current in it. The flux through the circuit, due to any current in it, is proportional to the current (assuming that no iron is present), and is therefore $L\,C$; and when the current changes, the rate of decrease of this is $-L\frac{dC}{dt}$, which is accordingly the e.m.f. due to the change of current. The minus sign indicates that this e.m.f. opposes change of current (just as the inertia of a fly-wheel opposes change of angular velocity). The work done against this internal e.m.f. in the elementary time dt is the product of $L\frac{dC}{dt}$ by the quantity of electricity $C\,dt$, that is, it is $L\,C\,d\,C$. This work is stored up in the circuit, and its total amount during the increase of the current from zero to any assigned value C_1 is $\frac{1}{2}L\,C_1^2$, as we see at once by integrating the above expression. If the external source of current ceases, and the circuit is left to itself, this stored-up energy runs down into heat while the current is diminishing to zero. The current thus maintained after the external source is cut off is well known under the name of the "extra current."

345. **Energy stored up in Two Circuits.**—Next consider two neighbouring circuits, still supposing that no iron is present. Let L_1 L_2 be their coefficients of self-induction, M their coefficient of mutual induction, and C_1 C_2 the currents at time t in the two circuits.

The flux through the first circuit due to the current in the second is $M\,C_2$, and the rate of decrease of this is $-M\frac{dC_2}{dt}$, which is accordingly the e.m.f. in the first due to change of current in the second. Also, as we have seen, $-L_1\frac{dC_1}{dt}$ is the e.m.f. in the first due to change of its own current. The work done in forcing a

quantity $C_1 dt$ through the first circuit in opposition to these opposing e.m.f.s is $C_1 dt\,(L_1 \frac{dC_1}{dt} + M \frac{dC_2}{dt})$, that is,

$$L_1 C_1 dC_1 + M C_1 dC_2;$$

and in like manner the work done in forcing the quantity $C_2 dt$ through the second circuit is

$$L_2 C_2 dC_2 + M C_2 dC_1.$$

The sum of these two is the energy stored up in the two circuits jointly. It can be put in the form

$$d\left(\tfrac{1}{2} L_1 C_1^2 + M C_1 C_2 + \tfrac{1}{2} L_2 C_2^2\right).$$

Accordingly the whole energy stored up during the increase of the currents from zero to their actual values is

$$\tfrac{1}{2} L_1 C_1^2 + M C_1 C_2 + \tfrac{1}{2} L_2 C_2^2.$$

346. Formula for Coefficient of Induction.—It can be shown that, when no iron or other magnetic substance is present, the coefficient of mutual induction of two circuits is the sum of such term as [1]

$$\frac{d s_1\, d s_2 \cos\theta}{r}$$

$d s_1$ denoting any element of the first circuit, $d s_2$ any element of the second, θ the mutual inclination of these two elements, and r their mutual distance. Each element of the first is to be combined with each element of the second; so that if we divide each circuit into n elements, the number of terms in the summation will be n^2.

The same formula gives the coefficient of self-induction of a circuit when no magnetic substance is present, each element being taken as ds_1 and also as ds_2. This is true whether other circuits are present or not; for they do not affect the permanent flux due to the given circuit.

347. Influence of Iron on Coefficients of Induction.—The total flux of induction due to a given current flowing in a given circuit is greater (sometimes hundreds of times greater) when iron is present than when there is no iron. If the magnetization at a given point of the iron were exactly proportional to the current, L_1, L_2 and M would still be constants, though their values would not be the same as if there were no iron. This supposition, however, is not fulfilled, and L_1, L_2, and M are functions of C_1 and C_2. The most convenient meanings to be attached to the coefficients when iron is present are such that

$$L_1 \frac{dC_1}{dt} + M \frac{dC_2}{dt}$$

[1] For a very neat proof of this formula see Emtage's *Electricity and Magnetism*, p 172.

is the e.m.f. in opposition to C_1 in the first circuit, due to the changes of the two currents, in other words that $L_1 C_1 dC_1 + M C_1 dC_2 + M C_2 dC_1 + L_2 C_2 dC_2$ is the energy stored up in the two circuits in the time dt. This expression cannot now be integrated as before, since the coefficients are no longer constants.

348. Effect of Self-induction on Current of Alternating Dynamo.— In consequence of self-induction, the maxima of current-strength in the circuit of an alternating dynamo are not contemporaneous with the maxima of electromotive force, but lag behind them, and the strength of the maximum current is somewhat less than the quotient of the maximum e.m.f. by the resistance.

In applying calculation to this subject it is usual to assume that the e.m.f. is a simple harmonic function of the time. This gives the differential equation

$$CR + L\frac{dC}{dt} = E \sin \omega t; \qquad (1)$$

C denoting the current at time t, E the maximum e.m f., ω a constant equal to 2π divided by the periodic time, R the resistance of the entire circuit, and L its self-induction, which is regarded as constant.

The only variables in the equation are C and t; and we require to find C in terms of t.

The complete solution of (1) takes account of the transitory stage as well as of the permanent state. We shall confine our attention to the latter, and shall assume, from the general analogy of forced vibrations, that the current is simple harmonic with the same period as the e m.f. This can be expressed by writing

$$C = A \cos \omega t + B \sin \omega t; \qquad (2)$$

and we have to find the values of A and B. (2) gives $dC/dt = \omega(-A \sin \omega t + B \cos \omega t)$, and when this is substituted in (1) we have

$$AR \cos \omega t + BR \sin \omega t + L\omega(-A \sin \omega t + B \cos \omega t) = E \sin \omega t. \qquad (3)$$

As this is to be true for all values of t, we may equate coefficients of $\cos \omega t$ and also of $\sin \omega t$, thus obtaining

$$AR + BL\omega = 0, \qquad BR - AL\omega = E. \qquad (4)$$

These give

$$A = -\frac{L\omega E}{R^2 + L^2\omega^2}, \quad B = \frac{RE}{R^2 + L^2\omega^2};$$

and when (2) is transformed into $C = a \sin(\omega t - b)$ we shall have

$$a = \frac{E}{\sqrt{(R^2 + L^2\omega^2)}} \qquad \tan b = \frac{L\omega}{R}.$$

a here denotes the maximum value of C, and is obviously less than E / R. The quantity $\sqrt{(R^2+L^2\omega^2)}$, which takes the place of R in the denominator, is called the *impedance* of the circuit. It has sometimes been called the "apparent resistance."

Since $R/\sqrt{R^2+L^2\omega^2}$ is equal to $\cos b$, the result can be put in the form

$$C = \frac{E\cos b}{R}\sin(\omega t - b), \qquad (5)$$

which brings out the fact that self-induction affects e.m.f., not resistance, and shows the precise relation between the reduction of e.m.f. and the retardation of phase.

When the alternations are excessively rapid, another effect of self-induction becomes important—the current is more impeded in the central than in the circumferential parts of a wire, so that it is practically confined to the circumferential portions, and the central portion is useless. For this reason copper tubes are employed instead of solid conductors for conveying the alternate currents from Deptford to London.

349. Energy in Surrounding Medium.—According to Maxwell's theory, which is a development of Faraday's, electrical and magnetic effects are propagated by means of the elastic properties of the ether and of the substances which it pervades. The magnetic or magnetizing force H is to be regarded as stress, and the induction B as strain (that is, distortion). By a general law in the theory of elasticity, the energy in a unit of volume of the medium is $\frac{1}{2}$ H B, which may also be written, $\frac{1}{2}\mu H^2$, or $\frac{1}{2} B^2/\mu$. Thus, for a given value of H the energy is much greater in iron than in air, but for a given value of B it is much less. Attractions and repulsions are to be regarded as differences of pressure of the medium against the two sides of a body. The potential energy of a system of magnets is the energy of distortion of the medium in the whole of the magnetic field, and the same remark applies to what we have called the energy stored up in a circuit or in a pair of circuits. It is not really stored up in the wires which compose the circuits, but in the surrounding medium to as great a distance as the influence of the currents extends.

350. Effects of Iron on Line-integral of H and on Flux of B.— The following distinctions must be carefully noted:—

We pointed out in § 331 that when the surrounding medium is air, the line-integral of H or B (which are the same in air) along any

closed curve singly linked with a circuit carrying a current C is $4\pi C$. When iron is present, it affects the line-integral of B, and makes it different for different paths, but does not affect the line-integral of H, which is still equal to $4\pi C$, both for paths which pass through the iron and for paths which do not. This conclusion, which is supported by the reasoning of § 337, is believed to be rigorously true. (See Maxwell, § 499, 2nd edition.)

On the other hand, Faraday's experiments, described in Series XXIX. of his "Researches," led him to the conclusion that, in a field produced by hard-steel magnets, the introduction of iron merely serves to attract lines of induction to the iron from other parts of the field, without altering the whole flux. His experiments (see § 3221 of "Researches") indicate that there is a slight increase, and for magnets of ordinary steel not specially hardened a very considerable increase, but quite trifling in comparison with the enormous increase which occurs in fields due to currents.

351.—Faraday measured the whole flux of induction due to a magnet, by employing a loop of insulated wire just large enough to be passed over the magnet and slipped along it, the two ends of the wire being connected with a ballistic galvanometer. When the loop surrounds the magnet in the middle all the lines of induction are to be regarded as passing through it, and when the loop is at a great distance from the magnet none of them pass through it.

By the principles laid down in § 343, if R denote the joint resistance of the wire and galvanometer, and n the number of lines girdled by the loop at time t, the e.m.f. at any instant[1] is $-dn/dt$, and this must be equal to C R; hence we find (neglecting sign) that the whole quantity of electricity which passes, and which is measured by the swing of the needle, is equal to n/R.

Faraday found the swing of the needle produced by moving the loop from any one position to any other to be independent of the path taken; hence he inferred that every line of induction is continuous—in other words, that the total flux of induction is the same across every complete section of the flux.

352. Measurement of μ.—The same principle has been employed in modern observations on the magnetization of iron by currents. In one method, a wire, through which a current can be sent in either direction by means of a battery and commutator, takes several turns

[1] If the circuit has self-induction, an additional term is introduced which vanishes in the integration, and the conclusion stated in the text remains true

round a ring of the material to be tested; and another wire, with its ends connected with a ballistic galvanometer, takes a few turns round another part of the same ring. When the primary current is passing, the flux of induction which it produces in the ring is girdled by the secondary wire; and when the primary current is reversed this flux is changed into an equal flux in the opposite direction. The quantity of electricity which passes through the galvanometer at each reversal, multiplied by the resistance R of the secondary circuit, is therefore double of the flux in the ring, if the secondary circuit is singly linked with the ring, and is $2n$ times the flux in the ring if the secondary circuit makes n turns. The value of B in the ring is obtained by dividing this flux by the sectional area. Again, if the primary wire makes N turns round the ring, and C denote the strength of the current in it, the line-integral of magnetizing force due to the current is $4\pi NC$, and this, divided by the length of the ring measured at the middle of its thickness, gives H. Then B divided by H is the value of the permeability μ.

353. Lifting Power.—A rough determination of μ can be made by observing the mutual attraction of two electromagnets.

If an iron ring with a coil of wire wound upon it is cut across a diameter without severing the wire, and is thus converted into two half-rings round which one and the same current can be sent, the force with which they attract each other can be computed by regarding the cuts as transverse crevasses with their sides in contact. If A denote the area of one of the sections, the quantity of free magnetism on one face of it is AI, and this is to be multiplied by the intensity of force B in a transverse crevasse, minus that part of it, namely, $2\pi I$ or $\frac{1}{2}(B-H)$, which is due to the face we are considering. Hence $\frac{1}{2}(B+H)AI$ is the force with which each side is urged towards the other. Since I is $\frac{1}{4\pi}(B-H)$ we thus obtain

$$\frac{1}{8\pi}(B^2-H^2)$$

as the expression for the lifting power per unit of area. The lifting power when a known current is passing can be determined by direct observation, and H can be determined from the known value of the current and length of the ring, as explained in the preceding section. B can then be computed from the above formula. If C.G.S. units are employed, the lifting power must be expressed in dynes per square centimetre. When B is not uniform over the section,

the formula gives the mean value of B^2, which is greater than the square of the mean value of B; hence there is danger of over-estimation of B by this method.

The lifting powers of any two magnets similar in shape and similarly magnetized are directly as their areas, that is, as the squares of their lengths or as the squares of the cube roots of their weights.

354. **Hertz's Experiments.**—Maxwell maintains that light, electricity, and magnetism are all affections of one and the same medium, and that light is an electromagnetic phenomenon. This theory is supported by the experimental fact that the velocity of light is represented, in any consistent system of units, by the same number which represents the ratio of the electromagnetic to the electrostatic unit of electricity; and it has received further confirmation from the remarkable experiments of Professor Hertz, in which vibrations undoubtedly electromagnetic, and having a frequency intermediate between those of light and sound, are shown to be propagated through air and other media with a velocity which, as near as it can be measured, is that of light. To render them intelligible we must premise:

1st. That the discharge of an electrified conductor is very often of an oscillatory character. The tendency of a current, when once started, to continue, causes the conductor to be more than discharged, that is, to acquire for the moment a charge opposite in sign to that which it originally had, but smaller. The whole discharge thus consists of a rapid succession of gradually diminishing discharges in opposite directions.

2nd. That these alternate discharges, being of the nature of currents, tend to induce a rapid succession of alternate currents in any conductor in their neighbourhood.

3rd. That, according to the electromagnetic theory of light, insulators are transparent and conductors opaque. As a matter of fact, all the metals are highly opaque, and some of the best insulators are highly transparent. There is reason to believe that those insulators which are opaque to ordinary light are transparent to rays of much longer wave-length.

In Hertz's experiments the influencing system consisted of a pair of similar conductors arranged in a horizontal line, and discharging to one another by a pair of knobs, the discharges being brought about by connecting the two conductors with the two terminals of

a Ruhmkorff coil. We shall refer to this influencing system as the "primary."

Another conductor or pair of similar conductors, which we shall call the "secondary," was also employed, and was intended to show the influence of the primary. When it consisted of a pair of similar conductors these were placed in a line parallel to that occupied by the primary. When it consisted of a single conductor its form was that of a ring, with a gap at one place. In both cases the two neighbouring ends terminated in knobs very near together, their distance being regulated by a fine adjustment. The experiments consisted in getting sparks to pass between these knobs in response to the sparks of the primary.

To make the secondary more sensitive to the influence of the primary, advantage was taken of the principle known in Acoustics as Resonance, that is to say, the secondary was made of such dimensions that its natural period of electric oscillation was identical with the period of the rapid succession of alternate discharges which made up one visible discharge of the primary. This period was very much longer than the periods of ordinary luminous vibrations, but very much shorter than the periods of sonorous vibrations. The following were some of the results obtained, and they have since been verified by other observers.

The secondary, when held in a proper position, gave sparks in response to those of the primary, even at distances of several yards, and even in adjoining rooms or rooms above or below.

When a large sheet of metal was stretched across the further end of the room (a very long one), to serve as a reflector, and the secondary was held at different distances from it, a series of points alternately of maximum and minimum effect could be traced, just as nodes and antinodes can be traced when the sonorous waves produced by a tuning-fork are reflected from a wall. The inference is that the effect is propagated in waves, and that these waves can be reflected. From the wave-length as thus determined, and from the oscillation period, which could be calculated independently, the velocity of propagation of the waves was calculated, and it agreed with the velocity of light.

The fact of the reflection of the waves by a sheet of metal having been thus established, two sheets of zinc were bent into parabolic form to act as reflectors, and were placed facing each other at distant points in a room, the line of foci of each being vertical.

A suitable "primary" and "secondary" of linear form were constructed, and placed in these two lines of foci, with results which clearly showed a convergence of influence on the second line.

A gigantic prism of pitch was then placed in the path of the waves instead of the second reflector, and was found to bend the path of the waves to one side, just as a glass prism refracts light

APPENDIX.

ELECTRICAL AND MAGNETIC UNITS.

UNITS AND DERIVED UNITS.

(1.) The numerical value of a concrete quantity is its ratio to a particular unit of the same kind; the selection of this unit being always more or less arbitrary.

(2.) One kind of quantity may, however, be so related to two or more others, as to admit of being specified in terms of units of these other kinds. For example, of the three kinds of quantity, called distance (or length), time, and velocity, any one is capable of being expressed in terms of the other two. Velocity can be specified (as regards amount) by stating the distance passed over in a specified time. Distance can be specified by stating the velocity required for describing it in a specified time, and time can be specified by stating the distance described with a specified velocity.

Force, distance, and work are in like manner three kinds of quantity, of which any two are just sufficient to specify the third.

(3.) Calculation is greatly facilitated by employing as few original or underived units as possible. These should be of kinds admitting of easy and accurate comparison; and all other units should be derived from them by the simplest modes of derivation which are available.

DIMENSIONS.

(4.) Velocity is proportional directly to distance described, and inversely to the time of its description; and is independent of all other elements. This is expressed, by saying that *the dimensions of velocity* are $\frac{\text{distance}}{\text{time}}$ or $\frac{\text{length}}{\text{time}}$.

Again, if we define the unit of velocity to be that with which unit distance would be described in unit time, the real magnitude of the unit of velocity will depend upon the units of length and time selected, being proportional directly to the real magnitude of the former, and inversely to the real magnitude of the latter. This is expressed by saying that *the dimensions of the unit of velocity* are $\frac{\text{length}}{\text{time}}$. Both forms of expression are convenient; and the ideas which they are intended to express are logically equivalent.

MECHANICAL UNITS.

(5.) All electrical and magnetic units can be derived from units of length, mass, and time. We shall denote length by l, mass by m, and time by t.

(6.) The unit of *velocity* is the velocity with which unit length is described in unit time. Its dimensions are $\frac{l}{t}$.

(7.) The unit of *acceleration* is the acceleration which gives unit increase of velocity in unit time. Its dimensions are $\frac{\text{velocity}}{\text{time}}$ or $\frac{l}{t^2}$.

(8.) The unit *force* is that which acting on unit mass produces unit acceleration. Its dimensions are mass $\times$ acceleration, or $\frac{ml}{t^2}$.

(9.) The unit of *work* is the work done by unit force working through unit length. Its dimensions are force $\times$ length, or $\frac{ml^2}{t^2}$.

(10.) The unit of *kinetic energy* is the kinetic energy of *two* units of mass moving with unit velocity (according to the formula $\frac{1}{2}mv^2$). Its dimensions are mass $\times$ (velocity)^2, or $\frac{ml^2}{t^2}$, and are the same as the dimensions of work. It might appear simpler to make it the energy of *one* unit of mass moving with unit velocity; but if this change were made, it would be necessary either to halve the unit of work, or else to make kinetic energy double of the work which produced it. Either of these alternatives would involve greater inconvenience and complexity than the selection made above.

ELECTRO-STATIC UNITS.

(11.) Let q denote *quantity* of electricity measured statically, so that the mutual repulsion of two equal quantities q at distance l,

is $\frac{q^2}{l^2}$. This being equal to a force, the dimensions of q^2 must be (length)^2 × force, or $\frac{m l^3}{t^2}$, and the dimensions of q must be $\frac{m^{\frac{1}{2}} l^{\frac{3}{2}}}{t}$.

(12.) Let V denote *difference of potential*. Then the work required to raise a quantity q through a difference of potential V, is q V. The dimensions of V are therefore $\frac{\text{work}}{q}$, or $\frac{m l^2}{t^2} \frac{t}{m^{\frac{1}{2}} l^{\frac{3}{2}}}$, or $\frac{m^{\frac{1}{2}} l^{\frac{1}{2}}}{t}$. The dimensions of potential are of course the same as those of difference of potential.

(13.) The *capacity* of a conductor is the quotient of the quantity of electricity with which it is charged, by the potential which this charge produces in the conductor. The dimensions of capacity are therefore $\frac{m^{\frac{1}{2}} l^{\frac{3}{2}}}{t} \cdot \frac{t}{m^{\frac{1}{2}} l^{\frac{1}{2}}}$, or simply l. In fact, as we have seen (§ 58), the capacity of a spherical conductor is equal to its radius.

MAGNETIC UNITS.

(14.) Let P denote the numerical value of a *pole* (or the strength of a pole). Then, since two equal poles P at distance l repel each other with the force $\frac{P^2}{l^2}$, which must be of the dimensions $\frac{m l}{t^2}$, the dimensions of P are $\frac{m^{\frac{1}{2}} l^{\frac{3}{2}}}{t}$.

(15.) Let I denote the *intensity of* a magnetic *field*. Then, a pole P in this field is acted on with a force P I. This must be of the dimensions $\frac{m\, l}{l^2}$. Hence, the dimensions of I are $\frac{m\, l}{t^2} \cdot \frac{t}{m^{\frac{1}{2}} l^{\frac{3}{2}}}$, or $\frac{m^{\frac{1}{2}}}{l^{\frac{1}{2}} t}$.

(16.) Let M denote the *moment* of a magnet. Since it is the product of the strength of a pole by the distance between two poles, its dimensions are $\frac{m^{\frac{1}{2}} l^{\frac{5}{2}}}{t}$.

(17.) Intensity of *magnetization* is the quotient of moment volume. Its dimensions are therefore $\frac{M}{l^3}$ or $\frac{m^{\frac{1}{2}}}{l^{\frac{1}{2}} t}$. These are the as the dimensions of intensity of field.

(18.) When a magnetic substance is placed in a magnetic field, it is magnetized by induction; and the ratio of the induced magnetization to the intensity of the field is called the *magnetic susceptibility* of the substance. The susceptibility usually increases with the intensity of the field up to a certain point at which it attains

a maximum, and then diminishes indefinitely. For diamagnetic substances, this coefficient is negative, that is to say, the induced polarity is reversed, end for end, as compared with that of a paramagnetic substance placed in the same field.

(19.) The work required to move a pole P from one point to another, is the product of P by the difference of the magnetic potentials of the two points. Hence, the dimensions of *magnetic potential* are $\frac{ml^2}{t^2}\,\frac{t}{m^{\frac{1}{2}}l^{\frac{3}{2}}}$ or $\frac{m^{\frac{1}{2}}l^{\frac{1}{2}}}{t}$.

ELECTROMAGNETIC UNITS.

(20.) A *current* C flowing along a circular arc, produces at the centre of the circle an intensity of field equal to C multiplied by length of arc divided by square of radius. Hence, C divided by a length is equal to a field-intensity, the dimensions of which are $\frac{m^{\frac{1}{2}}}{l^{\frac{1}{2}}t}$, and the dimensions of C are $\frac{m^{\frac{1}{2}}l^{\frac{1}{2}}}{t}$.

(21.) The *quantity* Q of electricity conveyed by a current is the product of the current by the time that it lasts. Its dimensions are therefore $m^{\frac{1}{2}}l^{\frac{1}{2}}$.

(22.) The work done in urging a quantity Q by an electromotive force E is E Q, hence the dimensions of *electromotive force* are $\frac{ml^2}{t^2}\,\frac{1}{m^{\frac{1}{2}}l^{\frac{1}{2}}}$ or $\frac{m^{\frac{1}{2}}l^{\frac{3}{2}}}{t^2}$; and as electromotive force is difference of potential, these are also the dimensions of *potential*.

(23.) The *capacity* of a conductor is the quotient of quantity of electricity by potential; its dimensions are therefore $\frac{t^2}{l}$.

(24.) The *resistance* R of a circuit is, by Ohm's law, equal to $\frac{\text{E}}{\text{C}}$. Its dimensions are therefore $\frac{m^{\frac{1}{2}}l^{\frac{3}{2}}}{t^2}\,\frac{t}{m^{\frac{1}{2}}l^{\frac{1}{2}}}$ or $\frac{l}{t}$, and are the same as the dimensions of velocity.

(25.) A coefficient of self-induction or of mutual induction is an electromotive force divided by the rate of increase of a current. Its dimensions are therefore $\frac{\text{e.m.f.}}{\text{current}}$ time = resistance × time = length.

COMPARISON OF THE TWO SETS OF UNITS.

(26.) On comparing the dimensions of the same element as measured according to the two systems, it will be observed that they are not identical. The dimensions of quantity of electricity, for

example, in the first system, are to its dimensions in the second, as l to t; and the dimensions of capacity are as l^2 to t^2.

Notwithstanding this difference of dimensions, two quantities of electricity which are equal when compared statically, are also equal when compared magnetically, or if one be double of the other when compared statically, it will also be double of the other when compared magnetically.

(27.) An illustration from a somewhat more familiar department may assist the reader in convincing himself that it is possible for one and the same kind of quantity to have different dimensions according to the line of derivation employed. It is well known that uniform spheres attract each other with a force which is directly as the product of their masses, and inversely as the square of the distance between their centres. If this law were made to furnish the unit of force, the dimensions of force would be $\frac{m^2}{l^2}$, instead of $\frac{ml}{t^2}$, as previously found.

If gravitational, electrical, and magnetic attractions are reducible to ordinary mechanical principles, all these differences of dimensions must be removable. Professor Rücker has shown that, by introducing specific inductive capacity (K) and magnetic permeability (μ) into dimensional formulæ, the electrostatic and electromagnetic dimensions can be brought into agreement by assuming that $K\mu$ is the reciprocal of the square of a velocity. (See our *C.G.S. System of Units*, 4th edition, p. 208.)

(28.) Derived units are often called *absolute* units; but it seems an abuse of language to define a unit by its *relation* to other arbitrary units, and then call it *absolute.*

(29.) By the general consent of the scientific world the Centimetre, Gramme, and Second are adopted as the basis of all derived units; and the units thence derived are distinguished by the initial letters C. G. S. prefixed.

The following units are employed for commercial purposes by the genèral consent of practical electricians.

Ohm $= 10^9$ C.G.S. units of resistance, electromagnetic system.

Volt $= 10^8$ C.G.S. units of e.m.f., electromagnetic system.

Ampere = current given by e.m.f. of 1 volt through resistance of 1 ohm =

$\frac{1}{10}$ C.G.S.

Coulomb = quantity conveyed by 1 ampere in 1 sec. $= \frac{1}{10}$ C.G.S.

Farad = capacity of a condenser which requires 1 coulomb to charge it to 1 volt $= \left(\frac{1}{10}\right)^9$ C.G.S.

Microfarad = one-millionth of farad $= \left(\frac{1}{10}\right)^{15}$ C.G.S.

The microfarad is the unit actually employed, the farad being much too large.

Watt = rate of working of 1 ampere driven by 1 volt, or of 1 ampere flowing through 1 ohm $= 10^7$ C.G.S.

Kilowatt = 1000 watts; whereas 1 horse-power = 746 watts.

Joule = work done by 1 ampere driven by 1 volt $= 10^7$ ergs.

The self-induction (or coefficient of self-induction) of a circuit in which increase of current at the rate of 1 ampere per second produces a reverse e.m.f. of 1 volt is sometimes called a Secohm, sometimes a Quadrant. It is equal to 10^9 centimetres, which is approximately a quadrant of a meridian.

It was agreed in 1892 that the ohm should be represented by 14·4521 grammes of pure mercury at 0° C., in the form of a column of uniform section 106·3 centimetres high.

That the e.m.f. of a Clark cell at 15° C. be taken as 1·434 volt.

That ·001118 be adopted as the number of grammes of silver deposited per second from a neutral solution of nitrate of silver by a current of 1 ampere.

(30.) Ratio of the Two Units of Quantity is the Velocity of Light.—Let the units of length, mass, and time in any other system be respectively equal to

$$l \text{ centimetres}, \qquad m \text{ grammes}, \qquad t \text{ seconds}.$$

Then the new electromagnetic unit of *quantity* will be $m^{\frac{1}{2}} l^{\frac{1}{2}}$ times the C.G.S. electromagnetic unit; and the new electrostatic unit of quantity will be $m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-1}$ times the C.G.S. electrostatic unit. If the two new units of quantity are *equal*, we shall have the following relation between the two C.G.S. units, namely—

$$m^{\frac{1}{2}} l^{\frac{1}{2}} \text{ electromagnetic units} = m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-1} \text{ electrostatic units};$$

that is,

$$\frac{\text{C.G.S. electromagnetic unit}}{\text{C.G.S. electrostatic unit}} = \frac{l}{t}.$$

But $\frac{l}{t}$ is clearly the value, in centimetres per second, of that velocity which would be called unity in the new system. This is a definite concrete velocity; and its numerical value will always be equal to the ratio of the electromagnetic to the electrostatic unit of quantity, whatever units of mass, length, and time are employed. Denoting

the numerical value of this velocity by v, it can be shown, by reasoning similar to the above, that the electrostatic units of potential and resistance are respectively v times and v^2 times the corresponding electromagnetic units, and that the electromagnetic unit of capacity is v^2 times the electrostatic unit.

From numerous experiments in which the same physical magnitude (quantity of electricity, difference of potential, or capacity), was measured both statically and magnetically, it appears that this velocity is (within the limits of experimental error) identical with the velocity of light.

EXAMPLES.

1. Sum to infinity the two geometric series of § 67, which express the quantities successively discharged from the two coatings.

2. Compare the energy of discharge of a single Leyden-jar with that of a battery of n such jars,

(1) When the charge of the single jar is equal to that of the battery,

(2) When both are charged to the same potential.

3. A series of n Leyden-jars are charged by cascade, the knob of the first being in connection with the conductor of the machine, the potential of which is V. If the first jar be disconnected from the machine and from the second jar, and its outer coating be connected with the earth, what will be the potential of its inner coating?

4. Show that the spark obtained by connecting the knob of the first jar in a cascade arrangement with the outer coating of the last, will be stronger than that which would be obtained by connecting the knob and outer coating of the first jar, if this jar were charged in the ordinary way with the same quantity of electricity which it has in the cascade arrangement.

5. If n similar jars are charged by cascade, and then connected so as to form an ordinary Leyden-battery, compare the energy of the discharge obtained by connecting the two coatings of the battery with that of the discharge which would have been obtained by connecting the inner coating of the first jar with the outer coating of the last in the cascade arrangement.

6. If n jars of unequal capacities C_1, C_2, ...C_n are charged by cascade, and the potentials of their coatings are V_0, V_1, V_2, ...V_n, show that the total energy is $\frac{1}{2}(V_n - V_0)^2$ divided by $\Sigma\ 1/C$.

7. An electrified body is fixed at the distance of a few inches from the knob of an uncharged gold-leaf electroscope, and produces divergence of the leaves. How will this divergence be affected,

(1) By introducing a wire-gauze shade in connection with the earth between the charged body and the electroscope?

(2) By placing a wire-gauze shade in connection with the earth so as to inclose both the charged body and the electroscope?

8. A metallic vessel A is filled with shot, which run out, one by one, along a metal tube terminating near to an insulated positively charged conductor, but not so near that a spark can pass. The shot drop from the tube into a second metallic vessel B. What will be the final electrical condition of each of the vessels;

(1) If both are insulated?

(2) If B is insulated, but not A?

(3) If A and B are connected together by a wire, but insulated from other bodies?

Show how the principle of the conservation of energy applies in each case.

9. Show that the gain or loss of potential energy in transferring a quantity Q of electricity from one conductor to another is $Q(V_1 - V_2)$, where V_1 is the arithmetical mean of the potentials of one of the two conductors before and after the transfer, and V_2 is the similar mean for the other conductor.

In the four following examples the skeletons are supposed to be made of uniform wire, and the resistance of one side (or one edge) is called unity.

10. Find the resistance between opposite corners of a skeleton square.

11. Find the resistance between two corners of a skeleton equilateral triangle.

12. Find the resistance between two opposite corners of a skeleton cube.

13. Find the resistance between two corners of a skeleton regular tetrahedron.

14. Investigate a formula for the resistance of a wire in terms of its length, mass, density, and specific resistance.

15. The terminals of a battery of three cells are connected by a wire of resistance R; and it is found that when the terminals of a fourth cell similar to the cells of the battery are connected with the terminals of the battery, the current through R is not altered. Compare R with the resistance of one cell.

16. A battery consists of ten similar cells arranged in a series, and the circuit is completed by a wire 10 ft. long. If this wire, at a point 2 ft. from the first terminal, is allowed to touch the binding-screw which connects the second and third cells, show whether there will be any alteration in the current in either part of the wire.

17. The terminals of a battery are connected by a wire of ten thousand units' resistance. Compare the indications of an electrometer when its electrodes are joined (1) to the terminals of the battery, (2) to points in the wire separated by eight thousand units' resistance, (3) to points separated by three thousand units' resistance.

18. A galvanic battery of ten cells has its ends joined by a wire 100 ft. long with a resistance five times that of the battery. Also the junction of the third and fourth cells is in communication with the earth. Find the potentials of the other junctions, and of the terminals of the battery.

19. Investigate the effect on the current of a battery of five cells, with an external resistance equal to that of one cell,

(1) When half the liquid in one of the cells is removed,

(2) When, for the zinc of one cell another metal is substituted, such that this cell by itself would produce a current only half that of one of the other cells by itself.

20. A battery is to be constructed with plates of a given aggregate area, and at a given distance apart, for the purpose of heating a wire whose resistance is given. Find the number of cells which will heat the wire most.

21. A and B are two batteries constructed of the same materials. The resistance of A is to that of B as $a : b$, and the current in A is to that in B as $i : j$, the external resistances being negligible. Find the ratio of the amount of zinc consumed in A to that consumed in B in the same time.

22. The current from a battery of ten equal elements, passes through a voltameter, and evolves 50 cc. of hydrogen per minute. Fifty metres of wire are now introduced, in addition, into the circuit, and the volume of hydrogen now evolved in the voltameter per minute is 30 cc. If the resistance of 2·5 metres of the introduced wire be the unit of resistance, and the unit of current be that which evolves 1 cc. of hydrogen per minute, what is the electromotive force of one element of the battery?

23. Show that a given battery generates a maximum quantity of heat in the connecting wire when this wire has the same resistance as the battery.

24. A straight horizontal copper bar, in electric communication with the earth by chains hanging vertically from its two ends, is carried in a horizontal plane in the four following ways,

(1) The bar is in the magnetic meridian, and is carried in the direction of its own length.

(2) It is perpendicular to the meridian, and is carried in the direction of its own length.

(3) It is in the meridian, and is carried perpendicular to its own length.

(4) It is perpendicular to the meridian, and is carried perpendicular to its own length.

Compare the currents (if any) generated by the motion in the four cases.

25. A metal ring rotates uniformly round a horizontal diameter, which is perpendicular to the magnetic meridian. In what parts of the revolution is the induced current strongest, and in what parts does it vanish?

26. Compare the currents in the preceding question with those induced by rotating the ring with the same velocity round a vertical diameter.

27. Compare the electromotive forces generated, in two rings of different radii, by rotating with equal angular velocities round parallel diameters.

28. Prove that the distance of a fault in a submarine cable is to the whole length of the cable as

$$S - \sqrt{(T - S)(R - S)} \text{ to } R,$$

S denoting the resistance of the faulty cable when connected with earth at the further end,

T its resistance when insulated at the further end,

R the resistance of the cable before it was faulty. The fault is supposed to consist in a loss of insulation, and all parts of the cable except the fault are supposed to be perfectly insulated.

29. A copper disc revolves in a field produced by a current in a fixed coil surrounding the circumference of the disc. In a wire in circuit with the coil two points are found which have a difference of potential equal to that between the centre and circumference of the disc. Show that the resistance of the wire between these points is M/T, M denoting the coefficient of mutual induction of the coil and the circumference of the disc, and T the time of rotation. [This is Lorenz's method of absolute measurement of resistance.]

ANSWERS.

Ex. 1. Q, mQ. Ex. 2. Energy of discharge of single jar is n times that of battery in case (1), and $\frac{1}{n}$ in case (2). Ex. 3. $\frac{V}{n}$.

Ex. 4. Since Q is to equal Q′, the charges and potentials in the cascade arrangement are n times as great as in the case supposed in § 77, Hence the energy of the spark is n^2 times as great. It is therefore n times the energy of the spark of the single jar. Ex. 5. The same.

Ex. 6. $Q = C_1(V_1 - V_0) = (V_1 - V_0) - 1/C_1 = (V_2 - V_1) \div 1/C_2 = \&c. = (V_n - V_0) \div \Sigma\, 1/C$. Energy $= \frac{1}{2} Q(V_1 - V_0) + \frac{1}{2} Q(V_2 - V_1) + \&c. = \frac{1}{2} Q(V_n - V_0)$.

Ex. 7. In case (1) the electroscope will be completely screened from the influence of the electrified body, and the leaves will collapse. In case (2) electricity of the opposite sign to that of the charged body will be induced on the shade, and the divergence of the leaves will be diminished.

Ex. 8. (1) A will become positively and B negatively charged, the shots acting as carriers of negative electricity from A to B. (2) B will become negatively charged, more quickly than in case (1). (3) The negative carried from A to B by the shots returns from B to A by the wire as fast as it is supplied.

In (1) and (2) the descent of the shots is opposed by electrical forces. Hence they fall more gently, and generate less heat by concussion, than they would in the absence of electricity.

Ex. 10. 1. Ex. 11. $\frac{2}{3}$. Ex. 12. $\frac{5}{8}$. Ex. 13. $\frac{1}{2}$. Ex. 14. $\frac{\rho l^2 d}{m}$.

Ex. 15. R is $1\frac{1}{2}$ times the resistance of one cell.

Ex. 16. None; for the two points which touch had the same potential before contact.

Ex. 17. As 10, 8, and 3.

Ex. 18. First terminal $\frac{15}{6}e$, last terminal $-\frac{35}{6}e$, junctions $\frac{10}{6}e$, $\frac{5}{6}e$, 0, $-\frac{5}{6}e$, $-\frac{10}{6}e$, &c., e being electromotive force of one cell.

Ex. 19. The current will be diminished (1) as 7 to 6, (2) as 10 to 9.

Ex. 20. Let R denote the resistance of the wire, and r the resistance of the battery when consisting of a single cell. The number of cells n must be that which gives the smallest difference between nr and $\frac{R}{n}$.

Ex. 21. As $a i^2$ to $b j^2$.

Ex. 22. $150 + \frac{1}{10}e'$, where e' denotes the reverse electromotive force of the voltameter.

Ex. 23. The heat per unit time can be shown to be E^2 divided by $R + 2B + B^2/R$, and is greatest when $R + B^2/R$ is least, that is, when $R = B^2/R$.

Ex 24. No current in cases (1), (2), and equal currents in (3), (4). The currents are due solely to the vertical magnetic force, for though the lines of horizontal force are cut by the chains in (2) and (3), the electromotive forces are *up* both chains, or *down* both, and so destroy each other.

Ex. 25. Strongest when the plane of the ring is parallel to the dipping-needle. Vanishes when plane of ring is perpendicular to dipping-needle.

Ex. 26. As total intensity to horizontal intensity, or as 1 to cosine of dip.

Ex. 27. As the squares of the radii, or directly as the areas inclosed.

Ex. 29. M C is the number of lines that go through the disc. These are all cut once in time T. Therefore e.m.f. = MC/T. But it also equals C R.

PART IV.

SOUND AND LIGHT.

ACOUSTICS.

CHAPTER I.

PRODUCTION AND PROPAGATION OF SOUND.

1. **Sound is a Vibration.**—Sound, as directly known to us by the sense of hearing, is an impression of a peculiar character, very broadly distinguished from the impressions received through the rest of our senses, and admitting of great variety in its modifications. The attempt to explain the physiological actions which constitute hearing forms no part of our present design. The business of physics is rather to treat of those external actions which constitute sound, considered as an objective existence external to the ear of the percipient.

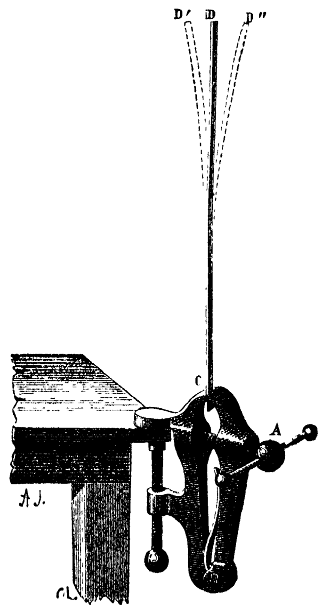


Fig 1.—Vibration of Straight Spring.

It can be shown, by a variety of experiments, that sound is the result of vibratory movement. Suppose, for example, we fix one end C of a straight spring CD (Fig. 1) in a vice A, then draw the other end D aside into the position D′, and let it go. In virtue of its elasticity the spring will return to its original position; but the kinetic energy which it acquires in returning is sufficient to carry it to a nearly equal distance on the other side; and it thus swings alternately from one side to the other through distances very gradually diminishing, until at last it comes to rest. Such movement is called vibratory. The motion from D′ to D″, or from D″ to D′, is called a *single vibration.* The two together constitute a

double or *complete vibration;* and the time of executing a complete vibration is the *period* of vibration. The *amplitude* of vibration for any point in the spring is the distance of its middle position from one of its extreme positions. These terms have been already employed in Part I. in connection with the movements of pendulums to which indeed the movements of vibrating springs bear an extremely close resemblance. The property of isochronism, which approximately characterizes the vibrations of the pendulum, also belongs to the spring, the approximation being usually so close that the period may practically be regarded as altogether independent of the amplitude.

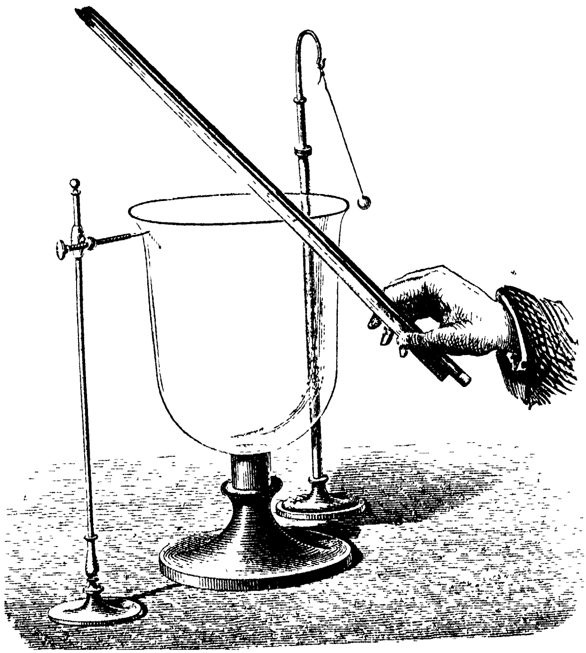


Fig. 2.—Vibration of Bell.

When the spring is long, the extent of its movements may generally be perceived by the eye. In consequence of the persistence of impressions, we see the spring in all its positions at once; and the edges of the space moved over are more conspicuous than the central parts, because the motion of the spring is slowest at its extreme positions.

As the spring is lowered in the vice, so as to shorten the vibrating portion of it, its movements become more rapid, and at the same time

more limited, until, when it is very short, the eye is unable to detect any sign of motion. But where sight fails us, hearing comes to our aid. As the vibrating part is shortened more and more, it emits a musical note, which continually rises in pitch; and this effect continues after the movements have become much too small to be visible.

It thus appears that a vibratory movement, if sufficiently rapid, may produce a sound. The following experiments afford additional illustration of this principle, and are samples of the evidence from which it is inferred that vibratory movement is essential to the production of sound.

Vibration of a Bell.—A point is fixed on a stand, in such a position as to be nearly in contact with a glass bell (Fig. 2). If a rosined fiddle-bow is then drawn over the edge of the bell, until a

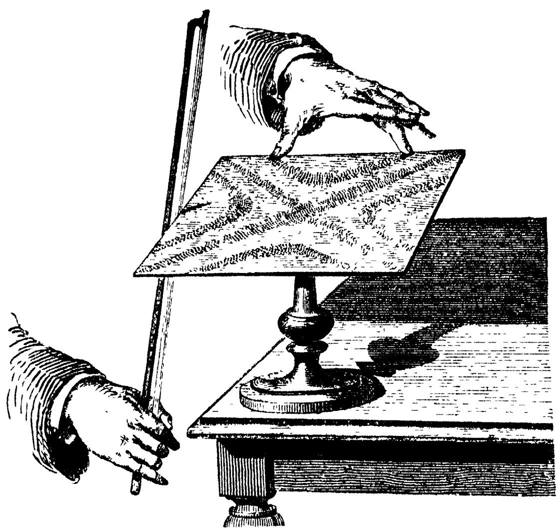


Fig 3 —Vibration of Plate

musical note is emitted, a series of taps are heard, due to the striking of the bell against the point. A pith-ball, hung by a thread, is driven out by the bell, and kept in oscillation as long as the sound continues. By lightly touching the bell, we may feel that it is vibrating; and if we press strongly, the vibration and the sound will both be stopped.

Vibration of a Plate.—Sand is strewn over the surface of a horizontal plate (Fig. 3), which is then made to vibrate by drawing a

bow over its edge. As soon as the plate begins to sound, the sand dances, leaves certain parts bare, and collects in definite lines, which are called *nodal lines.* These are, in fact, the lines which separate portions of the plate whose movements are in opposite directions. Their position changes whenever the plate changes its note.

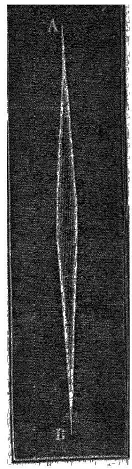


Fig 4
Vibration of String

The vibratory condition of the plate is also manifested by another phenomenon, opposite—so to speak—to that just described. If very fine powder, such as lycopodium, be mixed with the sand, it will not move with the sand to the nodal lines, but will form little heaps in the centre of the vibrating segments; and these heaps will be in a state of violent agitation, with more or less of gyratory movement, as long as the plate is vibrating. This phenomenon, after long baffling explanation, was shown by Faraday to be due to indraughts of air, and ascending currents, brought about by the movements of the plate. In a moderately good vacuum, the lycopodium goes with the sand to the nodal lines.

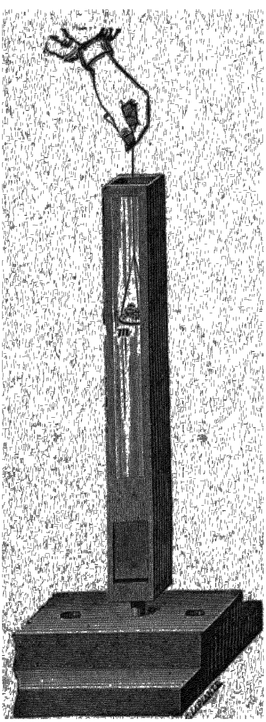
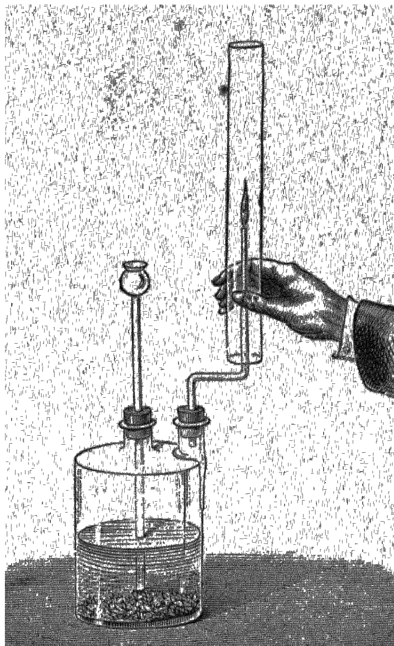


Fig 5.—Vibration of Air.

Vibration of a String.—When a note is produced from a musical string or wire, its vibrations are often of sufficient amplitude to be detected by the eye. The string thus assumes the appearance of an elongated spindle (Fig. 4).

Vibration of the Air.—The sonorous body may sometimes be air, as in the case of organ-pipes, which we shall describe in a later chapter. It is easy to show by experiment that when a pipe *speaks*, the air within it is vibrating. Let one side of the tube be of glass, and let a small membrane *m*, stretched over a frame, be strewed with sand, and lowered into the pipe. The sand will be thrown into violent agitation, and the rattling of the grains, as they fall back on the membrane, is loud enough to be distinctly heard.

Singing Flames.—An experiment on the production of musical sound by flame, has long been known under the name of the *chemical harmonica.* An apparatus for the production of hydrogen gas (Fig. 6) is furnished with a tube, which tapers off nearly to a point at its upper end, where the gas issues and is lighted. When a tube, open at both ends, is held so as to surround the flame, a musical tone is heard, which varies with the dimensions of the tube, and often attains considerable power. The sound is due to the vibration of the air and products of combustion within the tube; and on observing the reflection of the flame in a mirror rotating about a vertical axis, it will be seen that the flame is alternately rising and falling, its successive images, as drawn out into a horizontal series by the rotation of the mirror, resembling a number of equidistant tongues of flame, with depressions between them. The experiment may also be performed with ordinary coal-gas.



Chemical Harmonica

Trevelyan Experiment.—A fire-shovel (Fig. 7) is heated, and balanced upon the edges of two sheets of lead fixed in a vice; it is then seen to execute a series of small oscillations—each end being alternately raised and depressed—and a sound is at the same time emitted. The oscillations are so small as to be scarcely perceptible in themselves; but they can be rendered very obvious by attaching to the shovel a small silvered mirror, on which a beam of light is directed. The reflected light can be made to form an image upon a screen, and this image is seen to be in a state of oscillation as long as the sound is heard.

The movements observed in this experiment are due to the sudden expansion of the cold lead. When the hot iron comes in contact with

it, a protuberance is instantly formed by dilatation, and the iron is thrown up. It then comes in contact with another portion of the

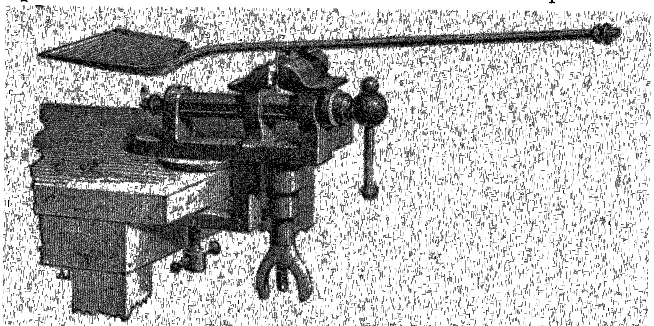


Fig. 7 —Trevelyan Experiment.

lead, where the same phenomenon is repeated while the first point cools. By alternate contacts and repulsions at the two points, the shovel is kept in a continual state of oscillation, and the regular succession of taps produces the sound.

The experiment is more usually performed with a special instrument invented by Trevelyan, and called a *rocker*, which, after being heated and laid upon a block of lead, rocks rapidly from side to side, and yields a loud note.

2. Distinctive Character of Musical Sound.—It is not easy to draw a sharp line of demarcation between musical sound and mere noise. The name of noise is usually given to any sound which seems unsuited to the requirements of music.

This unfitness may arise from one or the other of two causes. Either,

1. The sound may be unpleasant from containing discordant elements which jar with one another, as when several consecutive keys on a piano are put down together. Or,

2. It may consist of a confused succession of sounds, the changes being so rapid that the ear is unable to identify any particular note. This kind of noise may be illustrated by sliding the finger along a violin-string, while the bow is applied.

All sounds may be resolved into combinations of elementary musical tones occurring simultaneously and in succession. Hence the study of musical sounds must necessarily form the basis of acoustics.

Every sound which is recognized as musical is characterized by what may be called smoothness, evenness, or regularity; and the physical cause of this regularity is to be found in the accurate

periodicity of the vibratory movements which produce the sound By *periodicity* we mean the recurrence of precisely similar states at equal intervals of time, so that the movements exactly repeat themselves; and the time which elapses between two successive recurrences of the same state is called the *period* of the movements.

Practically, musical and unmusical sounds often shade insensibly into one another. The tones of every musical instrument are accompanied by more or less of unmusical noise. The sounds of bells and drums have a sort of intermediate character; and the confused assemblage of sounds which is heard in the streets of a city blends at a distance into an agreeable hum.

3. **Vehicle of Sound.**—The origin of sound is always to be found in the vibratory movements of a sonorous body; but these vibratory movements cannot bring about the sensation of hearing unless there be a medium to transmit them to the auditory apparatus. This medium may be either solid, liquid, or gaseous, but it is necessary that it be elastic. A body vibrating in an absolute vacuum, or in a medium utterly destitute of elasticity, would fail to excite our sensations of hearing. This assertion is justified by the following experiments:—

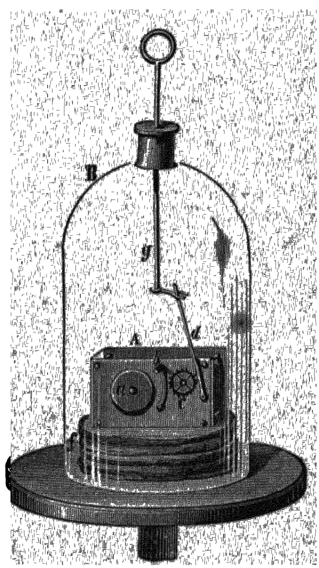


Fig 8.—Sound in Exhausted Receiver.

1. Under the receiver of an air-pump is placed a clock-work arrangement for producing a number of strokes on a bell. It is placed on a thick cushion of felt, or other inelastic material, and the air in the receiver is exhausted as completely as possible. If the clock-work is then started by means of the handle *g*, the hammer will be seen to strike the bell, but the sound will be scarcely audible. If hydrogen be introduced into the vacuum, and the receiver be again exhausted, the sound will be much more completely extinguished, being heard with difficulty even when the ear is placed in contact with the receiver. Hence it may fairly be concluded that if the receiver could be perfectly exhausted, and a perfectly inelastic support could be found for the bell, no sound at all would be emitted.

2. The experiment may be varied by using a glass globe, furnished with a stop-cock, and having a little bell suspended within it by a thread. If the globe is exhausted of air, the sound of the bell will be scarcely audible. The globe may be filled with any kind of gas, or with vapour either saturated or non-saturated, and it will thus be found that all these bodies transmit sound.

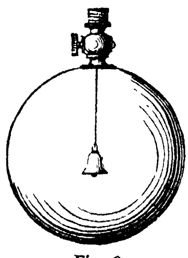


Fig. 9.
Globe with Stop-cock

Sound is also transmitted through liquids, as may easily be proved by direct experiment. Experiment, however, is scarcely necessary for the establishment of the fact, seeing that fishes are provided with auditory apparatus, and have often an acute sense of hearing.

As to solids, some well-known facts prove that they transmit sound very perfectly. For example, light taps with the head of a pin on one end of a wooden beam, are distinctly heard by a person with his ear applied to the other end, though they cannot be heard at the same distance through the air. This property is sometimes employed as a test of the soundness of a beam, for the experiment will not succeed if the intervening wood is rotten, rotten wood being very inelastic.

The *stethoscope* is an example of the transmission of sound through solids. It is a cylinder of wood, with an enlargement at each end, and a perforation in its axis. One end is pressed against the chest of the patient, while the observer applies his ear to the other. He is thus enabled to hear the sounds produced by various internal actions, such as the beating of the heart and the passage of the air through the tubes of the lungs. Even simple *auscultation,* in which the ear is applied directly to the surface of the body, implies the transmission of sound through the walls of the chest.

By applying the ear to the ground, remote sounds can often be much more distinctly heard; and it is stated that savages can in this way obtain much information respecting approaching bodies of enemies.

We are entitled then to assert that *sound, as it affects our organs of hearing, is an effect which is propagated, from a vibrating body, through an elastic and ponderable medium.*

4. **Mode of Propagation of Sound.**—We will now endeavour to explain the action by which sound is propagated.

Let there be a plate *a* vibrating opposite the end of a long tube, and let us consider what happens during the passage of the plate

from its most backward position a'', to its most advanced position a'. This movement of the plate may be divided in imagination into a number of successive parts, each of which is communicated to the layer of air close in front of it, which is thus compressed, and, in its

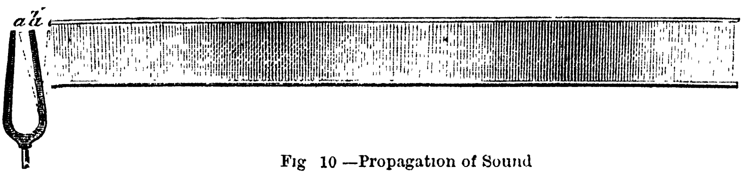


Fig 10 —Propagation of Sound

endeavour to recover from this compression, reacts upon the next layer, which is thus in its turn compressed. The compression is thus passed on from layer to layer through the whole tube, much in the same way as, when a number of ivory balls are laid in a row, if the first receives an impulse which drives it against the second, each ball will strike against its successor and be brought to rest.

The compression is thus passed on from layer to layer through the tube, and is succeeded by a rarefaction corresponding to the backward movement of the plate from a' to a''. As the plate goes on vibrating, these compressions and rarefactions continue to be propagated through the tube in alternate succession. The greatest compression in the layer immediately in front of the plate, occurs when the plate is at its middle position in its forward movement, and the greatest rarefaction occurs when it is in the same position in its backward movement. These are also the instants at which the plate is moving most rapidly.[1] When the plate is in its most advanced position, the layer of air next to it, A (Fig. 11) will be in its natural state, and another layer at A_1, half a wave-length further on, will also be in its natural state, the pulse having travelled from A to A_1, while the plate was moving from a'' to a'. At intervening points between A and A_1, the layers will have various amounts of compression corresponding to the different positions of the plate in its forward movement. The greatest compression is at C, a quarter of a wave-length in advance of A, having travelled over this distance while the plate was advancing from a to a'. The compressions at D and D_1 repre-

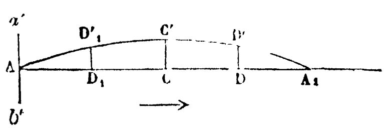


Fig. 11 —Graphical Representation

[1] See § 5, also Note A at the end of this chapter.

sent those which existed immediately in front of the plate when it had advanced respectively one-fourth and three-fourths of the distance from *a″* to *a′*, and the curve A C′ A_1 is the graphical representation both of condensation and velocity for all points in the air between A and A_1.

If the plate ceased vibrating, the condition of things now existing in the portion of air A A_1 would be transferred to successive portions of air in the tube, and the curve A C′ A_1 would, as it were, slide onward through the tube with the velocity of sound, which is about 1100 feet per second. But the plate, instead of remaining permanently at *a′*, executes a backward movement, and produces rarefactions and retrograde velocities, which are propagated onwards in the same manner as the condensations and forward velocities. A complete wave of the undulation is accordingly represented by the curve A E′ A_1 C′ A_2 (Fig. 12), the portions of the curve below the line of

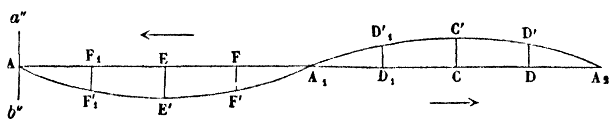


Fig. 12.—Graphical Representation of Complete Wave.

abscissas being intended to represent rarefactions and retrograde velocities. If we suppose the vibrating plate to be rigidly connected with a piston which works air-tight in the tube, the velocities of the particles of air in the different points of a wave-length will be identical with the velocities of the piston at the different parts of its motion.

The wave-length A A_2 is the distance that the pulse has travelled while the vibrating plate was moving from its most backward to its most advanced position, and back again. During this time, which is called the *period* of the vibrations, each particle of air goes through its complete cycle of changes, both as regards motion and density. The period of vibration of any particle is thus identical with that of the vibrating plate, and is the same as the time occupied by the waves in travelling a wave-length. Thus, if the plate be one leg of a common A tuning-fork, making 435 complete vibrations per second, the period will be $\frac{1}{435}$th of a second, and the undulation will travel in this time a distance of $\frac{1100}{435}$ feet, or 2 feet 6 inches, which is therefore the wave-length in air for this note. If the plate continues to vibrate in a uniform manner, there will be a continual series of equal

and similar waves running along the tube with the velocity of sound. Such a succession of waves constitutes an undulation. Each wave consists of a condensed portion, and a rarefied portion, which are distinguished from each other in Fig. 10 by different tints, the dark shading being intended to represent condensation.

5. **Nature of Undulations.**—The possibility of condensations and rarefactions being propagated continually in one direction, while each particle of air simply moves backwards and forwards about its original position, is illustrated by Fig. 13, which represents, in an

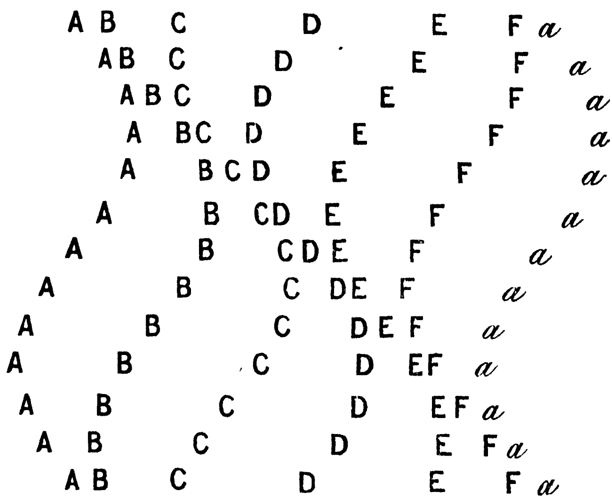


Fig 13—Longitudinal Vibration

exaggerated form, the successive phases of an undulation propagated through 7 particles A B C D E F a originally equidistant, the distance from the first to the last being one wave-length of the undulation. The diagram is composed of thirteen horizontal rows, the first and last being precisely alike. The successive rows represent the positions of the particles at successive times, the interval of time from each row to the next being $\frac{1}{12}$th of the period of the undulation.

In the first row A and a are centres of condensation, and D is a centre of rarefaction. In the third row B is a centre of condensation, and E a centre of rarefaction. In the fifth row the condensation and rarefaction have advanced by one more letter, and so on through the whole series, the initial state of things being

reproduced when each of these centres has advanced through a wave-length, so that the thirteenth row is merely a repetition of the first.

The velocities of the particles can be estimated by the comparison of successive rows. It is thus seen that the greatest forward velocity is at the centres of condensation, and the greatest backward velocity at the centres of rarefaction. Each particle has its greatest velocities, and greatest condensation and rarefaction, in passing through its mean position, and comes for an instant to rest in its positions of greatest displacement, which are also positions of mean density.

The distance between A and *a* remains invariable, being always a wave-length, and these two particles always agree in phase. Any other two particles represented in the diagram are always in different phases, and the phases of A and D, or B and E, or C and F, are always opposite; for example, when A is moving forwards with the maximum velocity, D is moving backwards with the same velocity.

The vibrations of the particles, in an undulation of this kind, are called *longitudinal;* and it is by such vibrations that sound is propagated through air. Fig. 14 illustrates the manner in which an undulation may be propagated by means of *transverse* vibrations, that is to say, by vibrations executed in a direction perpendicular to that in which the undulation advances. Thirteen particles A B C D E F G H I J K L *a* are represented in the positions which they occupy at successive times, whose interval is one-sixth of a period. At the instant first considered, D and J are the particles which are furthest displaced. At the end of the first interval, the wave has advanced two letters, so that F and L are now the furthest displaced. At the end of the next interval, the wave has advanced two letters further, and so on, the state of things at the end of the six intervals, or of one complete period, being the same as at the beginning, so that the seventh line is merely a repetition of the first. Some examples of this kind of wave-motion will be mentioned in later chapters.

6. Propagation in an Open Space.—When a sonorous disturbance occurs in the midst of an open body of air, the undulations to which it gives rise run out in all directions from the source. If the disturbance is symmetrical about a centre, the waves will be spherical; but this case is exceptional. A disturbance usually produces condensation on one side, at the same instant that it produces rarefaction on another. This is the case, for example, with a vibrating

plate, since, when it is moving towards one side, it is moving away from the other. These inequalities which exist in the neighbourhood of the sonorous body, have, however, a tendency to become less marked, and ultimately to disappear, as the distance is increased. Fig. 15 represents a diametral section of a series of spherical waves. Their mode of propagation has some analogy to that of the circular

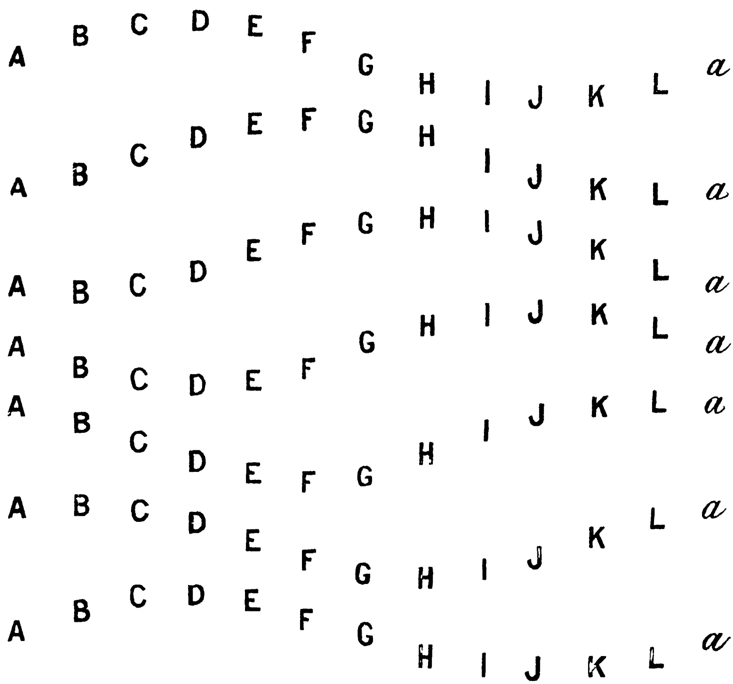


Fig 14—Transverse Vibration.

waves produced on water by dropping a stone into it; but the particles which form the waves of water rise and fall, whereas those which form sonorous waves merely advance and retreat, their lines of motion being always coincident with the directions along which the sound travels. In both cases it is important to remark that *the undulation does not involve a movement of transference.* Thus, when the surface of a liquid is traversed by waves, bodies floating on it rise and fall, but are not carried onward. This property is characteristic of undulations generally. *An undulation may be defined as a system of movements in which the several particles move to and fro, or round and round, about definite points, in such a*

manner as to produce the continued onward transmission of a condition, or series of conditions.

There is one important difference between the propagation of sound in a uniform tube and in an open space. In the former case, the layers of air corresponding to successive wave-lengths are of equal

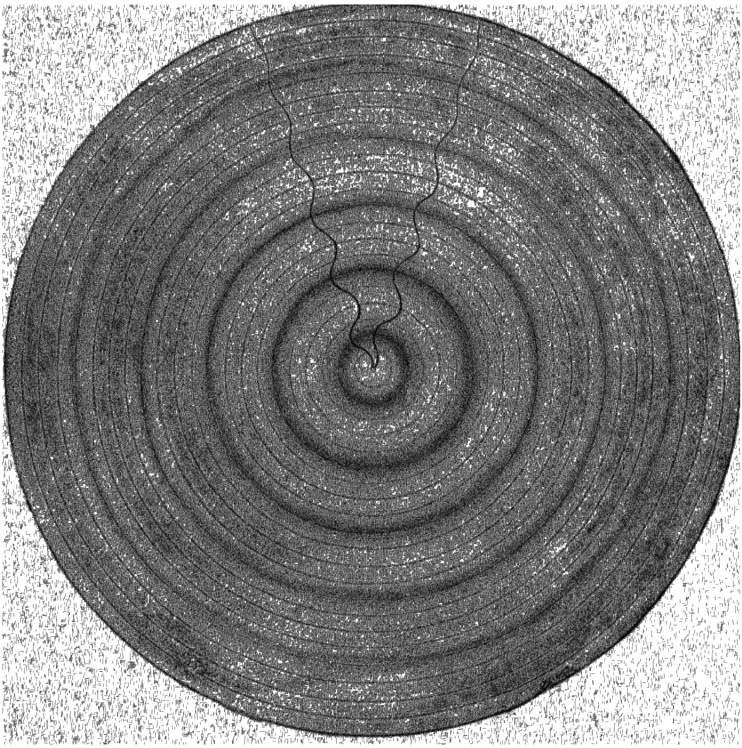


Fig 15 —Propagation in Open Space

mass, and their movements are precisely alike, except in so far as they are interfered with by friction. Hence sound is transmitted through tubes to great distances with but little loss of intensity, especially if the tubes are large.[1]

The same principle is illustrated by the ease with which a scratch

[1] Regnault, in his experiments on the velocity of sound, found that in a conduit ·108 of a metre in diameter, the report of a pistol charged with a gramme of powder ceased to be heard at the distance of 1150 metres. In a conduit of ·3^{m}, the distance was 3810^{m}. In the great conduit of the St. Michel sewer, of 1^{m}·10, the sound was made by successive reflections to traverse a distance of 10,000 metres without becoming inaudible.—*D.*

on a log of wood is heard at the far end, the substance of the log acting like the body of air within a tube.

In an open space, each successive layer has to impart its own condition to a larger layer; hence there is a continual diminution of amplitude in the vibrations as the distance from the source increases. This involves a continual decrease of loudness. An undulation involves the onward transference of energy; and the amount of energy which traverses, in unit time, any closed surface described about the source, must be equal to the energy which the source emits in unit time. Hence, by the reasoning which we employed in the case of radiant heat (§ 190, Part II.), it follows that the intensity of sonorous energy diminishes according to the law of inverse squares.

The energy of a particle executing simple vibrations in obedience to forces of elasticity, varies as the square of the amplitude of its excursions; for, if the amplitude be doubled, the distance worked through, and the mean working force, are both doubled, and thus the work which the elastic forces do during the movement from either extreme position to the centre is quadrupled. This work is equal to the energy of the particle in any part of its course. At the extreme positions it is all in the shape of potential energy; in the middle position it is all in the shape of kinetic energy; and at intermediate points it is partly in one of these forms, and partly in the other.

It can be shown that exactly half the energy of a complete wave is kinetic, the other half being potential.

7. **Dissipation of Sonorous Energy.**—The reasoning by which we have endeavoured to establish the law of inverse squares, assumes that onward propagation involves no loss of sonorous energy. This assumption is not rigorously true, inasmuch as vibration implies friction, and friction implies the generation of heat, at the expense of the energy which produces the vibrations. Sonorous energy must therefore diminish with distance somewhat more rapidly than according to the law of inverse squares. All sound, in becoming extinct, becomes converted into heat.

This conversion is greatly promoted by defect of homogeneity in the medium of propagation. In a fog, or a snow-storm, the liquid or solid particles present in the air produce innumerable reflections, in each of which a little sonorous energy is converted into heat.

8. Velocity of Sound in Air.—The propagation of sound through an elastic medium is not instantaneous, but occupies a very sensible time in traversing a moderate distance. For example, the flash of a gun at the distance of a few hundred yards is seen some time before the report is heard. The interval between the two impressions may be regarded as representing the time required for the propagation of the sound across the intervening distance, for the time occupied by the propagation of light across so small a distance is inappreciable.

It is by experiments of this kind that the velocity of sound in air has been most accurately determined. Among the best determinations may be mentioned that of Lacaille, and other members of a commission appointed by the French Academy in 1738; that of Arago, Bouvard, and other members of the Bureau de Longitudes in 1822; and that of Moll, Vanbeek, and Kuytenbrouwer in Holland, in the same year. All these determinations were obtained by firing cannon at two stations, several miles distant from each other, and noting, at each station, the interval between seeing the flash and hearing the sound of the guns fired at the other. If guns were fired only at one station, the determination would be vitiated by the effect of wind blowing either with or against the sound. The error from this cause is nearly eliminated by firing the guns alternately at the two stations, and still more completely by firing them simultaneously This last plan was adopted by the Dutch observers, the distance of the two stations in their case being about nine miles. Regnault has quite recently repeated the investigation, taking advantage of the important aid afforded by modern electrical methods for registering the times of observed phenomena. All the most careful determinations agree very closely among themselves, and show that the velocity of sound through air at 0° C. is about 332 metres, or 1090 feet per second.[1] The velocity increases with the temperature, being proportional to *the square root of the absolute temperature by air thermometer* (§ 50, Part II.). If t denote the ordinary Centigrade tempera-

[1] A recent determination by Mr. Stone at the Cape of Good Hope is worthy of note as being based on the comparison of observations made through the sense of hearing alone. It had previously been suggested that the two senses of sight and hearing, which are concerned in observing the flash and report of a cannon, might not be equally prompt in receiving impressions (Airy on *Sound*, p. 131). Mr. Stone accordingly placed two observers —one near a cannon, and the other at about three miles distance; each of whom on hearing the report, gave a signal through an electric telegraph. The result obtained was in precise agreement with that stated in the text.

ture, and α the coefficient of expansion ·00366, the velocity of sound through air at any temperature is given by the formula

$$332\sqrt{1+\alpha t} \text{ in metres per second, or}$$
$$1090\sqrt{1+\alpha t} \text{ in feet per second.}$$

The actual velocity of sound from place to place on the earth's surface is found by compounding this velocity with the velocity of the wind.

There is some reason, both from theory and experiment, for believing that very loud sounds travel rather faster than sounds of moderate intensity.

9. **Theoretical Computation of Velocity.**—By applying the principles of dynamics to the propagation of undulations,[1] it is computed that the velocity of sound through air must be given by the formula

$$v=\sqrt{\frac{E}{D}}. \qquad (1)$$

D denoting the density of the air, and E its coefficient of elasticity, as measured by the quotient of pressure applied by compression produced.

Let P denote the pressure of the air in units of force per unit of area; then, if the temperature be kept constant during compression, a small additional pressure p will, by Boyle's law, produce a compression equal to $\frac{p}{P}$, and the value of E, being the quotient of p by this quantity, will be simply P.

On the other hand, if no heat is allowed either to enter or escape, the temperature of the air will be raised by compression, and additional resistance will thus be encountered. In this case, as shown in Part II., the coefficient of elasticity will be P κ, κ denoting the ratio of the two specific heats, which for air and simple gases is about 1·41.

It thus appears that the velocity of sound in air cannot be less than $\sqrt{\frac{P}{D}}$ nor greater than $\sqrt{1{\cdot}41\,\frac{P}{D}}$. Its actual velocity, as determined by observation, is identical, or practically identical, with the latter of these limiting values. Hence we must infer that the compressions and extensions which the particles of air undergo in transmitting sound are of too brief duration to allow of any sensible transference of heat from particle to particle.

This conclusion is confirmed by another argument due to Professor

[1] See note B at the end of this chapter.

Stokes. If the inequalities of temperature due to compression and expansion were to any sensible degree smoothed down by conduction and radiation, this smoothing down would diminish the amount of energy available for wave-propagation, and would lead to a falling off in intensity incomparably more rapid than that due to the law of inverse squares.

10. Numerical Calculation.—The following is the actual process of calculation for perfectly dry air at 0° C., the centimetre, gramme, and second being taken as the units of length, mass, and time.

The density of dry air at 0°, under the pressure of 1033 grammes per square centimetre, at Paris, is ·001293 of a gramme per cubic centimetre. But the gravitating force of a gramme at Paris is 981 dynes. The density ·001293 therefore corresponds to a pressure of 1033 × 981 dynes per sq. cm.; and the expression for the velocity in centimetres per second is

$$v = \sqrt{1{\cdot}41\,\frac{\mathrm{P}}{\mathrm{D}}} = \sqrt{1{\cdot}41\,\frac{1033 \times 981}{{\cdot}001293}} = 33240 \text{ nearly};$$

that is, 332·4 metres per second, or 1093 feet per second.

11. Effects of Pressure, Temperature, and Moisture.—The velocity of sound is independent of the height of the barometer, since changes of this element (at constant temperature) affect P and D in the same direction, and to the same extent.

For a given density, if P_0 denote the pressure at 0°, and α the coefficient of expansion of air, the pressure at t° Centigrade is $\mathrm{P}_0\,(1 + \alpha\, t)$, the value of α being about $\frac{1}{273}$.

Hence, if the velocity at 0° be 1090 feet per second, the velocity at t° will be $1090\,\sqrt{1 + \frac{t}{273}}$. At the temperature 50° F. or 10° C., which is approximately the mean annual temperature of this country, the value of this expression is about 1110, and at 86° F. or 30° C. it is about 1148. The increase of velocity is thus about a foot per second for each degree Fahrenheit.

The humidity of air has some influence on the velocity of sound, inasmuch as aqueous vapour is lighter than air; but the effect is comparatively trifling, at least in temperate climates. At the temperature 50° F., air saturated with moisture is less dense than dry air by about 1 part in 220, and the consequent increase of velocity cannot be greater than about 1 part in 440, which will be between 2 and 3 feet per second. The increase should, in fact, be somewhat

less than this, inasmuch as the value of k (the ratio of the two specific heats) appears to be only 1·31 for aqueous vapour.[1]

12. Newton's Theory, and Laplace's Modification.—The earliest theoretical investigation of the velocity of sound was that given by Newton in the *Principia* (book 2, section 8). It proceeds on the tacit assumption that no changes of temperature are produced by the compressions and extensions which enter into the constitution of a sonorous undulation; and the result obtained by Newton is equivalent to the formula

$$v = \sqrt{\frac{P}{D}};$$

But H the height of a homogeneous atmosphere is P/gD (see Part I.), and $\sqrt{gH}$ is the velocity which a heavy body would acquire in falling through a height $\frac{1}{2}$ H; hence the velocity of sound in air is, according to Newton, the same as *the velocity which would be acquired by falling in vacuo through half the height of a homogeneous atmosphere.* This, in fact, is the form in which Newton states his result.[2]

Newton himself was quite aware that the value thus computed theoretically was too small, and he throws out a conjecture as to the cause of the discrepancy; but the true cause was first pointed out by Laplace, as depending upon increase of temperature produced by compression, and decrease of temperature produced by expansion.

13. Velocity in Gases generally.—The same principles which apply to air apply to gases generally; and since for all simple gases the ratio of the two specific heats is 1·41, the velocity of sound in any simple gas is $\sqrt{1{\cdot}41\,\frac{P}{D}}$, D denoting its absolute density at the pressure P. Comparing two gases at the same pressure, we see that the velocities of sound in them will be inversely as the square roots of their absolute densities; and this will be true whether the temperatures of the two gases are the same or different.

14. Velocity of Sound in Liquids.—The velocity of sound in water was measured by Colladon, in 1826, at the Lake of Geneva. Two boats were moored at a distance of 13,500 metres (between 8 and 9 miles). One of them carried a bell, weighing about 140 lbs., immersed in the lake. Its hammer was moved by an external lever, so arranged as to ignite a small quantity of gunpowder at the instant

[1] Rankine on the *Steam Engine*, p. 320.

[2] Newton's investigation relates only to *simple* waves; but if these have all the same velocity (as Newton shows), this must also be the velocity of the complex wave which they compose. Hence the restriction is only apparent.

of striking the bell. An observer in the other boat was enabled to hear the sound by applying his ear to the extremity of a trumpet-shaped tube (Fig. 16), having its lower end covered with a membrane and facing towards the direction from which the sound proceeded. By noting the interval between seeing the flash and hearing the sound, the velocity with which the sound travelled through the water was determined. The velocity thus computed was 1435 metres per second, and the temperature of the water was 8°·1 C.

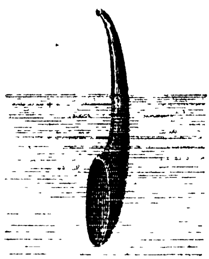


Fig. 16

Formula (1) of § 9 holds for liquids as well as for gases.

The resistance of water to compression is about $2{\cdot}1 \times 10^{10}$ dynes per sq. cm., and the correcting factor for the heat of compression, as calculated in Part II., is 1·0012, which may be taken as unity. The density is also unity. Hence we have

$$v = \sqrt{\frac{E}{D}} = \sqrt{(2{\cdot}1 \times 10^{10})} = 144914 \text{ cm. per sec.},$$

that is, about 1449 metres per second; which agrees sufficiently well with the experimental determination.

Wertheim has measured the velocity of sound in some liquids by an indirect method, which will be explained in a later chapter. He finds it to be 1160 metres per second in ether and alcohol, and 1900 in a solution of chloride of calcium.

15. Velocity of Sound in Solids.—The velocity of sound in cast-iron was determined by Biot and Martin by means of a connected series of water-pipes, forming a conduit of a total length of 951 metres. One end of the conduit was struck with a hammer, and an observer at the other end heard two sounds, the first transmitted by the metal, and the second by the air, the interval between them being 2·5 seconds. Now the time required for travelling this distance through air, at the temperature of the experiment (11° C.), is 2·8 seconds. The time of transmission through the metal was there-
ɔnd, which is at the rate of 3170 metres per second.
to be remarked, that the transmitting body was not
ass of iron, but a series of 376 pipes, connected to-
's of lead and tarred cloth, which must have consid-
the transmission of the sound. But in spite of this,
ıbout nine times as great as in air.

Wertheim, by the indirect methods above alluded to, measured the velocity of sound in a number of solids, with the following results, the velocity in air being taken as the unit of velocity:—

Lead,	3·974 to 4·120	Steel,	14·361 to 15·108
Tin,	7·338 to 7·480	Iron,	15·108
Gold,	5·603 to 6·424	Brass,	10·224
Silver,	7·903 to 8·057	Glass,	14·956 to 16·759
Zinc,	9·863 to 11·009	Flint Glass,	11·890 to 12·220
Copper,	11·167	Oak,	9·902 to 12·02
Platinum,	7·823 to 8·467	Fir,	12·49 to 17·26

16. Theoretical Computation.—The formula $\sqrt{\frac{E}{D}}$ serves for solids as well as for liquids and gases; but as solids can be subjected to many different kinds of strain, whereas liquids and gases can be subjected to only one, we may have different values of E, and different velocities of transmission of pulses for the same solid. This is true even in the case of a solid whose properties are alike in all directions (called an *isotropic* solid); but the great majority of solids are very far from fulfilling this condition, and transmit sound more rapidly in some directions than in others.

When the sound is propagated by alternate compressions and extensions running along a substance which is not prevented from extending and contracting laterally, the elasticity E becomes identical[1] with Young's modulus. On the other hand, if uniform spherical waves of alternate compression and extension spread outwards, symmetrically, from a point in the centre of an infinite solid, lateral extension and contraction will be prevented by the symmetry of the action. The effective elasticity is, in this case, greater than Young's modulus, and the velocity of sound will be increased accordingly.

The value of Young's modulus for copper is about 120×10^{10}, and the density of copper is about 8·8. Hence, for the velocity of sound through a copper rod, in centimetres per second, we have

$$v = \sqrt{\frac{E}{D}} = \sqrt{\frac{120 \times 10^{10}}{8 \cdot 8}} = 369300 \text{ nearly,}$$

or 3693 metres per second.

This is about 11·1 times the velocity in air.

[1] Subject to a very small correction for heat of compression, which we have discussed in Part II. under the head *adiabatic changes*. In the case of iron, the correcting factor is about 1·0023.

17. Reflection of Sound.—When sonorous waves meet a fixed obstacle they are reflected, and the two sets of waves—one direct, and the other reflected—are propagated just as if they came from two separate sources. If the reflecting surface is plane, waves di-

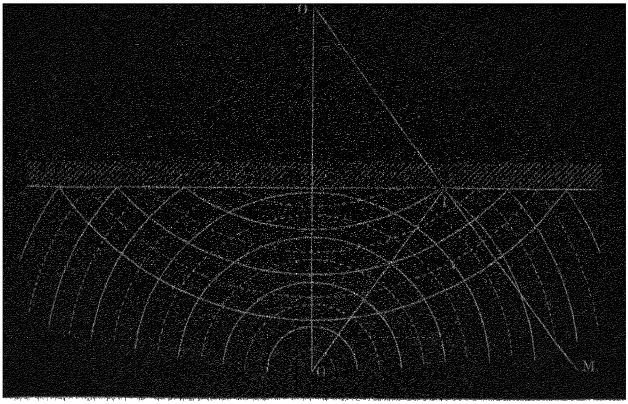


Fig. 17.—Reflection of Sound.

verging from any centre O (Fig. 17) in front of it are reflected so as to diverge from a centre O′ symmetrically situated behind it, and an ear at any point M in front hears the reflected sound as if it came from O′.

The direction from which a sound appears to the hearer to proceed is determined by the direction along which the sonorous pulses are propagated, and is always normal to the waves. A normal to a set of sound-waves may therefore conveniently be called a *ray* of sound.

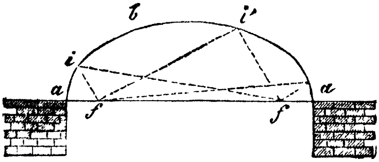


Fig 18.—Reflection from Elliptic Roof.

O I is a direct ray, and I M the corresponding reflected ray; and it is obvious, from the symmetrical position of the points O O′, that these two rays are equally inclined to the surface, or *the angles of incidence and reflection are equal.*

18. Illustrations of Reflection of Sound.—The reflection of sonorous waves explains some well-known phenomena. If *aba* (Fig. 18) be an elliptic dome or arch, a sound emitted from either of the foci *ff* will be reflected from the elliptic surface in such a direction as to pass through the other focus. A sound emitted from either focus

may thus be distinctly heard at the other, even when quite inaudible at nearer points. This is a consequence of the property, that lines drawn to any point on an ellipse from the two foci are equally inclined to the curve.

The experiment of the conjugate mirrors (Part II.) is also applicable to sound. Let a watch be hung in the focus of one of them (Fig. 19),

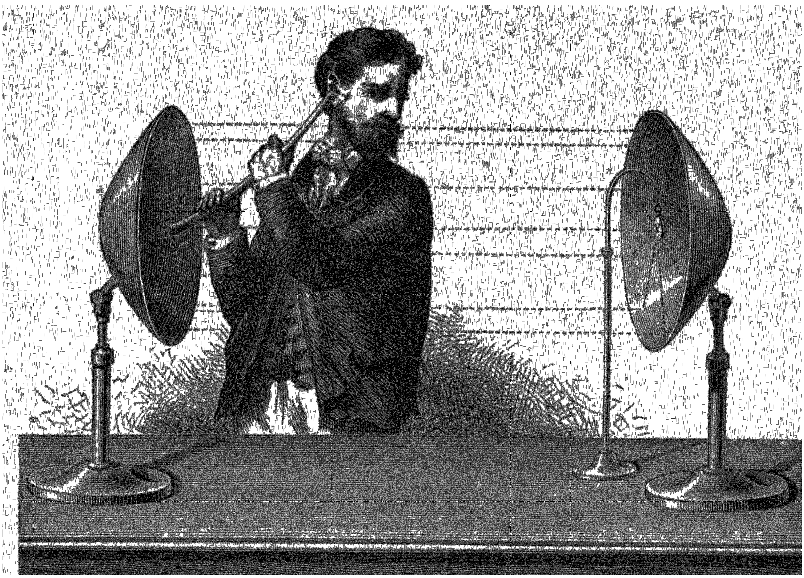


Fig 19.—Reflection of Sound from Conjugate Mirrors.

and let a person hold his ear at the focus of the other; or still better, to avoid intercepting the sound before it falls on the second mirror, let him employ an ear-trumpet, holding its further end at the focus. He will distinctly hear the ticking, even when the mirrors are many yards apart.[1]

19. **Echo.**—Echo is the most familiar instance of the reflection of sound. In order to hear the echo of one's own voice, there must be a distant body capable of reflecting sound directly back, and the number of syllables that an echo will repeat is proportional to the

[1] Sondhaus has shown that sound, like light, is capable of being *refracted*. A spherical balloon of collodion, filled with carbonic acid gas, acts as a sound-lens. If a watch be hung at some distance from it on one side, an ear held at the conjugate focus on the other side will hear the ticking. See also a later section on "Curved Rays of Sound" in the chapter on the "Wave Theory of Light."

distance of this obstacle. The sounds reflected to the speaker have travelled first over the distance O A (Fig. 20) from him to the reflecting body, and then back from A to O. Supposing five syllables to be pronounced in a second, and taking the velocity of sound as 1100 feet per second, a distance of 550 feet from the speaker to the reflecting body would enable the speaker to complete the fifth syllable before the return of the first; this is at the rate of 110 feet

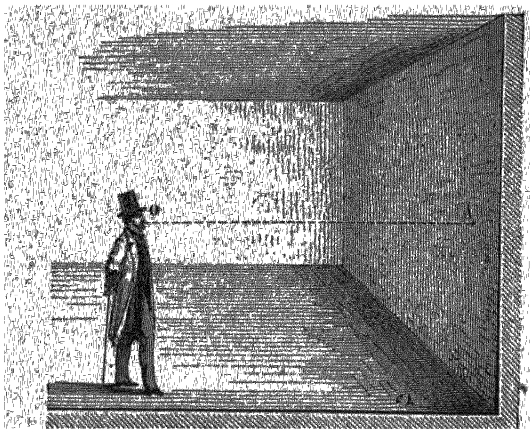


Fig 20 —Echo.

per syllable. At distances less than about 100 feet there is not time for the distinct reflection of a single syllable; but the reflected sound mingles with the voice of the speaker. This is particularly observable under vaulted roofs.

Multiple echoes are not uncommon. They are due, in some cases, to independent reflections from obstacles at different distances; in others, to reflections of reflections. A position exactly midway between two parallel walls, at a sufficient distance apart, is favourable for the observance of this latter phenomenon. One of the most frequently cited instances of multiple echoes is that of the old palace of Simonetta, near Milan, which forms three sides of a quadrangle. According to Kircher, it repeats forty times.

20. **Speaking and Hearing Trumpets.**—The complete explanation of the action of these instruments presents considerable difficulty. The speaking-trumpet (Fig. 21) consists of a long tube (sometimes 6 feet long), slightly tapering towards the speaker, furnished at this end with a hollow mouth-piece, which nearly fits the lips, and at

the other with a funnel-shaped enlargement, called the *bell*, opening out to a width of about a foot. It is much used at sea, and is found very effectual in making the voice heard at a distance. The explanation usually given of its action is, that the slightly conical form of the long tube produces a series of reflections in directions more and more nearly parallel to the axis; but this explanation fails to account for the utility of the *bell*, which experience has shown to be considerable. It appears from a theoretical investigation by Lord Rayleigh that the speaking-trumpet causes a greater total quantity of sonorous energy to be produced from the same expenditure of breath.[1]

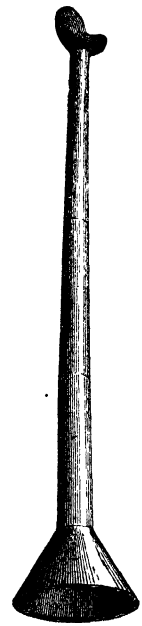


Fig 21. Speaking-trumpet.

Ear-trumpets have various forms, as represented in Fig. 22; having little in common, except that the external opening or *bell* is much larger than the end which is introduced into the ear. Membranes of gold-beaters' skin are sometimes stretched across their interior, in the positions indicated by the dotted lines in Nos. 4 and 5. No. 6 consists simply of a bell with such a membrane stretched across its outer end, while its inner end communicates with the ear by an indian-rubber tube with an ivory end-piece. These light membranes are peculiarly susceptible of impression from aerial vibrations. In Regnault's experiments above cited, it was found that membranes were affected at distances greater than those at which sound was heard.

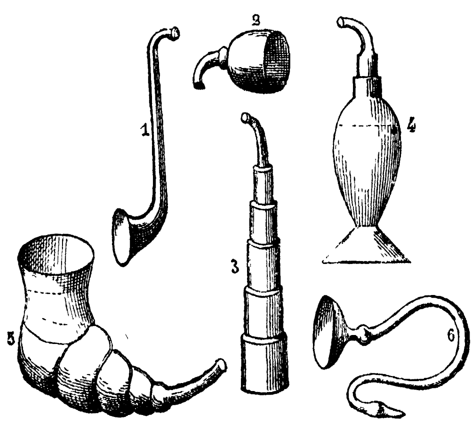


Fig 22 —Ear-trumpets

21. Interference of Sonorous Undulations.—When two systems of waves are traversing the same matter, the actual motion of each particle of the matter is the resultant of the motions due to each system separately. When these component motions are in the same direction the resultant is their

[1] *Theory of Sound*, vol. ii. p. 102.

sum; when they are in opposite directions it is their difference; and if they are equal, as well as opposite, it is zero. Very remarkable phenomena are thus produced when the two undulations have the same, or nearly the same wave-length; and the action which occurs in this case is called *interference.*

When two sonorous undulations of exactly equal wave-length and amplitude are traversing the same matter in the same direction, their phases must either be the same, or must everywhere differ by the same amount. If they are the same, the amplitude of vibration for each particle will be double of that due to either undulation separately. If they are opposite—in other words, if one undulation be half a wave-length in advance of the other—the motions which they would separately produce in any particle are equal and opposite, and the particle will accordingly remain at rest. Two sounds will thus, by their conjoint action, produce silence.

In order that the extinction of sound may be complete, the rarefied portions of each set of waves must be the *exact* counterparts of the condensed portions of the other set, a condition which can only be approximately attained in practice.

The following experiment, due to M. Desains, affords a very direct illustration of the principle of interference. The bottom of a wooden box is pierced with an opening, in which a powerful whistle fits. The top of the box has two larger openings symmetrically placed with respect to the lower one. The inside of the box is lined with felt, to prevent the vibrations from being communicated to the box, and to weaken internal reflection. When the whistle is sounded, if a membrane, with sand strewn on it, is held in various positions in the vertical plane which bisects, at right angles, the line joining the two openings, the sand will be agitated, and will arrange itself in nodal lines. But if it is carried out of this plane, positions will be found, at equal distances on both sides of it, at which the agitation is scarcely perceptible. If, when the membrane is in one of these positions, we close one of the two openings, the sand is again agitated, clearly showing that the previous absence of agitation was due to the interference of the undulations proceeding from the two orifices.

In this experiment the proof is presented to the eye. In the following experiment, which is due to M. Lissajous, it is presented to the ear. A circular plate, supported like the plate in Fig. 3, is made to vibrate in sectors separated by radial nodes. The number of sectors will always be even, and adjacent sectors will vibrate

in opposite directions. Let a disk of card-board of the same size be divided into the same number of sectors, and let alternate sectors be cut away, leaving only enough near the centre to hold the remaining sectors together. If the card be now held just over the vibrating disk, in such a manner that the sectors of the one are exactly over those of the other, a great increase of loudness will be observed, consequent on the suppression of the sound from alternate sectors; but if the card-board disk be turned through the width of half a sector, the effect no longer occurs. If the card is made to rotate rapidly in a continuous manner, the alterations of loudness will form a series of beats.

It is for a similar reason that, when a large bell is vibrating, a person in its centre hears the sound as only moderately loud, while within a short distance of some portions of the edge the loudness is intolerable.

22.. **Interference of Direct and Reflected Waves.**[1] **Nodes and Antinodes.**—Interference may also occur between undulations travelling in opposite directions; for example, between a direct and a reflected system. When waves proceeding along a tube meet a rigid obstacle, forming a cross section of the tube, they are reflected directly back again, the motion of any particle close to the obstacle being compounded of that due to the direct wave, and an equal and opposite motion due to the reflected wave. The reflected waves are in fact the images (with reference to the obstacle regarded as a plane mirror) of the waves which would exist in the prolongation of the tube if the obstacle were withdrawn. At the distance of half a wave-length from the obstacle the motions due to the direct and reflected waves will accordingly be equal and opposite, so that the particles situated at this distance will be permanently at rest; and the same is true at the distance of any number of half wave-lengths from the obstacle. The air in the tube will thus be divided into a number of vibrating segments separated by nodal planes or cross sections of no vibration arranged at distances of half a wave-length apart. One of these nodes is at the obstacle itself. At the centres of the vibrating segments—that is to say, at the distance of a quarter wave-length *plus* any number of half wave-lengths from the obstacle or from any node—the velocities due to the direct and reflected waves will be equal and in the same direction, and the amplitude of vibration will accordingly be double of that due to the direct wave alone. These

[1] See note C, page 31.

are the sections of greatest disturbance as regards change of place. We shall call them *antinodes*. On the other hand, it is to be remembered that motion *with* the direct wave is motion *against* the reflected waves, and *vice versa*, so that (§ 4) at points where the velocities due to both have the same absolute direction they correspond to condensation in the case of one of these undulations, and to rarefaction in the case of the other. Accordingly, these sections of maximum movement are the places of no change of density; and on the other hand, the nodes are the places where the changes of density are greatest. If the reflected undulation is feebler than the direct one, as will be the case, for example, if the obstacle is only imperfectly rigid, the destruction of motion at the nodes and of change of density at the antinodes will not be complete; the former will merely be places of minimum motion, and the latter of minimum change of density.

Direct experiments in verification of these principles, a wall being the reflecting body, were conducted by Savart, and also by Seebeck, the latter of whom employed a testing apparatus called the acoustic pendulum. It consists essentially of a small membrane stretched in a frame, from the top of which hangs a very light pendulum, with its bob resting against the centre of the membrane. In the middle portions of the vibrating segments the membrane, moving with the air on its two faces, throws back the pendulum, while it remains nearly free from vibration at the nodes.

Regnault made extensive use of the acoustic pendulum in his experiments on the velocity of sound. The pendulum, when thrown back by the membrane, completed an electric circuit, and thus effected a record of the instant when the sound arrived.

23. Beats Produced by Interference.—When two notes which are not quite in unison are sounded together, a peculiar palpitating effect is produced;—we hear a series of bursts of sound, with intervals of comparative silence between them. The bursts of sound are called *beats*, and the notes are said to *beat* together. If we have the power of tuning one of the notes, we shall find that as they are brought more nearly into unison, the beats become slower, and that, as the departure from unison is increased, the beats become more rapid, till they degenerate first into a rattle, and then into a discord. The effect is most striking with deep notes.

These beats are completely explained by the principle of interference. As the wave-lengths of the two notes are slightly different,

while the velocity of propagation is the same, the two systems of waves will, in some portions of their course, agree in phase, and thus strengthen each other; while in other parts they will be opposite in phase, and will thus destroy each other. Let one of the notes, for example, have 100 vibrations per second, and the other 101. Then, if we start from an instant when the maxima of condensation from the two sources reach the ear together, the next such conjunction will occur exactly a second later. During the interval the maxima of one system have been gradually falling behind those of the other, till, at the end of the second, the loss has amounted to one wave-length. At the middle of the second it will have amounted to half a wave-length, and the two sounds will destroy each other. We shall thus have one beat and one extinction in each second, as a consequence of the fact that the higher note has made one vibration more than the lower. In general, the frequency of beats is the difference of the frequencies of vibration of the beating notes.

NOTE A. § 4.

That the particles which are moving forward are in a state of compression, may be shown in the following way:—Consider an imaginary cross section travelling forward through the tube with the same velocity as the undulation. Call this velocity v, and the velocity of any particle of air u. Also let the density of any particle be denoted by ρ. Then u and ρ remain constant for the imaginary moving section, and the mass of air which it traverses in its motion per unit time is $(v-u)\,\rho$. As there is no permanent transfer of air in either direction through the tube, the mass thus traversed must be the same as if the air were at rest at its natural density. Hence the value of $(v-u)\,\rho$ is the same for all cross sections; whence it follows, that where u is greatest ρ must be greatest, and where u is negative ρ is less than the natural density.

If ρ_0 denote the natural density, we have $(v-u)\,\rho = v\,\rho_0$, whence $\frac{u}{v}=\frac{\rho-\rho_0}{\rho}$; that is to say, *the ratio of the velocity of a particle to the velocity of the undulation is equal to the condensation existing at the particle.* If u is negative—that is to say, if the velocity be retrograde—its ratio to v is a measure of the rarefaction.

From this principle we may easily derive a formula for the velocity of sound, bearing in mind that u is always very small in comparison with v.

For, consider a thin lamina of air whose thickness is δx, and let δu, $\delta\rho$, and δp be the excesses of the velocity, density, and pressure on the second side of the lamina above those on the first at the same moment. The above equation, $(v-u)\,\rho = v\,\rho_0$, gives $(v-u)\,\delta\rho - \rho\,\delta u = 0$, whence $\frac{\delta u}{\delta\rho}=\frac{v-u}{\rho}$, or, since u may be neglected in comparison with v,

$$\frac{\delta u}{\delta\rho}=\frac{v}{\rho}.$$

The time which the moving section occupies in traversing the lamina is $\frac{\delta x}{v}$, and in this time the velocity of the lamina changes by the amount $-\delta u$, since the velocity on the

second side of the lamina is $u+\delta u$ at the beginning and u at the end of the time. The force producing this change of velocity (if the section of the tube be unity) is $-\delta p$, and must be equal to the quotient of change of momentum by time, that is to $-\rho\,\delta x\,.\,\delta u \div \delta x/v$, or to $-\rho v\,\delta u$. Hence $\frac{\delta p}{\delta u}=\rho v$. But from above $\frac{\delta u}{\delta \rho}=\frac{v}{\rho}$. Hence by multiplication, $v^2=\frac{\delta p}{\delta \rho}$; that is, ultimately,

$$v^2=\frac{dp}{d\rho};$$

which is the simplest formula for the velocity of sound in a gas or liquid. It is equivalent to the formula $v^2=E/\rho$, (ρ being the same as D) of page 17, for, denoting the volume of unit mass by V, the compression is $-dV/V$ and the increase of pressure is the product of E by this compression. Thus we have

$$dp=-E\,\frac{dV}{V}\quad,\quad d\rho=d\,\frac{1}{V}=-\frac{dV}{V^2},$$

$$\text{whence } \frac{dp}{d\rho}=EV=\frac{E}{\rho}.$$

Note B. § 9.

The following is the usual investigation of the velocity of transmission of sound through a uniform tube filled with air, friction being neglected: Let x denote the original distance of a particle of air from a fixed section, and $x+y$ its distance at time t, so that y is the displacement of the particle from the position of equilibrium. Then a particle which was originally at distance $x+\delta x$ will at time t be at the distance $x+\delta x+y+\delta y$; and the thickness of the intervening lamina, which was originally δx, is now $\delta x+\delta y$. Its compression is therefore $-\delta y/\delta x$ or ultimately $-dy/dx$, and the increase of pressure is $-E\,dy/dx$, E denoting the coefficient of elasticity. The excess of pressure behind a lamina δx above the pressure in front is $\frac{d}{dx}\left(E\frac{dy}{dx}\right)\delta x$, or $E\,\frac{d^2y}{dx^2}\,\delta x$; and if ρ denote the original density of the air, the acceleration of the lamina will be the quotient of this expression by $\rho\,\delta x$. But this acceleration is $\frac{d^2y}{dt^2}$. Hence we have the equation

$$\frac{d^2y}{dt^2}=\frac{E}{\rho}\,\frac{d^2y}{dx^2};$$

the integral of which is

$$y=F(x-vt)+f(x+vt);$$

where v denotes $\sqrt{\frac{E}{\rho}}$, and F, f denote any functions whatever.

The term $F(x-vt)$ represents a wave, of the form $y=F(x)$, travelling forwards with velocity v; for it has the same value for $t_1+\delta t$ and $x_1+v\,.\,\delta t$ as for t_1 and x_1. The term $f(x+vt)$ represents a wave, of the form $y=f(x)$, travelling backwards with the same velocity. For waves travelling in one direction only we have either $y=F(x-vt)$ or $y=f(x+vt)$. The former gives $\frac{dy}{dt}=-v\,\frac{dy}{dx}$, the latter $\frac{dy}{dt}=v\,\frac{dy}{dx}$. These results confirm the first two paragraphs of Note A.

It is shown in Part II. that in adiabatic compression of a gas the coefficient of elasticity is κp, κ denoting the ratio of the specific heat at constant pressure to that at constant volume: Substituting this value for E, we have another useful formula

$$v=\sqrt{\kappa\,\frac{p}{\rho}}.$$

NOTE C. § 22.

The production of nodes and antinodes by two sets of equal waves travelling in opposite directions can be explained as follows:—

A series of simple harmonic waves travelling in the positive direction is expressed by the formula

$$y_1 = a \sin \frac{2\pi}{\lambda}(x - vt), \text{ or } y_1 = a \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right), \tag{1}$$

λ denoting the wave-length, T the period, and y_1 the displacement at time t of the particle whose original distance from the origin was x, this displacement being transverse or longitudinal according to the kind of waves considered.

A series of waves equal to the former but travelling in the negative direction is expressed by

$$y_2 = a \sin \frac{2\pi}{\lambda}(x + vt), \text{ or } y_2 = a \sin 2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right), \tag{2}$$

Let y denote the resultant displacement $y_1 + y_2$, we have

$$y = a \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + a \sin 2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right) = 2a \sin 2\pi\frac{x}{\lambda}\cos 2\pi\frac{t}{T}.$$

Hence y will be permanently zero at the points for which $\sin 2\pi x/\lambda$ vanishes, that is, the points at which x is either zero or a multiple of $\frac{1}{2}\lambda$. These are the nodes.

From the equation
$$y = 2a \sin 2\pi\frac{x}{\lambda} \cdot \cos 2\pi\frac{t}{T}, \tag{3}$$

we deduce
$$\frac{dy}{dx} = \frac{4\pi a}{\lambda}\cos 2\pi\frac{x}{\lambda} \cdot \cos 2\pi\frac{t}{T}, \tag{4}$$

which shows that $\frac{dy}{dx}$ will be permanently zero at the points for which $\cos 2\pi x/\lambda$ vanishes, that is, the points for which x is $\frac{1}{4}\lambda$, or $\frac{1}{4}\lambda$ *plus* a multiple of $\frac{1}{2}\lambda$. These are the antinodes

The motion represented by equation (3) is called *stationary undulation.* The above investigation shows that a stationary undulation can be resolved into two travelling undulations.

NOTE D. § 23.

The following is the mathematical investigation of beats for two systems of waves of equal amplitude but slightly different wave-length and period, travelling with the same velocity.

Denote $x - vt$ by θ, $\frac{2\pi}{\lambda_1}$ by m_1, and $\frac{2\pi}{\lambda_2}$ by m_2, λ_1 λ_2 being the wave-lengths of the two systems, and let their common amplitude be a. Then the resultant of the two sets is represented by

$$\begin{aligned} y &= a \sin m_1\theta + a \sin m_2\theta \\ &= 2a \sin \tfrac{1}{2}(m_1 + m_2)\,\theta . \cos \tfrac{1}{2}(m_1 - m_2)\,\theta. \end{aligned}$$

By hypothesis $m_1 - m_2$ is very small compared with $m_1 + m_2$; hence the factor $\cos \frac{1}{2}(m_1 - m_2)\,\theta$ remains nearly constant for an increment of θ which causes $\frac{1}{2}(m_1 + m_2)\,\theta$ to increase by 2π. The expression therefore represents a series of waves having a wave-length intermediate between λ_1 and λ_2 (since $\frac{1}{2}(m_1 + m_2)$ is intermediate between m_1 and m_2), and having an amplitude $2a \cos \frac{1}{2}(m_1 - m_2)\,\theta$ which gradually varies between the limits zero and $2a$.

CHAPTER II.

NUMERICAL EVALUATION OF SOUND.

24. Qualities of Musical Sound.—Musical tones differ one from another in respect of three qualities;—loudness, pitch, and character.

Loudness.—The loudness of a sound considered subjectively is the intensity of the sensation with which it affects the organs of hearing. Regarded objectively, it depends, in the case of sounds of the same pitch and character, upon the energy of the aerial vibrations in the neighbourhood of the ear, and is proportional to the square of the amplitude.

Our auditory apparatus is, however, so constructed as to be more susceptible of impression by sounds of high than of low pitch. A bass note must have much greater energy of vibration than a treble note, in order to strike the ear as equally loud. The intensity of sonorous vibration at a point in the air is therefore not an absolute measure of the intensity of the sensation which will be received by an ear placed at the point.

The word loud is also frequently applied to a source of sound, as when we say a loud voice, the reference being to the loudness as heard at a given distance from the source. The diminution of loudness with increase of distance according to the law of inverse squares is essentially connected with the proportionality of loudness to square of amplitude.

Pitch.—Pitch is the quality in respect of which an acute sound differs from a grave one; for example, a treble note from a bass note. All persons are capable of appreciating differences of pitch to some extent, and the power of forming accurate judgments of pitch constitutes what is called a *musical ear.*

Physically, pitch depends solely on *frequency of vibration,* that is to say, on the number of vibrations executed per unit time. In

ordinary circumstances this frequency is the same for the source of sound, the medium of transmission, and the drum of the ear of the person hearing; and in general the transmission of vibrations from one body or medium to another produces no change in their frequency. The *second* is universally employed as the unit of time in treating of sonorous vibrations; so that *frequency* means *number of vibrations per second.* Increase of frequency corresponds to elevation of pitch.

Period and *frequency* are reciprocals. For example, if the period of each vibration is $\frac{1}{100}$ of a second, the number of vibrations per second is 100. Period therefore is an absolute measure of pitch, and the longer the period the lower is the pitch.

The wave-length of a note in any medium is the distance which sound travels in that medium during the period corresponding to the note. Hence wave-length may be taken as a measure of pitch, provided the medium be given; but, in passing from one medium to another, wave-length varies directly as the velocity of sound. The wave-length of a given note in air depends upon the temperature of the air, and is shortened in transmission from the heated air of a concert-room to the colder air outside, while the pitch undergoes no change.

If we compare a series of notes rising one above another by what musicians regard as equal differences of pitch, their frequencies will not be equidifferent, but will form an increasing geometrical progression, and their periods (and wave-lengths in a given medium) will form a decreasing geometrical progression.

Character.—Musical sounds may, however, be alike as regards pitch and loudness, and may yet be easily distinguishable. We speak of the *quality* of a singer's voice, and the *tone* of a musical instrument; and we characterize the one or the other as rich, sweet, or mellow; on the one hand: or as poor, harsh, nasal, &c., on the other. These epithets are descriptive of what musicians call *timbre*—a French word literally signifying *stamp.* German writers on acoustics denote the same quality by a term signifying *sound-tint.* It might equally well be called *sound-flavour.* We adopt *character* as the best English designation.

Physically considered, as wave-length and wave-amplitude fall under the two previous heads, *character* must depend upon the only remaining point in which aerial waves can differ—namely their *form,* meaning by this term the law according to which the velo-

cities and densities change from point to point of a wave. This subject will be more fully treated in Chapter IV. Every musical sound is more or less mingled with non-musical noises, such as puffing, scraping, twanging, hissing, rattling, &c. These are not comprehended under *timbre* or *character* in the usage of the best writers on acoustics. The gradations of loudness which characterize the commencement, progress, and cessation of a note, and upon which musical effect often greatly depends, are likewise excluded from this designation. In distinguishing the sounds of different musical instruments, we are often guided as much by these gradations and extraneous accompaniments as by the character of the musical tones themselves.

25. **Musical Intervals.**—When two notes are heard, either simultaneously or in succession, the ear experiences an impression of a special kind, involving a perception of the relation existing between them as regards difference of pitch. This impression is often recognized as identical where absolute pitch is very different, and we express this identity of impression by saying that the *musical interval* is the same.

Each musical interval, thus recognized by the ear as constituting a particular relation between two notes, is found to correspond to a particular *ratio* between their frequencies of vibration. Thus the *octave*, which of all intervals is that which is most easily recognized by the ear, is the relation between two notes whose *frequencies* are as 1 to 2, the upper note making twice as many vibrations as the lower in any given time.

It is the musician's business so to combine sounds as to awaken emotions of the peculiar kind which are associated with works of art. In attaining this end he employs various resources, but musical intervals occupy the foremost place. It is upon the judicious employment of these that successful composition mainly depends.

26. **Gamut.**—The *gamut* or *diatonic scale* is a series of eight notes having certain definite relations to one another as regards frequency of vibration. The first and last of the eight are at an interval of an octave from each other, and are called by the same name; and by taking in like manner the octaves of the other notes of the series, we obtain a repetition of the gamut both upwards and downwards, which may be continued over as many octaves as we please.

The notes of the gamut are usually called by the names

Do	Re	Mi	Fa	Sol	La	Si	Do_2

and their vibration-frequencies are proportional to the numbers

1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

or, clearing fractions, to

24	27	30	32	36	40	45	48

The intervals from Do to each of the others in order are called a *second,* a *major third,* a *fourth,* a *fifth,* a *sixth,* a *seventh,* and an *octave* respectively. The interval from La to Do_2 is called a *minor third,* and is evidently represented by the ratio $\frac{6}{5}$.

The interval from Do to Re, from Fa to Sol, or from La to Si, is represented by the ratio $\frac{9}{8}$, and is called a *major tone.* The interval from Re to Mi, or from Sol to La, is represented by the ratio $\frac{10}{9}$, and is called a *minor tone.* The interval from Mi to Fa, or from Si to Do_2, is represented by the ratio $\frac{16}{15}$, and is called a *limma.* As the square of $\frac{16}{15}$ is a little greater than $\frac{9}{8}$, a limma is rather more than half a major tone.

The intervals between the successive notes of the gamut are accordingly represented by the following ratios[1]:—

Do		Re		Mi		Fa		Sol		La		Si		Do_2
	$\frac{9}{8}$		$\frac{10}{9}$		$\frac{16}{15}$		$\frac{9}{8}$		$\frac{10}{9}$		$\frac{9}{8}$		$\frac{16}{15}$	

Do (with all its octaves) is called the *key-note* of the piece of music, and may have any pitch whatever. In order to obtain perfect harmony, the above ratios should be accurately maintained whatever the key-note may be.

27. **Tempered Gamut.**—A great variety of keys are employed in music, and it is a practical impossibility, at all events in the case of instruments like the piano and organ, which have only a definite set of notes, to maintain these ratios strictly for the whole range of possible key-notes. Compromise of some kind becomes necessary, and different systems of compromise are called different *temperaments* or different *modes of temperament.* The temperament which is most in favour in the present day is the simplest possible, and is called *equal temperament,* because it favours no key above another, but makes the tempered gamut exactly the same for all. It ignores the

[1] The logarithmic differences, which are accurately proportional to the intervals, are approximately as under, omitting superfluous zeros.

Do		Re		Mi		Fa		Sol		La		Si		Do
	51		46		28		51		46		51		28	

difference between major and minor tones, and makes the limma exactly half of either. The interval from Do to Do_2 is thus divided into 5 tones and 2 semitones, a tone being $\frac{1}{6}$ of an octave, and a semitone $\frac{1}{12}$ of an octave. The ratio of frequencies corresponding to a tone will therefore be the sixth root of 2, and for a semitone it will be the 12th root of 2.

The difference between the natural and the tempered gamut for the key of C is shown by the following table, which gives the number of complete vibrations per second for each note of the middle octave of an ordinary piano:—

	Tempered Gamut.	Natural Gamut.		Tempered Gamut.	Natural Gamut.
C . .	258·7	258·7	G . .	387·6	388·0
D . .	290·3	291·0	A . .	435·0	431·1
E . .	325·9	323·4	B . .	488·2	485·0
F . .	345·3	344·9	C . .	517·3	517·3

The absolute pitch here adopted is that of the Paris Conservatoire, and is fixed by the rule that A (the middle A of a piano, or the A string of a violin) is to have 435 complete vibrations per second in the tempered gamut. This is rather lower than the concert-pitch which has prevailed in this country in recent years, but is probably not so low as that which prevailed in the time of Handel. It will be noted that the number of vibrations corresponding to C is approximately equal to a power of 2 (256 or 512). Any power of 2 accordingly expresses (to the same degree of approximation) the number of vibrations corresponding to one of the octaves of C.

The Stuttgard congress (1834) recommended 528 vibrations per second for C, and the C tuning-forks sold under the sanction of the Society of Arts are guaranteed to have this pitch. By multiplying the numbers 24, 27 . . . 48, in § 26, by 11, we shall obtain the frequencies of vibration for the natural gamut in C corresponding to this standard. What is generally called *concert-pitch* gives C about 538. The C of the Italian Opera is 546. Handel's C is said to have been $499\frac{1}{5}$.

28. Limits of Pitch employed in Music.—The deepest note regularly employed in music is the C of 32 vibrations per second which is emitted by the longest pipe (the 16-foot pipe) of most organs. Its wave-length in air at a temperature at which the velocity of sound is 1120 feet per second, is $\frac{1120}{32}=35$ feet. The highest note employed seldom exceeds A, the third octave of the A above defined. Its number of vibrations per second is $435 \times 2^3 = 3480$, and

its wave-length in air is about 4 inches. Above this limit it is difficult to appreciate pitch, but notes of at least ten times this number of vibrations are audible.

The average compass of the human voice is about two octaves. The deep F of a bass-singer has 87, and the upper G of the treble 775 vibrations per second. Voices which exceed either of these limits are regarded as deep or high.

29. **Minor Scale and Pythagorean Scale.**—The difference between a major and minor tone is expressed by the ratio $\frac{81}{80}$, and is called a *comma*. The difference between a minor tone and a limma is expressed by the ratio $\frac{25}{24}$, and is the smallest value that can be assigned to the somewhat indefinite interval denoted by the name *semitone*, the greatest value being the limma itself ($\frac{16}{15}$). The signs ♯ and ♭ (sharp and flat) appended to a note indicate that it is to be raised or lowered by a semitone. The major scale or gamut, as above given, is modified in the following way to obtain the minor scale:—

Do		Re		Mi♭		Fa		Sol		La♭		Si♭		Do_2
	$\frac{9}{8}$		$\frac{16}{15}$		$\frac{10}{9}$		$\frac{9}{8}$		$\frac{16}{15}$		$\frac{9}{8}$		$\frac{10}{9}$	

the numbers in the second line being the ratios which represent the intervals between the successive notes.

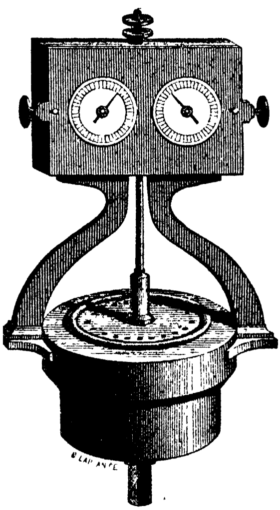
It is worthy of note that Pythagoras, who was the first to attempt the numerical evaluation of musical intervals, laid down a scheme of values slightly different from that which is now generally adopted. According to him, the intervals between the successive notes of the major scale are as follows:—

Do		Re		Mi		Fa		Sol		La		Si		Do
	$\frac{9}{8}$		$\frac{9}{8}$		$\frac{256}{243}$		$\frac{9}{8}$		$\frac{9}{8}$		$\frac{9}{8}$		$\frac{256}{243}$	

This scheme agrees exactly with the common system as regards the values of the fourth, fifth, and octave, and makes the values of the major third, the sixth, and the seventh each greater by a comma, while the small interval from *mi* to *fa*, or from *si* to *do*, is diminished by a comma. In the ordinary system, the prime numbers which enter the ratios are 2, 3, and 5; in the Pythagorean system they are only 2 and 3; hence the interval between any two notes of the Pythagorean scale can be expressed as the sum or difference of a certain number of octaves and fifths. In tuning a violin by making the intervals between the strings true fifths, the Pythagorean scheme is virtually employed.

30. Methods of Counting Vibrations. Siren.—The instrument which is chiefly employed for counting the number of vibrations corresponding to a given note, is called the *siren*, and was devised by Cagniard de Latour. It is represented in Figs. 23, 24, the former being a front, and the latter a back view.

There is a small wind-chest, nearly cylindrical, having its top pierced with fifteen holes, disposed at equal distances round the circumference of a circle. Just over this, and nearly touching it, is a movable circular plate, pierced with the same number of holes



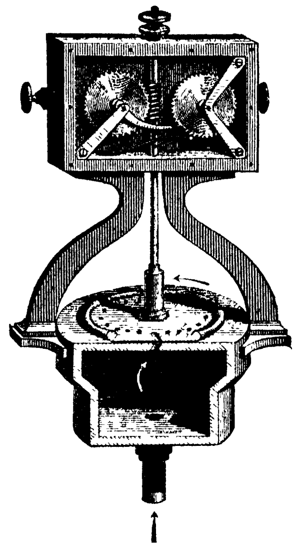


Fig. 23. Siren. Fig. 24.

similarly arranged, and so mounted that it can rotate very freely about its centre, carrying with it the vertical axis to which it is attached. This rotation is effected by the action of the wind, which enters the wind-chest from below, and escapes through the holes. The form of the holes is shown by the section in Fig. 24. They do not pass perpendicularly through the plates, but slope contrary ways, so that the air when forced through the holes in the lower plate impinges *upon one side* of the holes in the upper plate, and thus blows it round in a definite direction. The instrument is driven by means of the bellows shown in Fig. 34 (§ 45). As the rotation of one plate upon the other causes the holes to be alternately opened and closed, the wind escapes in successive puffs, whose frequency

depends upon the rate of rotation. Hence a note is emitted which rises in pitch as the rotation becomes more rapid.

The siren will sound under water, if water is forced through it instead of air; and it was from this circumstance that it derived its name.

In each revolution, the fifteen holes in the upper plate come opposite to those in the lower plate 15 times, and allow the compressed air in the wind-chest to escape; while in the intervening positions its escape is almost entirely prevented. Each revolution thus gives rise to 15 vibrations; and in order to know the number of vibrations corresponding to the note emitted, it is only necessary to have a means of counting the revolutions.

This is furnished by a counter, which is represented in Fig. 24. The revolving axis carries an endless screw, driving a wheel of 100 teeth, whose axis carries a hand traversing a dial marked with 100 divisions. Each revolution of the perforated plate causes this hand to advance one division. A second toothed-wheel is driven intermittently by the first, advancing suddenly one tooth whenever the hand belonging to the first wheel passes the zero of its scale. This second wheel also carries a hand traversing a second dial; and at each of the sudden movements just described this hand advances one division. Each division accordingly indicates 100 revolutions of the perforated plate, or 1500 vibrations. By pushing in one of the two buttons which are shown, one on each side of the box containing the toothed-wheels, we can instantaneously connect or disconnect the endless screw and the first toothed-wheel.

In order to determine the number of vibrations corresponding to any given sound which we have the power of maintaining steadily, we fix the siren on the bellows, the screw and wheel being disconnected, and drive the siren until the note which it emits is judged to be in unison with the given note. We then, either by regulating the pressure of the wind, or by employing the finger to press with more or less friction against the revolving axis, contrive to keep the note of the siren constant for a measured interval of time, which we observe by a watch. At the commencement of the interval we suddenly connect the screw and toothed-wheel, and at its termination we suddenly disconnect them, having taken care to keep the siren in unison with the given sound during the interval. As the hands do not advance on the dials when the screw is out of connection with the wheels, the readings before and after the measured interval of

time can be taken at leisure. Each reading consists of four figures, indicating the number of revolutions from the zero position, units and tens being read off on the first dial, and hundreds and thousands on the second. The difference of the two readings is the number of revolutions made in the measured interval, and when multiplied by 15 gives the number of vibrations in the interval, whence the number of vibrations per second is computed by division.

31. **Graphic Method.**—In the hands of a skilful operator, with a good musical ear, the siren is capable of yielding very accurate determinations, especially if, by adding or subtracting the number of beats,

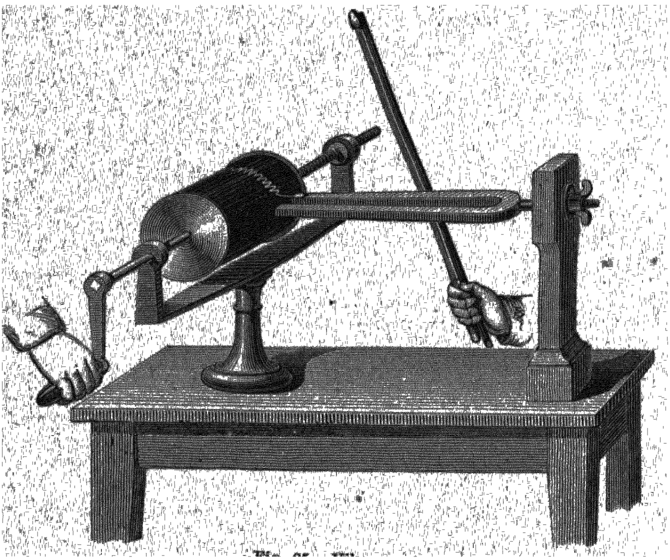


Fig. 25.—Vibroscope

correction be made for any slight difference of pitch between the siren and the note under investigation.

The vibrations of a tuning-fork can be counted, without the aid of the siren, by a graphical method, which does not call for any exercise of musical judgment, but simply involves the performance of a mechanical operation.

The tuning-fork is fixed in a horizontal position, as shown in Fig. 25, and has a light style, which may be of brass wire, quill, or bristle, attached to one of its prongs, by wax or otherwise. To receive the trace, a piece of smoked paper is gummed round a cylinder, which can be turned by a handle, a screw cut on the axis

causing it at the same time to travel endwise. The cylinder is placed so that the style barely touches the blackened surface. The fork is then made to vibrate by bowing it, and the cylinder is turned. The result is a wavy line traced on the blackened surface, and the number of wave-forms (each including a pair of bends in opposite directions) is the number of vibrations. If the experiment lasts for a measured interval of time, we have only to count these wave-forms, and divide by the number of seconds, in order to obtain the number of vibrations per second for the note of the tuning-fork. By plunging the paper in ether, the trace will be fixed, so that the paper may be laid aside, and the vibrations counted at leisure. The apparatus is called the *vibroscope*, and was invented by Duhamel.

M. Léon Scott invented an instrument called the *phonautograph*, which is adapted to the graphical representation of sounds in

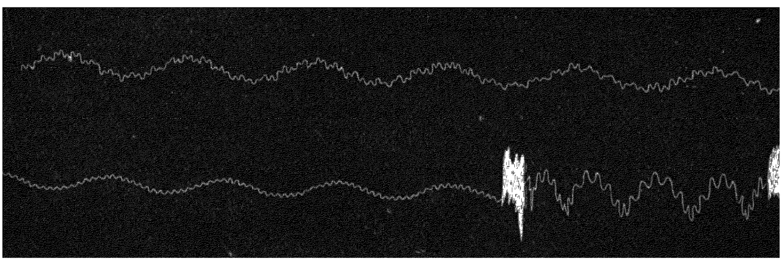


Fig 26.—Traces by Phonautograph

general. The style, which is very light, is attached to a membrane stretched across the smaller end of what may be called a large ear-trumpet. The membrane is agitated by the aerial waves proceeding from any source of sound, and the style leaves a record of these agitations on a blackened cylinder, as in Duhamel's apparatus. Fig. 26 represents the traces thus obtained from the sound of a tuning-fork in three different modes of vibration.

32. Tonometer.—When we have determined the frequency of vibration for a particular tuning-fork, that of another fork, nearly in unison with it, can be deduced by making the two forks vibrate simultaneously, and counting the beats which they produce.

Scheibler's *tonometer*, which is constructed by Koenig of Paris, consists of a set of 65 tuning-forks, such that any two consecutive forks make 4 beats per second, and consequently differ in pitch by

4 vibrations per second. The lowest of the series makes 256 vibrations, and the highest 512, thus completing an octave. Any note within this range can have its vibration-frequency at once determined, with great accuracy, by making it sound simultaneously with the fork next above or below it, and counting beats.

With the aid of this instrument, a piano can be tuned with certainty to any desired system of temperament, by first tuning the notes which come within the compass of the tonometer, and then proceeding by octaves.

In the ordinary methods of tuning pianos and organs, temperament is to a great extent a matter of chance; and a tuner cannot attain the same temperament in two successive attempts.

33. **Pitch modified by Relative Motion.**—We have stated in § 24 that, in ordinary circumstances, the frequency of vibration in the source of sound, is the same as in the ear of the listener, and in the intervening medium. This identity, however, does not hold if the source of sound and the ear of the listener are approaching or receding from each other. Approach of either to the other produces increased frequency of the pulses on the ear, and consequent elevation of pitch in the sound as heard; while recession has an opposite effect. Let n be the number of vibrations performed in a second by the source of the sound, v the velocity of sound in the medium, and a the relative velocity of approach. Then the number of waves which reach the ear of the listener in a second, will be n *plus* the number of waves which cover a length a, that is (since n waves cover a length v), will be $n+\frac{a}{v}n$ or $\frac{v+a}{v}n$.

The following investigation is more rigorous. Let n denote the number of vibrations made by the source per second, v the velocity of sound in the medium, a, a', m the velocities of the observer, the source, and the medium respectively, resolved in the direction from the source to the observer, so that those velocities are reckoned positive which have the same direction as that in which the sound in question travels to the observer.

Then $v+m$ is the absolute velocity of the sound which comes from the source to the observer, and $v+m-a'$ is its velocity relative to the source; its wave-length is therefore $(v+m-a')/n$. Its velocity relative to the observer is $v+m-a$, and the number of waves which reach him per second, being the quotient of this by the wave-length, is $\frac{v+m-a}{v+m-a'}n$.

Careful observation of the sound of a railway whistle, as an express train dashes past a station, has confirmed the fact that the sound as heard by a person standing at the station is higher while the train is approaching than when it is receding. A speed of about 40 miles an hour will sharpen the note by a semitone in approaching, and flatten it by the same amount in receding, the natural pitch being heard at the instant of passing.[1]

[1] The best observations of this kind were those of Buys Ballot, in which trumpeters, with their instruments previously tuned to unison, were stationed, one on the locomotive, and others at three stations beside the line of railway. Each trumpeter was accompanied by musicians, charged with the duty of estimating the difference of pitch between the note of his trumpet and those of the others, as heard before and after passing.

CHAPTER III.

MODES OF VIBRATION.

34. **Longitudinal and Transverse Vibrations of Solids.**—Sonorous vibrations are manifestations of elasticity. When the particles of a solid body are displaced from their natural positions relative to one another by the application of external force, they tend to return, in virtue of the elasticity of the body. When the external force is removed, they spring back to their natural position, pass it in virtue of the velocity acquired in the return, and execute isochronous vibrations about it until they gradually come to rest. The isochronism of the vibrations is proved by the constancy of pitch of the sound emitted; and from the isochronism we can infer, by the aid of mathematical reasoning, that the restoring force increases directly as the displacement of the parts of the body from their natural relative position (Part I.).

The same body is, in general, susceptible of many different modes of vibration, which may be excited by applying forces to it in different ways. The most important of these are comprehended under the two heads of *longitudinal* and *transverse* vibrations.

In the former the particles of the body move to and fro in the direction along which the pulses travel, which is always regarded as the longitudinal direction, and the deformations produced consist in alternate compressions and extensions. In the latter the particles move to and fro in directions transverse to that in which the pulses travel, and the deformation consists in bending. To produce longitudinal vibrations, we must apply force in the longitudinal direction. To produce transverse vibration, we must apply force transversely.

35. **Transverse Vibrations of Strings.**—To the transverse vibrations of strings, instrumental music is indebted for some of its most

precious resources. In the violin, violoncello, &c., the strings are set in vibration by drawing a bow across them. The part of the bow which acts on the strings consists of hairs tightly stretched and rubbed with rosin. The bow adheres to the string, and draws it aside till the reaction becomes too great for the adhesion to overcome. As the bow continues to be drawn on, slipping takes place, and the mere fact of slipping diminishes the adhesion. The string accordingly springs back suddenly through a finite distance. It is then again caught by the bow, and the same action is repeated. In the harp and guitar, the strings are plucked with the finger, and then left to vibrate freely. In the piano the wires are struck with little hammers faced with leather. The pitch of the sound emitted in these various cases depends only on the string itself, and is the same whichever mode of excitation be employed.

36. **Laws of the Transverse Vibrations of Strings.**—It can be shown by an investigation closely analogous to that which gives the velocity of sound in air, that the velocity with which transverse vibrations travel along a perfectly flexible string is given by the formula

$$v = \sqrt{\frac{t}{m}}; \qquad (1)$$

t denoting the tension of the string, and m the mass of unit length of it. If m be expressed in grammes per centimetre of length, t should be in dynes (see Part I.), and the value obtained for v will be in centimetres per second. The sudden disturbance of any point in the string, causes two pulses to start from this point, and run along the string in opposite directions. Each of these, on arriving at the end of the free portion of the string, is reflected from the solid support to which the string is attached, and at the same time undergoes reversal as to side. It runs back, thus reversed, to the other end of the free portion, and there again undergoes reflection and reversal. When it next arrives at the origin of the disturbance it has travelled over just twice the length of the string; and as this is true of both the pulses, they must both arrive at this point together. At the instant of their meeting, things are in the same condition as when the pulses were originated, and the movements just described will again take place. The period of a complete vibration of the string is therefore the time required for a pulse to travel over twice its length; that is,

$$\frac{1}{n}=\frac{2l}{v}=2l\sqrt{\frac{m}{t}};$$

$$\text{or } n=\frac{1}{2l}\sqrt{\frac{t}{m}}; \tag{2}$$

l denoting the length of the string between its points of attachment, and n the number of vibrations per second.

This formula involves the following laws:—

1. When the length of the vibrating portion of the string is altered, without change of tension, the frequency of vibration varies inversely as the length.

2. If the tension be altered, without change of length in the vibrating portion, the frequency of vibration varies as the square root of the tension.

3. Strings of the same length, stretched with the same forces, have frequencies of vibration which are inversely as the square roots of their masses (or weights).

4. Strings of the same length and density, but of different thicknesses, will vibrate in the same time, if they are stretched with forces proportional to their sectional areas.

All these laws are illustrated (qualitatively, if not quantitatively) by the strings of a violin.

The first is illustrated by the fingering, the pitch being raised as the portion of string between the finger and the bridge is shortened.

The second is illustrated by the mode of tuning, which consists in tightening the string if its pitch is to be raised, or slackening the string if it is to be lowered.

The third law is illustrated by the construction of the bass string, which is wrapped round with metal wire, for the purpose of adding to its mass, and thus attaining slow vibration without undue slackness. The tension of this string is in fact greater than that of the string next it, though the latter vibrates more rapidly in the ratio of 3 to 2.

The fourth law is indirectly illustrated by the sizes of the first three strings. The treble string is the smallest, and is nevertheless stretched with much greater force than any of the others. The third string is the thickest, and is stretched with less force than any of the others. The increased thickness is necessary in order to give sufficient power in spite of the slackness of the string.

37. **Experimental Illustration: Sonometer.**—For the quantitative illustration of these laws, the instrument called the sonometer, represented in Fig. 27, is commonly employed. It consists essen-

tially of a string or wire stretched over a sounding-box by means of a weight. One end of the string is secured to a fixed point at one end of the sounding-box. The other end passes over a pulley, and carries weights which can be altered at pleasure. Near the two ends of the box are two fixed bridges, over which the cord passes. There is also a movable bridge, which can be employed for altering the length of the vibrating portion.

To verify the law of lengths, the whole length between the fixed bridges is made to vibrate, either by plucking or bowing; the mov-

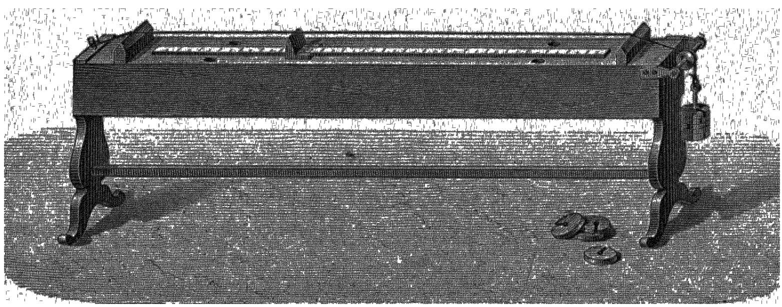


Fig 27 —Sonometer.

able bridge is then introduced exactly in the middle, and one of the halves is made to vibrate; the note thus obtained will be found to be the upper octave of the first. The frequency of vibration is therefore doubled. By making two-thirds of the whole length vibrate, a note will be obtained which will be recognized as the fifth of the fundamental note, its vibration-frequency being therefore greater in the ratio $\frac{3}{2}$. To obtain the notes of the gamut, we commence with the string as a whole, and then employ portions of its length represented by the fractions $\frac{8}{9}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{8}{15}$, $\frac{1}{2}$.

To verify the law independently of all knowledge of musical intervals, a light style may be attached to the cord, and caused to trace its vibrations on the vibroscope. This mode of proof is also more general, inasmuch as it can be applied to ratios which do not correspond to any recognized musical interval.

To verify the law of tensions, we must change the weight. It will be found that, to produce a rise of an octave in pitch, the weight must be increased fourfold.

To verify the third and fourth laws, two strings must be employed, their masses having first been determined by weighing them.

If the strings are thick, and especially if they are thick steel wires, their flexural rigidity has a sensible effect in making the vibrations quicker than they would be if the tension acted alone.

38. **Harmonics.**—Any person of ordinary musical ear may easily, by a little exercise of attention, detect in any note of a piano the presence of its upper octave, and of another note a fifth higher than this; these being the notes which correspond to frequencies of vibration double and triple that of the fundamental note. A highly trained ear can detect the presence of other notes, corresponding to still higher multiples of the fundamental frequency of vibration. Such notes are called *harmonics.*

When the vibration-frequency of one note is an exact multiple of that of another note, the former note is called a harmonic of the latter The notes of all stringed instruments contain numerous harmonics blended with the fundamental tones. Bells and vibrating plates have higher tones mingled with the fundamental tone; but these higher tones are not harmonics in the sense in which we use the word.

A violin string sometimes fails to yield its fundamental note, and gives the octave or some other harmonic instead. This result can be brought about at pleasure, by lightly touching the string at a properly-selected point in its length, while the bow is applied in the usual way. If touched at the middle point of its length, it gives the octave. If touched at one-third of its length from either end, it gives the fifth above the octave. The law is, that if touched at $\frac{1}{n}$ of its length[1] from either end, it yields the harmonic whose vibration-frequency is n times that of the fundamental tone. The string in these cases divides itself into a number of equal vibrating-segments, as shown in Fig. 28.

The division into segments is often distinctly *visible* when the string of a sonometer is strongly bowed, and its existence can be verified, when less evident, by putting paper riders on different parts of the string. These (as shown in the figure) will be thrown off by the vibrations of the string, unless they are placed accurately at the nodal points, in which case they will retain their seats. If two strings tuned to unison are stretched on the same sonometer, the vibration of the one induces similar vibrations in the other; and the experiment of the riders may be varied, in a very instructive way,

[1] Or at $\frac{m}{n}$ of its length, if m be prime to n.

by bowing one string and placing the riders on the other. This is an instance of a general principle of great importance—that a vibrating body communicates its vibrations to other bodies which are capable of vibrating in unison with it. The propagation of a sound may indeed be regarded as one grand vibration in unison; but, besides the general waves of *propagation*, there are waves of *re-*

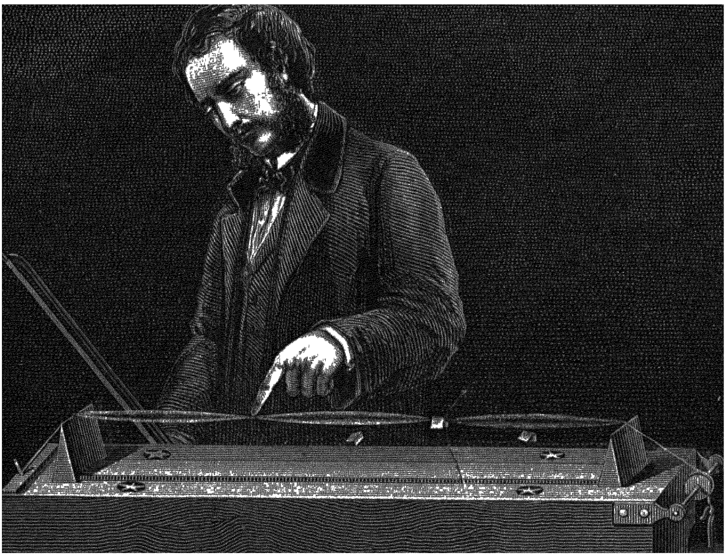


Fig. 28.—Production of a Harmonic

inforcement, due to the synchronous vibrations of limited portions of the transmitting medium. This is the principle of resonance.

39. **Resonance.**—By applying to a pendulum originally at rest a series of very feeble impulses, at intervals precisely equal to its natural time of vibration, we shall cause it to swing through an arc of considerable magnitude.

The same principle applies to a body capable of executing vibrations under the influence of its own elasticity. A series of impulses keeping time with its own natural period may set it in powerful vibration, though any one of them singly would have no appreciable effect.

Some bodies, such as strings and confined portions of air, have definite periods in which they can vibrate freely when once started;

and when a note corresponding to one of these periods is sounded in their neighbourhood, they readily take it up and emit a note of the same pitch themselves.

Other bodies, especially thin pieces of dry straight-grained deal, such as are employed for the faces of violins and the sounding-boards of pianos, are capable of vibrating, more or less freely, in any period lying between certain wide limits. They are accordingly set in vibration by all the notes of their respective instruments; and by the large surface with which they act upon the air, they contribute in a very high degree to increase the sonorous effect. All stringed instruments are constructed on this principle; and their quality mainly depends on the greater or less readiness with which they respond to the vibrations of the strings.

All such methods of reinforcing a sound must be included under *resonance;* but the word is often more particularly applied to the reinforcement produced by masses of air.

40. Longitudinal Vibrations of Strings.—Strings or wires may also be made to vibrate *longitudinally,* by rubbing them, in the direction of their length, with a bow or a piece of chamois leather covered with rosin. The sounds thus obtained are of much higher pitch than those produced by transverse vibration.

In the case of the fundamental note, each of the two halves A C, C B (Fig. 29), is alternately extended and compressed, one being

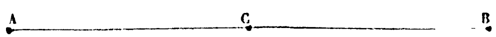


Fig 29 --Longitudinal Vibration. First Tone.

extended while the other is compressed. At the middle point C there is no extension or compression, but there is greater amplitude of movement than at any other point. The amplitudes diminish in passing from C towards either end, and vanish at the ends, which are therefore nodes. The extensions and compressions, on the other hand, increase as we travel from the middle towards either end, and obtain their greatest values at the ends.

But the string may also divide itself into any number of separately-vibrating segments, just as in the case of transverse vibrations. Fig. 30 represents the motions which occur when there are three such segments, separated by two nodes D, E. The upper portion of the figure is true for one-half of the period of vibration, and the lower portion for the remaining half.

The frequency of vibration, for longitudinal as well as for transverse vibrations, varies inversely as the length of the vibrating string, or segment of string. We shall return to this subject in § 51.

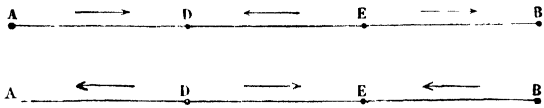


Fig. 30.—Longitudinal Vibration Third Tone

41. Stringed Instruments.—Only the transversal vibrations of strings are employed in music. In the violin and violoncello there are four strings, each being tuned a fifth above the next below it; and intermediate notes are obtained by fingering, the portion of string between the finger and the bridge being the only part that is free to vibrate. The bridge and sounding-post serve to transmit the vibrations of the strings to the body of the instrument. In the piano there is also a bridge, which is attached to the sounding-board, and communicates to it the vibrations of the wires.

42. Transversal Vibrations of Rigid Bodies: Rods, Plates, Bells.—We shall not enter into detail respecting the laws of the transverse vibrations of rigid bodies. The relations of their overtones to their fundamental tones are usually of an extremely complex character, and this fact is closely connected with the unmusical or only semi-musical character of the sounds emitted.

When one face of the body is horizontal, the division into separate vibrating segments can be rendered visible by a method devised by Chladni, namely, by strewing sand on this face. During the vibration, the sand, as it is tossed about, works its way to certain definite lines, where it comes nearly to rest. These nodal lines must be regarded as the intersections of internal nodal surfaces with the surface on which the sand is strewed, each nodal surface being the boundary between parts of the body which have opposite motions.

The figures composed by these nodal lines are often very beautiful, and quite startling in the suddenness of their production. Chladni and Savart published the forms of a great number. A complete theoretical explanation of them would probably transcend the powers of the greatest mathematicians.

Bells and bell-glasses vibrate in segments, which are never less than four in number, and are separated by nodal lines meeting in the middle of the crown. They are well shown by putting water in a

bell-glass, and bowing its edge. The surface of the water will immediately be covered with ripples, one set of ripples proceeding from each of the vibrating segments. The division into any possible number of segments may be effected by pressing the glass with the fingers in the places where a pair of consecutive nodes ought to be formed, while the bow is applied to the middle of one of the segments. The greater the number of segments the higher will be the note emitted.

43. **Tuning-fork.**—Steel rods, on account of their comparative freedom from change, are well suited for standards of pitch. The tuning-fork, which is especially used for this purpose, consists essentially of a steel rod bent double, and attached to a handle of the same material at its centre. Besides the fundamental tone, it is capable of yielding two or three overtones, which are very much higher in pitch; but these are never used for musical purposes. If the fork is held by the handle while vibrating, its motion continues for a long time, but the sound emitted is too faint to be heard except by holding the ear near it. When the handle is pressed against a table, the latter acts as a sounding-board, and communicates the vibrations to the air, but it also causes the fork to come much more speedily to rest. For the purposes of the lecture-room the fork is often mounted on a sounding-box (Fig. 31), which should be separated from the table by two pieces of india-rubber tubing. The box can then vibrate freely in unison with the fork, and the sound is both loud and lasting. The vibrations are usually excited either by bowing the fork or by drawing a piece of wood between its prongs.

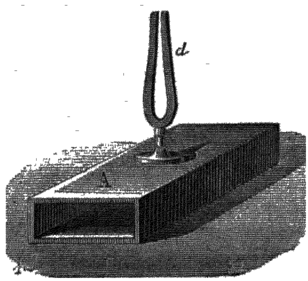


Fig. 31.—Fork on Sounding-box.

The pitch of a tuning-fork varies slightly with temperature, becoming lower as the temperature rises. This effect is due in some trifling degree to expansion, but much more to the diminution of elastic force.

44. **Law of Linear Dimensions.**—The following law is of very wide application, being applicable alike to solid, liquid, and gaseous bodies: —*When two bodies differing in size, but in other respects similar and similarly circumstanced, vibrate in the same mode, their vibration-periods are directly as their linear dimensions.* Their vibra-

tion-frequencies are consequently in the inverse ratio of their linear dimensions.

In applying the law to the transverse vibrations of strings, it is to be understood that the stretching force per unit of sectional area is constant. In this case the velocity of a pulse (§ 36) is constant, and the period of vibration, being the time required for a pulse to travel over twice the length of the string, is therefore directly as the length.

45. Organ-pipes.—In organs, and wind-instruments generally, the sonorous body is a column of air confined in a tube. To set this air

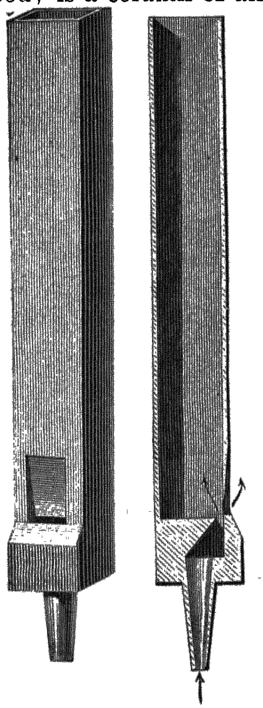


Fig. 32.—Block Pipe.

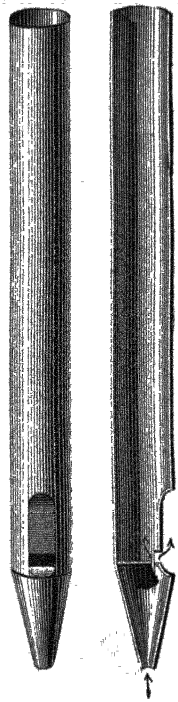


Fig 33 —Flue Pipe.

in vibration some kind of mouth-piece must be employed. That which is most extensively used in organs is called the *flute mouth-piece,*[1] and is represented, in conjunction with the pipe to which it is attached, in Figs. 32, 33. It closely resembles the mouth-piece of

[1] This is not the trade name. English organ-builders have no generic name for this mouth-piece.

an ordinary whistle. The air from the bellows arrives through the conical tube at the lower end, and, escaping through a narrow slit, grazes the edge of a wedge placed opposite. A rushing noise is thus produced, which contains, among its constituents, the note to which the column of air in the pipe is capable of resounding; and as soon as this resonance occurs, the pipe speaks. Fig. 32 represents a wooden and Fig. 33 a metal organ-pipe, both of them being furnished with flute mouth-pieces. The two arrows in the sections are intended to suggest the two courses which the wind may take as it issues from the slit, one of which it actually selects to the exclusion of the other.

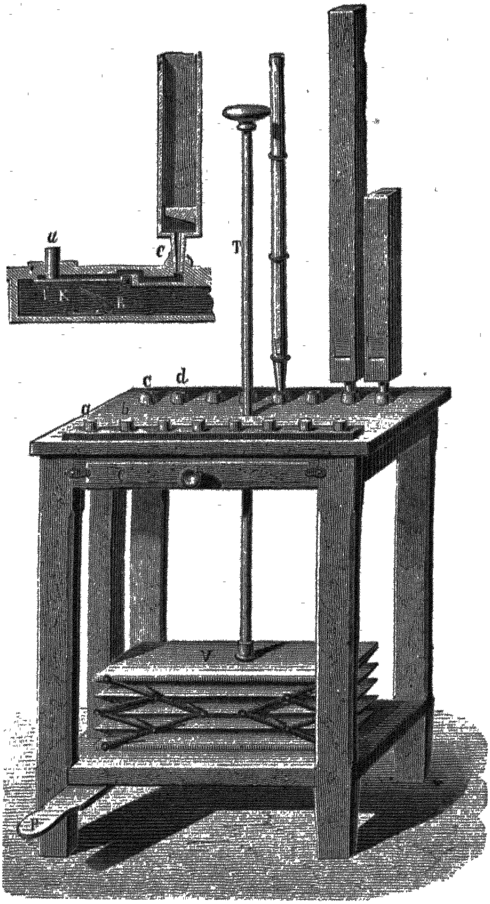


Fig. 34.—Experimental Organ.

The arrangements for admitting the wind to the pipes by putting down the keys are shown in Fig. 34. The bellows V are worked by the treadle P. The force of the blast can be increased by weighting the top of the bellows, or by pressing on the rod T. The air passes up from the bellows, through a large tube shown at one end, into a reservoir C, called the wind-chest. In the top of the wind-chest there are numerous openings *c*, *d*, &c., in which the tubes are to be fixed. The sectional drawing in the upper part of the figure shows the internal communications. A plate K, pressed up by a spring R, cuts off the tube *c* from the wind-chest, until the pin *a*

is depressed. The putting down of this pin lowers the plate, and admits the wind. This description only applies to the experimental organs which are constructed for lecture illustration. In real organs the pressure of the wind in the bellows is constant; and as this pressure would be too great for most of the pipes, the several apertures of admission are partially plugged, to diminish the force of the blast.

46. **The Air is the Sonorous Body.**—It is easily shown that the sound emitted by an organ-pipe depends, mainly at least, on the dimensions of the inclosed column of air, and not on the thickness or material of the pipe itself. For let three pipes, one of wood, one of copper, and the other of thick card, all of the same internal dimensions, be fixed on the wind-chest. On making them speak, it will be found that the three sounds have exactly the same pitch, and but slight difference in character. If, however, the sides of the tube are *excessively* thin, their yielding has a sensible influence, and the pitch of the sound is modified.

47. **Law of Linear Dimensions.**—The law of linear dimensions, stated in § 44 as applying to the vibrations of similar solid bodies, applies to gases also. Let two box-shaped pipes (Fig. 35) of precisely similar form, and having their linear dimensions in the ratio of 2 : 1, be fixed on the wind-chest; it will be found, on making them speak, that the note of the small one is an octave higher than the other;—showing double frequency of vibration.

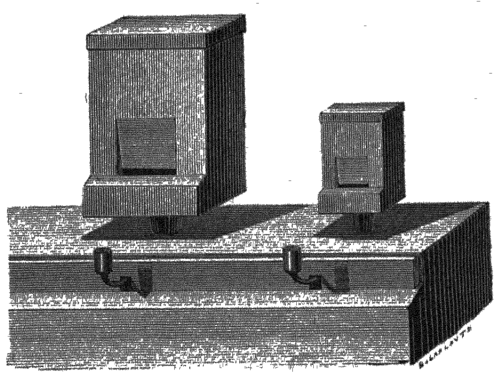


Fig. 35.—Law of Linear Dimensions

48. **Bernoulli's Laws.**—The law just stated applies to the comparison of similar tubes of any shape whatever. When the length of a tube is a large multiple of its diameter, the note emitted is nearly independent of the diameter, and depends almost entirely on the length. The relations between the fundamental note of such a tube and its overtones were discovered by Daniel Bernoulli, and are as follows:—

I. *Overtones of Open Pipes.*—Let the pipe B (Fig. 36), which is

open at the upper end, be fixed on the wind-chest; let the corresponding key be put down, and the wind gradually turned on, by means of the cock below the mouth-piece. The first note heard will be feeble and deep; it is the fundamental note of the pipe. As the wind is gradually turned full on, and increasing pressure afterwards applied to the bellows, a series of notes will be heard, each higher than its predecessor. These are the overtones of the pipe. They are the harmonics of the fundamental note; that is to say, if 1 denote the frequency of vibration for the fundamental tone, the frequencies of vibration for the overtones will be approximately 2, 3, 4, 5 . . . respectively.

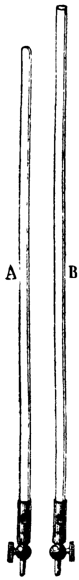


Fig. 36. Tubes for Overtones

II. *Overtones of Stopped Pipes.*—If the same experiment be tried with the pipe A, which is closed at its upper end; the overtones will form the series of odd harmonics of the fundamental note, all the even harmonics being absent; in other words, the frequencies of vibration of the fundamental tone and overtones will be approximately represented by the series of odd numbers 1, 3, 5, 7 . . .

It will also be found, that if both pipes are of the same length, the fundamental note of the stopped pipe is an octave lower than that of the open pipe.

49. Mode of Production of Overtones.—In the production of the overtones, the column of air in a pipe divides itself into vibrating segments, separated by nodal cross-sections. At equal distances on opposite sides of a node, the particles of air have always equal and opposite velocities, so that the air at the node is always subjected to equal forces in opposite directions, and thus remains unmoved by their action. The portion of air constituting a vibrating segment, sways alternately in opposite directions, and as the movements in two consecutive segments are opposite, two consecutive nodes are always in opposite conditions as regards compression and extension. The middle of a vibrating segment is the place where the amplitude of vibration is greatest, and the variation of density least. It may be called an *antinode.* The distance from one node to the next is half a wave-length, and the distance from a node to an antinode is a quarter of a wave-length. Both ends of an open pipe, and the end next the mouth-piece of a stopped pipe, are antinodes, being preserved from changes of density by their free communication with

the external air. At the closed end of a stopped pipe there must always be a node.

The swaying to and fro of the internodal portions of air between fixed nodal planes, is an example of *stationary undulation;* and the vibration of a musical string is another example. A stationary undulation may always be analysed into two component undulations equal and similar to one another, and travelling in opposite directions, their common wave-length being double of the distance from node to node (§ 22). These undulations are constantly undergoing reflection from the ends of the pipe or string, and, in the case of pipes, the reflection is opposite in kind according as it takes place from a closed or an open end. In the former case a condensation propagated towards the end is reflected as a condensation, the forward-moving particles being compelled to recoil by the resistance which they there encounter; and a rarefaction is, in like manner, reflected as a rarefaction. On the other hand, when a condensation arrives at an open end, the sudden opportunity for expansion which is afforded causes an outward movement in excess of that which would suffice for equilibrium of pressure, and a rarefaction is thus produced which is propagated back through the tube. A condensation is thus reflected as a rarefaction; and a rarefaction is, in like manner, reflected as a condensation.

The period of vibration of the fundamental note of a stopped pipe is the time required for propagating a pulse through four times the length of the pipe. For let a condensation be suddenly produced at the lower end by the action of the vibrating lip. It will be propagated to the closed end and reflected back, thus travelling over twice the length of the pipe. On arriving at the aperture where the lip is situated, it is reflected as a rarefaction. This rarefaction travels to the closed end and back, as the condensation did before it, and is then reflected from the aperture as a condensation. Things are now in their initial condition, and one complete vibration has been performed. The period of the movements of the lip is determined by the arrival of these alternate condensations and rarefactions; and the lip, in its turn, serves to divert a portion of the energy of the blast, and employ it in maintaining the energy of the vibrating column.

The wave-length of the fundamental note of a stopped pipe is thus four times the length of the pipe.

In an open pipe, a condensation, starting from the mouth-piece, is reflected from the other end as a rarefaction. This rarefaction, on

reaching the mouth-piece, is reflected as a condensation; and things are thus in their initial state after the length of the pipe has been traversed twice. The period of vibration of the fundamental note is accordingly the time of travelling over twice the length of the pipe; and its wave-length is twice the length of the pipe. In every case of longitudinal vibration, if the reflection is alike at both ends, the wave-length of the fundamental tone is twice the distance between the ends.

50. **Explanation of Bernoulli's Laws.**—In investigating the theoretical relations between the fundamental tone and overtones for a pipe of either kind, it is convenient to bear in mind that the distance from an open end to the nearest node is a quarter of a wave-length of the note emitted.

In the case of the open pipe the first or fundamental tone requires one node, which is at the middle of the length. The second tone requires two nodes, with half a wave-length between them, while each of them is a quarter of a wave-length from the nearest end. A quarter wave-length has thus only half the length which it had for the fundamental tone, and the frequency of vibration is therefore doubled.

The third tone requires three nodes, and the distance from either end to the nearest node is $\frac{1}{6}$ of the length of the pipe, instead of $\frac{1}{2}$ the length as in the case of the first tone. The wave-length is thus divided by 3, and the frequency of vibration is increased threefold. We can evidently account in this way for the production of the complete series of harmonics of the fundamental note.

In the case of the stopped pipe, the mouth-piece is always distant a quarter wave-length from the nearest node, and this must be distant an even number of quarter wave-lengths from the stopped end, which is itself a node.

For the fundamental tone, a quarter wave-length is the whole length of the pipe.

For the second tone, there is one node besides that at the closed end, and its distance from the open end is $\frac{1}{3}$ of the length of the pipe.

For the third tone, there are two nodes besides that at the closed end. The distance from the open end to the nearest node is therefore $\frac{1}{5}$ of the length of the pipe.

The wave-lengths of the successive tones, beginning with the fundamental, are therefore as $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7} \ldots$, and their vibration-frequencies are as $1, 3, 5, 7 \ldots$

Also, since the wave-length of the fundamental tone is four times the length of the pipe if stopped, and only twice its length if open, it is obvious that the wave-length is halved, and the frequency of vibration doubled, by unstopping the pipe.

No change of pitch, or only very slight change, will be produced by inserting a solid partition at a node, or by putting an antinode in free communication with the external air. These principles can be illustrated by means of the jointed pipe represented in Fig. 37.

Fig. 37 Jointed Pipe

51. **Application to Rods and Strings.**—The same laws which apply to open organ-pipes, also apply to the longitudinal vibrations of rods free at both ends, and to both the longitudinal and transverse vibrations of strings. In all these cases the overtones form the complete series of harmonics of the first or fundamental tone, and the period of vibration for this first tone is the time occupied by a pulse in travelling over twice the length of the given rod or string. In the case of longitudinal vibrations the velocity of a pulse is $\sqrt{\frac{M}{D}}$, M denoting the value of Young's modulus for the rod or string, and D its density. This is identical with the velocity of sound through the rod or string, and is independent of its tension. In the case of transverse pulses in a string (regarded as perfectly flexible), the formula for the velocity of transmission (1) § 36, may be written $\sqrt{\frac{F}{D}}$, F denoting the stretching force per unit of sectional area. The ratio of the latter velocity to the former is $\sqrt{\frac{F}{M}}$, which is always a small fraction, since $\frac{F}{M}$ expresses the fraction of itself by which the string is lengthened by the force F.

If a rod, free at both ends, is made to vibrate longitudinally, its nodes and antinodes will be distributed exactly in the same way as those of an open organ-pipe. The experiment can be performed by holding the rod at a node, and rubbing it with rosined chamois leather.

52. **Application to Measurement of Velocity in Gases.**—Let v denote the velocity of sound in a particular gas, in feet per second, λ the wave-length of a particular note in this gas in feet, and n the frequency of vibration for this note, that is the number of vibrations

per second which produce it. Then λ is the distance travelled in $\frac{1}{n}$ of a second, and the distance travelled in a second is

$$v = n\lambda.$$

For the same note, n is constant for all media whatever, and v varies directly as λ. The velocities of sound in two gases may thus be compared by observing the lengths of vibrating columns of the two gases which give the same note; or if columns of equal length be employed, the velocities will be directly as the frequencies of vibration, which are determined by observing the pitch of the notes emitted.

By these methods, Dulong, and more recently Wertheim, have determined the velocity of sound in several different gases. The following are Wertheim's results, in metres per second, the gases being supposed to be at 0° C.

Air,	331	Carbonic acid,	262
Oxygen,	317	Nitrous oxide,	262
Hydrogen,	1269	Olefiant gas,	314
Carbonic oxide,	337		

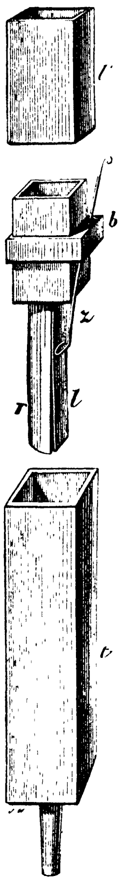


Fig. 38.—Reed-pipe.

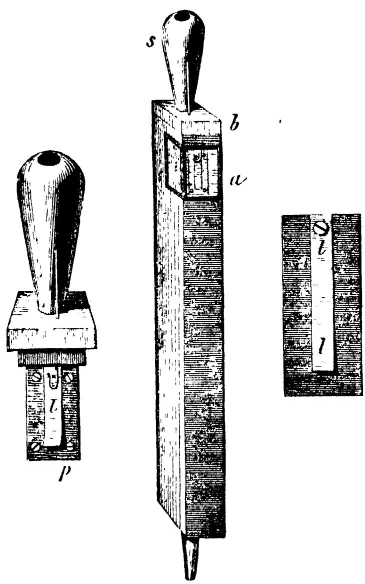


Fig 39.—Free Reed.

The same principle is applicable to liquids and solids; and it was by means of the longitudinal vibrations of rods that the velocities given in § 15 were ascertained.

53. Reed-pipes.—Instead of the flute mouth-piece above described, organ-pipes are often furnished with what is called a *reed*. A reed contains an elastic plate l (Figs. 38, 39) called the *tongue*, which, by its vibrations, alternately opens and closes or nearly closes an aperture through which the wind passes. In Fig. 38, the air from the bellows enters first

the lower part t of the pipe, and thence (when permitted by the tongue) passes through the channel[1] r into the upper part t'. The stiff wire z, movable with considerable friction through the hole b, limits the vibrating portion of the tongue, and is employed for tuning. Reed-pipes are often terminated above by a trumpet-shaped expansion.

A *striking reed* (Fig. 38) is one whose tongue closes the aperture by covering it. The tongue should be so shaped as not to strike along its whole length at once, but to roll itself down over the aperture. In the *free reed* (Fig. 39) the tongue can pass completely through.

The striking reed is generally preferred in organs, its peculiar character rendering it very effective by way of contrast. It is always used for the *trumpet* stop. Reed-pipes can be very strongly blown without breaking into overtones. Their pitch, however, if they are of the striking kind, is not independent of the pressure of the wind, but gradually rises as the pressure increases. Free reeds, which are used for harmoniums, accordions, and concertinas, do not change in pitch with change of pressure.

Elevation of temperature sharpens pipes with flute mouth-pieces, and flattens reed-pipes. The sharpening is due to the increased velocity of sound in hot air. The flattening is due to the diminished elasticity of the metal tongue. It is thus proved that the pitch of a reed-pipe is not always that due to the free vibration of the inclosed air, but may be modified by the action of the tongue.

54. **Wind-instruments.**—In all wind-instruments, the sound is originated by one of the two methods just described. With the flute-pipe must be classed the flute, the flageolet, and the Pandean-pipes. The clarionet, hautboy, and bassoon have reed mouth-pieces, the vibrating tongue being a piece of reed or cane. In the bugle, trumpet, and French-horn, which are mere tubes without keys, the lips of the performer act as the reed-tongue, and the notes produced are approximately the natural overtones. These, when of high order, are so near together, that a gamut can be formed by properly selecting from among them.

The fingering of the flute and clarionet, has the effect sometimes of altering the effective length of the vibrating column of air, and sometimes of determining the production of overtones. In the

[1] The piece r, which is approximately a half cylinder, is called the *reed* by organ-builders.

trombone and cornet-à-piston, the length of the vibrating column of air is altered. The harmonium, accordion, and concertina are reed instruments, the reeds employed being always of the free kind.

55. **Manometric Flames.**—Koenig, of Paris, constructs several forms of apparatus, in which the variations of pressure produced by vibrations of air in a pipe are rendered evident to the eye by their effect upon flames. One of these is represented in Fig. 40. Three small gas-burners are fixed at definite points in the side of a pipe, as represented in the figure. When the pipe gives its second tone, the central flame is at an antinode and remains unaffected, while the other two, being at nodes, are agitated or blown out. When it gives its first tone, the central flame, which is now at a node, is more powerfully affected than the others. The gas which supplies these burners is separated from the air in the pipe only by a thin membrane. When the pipe is made to speak, the flame at the node is violently agitated, in consequence of the changes of pressure on the back of the membrane, while those at the ventral points are scarcely affected. The agitation of the flame is a true vibration; and, when examined by the aid of a revolving mirror, presents the appearance of tongues of flame alternating with nearly dark spaces. If two pipes, one an octave higher than the other, are connected with the same gas flame, or with two gas flames which can be viewed in the same mirror, the tongues of flame corresponding to the upper octave are seen to be twice as numerous as the others.

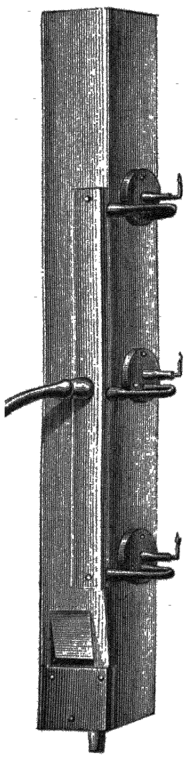


Fig. 40.—Manometric Flames.

CHAPTER IV.

ANALYSIS OF VIBRATIONS. CONSTITUTION OF SOUNDS.

56. **Optical Examination of Sonorous Vibrations.**—Sound is a special sensation belonging to the sense of hearing; but the vibrations which are its physical cause often manifest themselves to other senses. For instance, we can often feel the tremors of a sonorous body by touching it; we see the movements of the sand on a vibrating plate, the curve traced by the style of a vibroscope, &c. The aid which one sense can thus furnish in what seems the peculiar province of another is extremely interesting. M. Lissajous has devised a very beautiful optical method of examining sonorous vibrations, which we will briefly describe.

57. **Lissajous' Experiment.**—Suppose we introduce into a dark room (Fig. 41) a beam of solar rays, which, after passing through a lens L, is reflected, first, from a small mirror fixed on one of the branches of a tuning-fork D, and then from a second mirror M, which throws it on a screen E; we can thus, by proper adjustments, form upon the screen a sharp and bright image of the sun, which will appear as a small spot of light. As long as the apparatus remains at rest, we shall not observe any movement of the image; but if the tuning-fork vibrates, the image will move rapidly up and down along the line I, I′, producing, in consequence of the persistence of impressions, the appearance of a vertical line of light. If the tuning-fork remains at rest, but the mirror M is rotated through a small angle about a vertical axis, the image will move horizontally. Consequently, if both these motions take place simultaneousely, the spot of light will trace out on the screen a sinuous line, as represented in the figure, each S-shaped portion corresponding to one vibration of the tuning-fork.

Now, let the mirror M be replaced by a small mirror attached to

a second tuning-fork, which vibrates in a horizontal plane, as in

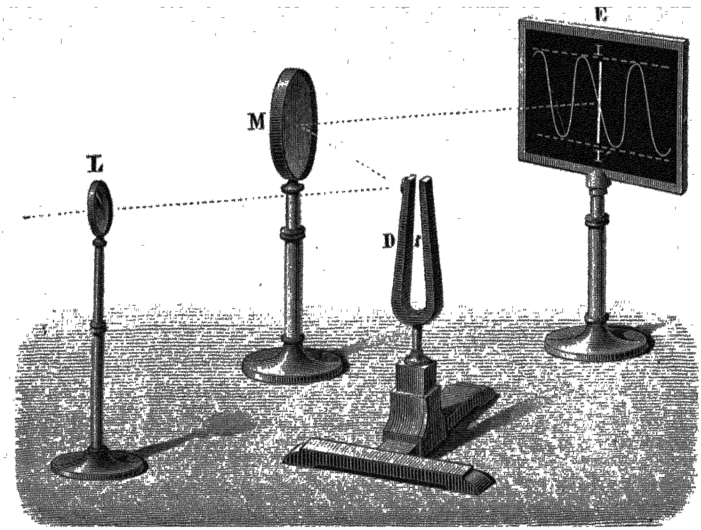


Fig. 41.—Principle of Lissajous' Experiment.

Fig. 42. If this fork vibrates alone, the image will move to and

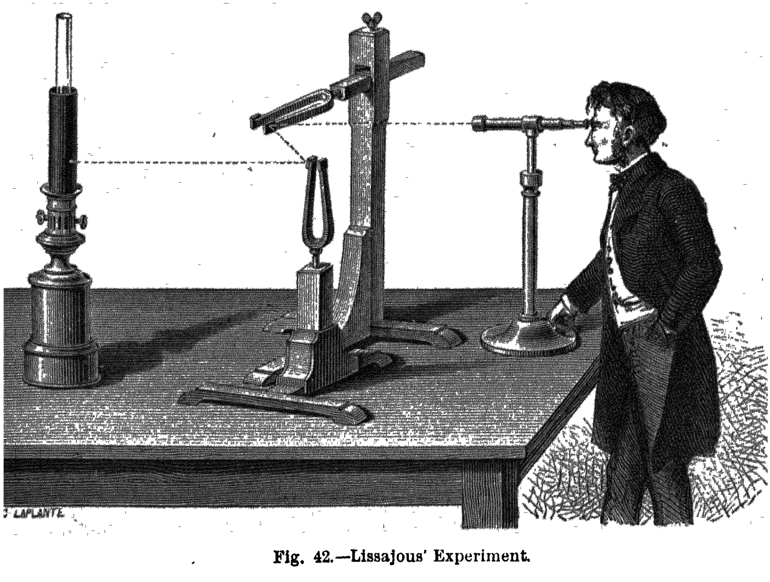


Fig. 42.—Lissajous' Experiment.

fro horizontally, presenting the appearance of a horizontal line of

light, which gradually shortens as the vibrations die away. If both forks vibrate simultaneously, the spot of light will rise and fall according to the movements of the first fork, and will travel left and right according to the movements of the second fork. The curve actually described, as the resultant of these two component motions, is often extremely beautiful. Some varieties of it are represented in Fig. 43.

Instead of throwing the curves on a screen, we may see them by looking into the second mirror, either with a telescope, as in Fig. 42,

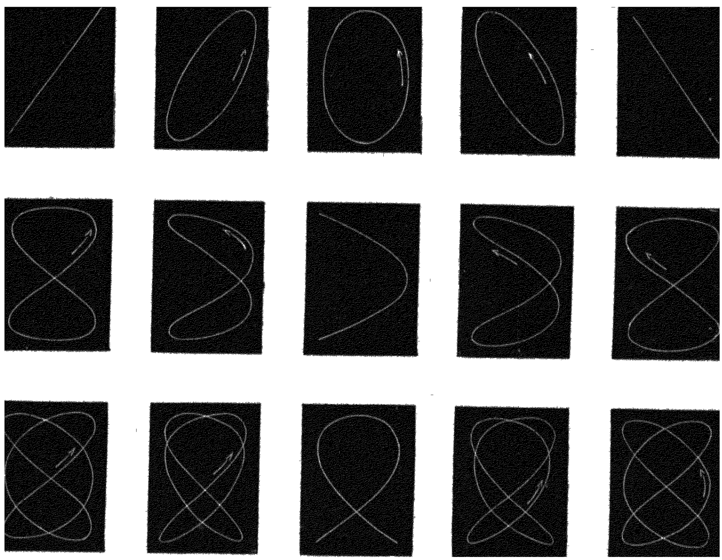


Fig 43 —Lissajous' Figures, Unison, Octave, and Fifth.

or with the naked eye. In this form of the experiment, a lamp surrounded by an opaque cylinder, pierced with a small hole just opposite the flame, as represented in the figure, is a very convenient source of light.

The movement of the image depends almost entirely on the angular movements of the mirrors, not on their movements of translation; but the distinction is of no importance, for, in the case of such small movements, the linear and angular changes may be regarded as strictly proportional.

Either fork vibrating alone would cause the image to execute *simple harmonic motion* (see Part I.), or, as it may conveniently

be called, *simple vibration;* so that the movement actually executed will be the resultant of two simple harmonic motions in directions perpendicular to each other.

Suppose the two forks to be in unison. Then the two simple harmonic motions will have the same period, and the path described will always be some kind of ellipse,[1] the circle and straight line being included as particular cases. It will be a straight line if both forks pass through their positions of equilibrium at the same instant. In order that it may be a circle, the amplitudes of the two simple harmonic motions must be equal, and one fork must be in a position of maximum displacement when the other is in the position of equilibrium.

If the unison were rigorous, the curve once obtained would remain unchanged, except in so far as its breadth and height became reduced by the dying away of the vibrations. But this perfect unison is never attained in practice, and the eye detects changes depending on differences of pitch too minute to be perceived by the ear. These changes are illustrated by the upper row of forms in Fig. 43, commencing, say, with the sloping straight line at the left hand, which gradually opens out into an ellipse, and afterwards contracts into a straight line, sloping the opposite way. It then retraces its steps, the motion being now in opposition to the arrows in the figure, and then repeats the same changes.

If the interval between the two forks is an octave, we shall obtain the curves represented in the second row;[2] if the interval is a fifth, we shall obtain the curves in the lowest row. In each case the order of the changes will be understood by proceeding from left to right,

[1] Employing horizontal and vertical co-ordinates, and denoting the amplitudes by a and b, we have, in the case of unison, $\frac{x}{a} = \sin\theta$, $\frac{y}{b} = \sin(\theta + \beta)$, where β denotes the difference of phase, and θ is an angle varying directly as the time. Eliminating θ, we obtain the equation to an ellipse, whose form and dimensions depend upon the given quantities, a, b, β.

[2] The middle curve in this row is a parabola, and corresponds to the elimination of θ between the equations $\frac{x}{a} = \cos 2\theta$, $\frac{y}{a} = \cos\theta$. The coefficient 2 indicates the double frequency of horizontal as compared with vertical vibrations.

The general equations to Lissajous' figures are $\frac{x}{a} = \sin m\theta$, $\frac{y}{b} = \sin(n\theta + \beta)$, where m and n are proportional to the frequencies of horizontal and vertical vibrations. The gradual changes from one figure to another depend on the gradual change of β, and all the figures can be inscribed in a rectangle, whose length and breadth are $2a$ and $2b$.

and then back again; but the curves obtained in returning will be inverted.

58. **Optical Tuning.**—By the aid of these principles, tuning-forks can be compared with a standard fork with much greater precision than would be attainable by ear. Fig. 44 represents a convenient

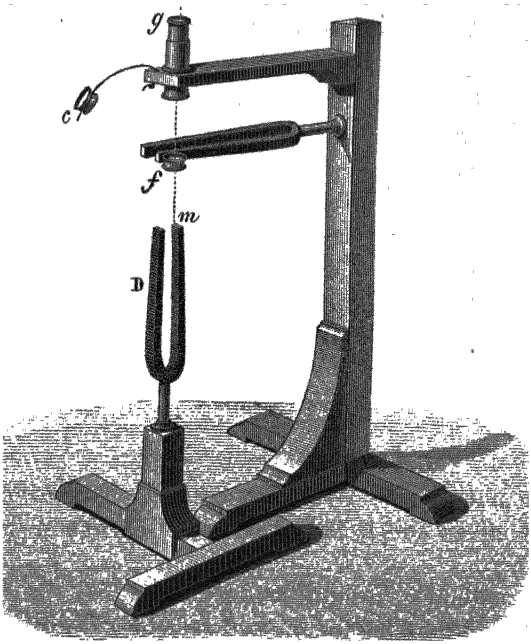


Fig. 44.—Optical Comparison of Tuning-forks.

arrangement for this purpose. A lens *f* is attached to one of the prongs of a standard fork, which vibrates in a horizontal plane; and above it is fixed an eye-piece *g*, the combination of the two being equivalent to a microscope. The fork to be compared is placed upright beneath, and vibrates in a vertical plane, the end of one prong being in the focus of the microscope. A bright point *m*, produced by making a little scratch on the end of the prong with a diamond, is observed through the microscope, and is illuminated, if necessary, by converging a beam of light upon it through the lens *c*. When the forks are set vibrating, the bright point is seen as a luminous ellipse, whose permanence of form is a test of the closeness of the unison. The ellipse will go through a complete cycle of changes in the time required for one fork to gain a complete vibration on the other.

59. **Other Modes of producing Lissajous' Figures.**—An arrangement devised in 1844 by Professor Blackburn, of Glasgow, then a student at Cambridge, affords a very easy mode of obtaining, by a slow motion, the same series of curves which, in the above arrangements, are obtained by a motion too quick for the eye to follow. A cord A B C (Fig. 45) is fastened at A and C, leaving more or less

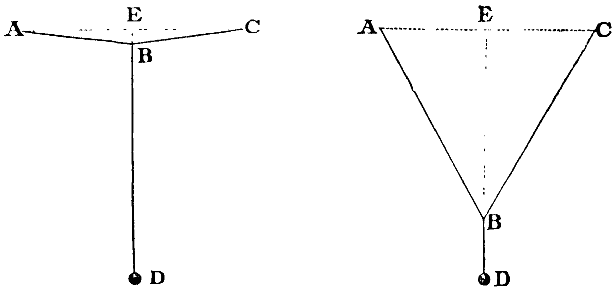


Fig. 45 —Blackburn's Pendulum.

slack, according to the curves which it is desired to obtain; and to any intermediate point B of the cord another string is tied, carrying at its lower end a heavy body D to serve as pendulum-bob.

If, when the system is in equilibrium, the bob is drawn aside in the plane of A B C and let go, it will execute vibrations in that plane, the point B remaining stationary, so that the length of the pendulum is B D. If, on the other hand, it be drawn aside in a plane perpendicular to the plane A B C, it will vibrate in this perpendicular plane, carrying the whole of the string with it in its motion, so that the length of the pendulum is the distance of the bob from the point E, in which the straight line A C is cut by D B produced. The frequencies of vibration in the two cases will be inversely as the square roots of the pendulum-lengths B D, E D.

If the bob is drawn aside in any other direction, it will not vibrate in one plane, but will perform movements compounded of the two independent modes of vibration just described, and will thus describe curves identical with Lissajous'. If the ratio of E D to B D is nearly equal to unity, as in the left-hand figure, we shall have curves corresponding to approximate unison. If it be approximately 4, as in the right-hand figure, we shall obtain the curves of the octave. Traces of the curves can be obtained by employing for the bob a

vessel containing sand, which runs out through a funnel-shaped opening at the bottom.[1]

The curves can also be exhibited by fixing a straight elastic rod at one end, and causing the other end to vibrate transversely. This was the earliest known method of obtaining them. If the flexural rigidity of the rod is precisely the same for all transverse directions, the vibrations will be executed in one plane; but if there be any inequality in this respect, there will be two mutually perpendicular directions possessing the same properties as the two principal directions of vibration in Blackburn's pendulum. A small bright metal knob is usually fixed on the vibrating extremity to render its path visible. The instrument constructed for this mode of exhibiting the figures is called a *kaleidophone*. In its best form (devised by Professor Barrett) the upper and lower halves of the rod (which is vertical) are flat pieces of steel, with their planes at right angles, and a stand is provided for clamping the lower piece at any point of its length that may be desired, so as to obtain any required combination.

60. **Character.**—*Character* or *timbre*, which we have already defined in § 24, must of necessity depend on the *form* of the vibration of the aerial particles by which sound is transmitted, the word *form* being used in the metaphorical sense there explained, for in the literal sense the form is always a straight line. When the changes of density are represented by ordinates of a curve, as in Fig. 12, the form of this curve is what is meant by the form of vibration.

The subject of *timbre* has been very thoroughly investigated in recent years by Helmholtz; and the results at which he has arrived are now generally accepted as correct.

The first essential of a musical note is, that the aerial movements which constitute it shall be strictly *periodic;* that is to say, that each vibration shall be exactly like its successor, or at all events, that, if there be any deviation from strict periodicity, it shall be so gradual as not to produce sensible dissimilarity between several consecutive vibrations of the same particle.

There is scarcely any proposition more important in its application

[1] Mr. Hubert Airy has obtained very beautiful traces by attaching a glass pen to the bob (see *Nature*, Aug. 17 and Sept. 7, 1871), and in Tisley's *harmonograph* the same result is obtained by means of two pendulums, one of which moves the paper and the other the pen.

to modern physical investigations than the following mathematical theorem, which was discovered by Fourier:—*Any periodic vibration executed in one line can be definitely resolved into simple vibrations, of which one has the same frequency as the given vibration, and the others have frequencies* 2, 3, 4, 5 . . . *times as great,* no fractional multiples being admissible. The theorem may be briefly expressed by saying that *every periodic vibration consists of a fundamental simple vibration and its harmonics.*

We cannot but associate this mathematical law with the experimental fact, that a trained ear can detect the presence of harmonics in all but the very simplest musical notes. The analysis which Fourier's theorem indicates, appears to be actually performed by the auditory apparatus.

The *constitution* of a periodic vibration may be said to be known if we know the ratios of the amplitudes of the simple vibrations which compose it; and in like manner the constitution of a sound may be said to be known if we know the relative intensities of the different elementary tones which compose it.

Helmholtz infers from his experiments that the *character* of a musical note depends upon its *constitution* as thus defined; and that, while change of intensity in any of the components produces a modification of character, change of phase has no influence upon it whatever. Lord Kelvin, in a paper "On Beats of Imperfect Harmonies,"[1] adduces strong evidence to show that change of phase has, in some cases at least, an influence on character.

The harmonics which are present in a note, usually find their origin in the vibrations of the musical instrument itself. In the case of stringed instruments, for example, along with the vibration of the string as a whole, a number of segmental vibrations are simultaneously going on. Fig. 46 represents curves obtained by the composition of the fundamental mode of vibration with another an octave higher. The broken lines indicate the forms which the string would assume if yielding only its fundamental note.[2] The continuous lines in the first and third figures are forms which a string may assume in its two positions of greatest displacement, when yielding the octave along with the fundamental, the time required for the

[1] *Proc. R. S. E.* 1878.

[2] The form of a string vibrating so as to give only one tone (whether fundamental or harmonic) is a curve of sines, all its ordinates increasing or diminishing in the same proportion, as the string moves.

string to pass from one of these positions to the other being the same as the time in which each of its two segments moves across and back

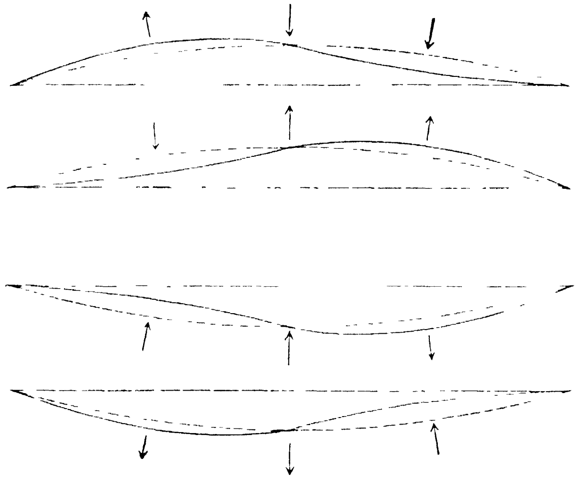


Fig 46 —String giving first Two Tones.

again. The second and fourth figures must in like manner be taken together, as representing a pair of extreme positions. The number of harmonics thus yielded by a pianoforte wire is usually some four or five; and a still larger number are yielded by the strings of a violin.

The notes emitted from wide organ-pipes with flute mouth-pieces are very deficient in harmonics. This defect is remedied by combining with each of the larger pipes a series of smaller pipes,[1] each yielding one of its harmonics. An ordinary listener hears only one note, of the same pitch as the fundamental, but much richer in character than that which the fundamental pipe yields alone. A trained ear can recognize the individual harmonics in this case as in any other.

[1] The stops called *open diapason* and *stop diapason* (consisting respectively of open and stopped pipes), give the fundamental tone, almost free from harmonics. The stop absurdly called *principal* gives the second tone, that is the octave above the fundamental. The stops called *twelfth* and *fifteenth* give the third and fourth tones, which are a twelfth (octave + fifth), and a fifteenth (double octave) above the fundamental. The fifth, sixth, and eighth tones are combined to form the stop called *mixture*.

As many of our readers will be unacquainted with the structure of organs, it may be desirable to state that an organ contains a number of complete instruments, each consisting of several octaves of pipes. Each of these complete instruments is called a *stop*, and is brought into use at the pleasure of the organist by pulling out a slide, by means of a knob-handle, on which the name of the stop is marked. To throw it out of use, he pushes in the slide. A large number of stops are often in use at once.

It is important to remark, that though the presence of harmonic subdivisions in a vibrating body necessarily produces harmonics in the sound emitted, the converse cannot be asserted. Simple vibrations, executed by a vibrating body, produce vibrations *of the same frequency* as their own, in any medium to which they are transmitted, but not necessarily *simple* vibrations. If they produce compound vibrations, these, as we have just seen, must consist of a fundamental simple vibration and its harmonics.

61. Helmholtz's Resonators.—Helmholtz derived material aid in his researches from an instrument devised by himself, and called a *resonator* or *resonance globe* (Fig. 47). It is a hollow globe of thin

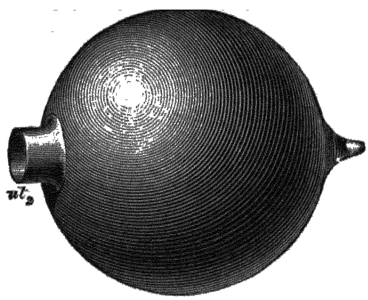


Fig. 47.—Resonator.

brass, with an opening at each end, the larger one serving for the admission of sound, while the smaller one is introduced into the ear. The inclosed mass of air has, like the column of air in an organ-pipe, a particular fundamental note of its own, depending upon its size; and whenever a note of this particular pitch is sounded in its neighbourhood, the inclosed air takes it up and intensifies it by resonance. In order to test the presence or absence of a particular harmonic in a given musical tone, a resonator, in unison with this harmonic, is applied to the ear, and if the resonator speaks it is known that the harmonic is present. These instruments are commonly constructed so as to form a series, whose notes correspond to the bass C of a man's voice, and its successive harmonics as far as the 10th or 12th.

Koenig has applied the principle of manometric flames to enable a large number of persons to witness the analysis of sounds by resonators. A series of 6 resonators, whose notes have frequencies proportional to 1, 2, 3, 4, 5, 6, are fixed on a stand (Fig. 48), and their smaller ends, instead of being applied to the ear, are connected each

with a separate manometric capsule, which acts on a gas jet. When the mirrors are turned, it is easy to see which of the flames vibrate while a sonorous body is passed in front of the resonators.

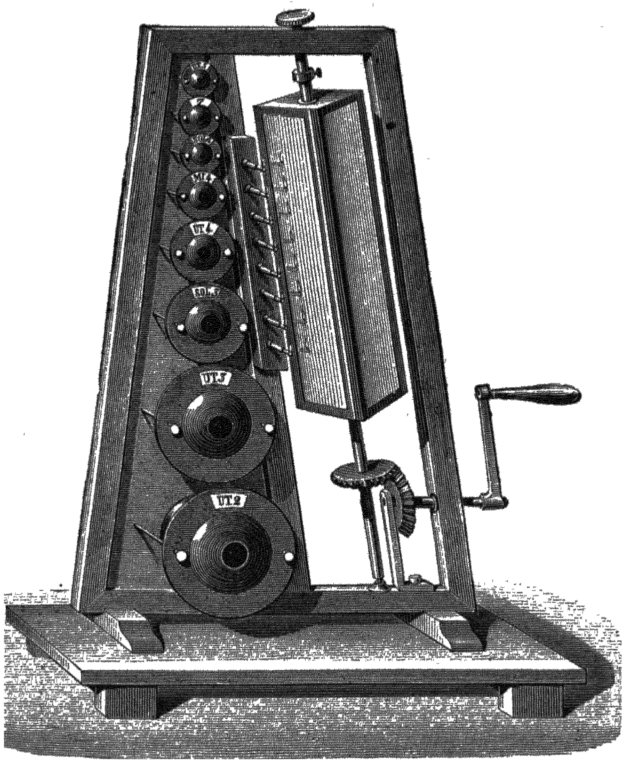


Fig. 48.—Analysis by Manometric Flames.

A simple tone, unaccompanied by harmonics, is dull and uninteresting, and, if of low pitch, is very destitute of penetrating quality.

Sounds composed of the first six elementary tones in fair proportion, are rich and sweet.

The higher harmonics, if sufficiently subdued, may also be present without sensible detriment to sweetness, and are useful as contributing to expression. When too loud, they render a sound harsh and grating; an effect which is easily explained by the discordant combinations which they form one with another; the 8th and 9th tones, for example, are at the same interval as the notes *Do* and *Re*.

62. **Vowel Sounds.**—The human voice is extremely rich in harmonics, as may be proved by applying the series of resonators to the

ear while the fundamental note is sung. The origin of the tones of the voice is in the vocal chords, which, when in use, form a diaphragm with a slit along its middle. The edges of this slit vibrate when air is forced through, and, by alternately opening and closing the passage, perform the part of a reed. The cavity of the mouth serves as a resonance chamber, and reinforces particular notes depending on the position of the organs of speech. It is by this resonance that the various vowel sounds are produced. The deepest pitch belongs to the vowel sound which is expressed in English by *oo* (as in *moon*), and the highest to *ee* (as in *screech*).

Willis in 1828[1] succeeded in producing the principal vowel sounds by a single reed fitted to various lengths of tube. Wheatstone, a few years later, made some advances in theory,[2] and constructed a machine by which nearly all articulate sounds could be imitated.

Excellent imitations of some of the vowel sounds can be obtained by placing Helmholtz's resonators, one at a time, on a free-reed pipe, the small end of the resonator being inserted in the hole at the top of the pipe.

The best determinations of the particular notes which are reinforced in the case of the several vowel sounds, have been made by Helmholtz, who employed several methods, but chiefly the two following:—

1. Holding resonators to the ear, while a particular vowel sound was loudly sung.

2. Holding vibrating tuning-forks in front of the mouth when in the proper position for pronouncing a given vowel; and observing which of them had their sounds reinforced by resonance.[3]

Helmholtz has verified his determinations synthetically. He employs a set of tuning-forks which are kept in vibration by the alternate making and unmaking of electro-magnets, the circuit being made and broken by the vibrations of one large fork of 64 vibrations per second. The notes of the other forks are the successive harmonics of this fundamental note. Each fork is accompanied by a

[1] *Cambridge Transactions*, vol. iii.

[2] *London and Westminster Review*, October, 1837.

[3] According to Koenig (*Comptes Rendus*, 1870) the notes of strongest resonance for the vowels *u*, *o*, *a*, *e*, *i*, as pronounced in North Germany, are the five successive octaves of B flat, commencing with that which corresponds to the space above the top line of the base clef. Willis, Helmholtz, and Koenig all agree as regards the note of the vowel *o*, which is very nearly that of a common A tuning-fork. They are also agreed respecting the note of *a* (as in *father*), which is an octave higher.

resonance-tube, which, when open, renders the note of the fork audible at a distance; and by means of a set of keys, like those of a piano, any of these tubes can be opened at pleasure. The different vowel-sounds can thus be produced by employing the proper combinations.

The same apparatus served for establishing the principle (§ 60), that the character of a musical sound depends mainly on *constitution,* irrespective of change of phase.

63. Phonograph.—Mr. Edison of New York has been successful in constructing an instrument which can reproduce articulate sounds spoken into it. The voice of the speaker is directed into a funnel, which converges the sonorous waves upon a diaphragm carrying a style. The vibrations of the diaphragm are impressed by means of this style upon a sheet of tin-foil, which is fixed on the outside of a cylinder to which a spiral motion is given as in the vibroscope (Fig. 25). After this has been done, the cylinder with the tin-foil on it is shifted back to its original position, the style is brought into contact

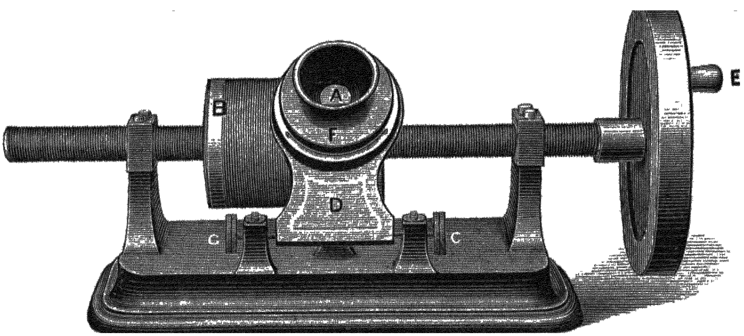


Fig 49.—Phonograph.

with the tin-foil as at first, and the cylinder is then turned as before. The indented record is thus passed beneath the style, and forces it and the attached diaphragm to execute movements resembling their original movements. The diaphragm accordingly emits sounds which are imitations of those previously spoken to it. Tunes sung into the funnel are thus reproduced with great fidelity, and sentences clearly spoken into it are reproduced with sufficient distinctness to be understood.

The instrument is represented in Fig. 49. By turning the handle E, which is attached to a massive fly-wheel, the cylinder B is made

to revolve and at the same time to travel longitudinally, as the axle on which it is mounted is a screw working in a fixed nut. The surface of the cylinder is also fluted screw-fashion, the distance between its flutings being the same as the distance between the threads on the axle. A is the diaphragm, of thin sheet-iron, having the style fixed to its centre but not visible in the figure. The diaphragm and funnel are carried by the frame D, which turns on a hinge at the bottom. C C are adjusting screws for bringing the style exactly opposite the centre of the groove on the cylinder, and another screw is provided beneath the frame D, for making the style project so far as to indent the tin-foil without piercing it. The tin-foil is put round the cylinder, and lightly fastened with cement, so that it can be quickly taken off and changed.

In another form of the instrument, the rotation of the cylinder is effected by means of a driving weight and governor, which give it a constant velocity. This is a great advantage in reproducing music, but is of little or no benefit for speech.

The above description applies to the instrument as first given to the public. In its more recent form, the tin-foil is replaced by a hollow cylinder of wax, which, after receiving the impression of the voice, can be slipped off and preserved for reproduction of the voice at any subsequent time. The diaphragm and style used for reproduction are lighter than those used for impression; and it is stated that words can be reproduced thousands of times from the same impression. The sounds reproduced are conveyed to the ear of the listener through flexible tubes terminating in ear-pieces.

CHAPTER V.

CONSONANCE, DISSONANCE, AND RESULTANT TONES.

64. **Concord and Discord.**—Every one not utterly destitute of musical ear is familiar with the fact that certain notes, when sounded together, produce a pleasing effect by their combination, while certain others produce an unpleasing effect. The combination of two or more notes, when agreeable, is called *concord* or *consonance;* when disagreeable, *discord* or *dissonance.* The distinction is found to depend almost entirely on difference of pitch, that is, on relative frequency of vibration; so that the epithets consonant and dissonant can with propriety be applied to intervals.

The following intervals are consonant: unison (1 : 1), octave (1 : 2), octave + fifth (1 : 3), double octave (1 : 4), fifth (2 : 3), fourth (3 : 4).

The major third (4 : 5) and major sixth (3 : 5), together with the minor third (5 : 6) and minor sixth (5 : 8), are less perfect in their consonance.

The second and the seventh, whether major or minor, are dissonant intervals, whatever system of temperament be employed, as are also an indefinite number of other intervals not recognized in music.

Besides the difference as regards pleasing or unpleasing effect, it is to be remarked that consonant intervals can be identified by the ear with much greater accuracy than those which are dissonant. Musical instruments are generally tuned by octaves and fifths, because very slight errors of excess or defect in these intervals are easily detected by the ear. To tune a piano by the mere comparison of successive notes would be beyond the power of the most skilful musician. A sharply marked interval is always a consonant interval.

65. Jarring Effect of Dissonance.—According to the theory propounded by Helmholtz, the unpleasant effect of a dissonant interval consists essentially in the production of beats. These have a jarring effect upon the auditory apparatus, which becomes increasingly disagreeable as the beats increase in frequency up to a certain limit (about 33 per second for notes of medium pitch), and becomes gradually less disagreeable as the frequency is still further increased. The sensation produced by beats is comparable to that which the eye experiences from the *bobbing* of a gas flame in a room lighted by it; but the frequency which entails the maximum of annoyance is smaller for the eye than for the ear, on account of the greater persistence of visible impressions. The annoyance must evidently cease when the succession becomes so rapid as to produce the effect of a continuous impression.

We have already (§ 23) described a mode of producing beats with any degree of frequency at pleasure; and this experiment is one of the main foundations on which Helmholtz bases his view.

66. Beats of Harmonics.—The beats in the experiment above alluded to, are produced by the imperfect unison of two notes, and indicate the number of vibrations gained by one note upon the other. Their existence is easily and completely explained by the considerations adduced in § 23. But it is well known to musicians, and easily established by experiment, that beats are also produced between notes whose interval is approximately an octave, a fifth, or some other consonance; and that, in these cases also, the beats become more rapid as the interval becomes more faulty.

These beats are ascribed by Helmholtz to the common harmonic of the two fundamental notes. For example, in the case of the fifth (2 : 3), the third tone of the lower note would be identical with the second tone of the upper, if the interval were exact; and the beats which occur are due to the imperfect unison consequent on the deviation from exact truth. All beats are thus explained as due to imperfect *unison*.

This explanation is not merely conjectural, but is established by the following proofs:—

1. When an arrangement is employed by which the fifth is made false by a known amount, the number of beats is found to agree with the above explanation. Thus, if the interval is made to correspond to the ratio 200 : 301, it is observed that there are 2 beats to every 200 vibrations of the lower note. Now the harmonics which

are in approximate unison are represented by 600 and 602, and the difference of these is 2.

2. When the resonator corresponding to this common harmonic is held to the ear, it responds to the beats, showing that this harmonic is undergoing variations of strength; but when a resonator corresponding to either of the fundamental notes is employed, it does not respond to the beats, but indicates steady continuance of its appropriate note.

3. By a careful exercise of attention, a person with a good ear can hear, without any artificial aids, that it is the common harmonic which undergoes variations of intensity, and that the fundamental notes continue steady.

67. **Beating Notes must be Near Together.**—In order that two simple tones may yield audible beats, it is necessary that the musical interval between them should be small, in other words, that the ratio of their frequencies of vibration should be nearly equal to unity. Two simple notes of 300 and 320 vibrations per second will yield 20 beats in a second, and will be eminently discordant, the interval between them being only a semitone (15 : 16), but simple notes of 40 and 60 vibrations per second will not give beats, the interval between them being a fifth (2 : 3). The wider the interval between two simple notes, the feebler will be their beats; and accordingly, for a given frequency of beats, the harshness of the effect increases with the nearness of the notes to each other on the musical scale.[1] By taking joint account of the number of beats and the nearness of the beating tones, Helmholtz has endeavoured to express numerically the severity of the discords resulting from the combination of the note C of 256 vibrations per second with any possible note lying within an octave on the upper side of it, a particular constitution (approximately that of the violin) being assumed for both notes. He finds a complete absence of discord for the intervals of unison, the octave, and the fifth, and very small amounts of discord for the fourth, the sixth, and the third. By far the worst discords are found for the intervals of the semitone and major seventh,

[1] The explanation adopted by Helmholtz is, that a certain part of the ear—the *membrana basilaris*—is composed of tightly stretched elastic fibres, each of which is attuned to a particular simple tone, and is thrown into vibration when this tone, or one nearly in unison with it, is sounded. Two tones in approximate unison, when sounded together, affect several fibres in common, and cause them to beat. Tones not in approximate unison affect entirely distinct sets of fibres, and thus cannot produce interference.

and the next worst are for intervals a little greater or less than the fifth.

68. **Imperfect Concord.**—When there is a complete absence of discord between two notes, they are said to form a perfect concord. The intervals unison, fifth, octave, octave + fifth, and the interval from any note to any of its harmonics, are of this class. The third, fourth, and sixth are instances of imperfect concord. Suppose, for example, that the two notes sounded together are C of 256 and E of 320 vibrations per second, the interval between these notes being a true major third (4 : 5); and suppose each of these notes to consist of the first six simple tones.

The first six multiples of 4 are

4, 8, 12, 16, 20, 24.

The first six multiples of 5 are

5, 10, 15, 20, 25, 30.

In searching for elements of discord, we select (one from each line) two multiples differing by unity.

Those which satisfy this condition are

4 and 5; 16 and 15; 24 and 25.

But the first pair (4 and 5) may be neglected, because their ratio differs too much from unity. Discordance will result from each of the two remaining pairs; that is to say, the 4th element of the lower of our two given notes is in discordance with the 3d element of the upper; and the 6th element of the lower is in discordance with the 5th element of the higher. To find the frequencies of the beats, we must multiply all these numbers by 64, since 256 is 4 times 64, and 320 is 5 times 64. Instead of a difference of 1, we shall then find a difference of 64, that is to say, the number of beats per second is 64 in the case of each of the two discordant combinations which we have been considering.

69. **Resultant Tones.**—Under certain conditions it is found that two notes, when sounded together, produce by their combination other notes, which are not constituents of either. They are called *resultant tones*, and are of two kinds, *difference-tones* and *summation-tones*. A difference-tone has a frequency of vibration which is the difference of the frequencies of its components. A summation-tone has a frequency of vibration which is the sum of the

frequencies of its components. As the components may either be fundamental tones or overtones, two notes which are rich in harmonics may yield, by their combination, a large number of resultant tones.

The difference-tones were observed in the last century by Sorge and by Tartini, and were, until recently, attributed to beats. The frequency of beats is always the difference of the frequencies of vibration of the two elementary tones which produce them; and it was supposed that a rapid succession of beats produced a note of pitch corresponding to this frequency.

This explanation, if admitted, would furnish an exception to what otherwise appears to be the universal law, that every *elementary tone* arises from a corresponding *simple vibration.*[1] Such an exception should not be admitted without necessity; and in the present instance it is not only unnecessary, but also insufficient, inasmuch as it fails to render any account of the summation-tones.

Helmholtz has shown, by a mathematical investigation, that when two systems of simple waves agitate the same mass of air, their mutual influence must, according to the recognized laws of dynamics, give rise to two derived systems, having frequencies which are respectively the sum and the difference of the frequencies of the two primary systems. Both classes of resultant tones are thus completely accounted for.

The resultant tones—especially the summation-tones, which are fainter than the others—are only audible when the primary tones are loud; for their existence depends upon small quantities of the second order, the amplitudes of the primaries being regarded (in comparison with the wave-lengths) as small quantities of the first order.

If any further proof be required that the difference tones are not due to the coalescence of beats, it is furnished by the fact that, under favourable conditions, the rattle of the beats and the booming of the difference-tones can both be heard together.

70. Beats due to Resultant Tones.—The existence of resultant tones serves to explain, in certain cases, the production of beats between notes which are wanting in harmonics. For example, if two *simple* sounds, of 100 and 201 vibrations per second respectively, are sounded together, one beat per second will be produced between

[1] The discovery of this law is due to Ohm.

the difference-tone of 101 vibrations and the primary tone of 100 vibrations. By the beats to which they thus give rise, resultant tones exercise an influence on consonance and dissonance.

Resultant tones, when sufficiently loud, are themselves capable of performing the part of primaries, and yielding what are called *resultant tones of the second order*, by their combination with other primaries. Several higher orders of resultant tones can, under peculiarly favourable circumstances, be sometimes detected.

OPTICS.

CHAPTER VI.

PROPAGATION OF LIGHT.

71. **Light.**—Light is the immediate external cause of our visual impressions. Objects, except such as are styled *self-luminous*, become invisible when brought into a dark room. The presence of something additional is necessary to render them visible, and that mysterious agent, whatever its real nature may be, we call *light*.

Light, like sound, is believed to consist in vibration; but it does not, like sound, require the presence of air or other gross matter to enable its vibrations to be propagated from the source to the percipient. When we exhaust a receiver, objects in its interior do not become less visible; and the light of the heavenly bodies is not prevented from reaching us by the highly vacuous spaces which lie between.

It seems necessary to assume the existence of a medium far more subtle than ordinary matter; a medium which pervades alike the most vacuous spaces and the interior of all bodies, whether solid, liquid, or gaseous; and which is so highly elastic, in proportion to its density, that it is capable of transmitting vibrations with a velocity enormously transcending that of sound.

This hypothetical medium is called *æther*. From the extreme facility with which bodies move about in it, we might be disposed to call it a subtle *fluid;* but the undulations which it serves to propagate are not such as can be propagated by fluids. Its elastic properties are rather those of a solid; and its waves are analogous to the pulses which travel along the wires of a piano rather than to the waves of extension and compression by which sound is propagated through air. *Luminous vibrations are transverse, while those of sound are longitudinal.*

A self-luminous body, such as a red-hot poker or the flame of a

candle, is in a peculiar state of vibration. This vibration is communicated to the surrounding æther, and is thus propagated to the eye, enabling us to see the body. In the majority of cases, however, we see bodies not by their own but by reflected light; and we are enabled to recognize the various kinds of bodies by the different modifications which light undergoes in reflection from their surfaces.

As all bodies can become sonorous, so also all bodies can become self-luminous. To render them so, it is only necessary to raise them to a sufficiently high temperature, whether by the communication of heat from a furnace, or by the passage of an electric current, or by causing them to enter into chemical combination. It is to chemical combination, in the active form of combustion, that we are indebted for all the sources of artificial light in ordinary use.

The vibrations of the æther are capable of producing other effects besides illumination. They constitute what is called radiant heat, and they are also capable of producing chemical effects, as in photography. Vibrations of high frequency, or short period, are the most active chemically. Those of low frequency or long period have usually the most powerful heating effects; while those which affect the eye with the sense of light are of moderate frequency.

72. **Rectilinear Propagation of Light.**—All the remarks which have been made respecting the relations between period, frequency, and wave-length, in the case of sound, are equally applicable to light, inasmuch as all kinds of luminous waves (like all kinds of sonorous waves) have sensibly the same velocity in air; but this velocity is many hundreds of thousands of times greater for light than for sound, and the wave-lengths of light are at the same time very much shorter than those of sound. Frequency, being the quotient of velocity by wave-length, is accordingly about a million of millions of times greater for light than for sound. The colour of lowest pitch is deep red, its frequency being about 400 million million vibrations per second, and its wave-length in air 760 millionths of a millimetre. The colour of highest pitch is deep violet; its frequency is about 760 million million vibrations per second, and its wave-length in air 400 millionths of a millimetre. It thus appears that the range of seëing is much smaller than that of hearing, being only about one octave.

The excessive shortness of luminous as compared with sonorous waves is closely connected with the strength of the shadows cast by a light, as compared with the very moderate loss of intensity produced by interposing an obstacle in the case of sound. Sound may,

for ordinary purposes, be said to be capable of turning a corner, and light to be only capable of travelling in straight lines. The latter fact may be established by such an arrangement as is represented in Fig. 50. Two screens, each pierced with a hole, are arranged so that these holes are in a line with the flame of a candle. An eye placed in this line, behind the screens, is then able to see the flame; but a slight lateral displacement, either of the eye, the candle, or either of the screens, puts the flame out of sight. It is to be noted that, in this experiment, the same medium (air) extends from the eye to the candle. We shall hereafter find that, when light has to pass from one medium to another, it is often bent out of a straight line.

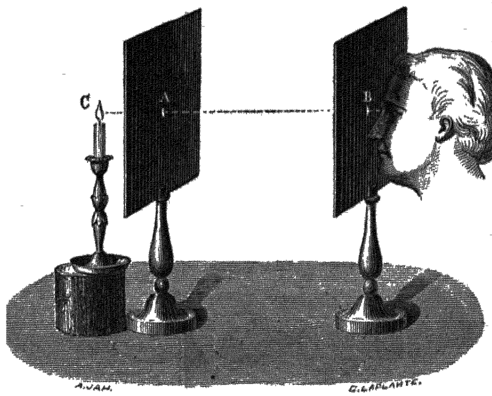


Fig 50.—Rectilinear Propagation.

We have said that the strength of light-shadows as compared with sound-shadows is connected with the shortness of luminous waves. Theory shows that, if light is transmitted through a hole or slit whose diameter is a very large multiple of the length of a light-wave, a strong shadow should be cast in all oblique directions; but that, if the hole or slit is so narrow that its diameter is comparable to the length of a wave, a large area not in the direct path of the beam will be illuminated. The experiment is easily performed in a dark room, by admitting sunlight through an exceedingly fine slit, and receiving it on a screen of white paper. The illuminated area will be marked with coloured bands, called diffraction-fringes; and if the slit is made narrower, these bands become wider.

On the other hand, Colladon, in his experiments on the transmission of sound through the water of the Lake of Geneva, established the presence of a very sharply defined sound-shadow in the water behind the end of a projecting wall.

For the present we shall ignore diffraction,[1] and confine our atten-

[1] See Chap. xiv.

tion to the numerous phenomena which result from the rectilinear propagation of light.

73. **Images produced by Small Apertures.**—If a white screen is placed opposite a hole in the shutter of a room otherwise quite dark,

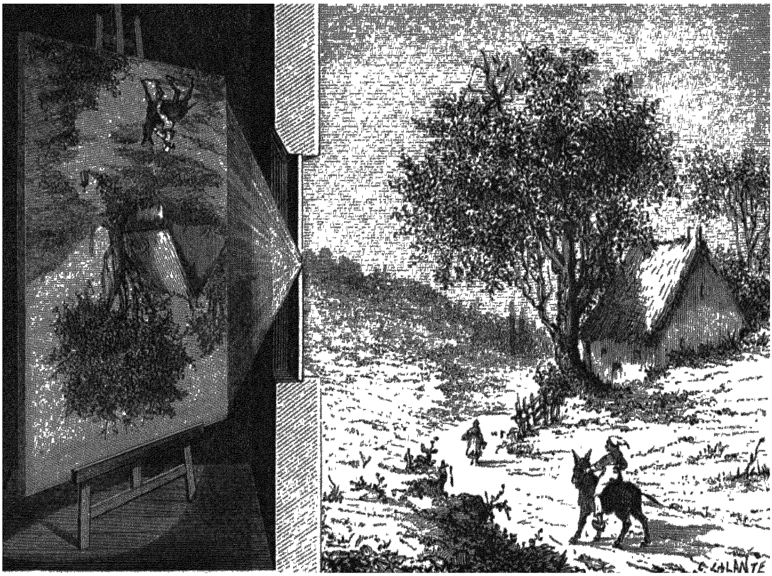


Fig 51.—Image formed by Small Aperture.

an inverted picture of the external landscape will be formed upon it, in the natural colours. The outlines will be sharper in proportion as the hole is smaller, and distant objects will be more distinctly represented than those which are very near.

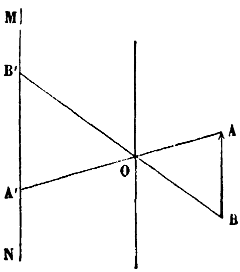


Fig. 52.—Explanation.

These results are easily explained. Consider, in fact, an external object A B (Fig. 52), and let O be the hole in the shutter. The point A sends rays in all directions into space, and among them a small pencil, which, after passing through the opening O, falls upon the screen at A′. A′ receives light from no other point but A, and A sends light to no part of the screen except A′. The colour and brightness of the spot A′ will accordingly depend upon the colour and brightness of A; in other words, A′ will be the

image of A. In like manner B′ will be the image of B, and points of the object between A and B will have their images between A′ and B′. An inverted image A′ B′ will thus be formed of the object A B.

As the image thus formed of an external point is not a point, but a spot, whose size increases with that of the opening, there must always be a little blurring of the outlines from the overlapping of the spots which represent neighbouring points; but this will be comparatively slight if the opening is very small.

An experiment, substantially the same as the above, may be performed by piercing a card with a large pin-hole, and holding it between

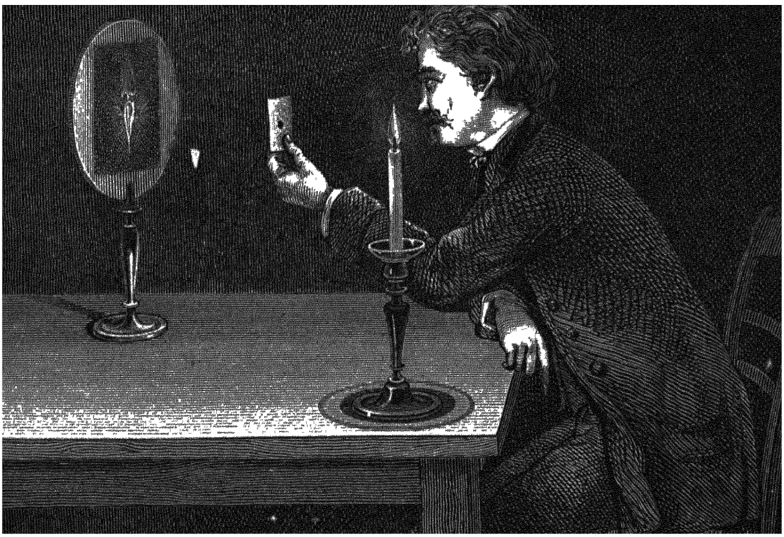


Fig 53.—Image formed by Hole in a Card.

a candle and a screen, as in Fig. 53. An inverted image of the candle will thus be formed upon the screen.

When the sun shines through a small hole into a room with the blinds down (Fig. 54), the cone of rays thus admitted is easily traced by the lighting up of the particles of dust which lie in its course. The image of the sun which is formed at its further extremity will be either circular or elliptical, according as the incidence of the rays is normal or oblique. Fine images of the sun are sometimes thus formed by the chinks of a venetian-blind, especially when the sun is low, and there is a white wall opposite to receive the

image. In these circumstances it is sometimes possible to detect the presence of spots on the sun by examining the image.

When the sun's rays shine through the foliage of a tree, the spots of light which they form upon the ground are always round or oval, whatever may be the shape of the interstices through which they have passed, provided always that these interstices are small. When the sun is undergoing eclipse, the progress of the eclipse can

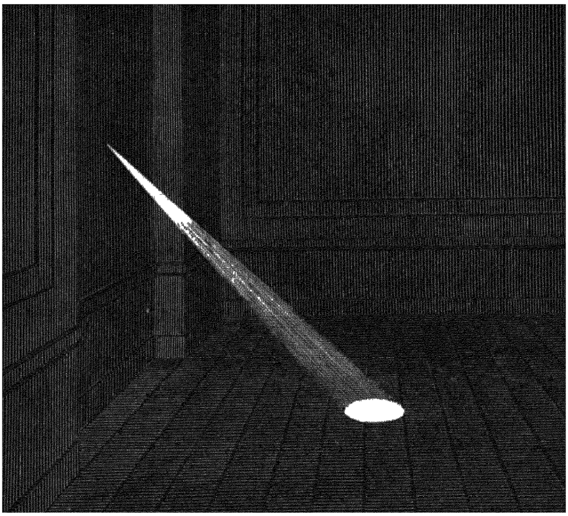


Fig 54.—Conical Sunbeam.

be traced by watching the shape of these images, which resembles that of the uneclipsed portion of the sun's disc.

74. Theory of Shadows.—The rectilinear propagation of light is the foundation of the geometry of shadows. Let the source of light be a luminous point, and let an opaque body be placed so as to intercept a portion of its rays. If we construct a conical surface touching the body all round, and having its vertex at the luminous point it is evident that all the space within this surface on the further side of the opaque body is completely screened from the rays. The cone thus constructed is called the shadow-cone, and its intersection with any surface behind the opaque body defines the shadow cast upon that surface. In the case which we have been supposing—that of a luminous point—the shadow-cone and the shadow itself will be sharply defined.

Actual sources of light, however, are not mere luminous points, but have finite dimensions. Hence some complication arises. Consider, in fact (Fig. 55), a luminous body situated between two opaque bodies, one of them larger, and the other smaller than itself. Conceive a cone touching the luminous body and either of the opaque bodies *externally*. This will be the cone of *total shadow*, or the cone of the *umbra*. All points lying within it are completely excluded from view of the luminous body. This cone narrows or enlarges as it recedes, according as the opaque body is smaller or larger than the luminous body. In the former case it terminates at a finite distance. In the latter case it extends to infinite distance.

Now conceive a double cone touching the luminous body and either of the opaque bodies *internally*. This cone will be wider than the cone of total shadow, and will include it. It is called the

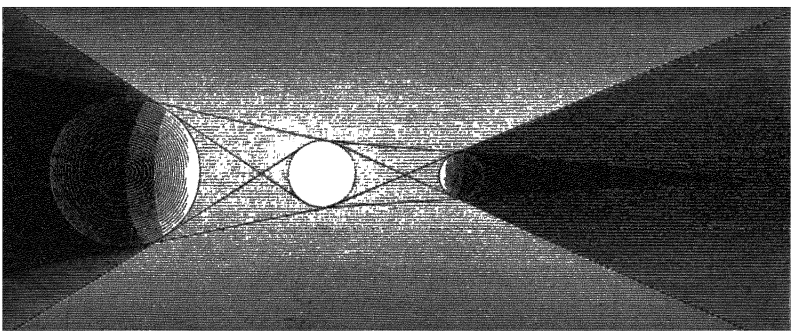


Fig. 55.—Umbra and Penumbra.

cone of *partial shadow*, or the cone of the *penumbra*. All points lying within it are excluded from the view of some portion of the luminous body, and are thus partially shaded by the opaque body. If they are near its outer boundary, they are very slightly shaded. If they are so far within it as to be near the total shadow, they are almost completely shaded. Accordingly, if the shadow of the opaque body is received upon a screen, it will not have sharply defined edges, but will show a gradual transition from the total shadow which covers a finite central area to a complete absence of shadow at the outer boundary of the penumbra. Thus neither the edges of the umbra nor those of the penumbra are sharply defined.

The umbra and penumbra show themselves on the surface of the

opaque body itself, the line of contact of the umbral cone being further back from the source of light than the line of contact of the penumbral cone. The zone between these two lines is in partial shadow, and separates the portion of the surface which is in total shadow from the part which is not shaded at all.

75. **Velocity of Light.**—Luminous undulations, unlike those of sound, advance with a velocity which may fairly be styled inconceivable, being about 300 million metres per second, or 186,000 miles per second. As the circumference of the earth is only 40 million metres, light would travel seven and a half times round the earth in a second.

Hopeless as it might appear to attempt the measurement of such an enormous velocity by mere terrestrial experiments, the feat has actually been performed, and that by two distinct methods. In Fizeau's experiments the distance between the two experimental stations was about 5½ miles. In Foucault's experiments the whole apparatus was contained in one room, and the movement of light within this room served to determine the velocity.

We will first describe Fizeau's experiment.

76. **Fizeau's Experiment.**—Imagine a source of light placed directly in front of a plane mirror, at a great distance. The mirror will send back a reflected beam along the line of the incident beam, and an observer stationed behind the source will see its image in the mirror as a luminous point.

Now imagine a toothed-wheel, with its plane perpendicular to the path of the beam, revolving uniformly in front of the source, in such a position that its teeth pass directly between the source of light and the mirror. The incident beam will be stopped by the teeth, as they successively come up, but will pass through the spaces between them. Now the velocity of the wheel may be such that the light which has thus passed through a space shall be reflected back from the mirror just in time to meet a tooth and be stopped. In this case it will not reach the observer's eye, and the image may thus become permanently invisible to him. From the velocity of the wheel, and the number of its teeth, it will be possible to compute the time occupied by the light in travelling from the wheel to the mirror, and back again. If the velocity of the wheel is such that the light is sometimes intercepted on its return, and sometimes allowed to pass, the image will appear steadily visible, in consequence of the persistence of impressions on the retina, but with a

loss of brightness proportioned to the time that the light is intercepted. The wheel employed by Fizeau had 720 teeth, the distance between the two stations was 8663 metres, and 12·6 revolutions per second produced disappearance of the image. The width of the teeth being equal to the width of the spaces, the time required to turn through the width of a tooth was $\frac{1}{2} \times \frac{1}{720} \times \frac{1}{12 \cdot 6}$ of a second, that is $\frac{1}{18144}$ of a second.

In this time the light travelled a distance of $2 \times 8663 = 17326$ metres. The distance traversed by light in a second would therefore be $17{,}326 \times 18{,}144 = 314{,}262{,}944$ metres. This determination of M. Fizeau's is believed to be somewhat in excess of the truth.

A double velocity of the wheel would allow the reflected beam to pass through the space succeeding that through which the incident beam had passed; a triple velocity would again produce total eclipse, and so on. Several independent determinations of the velocity of light may thus be obtained.

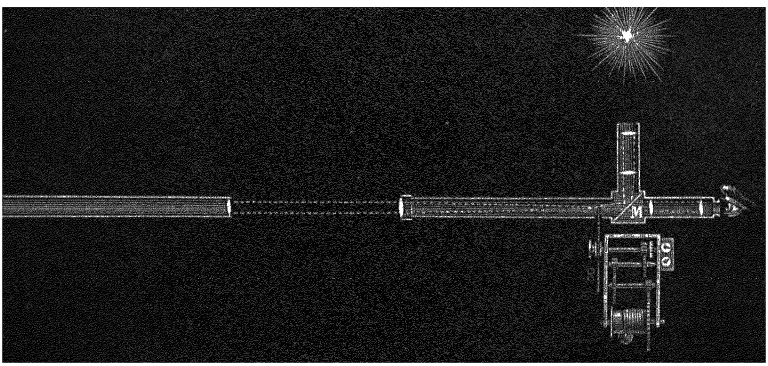


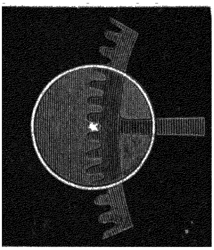
Fig. 56.—Fizeau's Experiment.

Thus far, we have merely indicated the principle of calculation. It will easily be understood that special means were necessary to prevent scattering of the light, and render the image visible at so great a distance. Fig. 56 will serve to give an idea of the apparatus actually employed.

A beam of light from a lamp, after passing through a lens, falls on a plate of unsilvered glass M, placed at an angle of 45°, by which it is reflected along the tube of a telescope; the object-glass of the telescope is so adjusted as to render the rays parallel on emergence, and in this condition they traverse the interval

between the two stations. At the second station they are collected by a lens, which brings them to a focus on the surface of a mirror, which sends them back along the same course by which they came. A portion of the light thus sent back to the glass plate M passes through it, and is viewed by the observer through an eye-piece.

The wheel R is driven by clock-work. Figs. 57, 58, 59 respec-



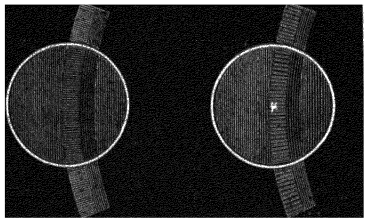


Fig. 57.—Wheel at Rest. Fig. 58.—Total Eclipse Fig. 59.—Partial Eclipse.

tively represent the appearance of the luminous point as seen between the teeth of the wheel when not revolving, the total eclipse produced by an appropriate speed of rotation, and the partial eclipse produced by a different speed.

More recently M. Cornu has carried out an extensive series of experiments on the same plan, with more powerful appliances, the distance between the two stations being 23 kilometres, and the extinctions being carried to the 21st order. His result is that the velocity of light (in millions of metres per second) is 300·33 in air, or 300·4 *in vacuo*.

77. **Foucault's Experiment.**—Foucault employed the principle of the rotating mirror, first adopted by Wheatstone in his experiments on the duration of the electric spark and the velocity of electricity (§ 36, 38, Part III.). The following was the construction of his original apparatus:—

A beam of light enters a room by a square hole, which has a fine platinum wire stretched across it, to serve as a mark; it is then concentrated by an achromatic lens, and, before coming to a focus, falls upon a plane mirror, revolving about an axis in its own plane. In one part of the revolution the reflected beam is directed upon a concave mirror, whose centre of curvature is in the axis of rotation, so that the beam is reflected back to the revolving mirror, and

thence back to the hole at which it first entered. Before reaching the hole, it has to traverse a sheet of glass, placed at an angle of 45°, which reflects a portion of it towards the observer's eye; and the image which it forms (an image of the platinum wire) is viewed through a powerful eye-piece. The image is only formed during a small part of each revolution; but when 30 turns are made per second, the appearance presented, in consequence of the persistence of impressions, is that of a permanent image occupying a fixed position. When the speed is considerably greater, the mirror turns through a sensible angle while the light is travelling from it to the concave mirror and back again, and a sensible displacement of the image is accordingly observed. The actual speed of rotation was from 700 to 800 revolutions per second.[1]

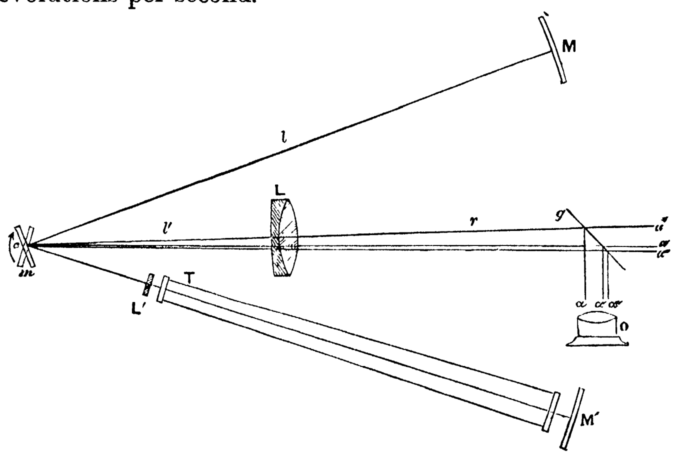


Fig 60.—Foucault's Experiment

On interposing a tube filled with water between the two mirrors, it was found that the displacement was increased, showing that a longer time was occupied in traversing the water than in traversing the same length of air.

This result, as we shall have occasion to point out later, is very important as confirming the undulatory theory and disproving the emission theory of light.

In Fig. 60, a is the position of the platinum wire, L is the achromatic lens, m the revolving mirror, c the axis of revolution, M

[1] It was found that, at this high speed, the amalgam at the back of ordinary looking-glasses was driven off by centrifugal force. The mirror actually employed was silvered in front with real silver.

the concave mirror, a' the image of the platinum wire, displaced from a in virtue of the rotation of the mirror; α, α' images of a, a', formed by the glass plate g, and viewed through the eye-piece O.

M′ is a second concave mirror, at the same distance as M from the revolving mirror; T is a tube filled with water, and having plane glass ends, and L′ a lens necessary for completing the focal adjustment; a'' and α'' are the images formed by the light which has traversed the water.[1]

Foucault's experiment, as thus described, was performed in 1850 very shortly after that of Fizeau, and was mainly designed for giving a relative determination of the velocities in air and water. Foucault subsequently adapted it to absolute measurement, and determined the velocity in air to be 298 million metres per second.

78. **Later Determinations.**—Professor Michelson of the United States carried out in 1879 and 1882 two excellent series of experiments, in which the lens L, which was 8 inches in diameter and of admirable quality, was placed not between the slit and the revolving mirror but between the revolving and the fixed mirror, in such a position that the sum of the distances of the slit and lens from the revolving mirror was a very little greater than the focal length of

[1] The distances are such that La and Lc+cM are conjugate focal distances with respect to the lens L. An image of the wire a is thus formed at M, and an image of this image is formed at a, the mirror being supposed stationary; and this relation holds not only for the central point of the concave mirror, but for any part of it on which the light may happen to fall at the instant considered.

Let l denote the distance cM between the revolving and the fixed mirror, l' the distance cL of the revolving mirror from the centre of the lens, r the distance aL of the platinum wire from the centre of the lens, n the number of revolutions per second, V the space traversed by light in a second, t the time occupied by light in travelling from one mirror to the other and back, θ the angle turned by the mirror in this time, and δ the angle subtended at the centre of the lens by the distance $a\,a'$ between the wire and its displaced image.

Then obviously $t=\frac{2l}{V}$, but also $t=\frac{\theta}{2\pi n}$; hence $V=\frac{4\pi n l}{\theta}$.

Now the distance between the two images (corresponding to a, a' respectively) at the back of the revolving mirror is $(l+l')\,\delta$, and is also $2\,\theta\,l$ (§ 98). Hence $\theta=\frac{(l+l')\,\delta}{2l}$, and $V=\frac{8\pi n l^2}{(l+l')\delta}$. The observed distance $a\,a'$ between the two images is equal to the distance between a, a', that is to $r\,\delta$. Calling this distance d, we have finally,

$$V=\frac{8\,\pi\,n\,l^2 r}{(l+l')\,d}.$$

the lens.[1] The image of the slit was accordingly formed at a very great distance on the other side of the lens, and it was at this distance that the fixed mirror M was placed. The focal length of the lens was 150 ft., and the distance between the two mirrors nearly 2000 ft. The measured deviation of the image of the slit from the slit itself was due to the angle through which the mirror turned while light travelled over twice this distance, or nearly 4000 ft.; and the distance of the slit from the mirror being about 30 ft., the deviation of the image from the slit amounted to more than 133 millimetres, whereas the deviation obtained by Foucault was less than 1 millimetre The results of different experiments were in extremely close accordance, and the velocity *in vacuo* finally deduced was 299·91 from the first series, and 299·85 from the second series.

Experiments on a still larger scale were made in 1882 by Professor Newcomb of the U. S. Naval Observatory, Washington, the distance between the revolving and the fixed mirror being in some of the observations 2550 metres, and in others 3720 metres. The method employed was in principle the same as in Foucault's original experiment. The revolving mirror was four-sided, like that in Fig. 48, p. 73, but made of polished steel, and driven by a blast of air impinging on vanes. The speed of revolution was measured by a self-recording apparatus which, by breaking an electric current, made a mark once in every 28 revolutions upon paper, on which a mark was also made every second by a chronograph.

The source of light was a slit illuminated by the rays of the sun reflected from a heliostat. The slit was in the principal focus of a collimating lens, so that a parallel beam of light fell upon the revolving mirror, and was reflected to and from the distant station. On its return it was reflected by the revolving mirror into an observing telescope furnished with two parallel wires near together in

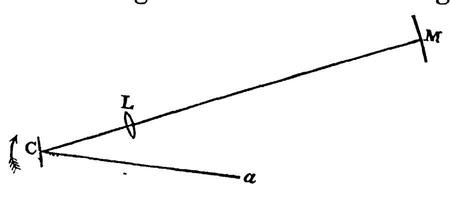


Fig 61

[1] The calculation in this case is rather simpler. L M and L C + C a (Fig. 61) are conjugate focal distances with respect to the lens L. While the light is going from C to M and back, the mirror turns through an angle θ, and the image which would otherwise be formed at a is formed at a', the distance $a a'$ being $2\theta . Ca$, θ is $2\pi n t$ as before, and t is $2CM/V$. Hence $a a' = 2Ca . 2\pi n . 2CM/V$ and $V = 8\pi n \frac{Ca . CM}{a a'}$.

the focus of its eye-piece, between which the image was made to fall. This telescope was so mounted that, while always directed centrally on the revolving mirror, it could be moved through about 4° on each side of the position of no deviation; and in actual use it was moved into such a position that the displaced image of the slit could be kept steadily between the two parallel wires, the regulation of the displacement being effected by means of two cords which governed the blast of air. The deviation amounted in some of the experiments to 3° on each side, the arrangements being such that the mirror could be turned either way. The result deduced was a velocity *in vacuo* of 299·86 million metres per second (about 186,300 miles per second), which, being practically identical with Michelson's, may be accepted as the most probable value.

79. Velocity of Light deduced from Observations of the Eclipses of Jupiter's Satellites.—The fact that light occupies a sensible time in travelling over celestial distances, was first established about 1675, by Roemer, a Danish astronomer, who also made the first computation of its velocity. He was led to this discovery by comparing the observed times of the eclipses of Jupiter's first satellite, as contained in records extending over many successive years.

The four satellites of Jupiter revolve nearly in the plane of the planet's orbit, and undergo very frequent eclipse by entering the cone of total shadow cast by Jupiter. The satellites and their eclipses are easily seen, even with telescopes of very moderate power, and being visible at the same absolute time at all parts of the earth's surface at which they are visible at all, they serve as signals for comparing local time at different places, and thus for determining longitudes. The first satellite (that is, the one nearest to Jupiter), from its more rapid motion and shorter time of revolution, affords both the best and the most frequent signals. The interval of time between two successive eclipses of this satellite is about 42½ hours, but was found by Roemer to vary by a regular law according to the position of the earth with respect to Jupiter. It is longest when the earth is increasing its distance from Jupiter most rapidly, and is

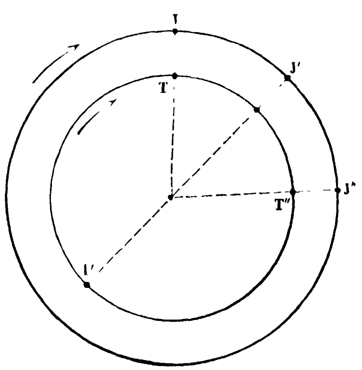


Fig. 62.—Earth and Jupiter.

shortest when the earth is diminishing its distance most rapidly. Starting from the time when the earth is nearest to Jupiter, as at T, J (Fig. 62), the intervals between successive eclipses are always longer than the mean value, until the greatest distance has been attained, as at T′ J′, and the sum of the excesses amounts to 16 min. 26·6 sec. From this time until the nearest distance is again attained, as at T″, J″, the intervals are always shorter than the mean, and the sum of the defects amounts to 16 min. 26·6 sec. It is evident, then, that the eclipses are visible 16 m. 26·6 s. earlier at the nearest than at the remotest point of the earth's orbit, in other words, that this is the time required for the propagation of light across the diameter of the orbit. Taking this diameter as 184 millions of miles,[1] we have a resulting velocity of about 186,500 miles per second.

80. Velocity of Light deduced from Aberration.—About fifty years after Roemer's discovery, Bradley, the English astronomer, employed the velocity of light to explain the astronomical phenomenon called *aberration*. This consists in a regular periodic displacement of the stars as seen from the earth, the period of the displacement being a year. If the direction in which the earth is moving in its orbit at any instant be regarded as the *forward* direction, every star constantly appears on the forward side of its true place, so that, as the earth moves once round its orbit in a year, each star describes in this time a small apparent orbit about its true place.

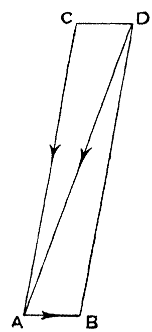


Fig 63
Aberration.

The phenomenon is explained in the same way as the familiar fact, that a shower of rain falling vertically, seems, to a person running forwards, to be coming in his face. The relative motion of the raindrops with respect to his body, is found by compounding the actual velocity of the drops (whether vertical or oblique) with a velocity equal and opposite to that with which he runs. Thus if A B (Fig. 63) represents the velocity with which he runs, and C A, the true velocity of the drops, the apparent velocity of the drops will be represented by D A. If a tube pointed along A D moves forward parallel to itself with the velocity A B, a drop entering at its upper end will pass through its whole length without wetting

[1]The sun's mean distance from the earth was, until recently, estimated at 95 millions of miles. It is now estimated at 92½ or 93 millions.

its sides, for while the drop is falling along D B (we suppose with uniform velocity) the tube moves along A B, so that the lower end of the tube reaches B at the same time as the rain-drop.

In like manner, if A B is the velocity of the earth, and C A the velocity of light, a telescope must be pointed along A D to see a star which really lies in the direction of A C or B D produced. When the angle B A C is a right angle (in other words, when the star lies in a direction perpendicular to that in which the earth is moving), the angle C A D, which is called the aberration of the star, is $20''\cdot5$, and the tangent of this angle is the ratio of the velocity of the earth to the velocity of light. Hence it is found by computation that the velocity of light is about ten thousand times greater than that with which the earth moves in its orbit. The latter is easily computed, if the sun's distance is known, and is about $18\frac{1}{2}$ miles per second. Hence the velocity of light is about 185,000 miles per second. It will be noted that both these astronomical methods of computing the velocity of light, depend upon the knowledge of the sun's distance from the earth, and that, if this distance is overestimated, the computed velocity of light will be too great in the same ratio.

Conversely, the velocity of light, as determined by Foucault's method, can be employed in connection either with aberration or the eclipses of the satellites, for computing the sun's distance; and the first correct determination of the sun's distance was, in fact, that deduced by Foucault from his own results.

81. Photometry.—Photometry is the measurement of the relative amounts of light emitted by different sources. The methods employed for this purpose all consist in determinations of the relative distances at which two sources produce equal intensities of illumination. The eye would be quite incompetent to measure the ratio of two unequal illuminations; but a pretty accurate judgment can be formed as to quality or inequality of illumination, at least when the surfaces compared are similar, and the lights by which they are illuminated are of the same colour. The law of inverse squares is always made the basis of the resulting calculations; and this law may itself be verified by showing that the illumination produced by one candle at a given distance is equal to that produced by four candles at a distance twice as great.

82. Bouguer's Photometer.—Bouguer's photometer consists of a semi-transparent screen, of white tissue paper, ground glass, or thin white porcelain, divided into two parts by an opaque partition at

right angles to it. The two lamps which are to be compared are placed one on each side of this partition, so that each of them illuminates one-half of the transparent screen. The distances of the two lamps are adjusted until the two portions of the screen, as seen

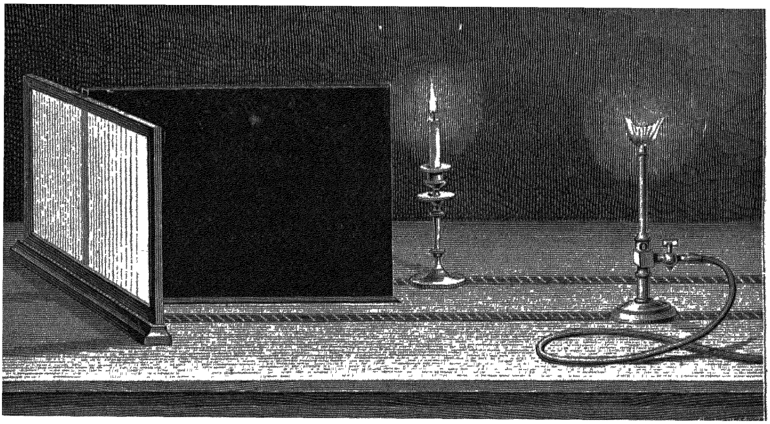


Fig. 64.—Bouguer's Photometer.

from the back, appear equally bright. The distances are then measured, and their squares are assumed to be directly proportional to the illuminating powers of the lamps.

83. Rumford's Photometer.—Rumford's photometer is based on the comparison of shadows. A cylindric rod is so placed that each of the two lamps casts a shadow of it on a screen; and the distances are adjusted until the two shadows are equally dark. As the shadow thrown by one lamp is illuminated by the other lamp, the comparison of shadows is really a comparison of illuminations.

84. Foucault's Photometer.—The two photometers just described are alike in principle. In each of them the two surfaces compared are illuminated each by one only of the sources of light. In Rumford's the remainder of the screen is illuminated by both. In Bouguer's it consists merely of an intervening strip which is illuminated by neither. If the partition is movable, the effect of moving it further from the screen will be to make this dark strip narrower until it disappears altogether; and if it be advanced still further, the two illuminated portions will overlap. In Foucault's photometer there is an adjusting screw, for the purpose of advancing the partition so far that the dark strip shall just vanish. The two illuminated

portions, being then exactly contiguous, can be more easily and certainly compared.

85. **Bunsen's Photometer.**—In the instruments above described the two sources to be compared are both on the same side of the screen, and illuminate different portions of it. Bunsen introduced the plan of placing the sources on opposite sides of the screen, and making the screen consist of two parts, one of them more translucent

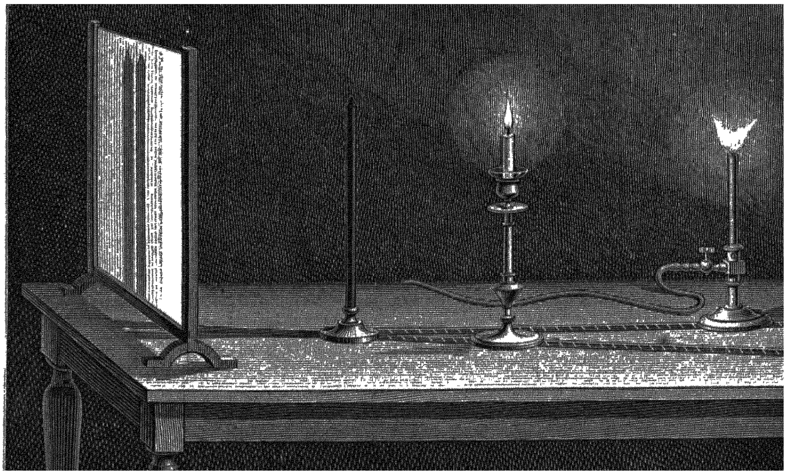


Fig. 65.—Rumford's Photometer.

than the other. In his original pattern the screen was a sheet of white paper, with a large grease spot in the centre. In Dr. Letheby's pattern it is composed of three sheets of paper, laid face to face, the middle one being very thin, and the other two being cut away in the centre, so that the central part of the screen consists of one thickness, and the outer part of three.

When such a screen is more strongly illuminated on one side than on the other, the more translucent part appears brighter than the less translucent when seen from the darker side, while the reverse appearance is presented on the brighter side. It is therefore the business of the observer so to adjust the distances that the central and circumferential parts appear equally bright. When they appear equally bright from one side they will also appear equally bright from the other; but as there is always some little difference of tint, the observer's judgment is aided by seeing both sides at once. This

is accomplished in Dr. Letheby's photometer by means of two mirrors, one for each eye, as represented in the accompanying ground-plan (Fig. 66).

s is the screen, and *m m* are the two mirrors, in which images *s′ s′* are seen by an observer in front. The frame which carries the screen and mirrors travels along a graduated bar AB, on which the distances of the screen from the two lights are indicated.

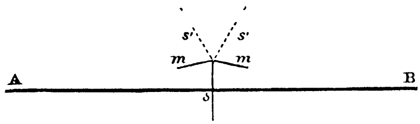


Fig 66 —Letheby's Photometer

In all delicate photometric observations, the eye should be shielded from direct view of the lights, and, as much as possible, from all extraneous lights. The objects to be compared should be brighter than anything else in the field of view.

86. Photometers for very Powerful Lights.—In comparing two very unequal lights, for example, a powerful electric light and a standard candle, it is scarcely possible to obtain an observing-room long enough for a direct comparison by any of the above methods. To overcome this difficulty a lens (either convex or concave), of short focal length, may be employed to form an image of the more powerful source near its principal focus. Then all the light which this source sends to the lens may be regarded as diverging from the image and filling a solid angle equal to that which the lens subtends at the image. In other words, the illuminations of the lens itself due to the source and the image are equal. Hence, if S and I are the distances of the source and image from the lens, the image is weaker than the source in the ratio of I^2 to S^2, and a direct comparison can be made between the light from the image and that from a standard candle. Thus, if a screen at distance D from the image has the same illumination from the image as from a candle at distance C on the other side, the image is equal to $\frac{D^2}{C^2}$ candles, and the source itself to $\frac{D^2}{C^2}\frac{S^2}{I^2}$ candles. A correction must, however, be applied to this result for the light lost by reflection at the surfaces of the lens.

CHAPTER VII.

REFLECTION OF LIGHT.

87. **Reflection.**—If a beam of the sun's rays A B (Fig. 67) be admitted through a small hole in the shutter of a dark room, and allowed to fall on a polished plane surface, it will be seen to continue its course in a different direction B C. This is an example of reflec-

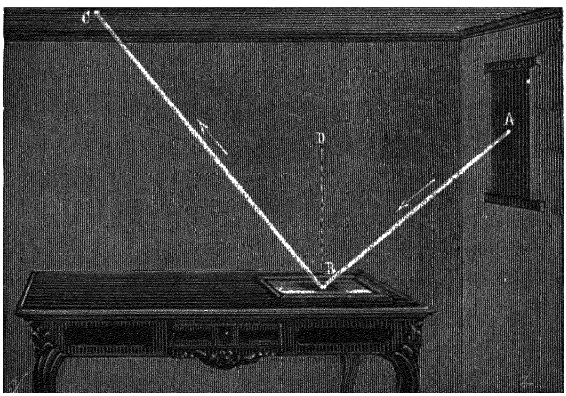


Fig. 67.—**Reflection of Light.**

tion. A B is called the incident beam, and B C the reflected beam. The angle A B D contained between an incident ray and the normal is called the angle of incidence; and the angle C B D contained between the corresponding reflected ray and the normal is called the angle of reflection. The plane A B D containing the incident ray and the normal is called the plane of incidence.

88. **Laws of Reflection.**—The reflection of light from polished surfaces takes place according to the following laws:—

1. The reflected ray lies in the plane of incidence.

2. The angle of reflection is equal to the angle of incidence.

These laws may be verified by means of the apparatus represented in Fig. 68. A vertical divided circle has a small polished plate fixed at its centre, at right angles to its plane, and two tubes travelling on its circumference with their axes always directed towards the centre. The zero of the divisions is the highest point of the circle, the plate being horizontal.

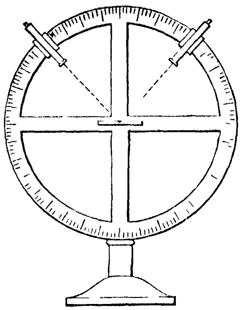


Fig 68.—Verification of Laws of Reflection

A source of light, such as the flame of a candle, is placed so that its rays shine through one of the tubes upon the plate at the centre. As the tubes are blackened internally, no light passes through except in a direction almost precisely parallel to the axis of the tube. The observer then looks through the other tube, and moves it along the circumference till he finds the position in which the reflected light is visible through it. On examining the graduations, it will be found that the two tubes are at the same distance from the zero point, on opposite sides. Hence the angles of incidence and reflection are equal. Moreover the plane of the circle is the plane of incidence, and this also contains the reflected rays. Both the laws are thus verified.

89. Artificial Horizon.—These laws furnish the basis of a method of observation which is frequently employed for determining the altitude of a star, and which, by the consistency of its results, furnishes a very rigorous proof of the laws.

A vertical divided circle (Fig. 69) is set in a vertical plane by proper adjustments. A telescope movable about the axis of the circle is pointed to a particular star, so that its line of collimation I′ S′ passes through the apparent place of the star. Another telescope,[1] similarly mounted on the other side of the circle, is directed downwards along the line I′R towards the image of the star as seen in a trough of mercury I. Assuming the truth of the laws of reflection as above stated, the altitude of the star is half the angle between the directions of the two telescopes; for the ray S I from the star to the mercury is parallel to the line S′ I′, by reason of the excessively great distance of the star; and since the rays S I, I R are equally inclined to the normal I N, which is a vertical line, the lines I′S, I′R are also equally inclined to the vertical, or, what is the same thing,

[1] In practice, a single telescope usually serves for both observations.

are equally inclined to a horizontal plane. A reflecting surface of mercury thus used is called a mercury horizon, or an *artificial*

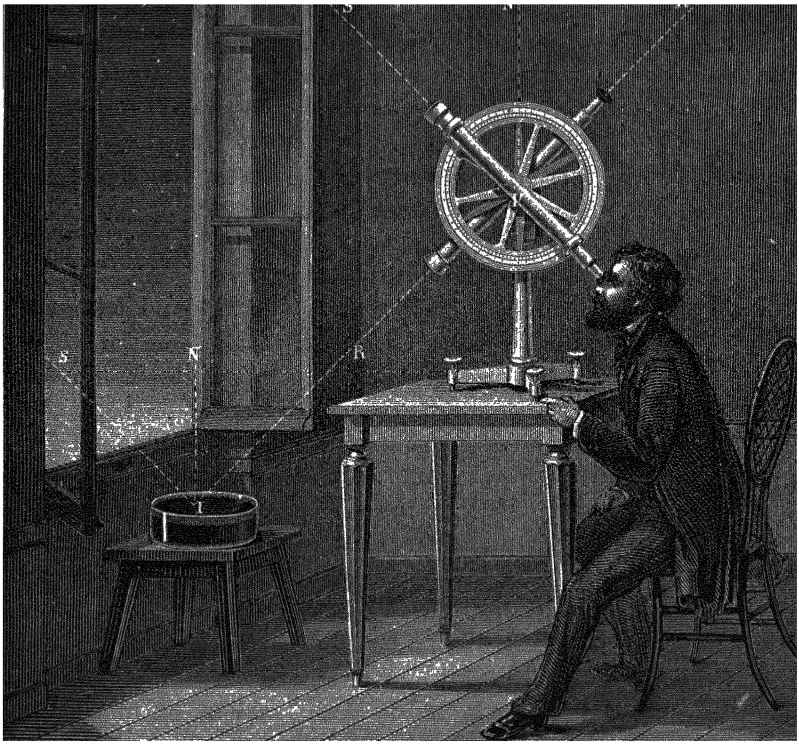


Fig. 69.—**Artificial Horizon.**

horizon. Observations thus made give even more accurate results than those in which the natural horizon presented by the sea is made the standard of reference.

90. Irregular Reflection.—The reflection which we have thus far been discussing is called *regular reflection.* It is more marked as the reflecting surface is more highly polished, and (except in the case of metals) as the incidence is more oblique. But there is another kind of reflection, in virtue of which bodies, when illuminated, send out light in all directions, and thus become visible. This is called *irregular reflection* or *diffusion.* Regular reflection does not render the reflecting body visible, but exhibits images of surrounding objects. A perfectly reflecting mirror would be itself unseen, and

actual mirrors are only visible in virtue of the small quantity of diffused light which they usually emit. The transformation of incident into diffused light is usually selective; so that, though the incident beam may be white, the diffused light is usually coloured. The power which a body possesses of making such selection constitutes its colour.

The word *reflection* is often used by itself to denote what we have here called *regular reflection*, and we shall generally so employ it.

91. **Mirrors.**—The mirrors of the ancients were of metal, usually of the compound now known as speculum-metal. Looking-*glasses* date from the twelfth century. They are plates of glass, coated at the back with an amalgam of quicksilver and tin, which forms the reflecting surface. This arrangement has the great advantage of excluding the air, and thus preventing oxidation. It is attended, however, with the disadvantage that the surface of the glass and the surface of the amalgam form two mirrors; and the superposition of the two sets of images produces a confusion which would be intolerable in delicate optical arrangements. The mirrors, or *specula* as they are called, of reflecting telescopes are usually made of *speculum-metal*, which is a bronze composed of about 32 parts of copper to 15 of tin. Lead, antimony, and arsenic are sometimes added. Of late years specula of glass coated in *front* with real silver have been extensively used; they are known as *silvered specula*. A coating of platinum has also been tried, but not with much success. The mirrors employed in optics are usually either *plane* or *spherical*.

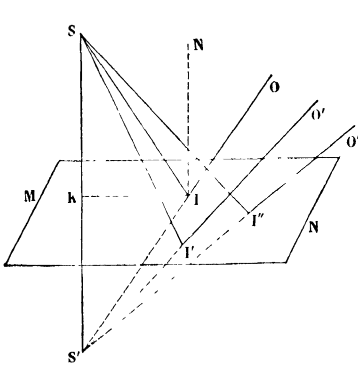


Fig 70.—Plane Mirror

92. **Plane Mirrors.**—By a plane mirror we mean any plane reflecting surface. Its effect, as is well known, is to produce, behind the mirror, images exactly similar, both in form and size, to the real objects in front of it. This phenomenon is easily explained by the laws of reflection.

Let M N (Fig. 70) be a plane mirror, and S a luminous point. Rays S I, S I′, S I″ proceeding from this point give rise to reflected rays I O, I′ O′, I″ O″; and each of these, if produced backwards, will meet the normal S K in a point S′, which is at the same distance behind the mirror that S is in front of

it.[1] The reflected rays have therefore the same directions as if they had come from S′, and the eye receives the same impression as if S′ were a luminous point.

Fig. 71 represents a pencil of rays emitted by the highest point of a candle-flame, and reflected from a plane mirror to the eye of an observer. The reflected rays are divergent (like the incident rays), and if produced backwards would meet in a point, which is the position of the image of the top of the flame.

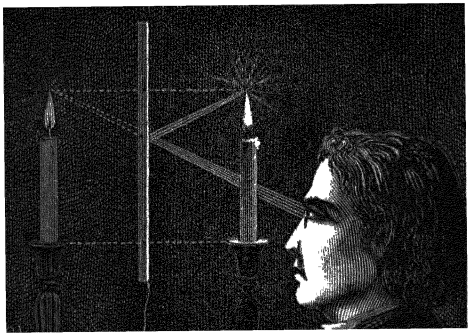


Fig. 71.—Image of Candle.

As an object is made up of points, these principles show that the image of an object formed by a plane mirror must be equal to the object, and symmetrically situated with respect to the plane of the mirror. For example, if AB (Fig. 72) is an object in front of the mirror, an eye placed at O will see the image of the point A at A′, the image of B at B′, and so on for all the other points of the object. The position of the image A′B′ depends only on the positions of the object and of the mirror, and remains stationary as the eye is moved about. It is possible, however, to find positions from which the eye will not see the image at all, the conditions of visibility being the same as if the image were a real object, and the mirror were an opening through which it could be seen.

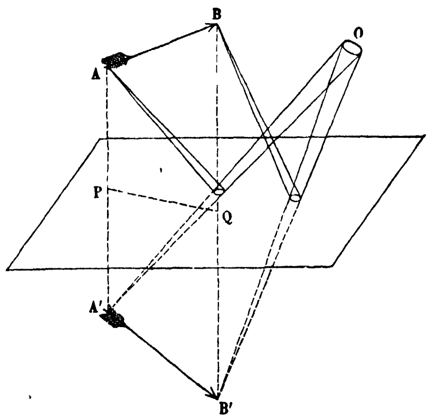


Fig. 72.—Incident and Reflected Pencils.

The images formed by a plane mirror are *erect.* They are not however exact duplicates of the objects from which they are formed,

[1] This is evident from the comparison of the two triangles S K I, S′ K I, bearing in mind that the angle N I S is equal to the alternate angle I S K, and N I O to K S′ I.

but differ from them precisely in the same way as the left foot or hand differs from the right. The image of a printed page is like the appearance of the page as seen through the paper from the back, or like the type from which the page was printed.

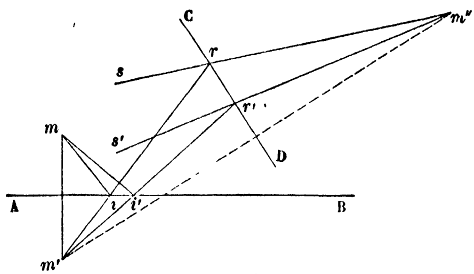


Fig. 73.—Reflection from two Mirrors.

93. Images of Images.—When rays from a luminous point m have been reflected from a mirror A B (Fig. 73), their subsequent course is the same as if they had come from the image m' at the back of the mirror. Hence, if they fall upon a second mirror C D, an image m'' of the first image will be formed at the back of the second mirror. If, after this, they undergo a third reflection, an image of m'' will be formed, and so on indefinitely. The figure shows the actual paths of two rays $m\,i\,r\,s$, $m\,i'\,r'\,s'$. They diverge first from

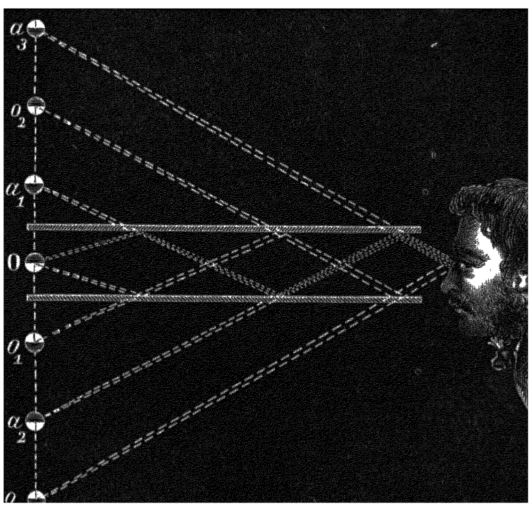


Fig. 74.—Parallel Mirrors

m, then from m', and lastly from m''. This is the principle of the multiple images formed by two or more mirrors, as in the following experiments.

94. Parallel Mirrors.—Let an object O be placed between two

parallel mirrors which face each other, as in Fig. 74. The first reflections will form images $a_1 o_1$. The second reflections will form images $a_2 o_2$ of the first images; and the third reflections will form images $a_3 o_3$ of the second images. The figure represents an eye receiving *the rays which form the third images,* and shows the paths which *these rays* have taken in their whole course from the object O to the eye. The rays by which the same eye sees the other images are omitted, to avoid confusing the figure. A long row of images can thus be seen at once, becoming more and more dim as they recede in the distance, inasmuch as each reflection involves a loss of light.

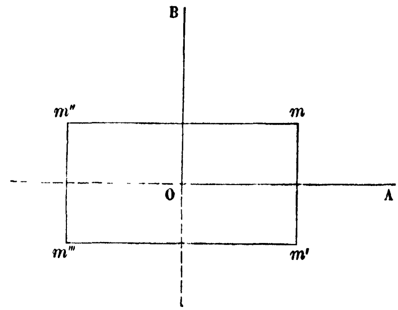


Fig 75.—Mirrors at Right Angles.

If the mirrors are truly parallel, all the images will be ranged in one straight line, which will be normal to the mirrors. If the mirrors are inclined at any angle, the images will be ranged on the circumference of a circle, whose centre

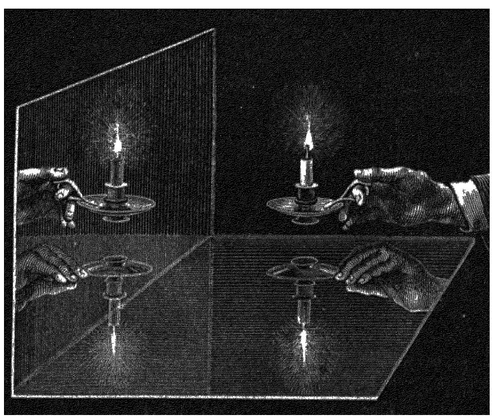


Fig 76 —Mirrors at Right Angles.

is on the line in which the reflecting surfaces would intersect if produced. This principle is sometimes employed as a means of adjusting mirrors to exact parallelism.

95. Mirrors at Right Angles.—Let two mirrors O A, O B (Fig. 75),

be set at right angles to each other, facing inwards, and let m be a luminous point placed between them. Images m' m'' will be formed by first reflections, and two coincident images will be formed at m''' by second reflections. No third reflection will occur, for the point m''', being behind the planes of both the mirrors, cannot be reflected in either of them. Counting the two coincident images as one, and also counting the object as one, there will be in all four images, placed at the four corners of a rectangle. Fig. 76 will give an idea of the appearance actually presented when one of the mirrors is vertical and the other horizontal. When both the mirrors are vertical, an observer sees his own image constantly bisected by their common section in a way which appears at first sight very paradoxical.

96. **Mirrors Inclined at 60 Degrees.**—A symmetrical distribution of images may be obtained by placing a pair of mirrors at any angle which is an aliquot part of 360°. If, for example, they be inclined at 60° to each other, the number of images, counting the object itself as one, will be six. Their position is illustrated by Fig. 77. The object is placed in the sector A C B. The images formed by first reflections are situated in the two neighbouring sectors B C A′, A C B′; the images formed by second reflections are in the sectors B′ C A″, A′ C B″, and these yield, by third reflections, two coincident images in the sector B″ C A″, which is vertically opposite to the sector A C B in which the object lies, and is therefore behind the planes of both mirrors, so that no further reflection can occur.

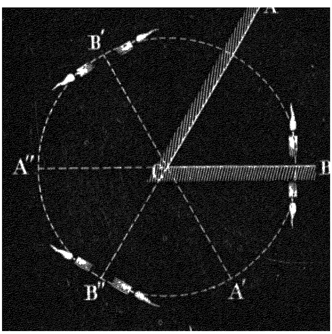


Fig 77.—Images in Kaleidoscope

97. **Kaleidoscope.**—The symmetrical distribution of images, obtained by two mirrors inclined at an angle which is an aliquot part of four right angles, is the principle of the *kaleidoscope*, an optical toy invented by Sir David Brewster. It consists of a tube containing two glass plates, extending along its whole length, and inclined at an angle of 60°. One end of the tube is closed by a metal plate, with the exception of a hole in the centre, through which the observer looks in; at the other end there are two plates, one of ground and the other of clear glass (the latter being next the eye), with a number of little pieces of coloured glass lying loosely between them. These

coloured objects, together with their images in the mirrors, form symmetrical patterns of great beauty, which can be varied by turning or shaking the tube, so as to cause the pieces of glass to change their positions.

A third reflecting plate is sometimes employed, the cross-section of the three forming an equilateral triangle. As each pair of plates produces a kaleidoscopic pattern, the arrangement is nearly equivalent to a combination of three kaleidoscopes.

The kaleidoscope is capable of rendering important aid to designers.

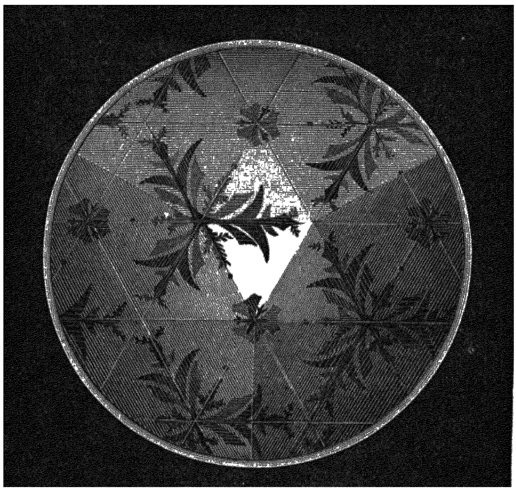


Fig. 78.—Kaleidoscopic Pattern.

Fig. 78 represents a pattern produced by the equilateral arrangement of three reflectors just described.

98. **Pepper's Ghost.**—Many ingenious illusions have been contrived, depending on the laws of reflection from plane surfaces. We shall mention two of the most modern.

In the *magic cabinet*, there are two vertical mirrors hinged at the two back corners of the cabinet, and meeting each other at a right angle, so as to make angles of 45° with the sides, and also with the back. A spectator seeing the images of the two sides, mistakes them for the back, which they precisely resemble; and performers may be concealed behind the mirrors when the cabinet appears empty. If one of the persons thus concealed raises his head above the mirrors, it will appear to be suspended in mid-air without a body

The striking spectral illusion known as *Pepper's Ghost* is produced by reflection from a large sheet of unsilvered glass, which is so arranged that the actors on the stage are seen through it, while other actors, placed in strong illumination, and out of the direct view of the spectators, are seen by reflection in it, and appear as ghosts on the stage.

99. Deviation produced by Rotation of Mirror.—Let A B (Fig. 79) represent a mirror perpendicular to the plane of the paper, and capable of being rotated about an axis through C, also perpendicular to the paper; and let I C represent an incident ray. When the mirror is in the position A B, perpendicular to I C, the ray will be reflected directly back upon its course; but when the mirror is turned through the acute angle A C A′, the reflected ray will take the direction C R, making with the normal C N an angle N C R, equal to the angle of incidence N C I. The deviation I C R of the reflected ray, produced by rotating the mirror, is therefore double of the angle I C N or A C A′, through which the mirror has been turned; and if, starting from the position A′B′, we turn the mirror through a further angle θ, the reflected ray C R will be turned through a further angle $2\,\theta$. It thus appears, that, *when a plane mirror is rotated in the plane of incidence, the direction of the reflected ray is changed by double the angle through which the mirror is turned.* Conversely, if we assign a constant direction C I to the reflected ray, the direction of the incident ray R C must vary by double the angle through which the mirror is turned.

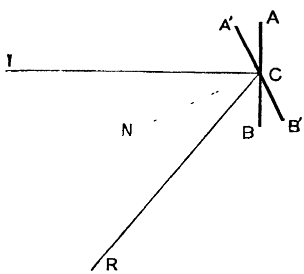


Fig 79 —Effect of rotating a Mirror

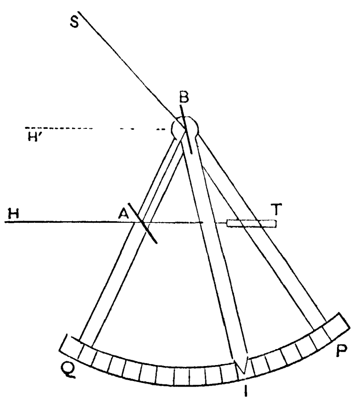


Fig 80 —Sextant

100. Hadley's Sextant.—The above principle is illustrated in the nautical instrument called the *sextant* or *quadrant,* which was invented by Newton, and reinvented by Hadley. It serves for measuring the angle between any two distant objects as seen from the station occupied by the observer. Its essential parts are represented in Fig. 80.

It has two plane mirrors A, B, one of which, A, is fixed to the frame of the instrument, and is only partially silvered, so that a distant object in the direction A H can be seen through the unsilvered part. The other mirror B is mounted on a movable arm B I, which carries an index I, traversing a graduated arc P Q. When the two mirrors are parallel, the index is at P, the zero of the graduations, and a ray H′ B incident on B parallel to H A, will be reflected first along B A, and then along A T, the continuation of H A. The observer looking through the telescope T thus sees, by two reflections, the same objects which he also sees directly through the unsilvered part of the mirror. Now let the index be advanced through an angle θ; then, by the principles of last section, the incident ray S B makes with H′ B, or H A, an angle $2\,\theta$. The angle between S B and H A would therefore be given by reading off the angle through which the index has been advanced, and doubling; but in practice the arc P Q is always graduated on the principle of marking half degrees as whole ones, so that the reading at I is the required angle 2θ. In using the instrument, the two objects which are to be observed are brought into apparent coincidence, one of them being seen directly, and the other by successive reflection from the two mirrors. This coincidence is not disturbed by the motion of the ship; but unpractised observers often find a difficulty in keeping both objects in the field of view. Dark glasses, not shown in the figure, are provided for protecting the eye in observations of the sun, and a vernier and reading microscope are provided instead of the pointer I.

101. Goniometers.—A goniometer is an instrument for measuring the angle between two plane faces either of a crystal or of a prism. It sometimes takes the form shown in Fig. 124, § 142. The measurement is usually made by means of reflections from the two faces. This may be done in either of the two following ways. For convenience of description we shall assume that the edge in which the two faces meet is vertical; in practice it may have any direction.

First method.—Observe in one of the two faces the reflection of an object at a few yards' distance, in the same horizontal plane with the prism; and by rotating the prism in this plane bring the image into apparent coincidence with some other object; or, if preferred, bring it upon the cross-wires of a fixed telescope. Then rotate the prism in the horizontal plane till the other face gives an image of the same object in the same position. The second face is now in or parallel to the position previously occupied by the first

face, and the angle through which the prism has been turned is the angle between one face and the other face produced. By subtracting it from 180° we obtain the required angle between the faces. The goniometer is furnished with a graduated circle on which the rotation is read off. The second object is very frequently a fixed image of the first object formed by a small plane mirror provided for the purpose. The best kind of object is a long vertical line.

Second method.—The goniometer must have a telescope (with cross-wires) which can travel round the graduated circle, while always directed towards its centre, where the prism stands. The prism is placed in such a position that rays from a distant object, or more conveniently from a slit in the focus of a collimating lens, fall upon both faces simultaneously. The telescope is first placed so as to receive on its cross-wires the image formed by reflection at one face, and is then moved past the base of the prism till it receives the image formed by reflection at the other face. The angle through which it has been moved is double the angle of the prism, as is obvious from Fig. 81, in which the directions of the incident and reflected rays are represented by lines marked with arrow-heads. The incident rays being parallel, $\alpha+\beta$ is the angle of the prism, and $2\alpha+2\beta$ is the angle between the reflected rays.

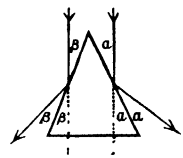


Fig 81 - Measurement of Angle of Prism

102. Spherical Mirrors.—By a spherical mirror is meant a mirror whose reflecting surface is a portion (usually a very small portion) of the surface of a sphere. The radius of the sphere is called the *radius of curvature* of the mirror, and the centre of the sphere is called the *centre of curvature.* If the outline of the mirror is circular, that point of the reflecting surface which is equidistant from all points of the circumference is called the *vertex* of the mirror.

When the incident rays are parallel to the axis of the mirror, the reflected rays converge to a point F (Fig. 82) called the *principal focus.* This law is rigorously true for parabolic mirrors (generated by the revolution of a parabola about its axis). For spherical mirrors it is only approximately true, but the approximation is very close if the mirror is only a small fraction of an entire sphere. In grinding and polishing the specula of large reflecting telescopes, the attempt is made to give them, as nearly as possible, the parabolic form. Parabolic mirrors are also frequently employed to reflect, in a definite direction, the rays of a lamp placed at the focus.

The name *secondary axis* is often applied to any line through the centre of curvature of a spherical mirror other than the principal axis.

103. Position of Focus.—Let R I (Fig. 83) be a ray incident upon the concave spherical mirror M N. Draw the radius C I.

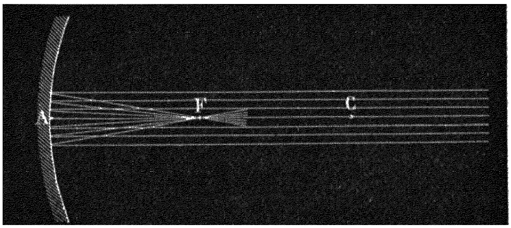


Fig. 82.—Principal Focus.

Then the angles of incidence and reflection (marked *i* and *r* in the figure) are equal, and if C O be a radius drawn parallel to the incident ray, and meeting the reflected ray in F, the angle O C I is equal to the alternate angle R I C. Thus, in the triangle F I C the angles at I and C are equal, hence F I = F C. But F I is ultimately equal to F O, therefore F O is ultimately equal to F C; that is, the ultimate position of F is midway between C and O. Thus *the*

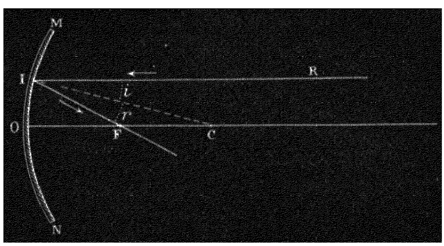


Fig. 83.

principal focus is midway between the centre of curvature and the vertex, and *rays parallel to any radius* of the mirror *will be reflected to a focus which bisects this radius.* Conversely, rays incident on the mirror from the middle point of any radius will be reflected parallel to the radius.

Strictly speaking, F O is less than F I and therefore than F C; and the inequality is greater as I departs further from O. Hence rays reflected from the circumferential portion of a spherical mirror are too convergent to concur exactly with those reflected from the

central portion. This deviation from exact concurrence is called *spherical aberration.*

104. Image of the Sun.—When the axis of a concave spherical mirror is directed towards the sun, an image of the sun is formed midway between the mirror and the centre of curvature. Any given point of the sun's disc sends parallel rays to all parts of the mirror, and these are reflected to a point which is the image of that particular point of the sun's disc, the aggregate of all these images makes up the image of the sun. They all lie on a spherical surface described about the centre of curvature, with a radius half that of the mirror, and the line joining any point of the sun to its own image passes through the centre of curvature. From this last property it follows that the diameters of the sun and of its image subtend equal angles at the centre of curvature. The magnitude of this angle is about half a degree.

As all the rays of the sun reflected from the mirror are collected into the area occupied by the image, the heating effect at the focus is very intense when the mirror is large compared with the image. Hence the name *focus*, literally meaning *hearth* or fireplace.

105. Conjugate Foci of Concave Mirror.—Let rays fall upon a concave mirror (Fig. 84) from a point P at a distance from the mirror greater than the radius of curvature. Draw the line P C O through the centre of curvature C to meet the mirror in O. Then any incident ray P I will be reflected so as to make the angle of reflection C I P′ equal to the angle of incidence C I P, hence we have I P : I P′ :: C P : P′C. But I P is ultimately equal to O P, and I P′ to O P′, hence ultimately

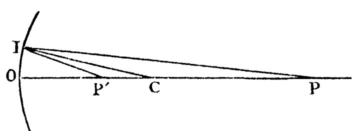


Fig 84

$$OP : OP' :: CP : P'C. \qquad (1)$$

Putting p for O P, p' for O P′, and r for O C, equation (1) becomes $p : p' :: p - r : r - p'$, whence $pr + p'r = 2pp'$, or, dividing by $pp'r$,

$$\frac{1}{p} + \frac{1}{p'} = \frac{2}{r} = \frac{1}{f}, \qquad (2)$$

where f denotes the focal length $r/2$. In applying this formula to various cases, the three quantities p, p', r are to be regarded as having the same sign when P, P′, C are on the same side of O.

106. Interchangeableness of Object and Image.—Rays incident from P′ will obviously be reflected to P; and hence the relation

between the two points P, P′ is mutual. They are therefore called *conjugate foci* or *conjugate points*. If a small object be placed at either of them its image will be formed at the other. It is a general principle in optics that an object can always change places with its image. We have here treated the object and image as indefinitely

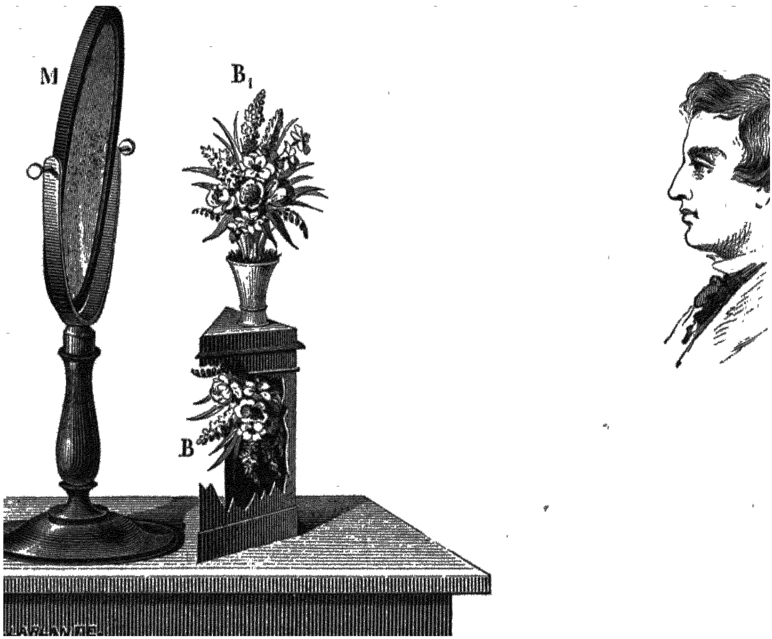


Fig. 85.—Experiment of Phantom Bouquet.

small. The image of an object of finite size is made up of the images of its several points.

107.—Equation (1) expresses that *the distances of conjugate points from the vertex of the mirror are in the same ratio as their distances from the centre of curvature.*

Equation (2) expresses that the radius of the mirror is the harmonic mean of the distances of any pair of conjugate points from the vertex. This can be more directly proved as a deduction from the fact that the angles O P I, O C I, O P′ I are in arithmetical progression, their common difference being C I P or its equal C I P′.

Again, let q, q' denote the distances of P, P′ from the centre of curvature C, reckoned positive when P, P′ are on the same side of C that O is. In the figure we have $CP = -q$, $CP' = q'$, $CO = r$, $OP =$

$r-q$, $\mathrm{O\,P'}=r-q'$; hence (1) becomes $r-q : r-q' : -q : q'$, whence $r q' + r q = 2 q q'$, or, dividing by $q q' r$,

$$\frac{1}{q} + \frac{1}{q'} = \frac{2}{r} = \frac{1}{f}, \qquad (3)$$

an equation of the same form as (2).

108. Experimental Illustrations.—Let a box, open on one side, be placed in front of a concave mirror (Fig. 85) at a distance about equal to its radius of curvature, and let an inverted bouquet be suspended within it, the open side of the box being next the mirror. By giving a proper inclination to the mirror, an image of the bouquet will be obtained in mid-air, just above the top of the box. As

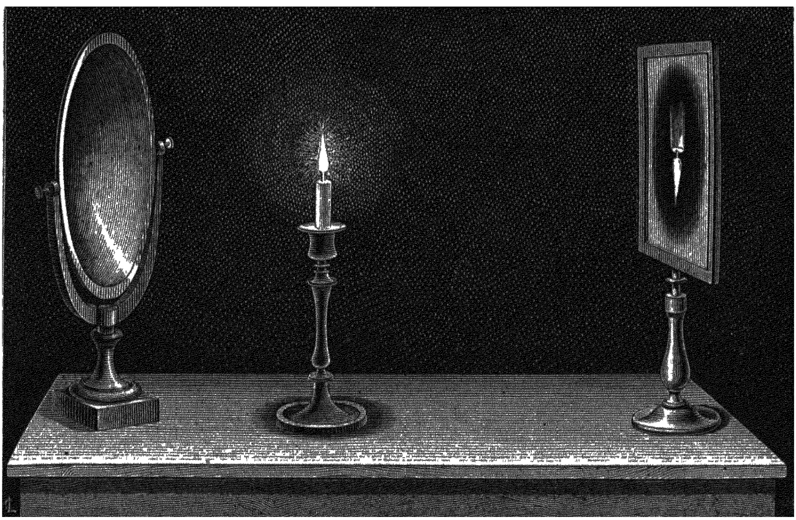


Fig. 86 —Image on Screen

the bouquet is inverted, its image is erect, and a real vase may be placed in such a position that the phantom bouquet shall appear to be standing in it. The spectator must be full in front of the mirror, and at a sufficient distance for all parts of the image to lie between his eyes and the mirror. When the colours of the bouquet are bright, the image is generally bright enough to render the illusion very complete. This experiment can only be seen by a few persons at once, since the observer's eye must be in a line with the image and the mirror. When an image is projected on a screen, it can be seen by a whole audience at once, if the room be darkened and the image be large and bright. Let a lighted candle, for example, be

placed in front of a concave mirror, at a distance exceeding the focal length, and let a screen be placed at the conjugate focus; an inverted image of the candle will be depicted on the screen. Fig. 86 represents the case in which the candle is at a distance less than the radius of curvature, and the image is accordingly magnified.

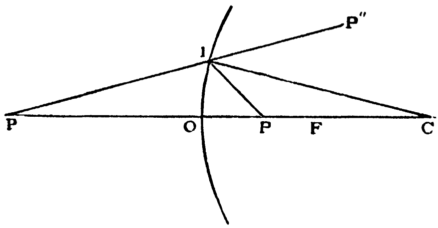


Fig 87 —Virtual Focus.

By this mode of operating, the formula for conjugate focal distances can be experimentally verified with considerable rigour, care being taken, in each experiment, to place the screen in the position which gives the most sharply defined image.

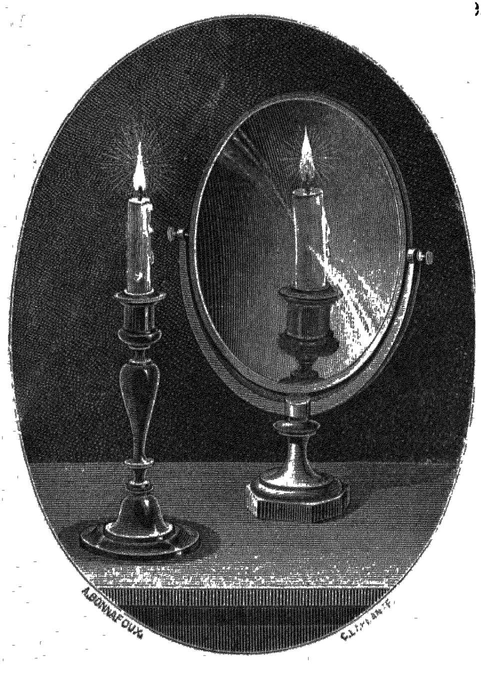


Fig. 88.—Virtual Image in Concave Mirror

s.—If rays fall upon a concave mirror from a point P (Fig. 87) whose distance from the mirror is less than the focal length O F, it is evident by comparison with a ray incident from F (which would be reflected parallel to O C), that the reflected ray if produced backwards will meet C O in a point P′ behind the mirror. Then, since I C bisects the exterior angle of the triangle P I P′, we have, as before, I P : I P′ :: C P : C P′, and ultimately O P : O P′ :: C P : C P′.

Putting $r = OC$, $p = OP$, $-p' = OP'$, we shall obtain the same equation as before.

P′ is called a *virtual* focus, because the reflected rays do not pass through it unless produced backwards, as distinguished from a *real* focus through which they actually pass. An

image composed of virtual foci is called a *virtual image*, and an image composed of real foci is called a *real image.*

The virtual image of a candle as seen in a concave mirror is illustrated by Fig. 88.

110. Pencils and their Foci.—A ray coming from P′ (Fig. 87) would not fall on the reflecting surface of the mirror at all; but a ray P″ I

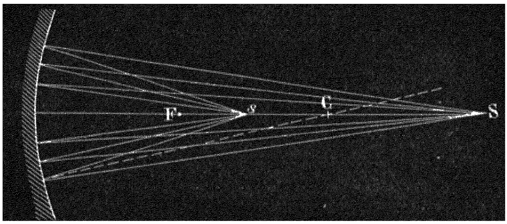


Fig 89 —Conjugate Foci

intercepted by the mirror on its way to P′ would be reflected to P. In this sense the relation between P and P′ is still a mutual relation. To avoid circumlocution the continuation of a ray either backwards or forwards is regarded as part of the ray itself. We can then assert that rays which pass through either one of two conjugate foci after reflection, pass through the other before incidence.

P and P′ are often called the *foci of the* incident and reflected *pencils*. A *pencil* means a solid cone of rays, and the vertex of the

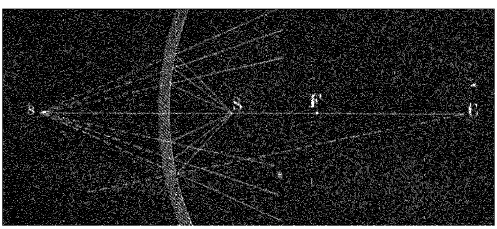


Fig 90 —Virtual Focus

cone is called their focus. When a pencil of rays undergoes reflection or refraction, the reflected or refracted rays are still usually spoken of as a pencil, even if they no longer pass accurately through one point.

Figs. 89, 90 exhibit the relation between two conjugate foci S and *s*. In the first figure both foci are real, in the second figure S is real and *s* virtual.

111. Construction for Conjugate Foci. Their Movements.—The focus conjugate to any given point can be found by the following construction, when the focal length is given:

Let A (Figs. 91, 92) be the given point. Draw an axis O F not passing through A, and take O F equal to the given focal length, so that F is the focus for incident rays parallel to F O.

Draw the incident ray A B parallel to F O, and the corresponding reflected ray B F. Also draw the incident ray A F meeting the

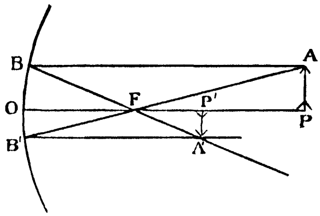


Fig. 91.

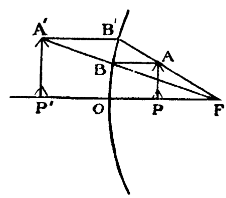


Fig 92.

mirror in B′, and the corresponding reflected ray B′A′ parallel to O F. The point A′ in which these two reflected rays meet must be the point conjugate to A.

Since the three parallel lines in the figure divide all lines in the same ratio, we have

$$FA : FB' :: FB : FA'.$$

But F B and F B′ are ultimately equal to F O, that is, to the focal length f. Hence, multiplying extremes and means, we have

$$FA \cdot FA' = f^2.$$

If we make A approach the axis O F by moving along the perpendicular A P, A′ will also approach the axis, and when A reaches P, A′ will reach a point P′ in the axis such that

$$FP \cdot FP' = f^2. \qquad (3)$$

P and P′ will be both on the same side of F. In the first figure they are both on the side remote from the mirror and both are real. In the second figure one is real and the other virtual.

Equation (3) shows that when F P is equal to F P′ their common value is f, that is, F C or F O. Hence the two conjugate foci coincide with one another at the centre of curvature, and also at the surface of the mirror. As they move away from these points of coincidence, their distances from F will vary in inverse proportion,

so that one of them moves up to F while the other moves to infinite distance.

112. Object, Image, and Centre of Curvature.—When the object is a circular arc described about the centre of curvature, the image will be another circular arc, the two arcs subtending the same angle at the centre. Their lengths will therefore be directly as their distances from C. When the arcs are small they may be regarded as straight lines.

Accordingly, in Figs. 93, 94, the object A B and its image A′ B′ are represented as straight. The construction in both figures is as follows:—Draw through A the incident ray A I parallel to the axis C O, and draw the reflected ray I F A′ meeting A C produced in A′, which will be the image of the point A. The point B′, which is the image of B, is found in the same way, and the joining line A′B′ is taken to be the image of the line AB.

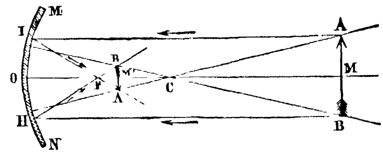


Fig 93

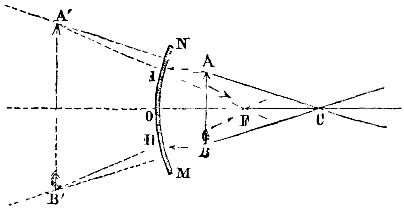


Fig 94

In both figures it is assumed that the line joining a point to its image passes through the centre of curvature, a property which can be inferred from the fact that a ray through C is normally incident, and is therefore reflected back upon itself.

It follows at once that when object and image lie on opposite sides of the centre of curvature the image is inverted, and when on the same side erect; the linear *dimensions of object and image*, measured in a direction perpendicular to the axis of the mirror, being in both cases (by similar triangles) *directly as the distances* of object and image *from the centre*.

Again, we have proved (§ 107) that the distances from the centre are proportional to the distances from the mirror. Hence we obtain a second rule, namely, that the linear *dimensions* are *directly as the distances of object and image from the mirror*.

When the object is at the centre of curvature, its inverted image is equal to it, because they are at the same distance from the mirror.

When the object touches the mirror, its image (which is erect and virtual) is equal to it, because they are at the same distance from the centre of curvature.

113.—Again, in Figs. 91, 92, since P A or O B may be regarded as the length of the object, and P′ A′ or O B′ as the length of the image, we have, by similar triangles—

$$\frac{\mathrm{AP}}{\mathrm{A'P'}}=\frac{\mathrm{OB}}{\mathrm{A'P'}}=\frac{\mathrm{OF}}{\mathrm{FP'}}=\frac{f}{\mathrm{FP'}};$$

and again,
$$\frac{\mathrm{AP}}{\mathrm{A'P'}}=\frac{\mathrm{AP}}{\mathrm{OB'}}=\frac{\mathrm{FP}}{\mathrm{OF}}=\frac{\mathrm{FP}}{f}.$$

Thus we have the two following additional rules:—

The linear *dimensions of the image* are *directly as the distance FP′ of the image from the principal focus,* and are equal to the dimensions of the object when this distance is f.

The linear *dimensions of the image* are *inversely as the distance FP of the object from the principal focus,* and are equal to those of the object when this distance is f.

These two rules can be deduced from the formula $\frac{1}{p}+\frac{1}{p'}=\frac{1}{f}$. For this gives $\frac{1}{p'}=\frac{1}{f}-\frac{1}{p}=\frac{p-f}{fp}$; whence $\frac{p}{p'}=\frac{p-f}{f}$; and by symmetry $\frac{p'}{p}=\frac{p'-f}{f}$. But p/p' is the ratio of the linear dimensions, $p-f$ is F P, and $p'-f$ is F P′.

114. **Convex Mirrors.**—Every diagram illustrating the formation of an image by a concave mirror, will also serve for a convex mirror by supposing the mirror to be indefinitely thin and polished on the convex instead of the concave side. For instance, in Fig. 94, A′ B′ may represent an object in front of a convex mirror of which C is the centre. The topmost line in the figure will represent a normally incident ray from A′, which is reflected back upon itself; and the next line will represent a ray incident from A′ in the direction of F, which will be reflected parallel to the axis C F. These two reflected rays, when produced backwards, meet in A, which is accordingly the position of the image of the point A′.

Again, interchanging the positions of object and image, A B may represent a real image formed by rays coming from the left hand, which have been concentrated by some optical arrangement; and then A′ B′ will represent a second real image, produced by interposing a convex mirror in the position shown.

In Fig. 93, either A B or A′ B′ may represent a real image formed by rays from the left hand, and A′ B′ or A B will then

represent the virtual image which will be seen by looking into the mirror.

Fig. 95 represents a convex mirror, with the same object at two different distances, and its image in each case found by construction. The figure shows that as the object moves further away, the image moves further from the mirror and nearer to the principal focus;

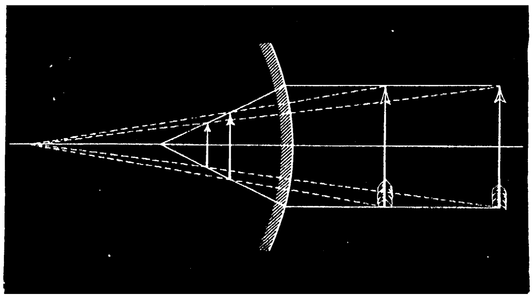


Fig 95.—Formation of Image in Convex Mirror

also that the magnitude of the image is directly as its distance from the principal focus.

115.—The image of a real object formed by a convex mirror always lies between the principal focus and the mirror, and is virtual, erect and diminished. The principal focus is the virtual image of a point at infinite distance.

When a convex mirror is interposed in the path of rays which are on their way to form a real image, it will form a second image. If the first image lies between the surface of the mirror and the principal focus, as A B, Fig. 94, the second image A′ B′ will be real and erect as compared with the first It will recede from the mirror and grow larger as the first moves towards the principal focus F, and will be at infinite distance when the first is at F.

When the first image is between the principal focus and the centre of curvature, as A′ B′, Fig 93, the second image A B will be inverted and magnified as compared with the first. When the first is beyond the centre, as A B in the same figure, the second will be A′ B′, and will be inverted and diminished as compared with the first. In both these cases the second image is virtual.

In the Cassegranian telescope a convex mirror is employed to form a real image, after the manner of A′ B′, Fig. 94.

116.—Since the same diagram serves for a convex as for a concave mirror, the same formulæ will also serve, with the interpretations which we have above indicated. In applying the formula

$$\frac{1}{p}+\frac{1}{p'}=\frac{1}{f},$$

in which p and p' are the distances of object and image respectively from the mirror, f stands for the distance of the principal focus from the mirror, and is to be considered opposite in sign to p when F and the object are on opposite sides of the mirror, as is always the case if the object is a material object and not an optical image.

Convex mirrors are seldom employed in optical instruments.

The silvered globes which are frequently used as ornaments, are examples of convex mirrors, and present to the observer at one view an image of nearly the whole surrounding landscape. As the part of the mirror in which he sees this image is nearly an entire hemisphere, the deformation of the image is very notable, straight lines being reflected as curves.

117. Cylindric Mirrors.—Much greater deformations are produced by cylindric mirrors. A cylindric mirror, when the axis of the cylinder is vertical, behaves like a plane mirror as regards the angular magnitude under which the height of the image is seen, and like a spherical mirror as regards the breadth of the image. If it be a convex cylinder, it causes bodies to appear unduly contracted horizontally in proportion to their heights.

118. Medical Applications.—Concave mirrors are frequently used to concentrate light upon an object for the purpose of rendering it more distinctly visible.

The *ophthalmoscope* is a small concave mirror, with a small hole in its centre, through which the observer looks from behind, while he directs a beam of reflected light from a lamp into the pupil of the patient's eye. In this way (with the help sometimes of a lens) the retina can be rendered visible, and can be minutely examined.

The *laryngoscope* consists of two mirrors. One is a small plane mirror, with a handle attached, at an angle of about 45° to its plane. This small mirror is held at the back of the patient's mouth, so that the observer, looking into it, is able by reflection to see down the patient's throat, the necessary illumination being supplied by a concave mirror, strapped to the observer's forehead, by means of which the light from a lamp is reflected upon the plane mirror, which again reflects it down the throat.

119. Difference between Image on Screen, and Image as seen in Mid-air. Caustics.—For the sake of simplicity we have made some statements regarding visible images which are not quite accurate; and we must now indicate the necessary corrections

Images thrown on a screen have a determinate position, and are really the loci of the conjugate foci of the points of the object, but this is not rigorously true of images seen directly. They change

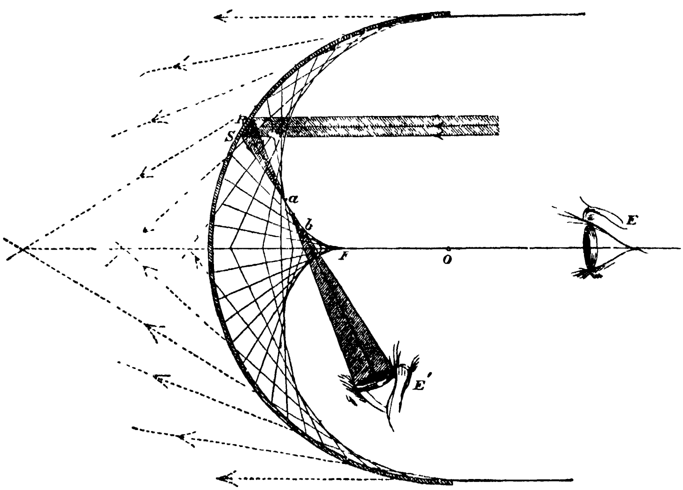


Fig 96 —Position of Image in Oblique Reflection

their position to some extent, according to the position of the observer.

The actual state of things is explained by Fig 96. The plane of the figure[1] is a principal plane (that is, a plane containing the principal axis) of a concave hemispherical mirror, and the incident rays are parallel to the principal axis. All the rays reflected in the plane of the figure touch a certain curve called a *caustic curve,* which has a cusp at F, the principal focus, and the direction in which the image is seen by an eye situated in the plane of the figure is determined by drawing from the eye a tangent to this caustic. If the eye be at E on the principal axis, the point of contact will be F, but when the rays are received obliquely, as at E′, it will be at a point *a* not lying in the direction of F. For an eye thus situated, *a* is called the *primary focus,* and the point where the tangent at *a*

[1]Figs. 96 and 114 are borrowed, by permission, from Mr. Osmund Airy's *Geometrical Optics.*

cuts the principal axis is called the *secondary focus.* When the eye is moved in the plane of the diagram, the apparent position of the image (as determined by its remaining in coincidence with a cross of threads or other mark) is the primary focus; and when the eye is moved perpendicular to the plane of the diagram, the apparent position of the image is the secondary focus.[1] If we suppose the diagram to rotate about the principal axis, it will still remain true in all positions, and the surface generated by this revolution of the caustic curve is the *caustic surface.* Its form and position vary with the position of the point from which the incident rays proceed; and it has a cusp at the focus conjugate to this point.

There is always more or less blurring, in the case of images seen obliquely (except in plane mirrors), by reason of the fact that the point of contact with the caustic surface is not the same for rays entering different parts of the pupil of the eye.

A caustic curve can be exhibited experimentally by allowing the rays of the sun or of a lamp to fall on a concave semicircular reflector, the reflected light being received on a sheet of white paper on which the reflector rests. The same effect may often be observed on the surface of a cup of tea, the reflector in this case being the inside of the cup.

The image of a luminous point received upon a screen is formed by all the rays which touch the corresponding caustic surface. The brightest and most distinct image will be formed at the cusp, which is, in fact, the conjugate focus; but there will be a border of fainter light surrounding it. This source of indistinctness in images is an example of *spherical aberration* (§ 103).

120. Oblique Reflection. Two Focal Lines.—If we attempt to throw upon a screen the image of a luminous point by means of a concave mirror very oblique to the incident rays, we shall find that no image can be obtained at all resembling a point; but that there are two positions of the screen in which the image becomes a line.

In the annexed figure (Fig. 97), which represents on a larger scale a portion of Fig. 96, *a c*, *b d* are rays from the highest and lowest points of the portion R S of the hemispherical mirror, which portion we suppose to be small in both its dimensions in comparison

[1] Since every ray incident parallel to the principal axis, is reflected through the principal axis. If the incident rays diverged from a point on the principal axis, they would still be reflected through the principal axis.

with the radius of curvature, and we may suppose the rest of the hemisphere to be removed, so that R S will represent a small concave mirror, receiving a pencil very obliquely.

Then, if a screen be held perpendicular to the plane of the diagram, at *m*, where the section of the pencil by the plane of the diagram is narrowest, a blurred line of light will be formed upon it, the length of the line being perpendicular to the plane of the diagram. This is called the *primary focal line.*

The *secondary focal line* is *c d*, which, if produced, passes through the centre of curvature of the mirror, and also through the point from which the incident light proceeds This line is very sharply formed upon a screen held so as to coincide with *c d* and to be perpendicular to the plane of the diagram. Its edges are much better defined than those of the primary line; and its position in space is also more definite. If the mirror is used as a burning-glass to collect the sun's rays, ignition will be more easily obtained at one of these lines than in any intermediate position.

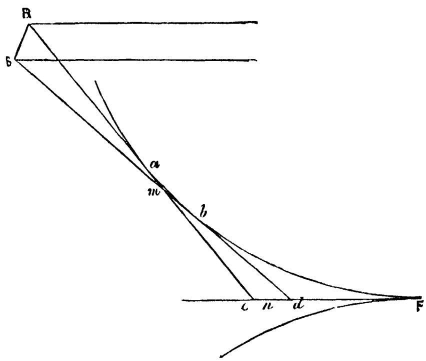


Fig 97 —Formation of Focal Lines

Focal lines can also be seen directly. In this case a small element of the mirror sends all its reflected rays to the eye, the rays from opposite sides of the element crossing each other at the focal lines, before they reach the eye. It is possible, in certain positions of the eye, to see either focal line at pleasure, by altering the focal adjustment of the eye; or the two may be seen with imperfect definition crossing each other at right angles. The experiment is easily made by employing a gas flame, turned very low, as the source of light. One line is in the plane of incidence, and the other is normal to this plane.

121.—A screen held perpendicular to the general direction of the pencil will receive an image which is narrow in one direction when held at the primary focus, and in the direction at right angles to this when held at the secondary focus; and these are the positions which exhibit the greatest concentration of light.

There is, of course, some intermediate section which has the same

breadth in both directions, and mathematicians have calculated its position; but it possesses no physical interest, as it exhibits no special concentration of light, and never asserts itself in optical experiments. It is called the "circle of least confusion," but its appearance as seen on a screen does not justify this name. It is very confused, and very unlike a circle.[1] The experiment can be tried by reflecting the sun's rays obliquely from a concave mirror.

[1] If we neglect the blurring of the primary line, we may describe the part of the pencil lying between the two lines as a tetrahedron of which the two lines are opposite edges; and no section of a tetrahedron is a circle.

CHAPTER VIII.

REFRACTION.

122. Refraction.—When a ray of light passes from one transparent medium to another, it undergoes a change of direction at the surface of separation, so that its course in the second medium makes an angle with its course in the first This changing of direction is called *refraction.*

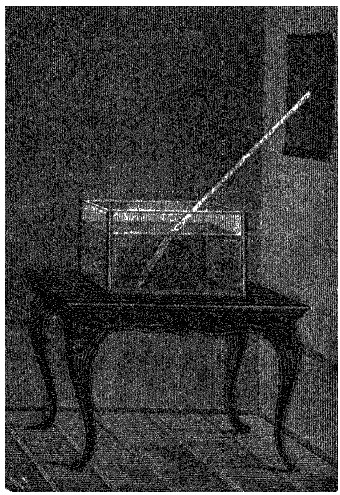


Fig 98 —Refraction

The phenomenon can be exhibited by admitting a beam of the sun's rays into a dark room, and receiving it on the surface of water contained in a rectangular glass vessel (Fig. 98). The path of the beam will be easily traced by its illumination of the small solid particles which lie in its course.

The following experiment is a well-known illustration of refraction:—A coin $m n$ (Fig. 99) is laid at the bottom of a vessel with opaque sides, and a spectator places himself so that the coin is just hidden from him by the side of the vessel; that is to say, so that the line m A in the figure passes just above his eye. Let water now be poured into the vessel, care being taken not to displace the coin. The bottom of the vessel will appear to rise, and the coin will come into sight. Hence a pencil of rays from m must have entered the spectator's eye. The pencil in fact undergoes a sudden bend at the surface of the water, and thus reaches the eye by a crooked course,

in which the obstacle A is evaded. If the part of the pencil in air be produced backwards, its rays will approximately meet in a point

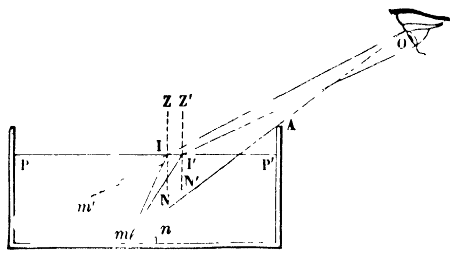


Fig. 99 —Experiment of Coin in Basin.

m', which is therefore the image of m. Its position is not correctly indicated in the figure, being placed too much to the left (§ 136).

The broken appearance presented by a stick (Fig. 100) when partly immersed in water in an oblique position, is similarly explained, the part beneath the water being lifted up by refraction.

123. Refractive Powers of Different Media.—In the experiments of the coin and stick, the rays, in leaving the water, are bent away from the normals Z I N, Z′ I′ N′ at the points of emergence; in the experiment first described (Fig. 98), on the other hand, the rays, in passing from air into water, are bent nearer to the normal. In every case the path which the rays pursue in going is the same as they would pursue in returning; and of the two media concerned, that in which the ray makes the smaller angle with the normal is said to have greater refractive power than the other, or to be more highly refracting.

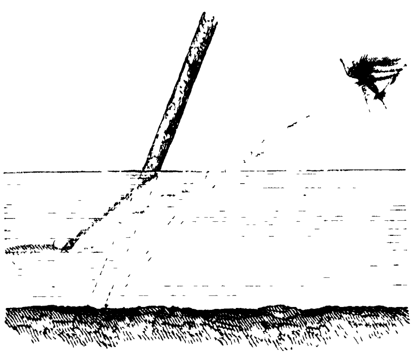


Fig. 100 —Appearance of Stick in Water.

Liquids have greater refractive power than gases, and as a general rule (subject to some exceptions in the comparison of dissimilar substances) the denser of two substances has the greater refracting power. Hence it has become customary, in enunciating some of the laws of optics, to speak of the *denser* medium and the *rarer* medium, when the more correct designations would be *more refractive* and *less refractive*.

124. Laws of Refraction.—The quantitative law of refraction was not discovered till quite modern times. It was first stated by Snell, a Dutch philosopher, and was made more generally known by Descartes, who has often been called its discoverer.

Let R I (Fig. 101) be a ray incident at I on the surface of separation of two media, and let I S be the course of the ray after refraction. Then the angles which R I and I S make with the normal are called the *angle of incidence* and the *angle of refraction* respectively; and the first law of refraction is that these angles lie in the same plane, or *the plane of refraction is the same as the plane of incidence.*

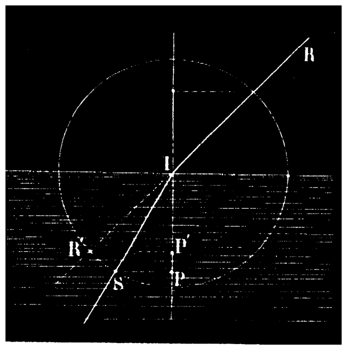


Fig 101 —Law of Refraction

The second law is that *the sines of the angles of incidence and refraction are in a constant ratio,* which is called the *index of refraction.* Describe a circle about the point of incidence I as centre (Fig. 101), and drop perpendiculars from the points where it cuts the rays on the normal. The ratio of these perpendiculars R′ P′, S P is the index of refraction.

The most convenient graphical construction is that which was given by Snell in his original statement of the law. We will illustrate it by the case in which the index is 4/3, which is its value for a ray going from air into water.

If the ray in air is given, lay off upon it 4 equal parts A B, B C, CD, DE (Fig. 102), starting from the point of incidence A. Through D draw a normal NDF, and on it find a point F whose distance from A is equal to AE. Then FA produced will be the path of the ray in water.

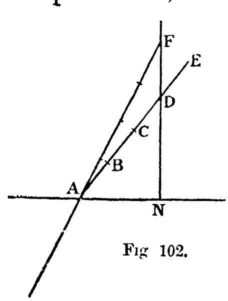


Fig 102.

If the ray in water is given, lay off upon its production in air 4 equal parts, and drop a normal FN from the extremity of the 4th part. Upon FN find a point D such that A D shall be equal to 3 of these parts. A D will be the path of the ray in air.

The same result would of course be obtained by employing the actual ray in water and the production of the air ray into the water.

In general the ratio AF / AD is to be made equal to the index of refraction; and this is in accordance with the law of sines; for AN / AF is the sine of the angle of refraction, and AN / AD is the sine of the angle of incidence.

125. Verification of the Law of Sines.—These laws can be verified by means of the apparatus represented in Fig. 103, which is very similar to that employed by Descartes. It has a vertical divided circle, to the front of which is attached a cylindrical vessel, half-filled with water or some other transparent liquid. The surface of the liquid must pass exactly through the centre of the circle. I is a movable mirror for directing a reflected beam of solar light on the centre O. The beam must be directed centrally through a short tube attached to the mirror, and to facilitate this adjustment the tube is furnished with a diaphragm with a hole in its centre. The arm O a is movable about the centre of the circle, and carries a vernier for measuring the angle of incidence. The ray undergoes refraction at O; and the angle of refraction is measured by means of a second arm O R, which is to be moved into such a position that the diaphragm of its tube receives the beam centrally. No refraction occurs at emergence, since the emergent beam is normal to the surfaces of the liquid and glass; the position of the arm accordingly indicates the direction of the refracted ray. The angles of incidence and refraction can be read off at the verniers carried by the two arms; and the ratio of their sines will be found constant. The sines can also be directly measured by employing sliding-scales as

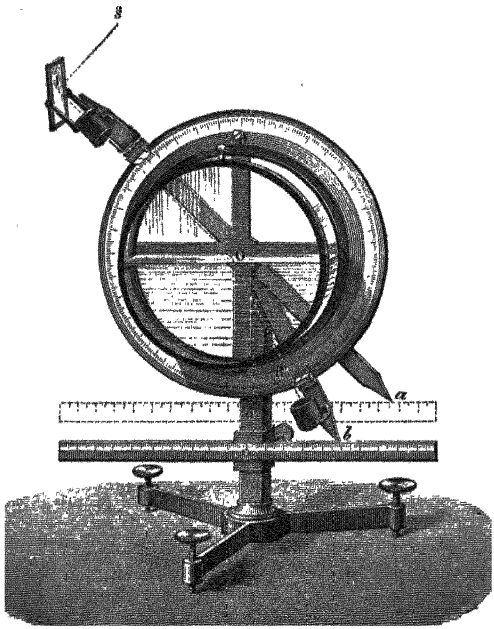


Fig. 103 —Apparatus for Verifying the Law.

indicated in the figure, the readings being taken at the extremity of each arm.

It would be easy to make a beam of light enter at the lower side of the apparatus, in a radial direction; and it would be found that the ratio of the sines was precisely the same as when the light entered from above. This is merely an instance of the general law, that the course of a returning ray is the same as that of a direct ray

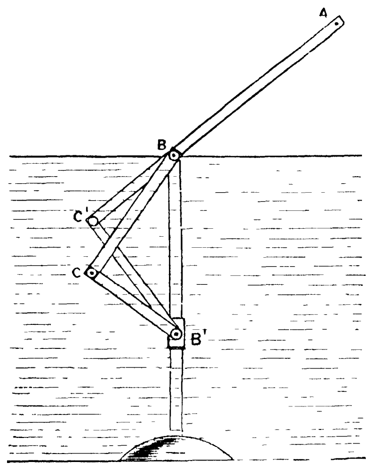


Fig 104 —Airy's Apparatus

126. Airy's Apparatus.—The following apparatus for the same purpose was invented, many years ago, by the late Sir George Airy. B′ is a slider travelling up and down a vertical stem. A C′ and B C are two rods pivoted on a fixed point B of the vertical stem. C′ B′ and C B′ are two other rods jointed to the former at C′ and C, and pivoted at their lower ends on the centre of the slider. B C is equal to B′ C′, and B C′ to B′ C. Hence the two triangles B C B′, B′ C′ B are equal to one another in all positions of the slider, their common side B B′ being variable, while the other two sides of each remain unchanged in length though altered in position.

The ratio $\frac{BC}{CB'}$ or $\frac{B'C'}{C'B}$ is made equal to the index of refraction of the liquid in which the observation is to be made. For water this ratio will be $\frac{4}{3}$. Then, if the apparatus is surrounded with water up to the level of B, A B C will be the path of a ray, and a stud at C will appear in the same line with studs at A and B, for we have

$$\frac{\sin C'BB'}{\sin CBB'} = \frac{\sin C'BB'}{\sin C'B'B} = \frac{C'B'}{C'B} = \frac{4}{3}.$$

127. Indices of Refraction.—The ratio of the sine of the angle of incidence to the sine of the angle of refraction when a ray passes from one medium into another, is called the *relative index of refraction* from the former medium to the latter. When a ray passes from vacuum into any medium this ratio is always greater than unity, and is called the *absolute index of refraction,* or simply the *index of refraction,* for the medium in question. The relative

index of refraction from any medium A into another B is always equal to the absolute index of B divided by the absolute index of A. The absolute index of air exceeds unity by a very small quantity, which can usually be neglected in dealing with solids or liquids: but strictly speaking, the relative index for a ray passing from air into a given substance must be multiplied by the absolute index for air, in order to obtain the absolute index of refraction for the substance.

The following table gives the indices of refraction of several substances:—

INDICES OF REFRACTION.[1]

Diamond,	2·44 to 2·755	Alcohol,	1·372
Sapphire,	1·794	Aqueous humour of eye,	1·337
Flint-glass,	1·576 to 1·642	Vitreous humour,	1·339
Crown-glass,	1·531 to 1·563	Crystalline lens, outer coat,	1·337
Rock-salt,	1·545	„ „ under coat,	1·379
Canada balsam,	1 540	„ „ central portion,	1·400
Bisulphide of carbon,	1·678	Sea water,	1·343
Linseed oil (sp. gr. ·932),	1·482	Pure water,	1 336
Oil of turpentine (sp. gr. ·885),	1·478	Air at 0° C and 760^{mm}	1·000294

128. Critical Angle.—We see, from the law of sines, that when the incident ray is in the less refractive of the two media, to every pos-

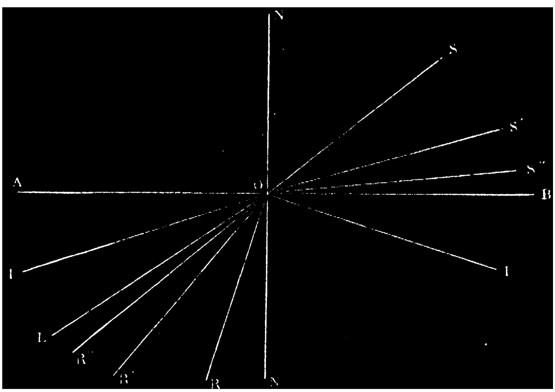


Fig. 105 —Critical Angle.

sible angle of incidence there is a corresponding angle of refraction. This, however, is not the case when the incident ray is in the more refractive of the two media. Let S O, S′ O, S″ O (Fig. 105) be incident rays in the less refractive medium, and O R, O R′, O R″ the

[1] The index of refraction is always greater for violet than for red (see Chap. xii.). The numbers in this table are to be understood as mean values.

corresponding refracted rays. There will be a particular direction of refraction O L corresponding to the angle of incidence of 90°. Conversely, incident rays R O, R′ O, R″ O, in the more refractive medium, will emerge in the directions O S, O S′, O S″, and the direction of emergence for the incident ray L O will be O B, which is coincident with the bounding surface.

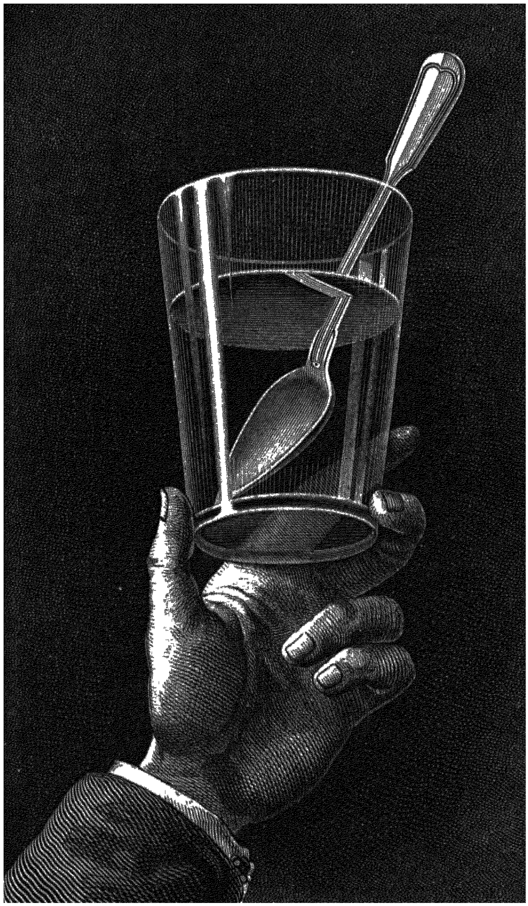


Fig 106 —Total Reflection.

The angle L O N is called the critical angle, and is easily computed when the relative index of refraction is given. For let μ denote this index (the incident ray being supposed to be in the less refractive medium), then we are to have

$$\frac{\sin 90^\circ}{\sin x} = \mu, \text{whence} \sin x = \frac{1}{\mu};$$

that is, *the sine of the critical angle is the reciprocal of the index of refraction.*

When the media are air and water, this angle is about 48° 30′. For air and different kinds of glass its value ranges from 38° to 41°.

If a ray, as I O, is incident in the more refractive medium, at an angle greater than the critical angle, the law of sines becomes nugatory, and experiment shows that such a ray undergoes internal reflection in the direction O I′, the angle of reflection being equal to the angle of incidence. Reflection occurring in these circumstances

is nearly perfect, and has received the name of *total reflection*. *Total reflection occurs when rays are incident in the more refractive medium at an angle greater than the critical angle.*

The phenomenon of total reflection may be observed in several familiar instances. For example, if a glass of water, with a spoon in it (Fig. 106), is held above the level of the eye, the under side of the surface of the water is seen to shine like a brilliant mirror, and the lower part of the spoon is seen reflected in it. Beautiful effects of the same kind may be observed in aquariums.

129. Camera Lucida.—The *camera lucida* is an instrument sometimes employed to facilitate the sketching of objects from nature. It acts by total reflection, and may have various forms, of which that proposed by Wollaston, and represented in Figs. 107, 108, is

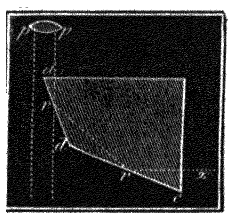


Fig. 107.—Section of Prism.

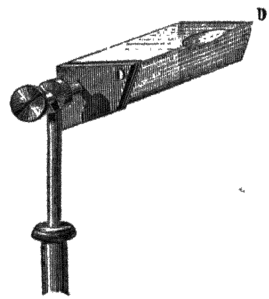


Fig. 108.—Camera Lucida.

one of the commonest. The essential part is a totally-reflecting prism with four angles, one of which is 90°, the opposite one 135°, and the other two each 67° 30′. One of the two faces which contain the right angle is turned towards the objects to be sketched. Rays incident normally on this face, as $x\ r$, make an angle greatly exceeding the critical angle with the normal to the face $c\ d$, and are totally reflected from it to the next face $d a$, whence they are again totally reflected to the fourth face, from which they emerge normally.[1] An eye placed so as to receive the emergent rays will see a virtual image in a direction at right angles to that in which the object lies. In practice, the eye is held over the angle a of the prism, in such a position that one-half of the pupil receives these reflected rays, while the other half receives light in a parallel direction outside the prism. The observer thus sees the reflected image projected on a real back-

[1] The use of having *two* reflections is to obtain an erect image. An image obtained by one reflection would be upside down.

ground, which consists of a sheet of paper for sketching. He is thus enabled to pass a pencil over the outlines of the image, pencil, image, and paper being simultaneously visible. It is very desirable that the image should lie in the plane of the paper, not only because the pencil point and the image will then be seen with the same focussing of the eye, but also because parallax is thus obviated, so that when the observer shifts his eye the pencil point is not displaced on the image. A concave lens, with a focal length of something less than a foot, is therefore placed close in front of the prism, in drawing distant objects By raising or lowering the prism in its stand (Fig 108), the image of the object to be sketched may be made to coincide with the plane of the paper.

The prism is mounted in such a way that it can be rotated either about a horizontal or a vertical axis, and its top is usually covered with a movable plate of blackened metal, having a semicircular notch at one edge, for the observer to look through.

The camera lucida is now little used except for drawing objects as seen in the microscope. For this purpose inversion is not objectionable, and a triangular prism giving only a single reflection is commonly employed instead of the quadrilateral one shown in the figure. A still simpler form, which is more convenient for use, consists of a thin plate of glass, which receives the rays at an incidence of about 45° and reflects them to the eye, while the paper is at the same time seen through the glass.

130.—Theoretical Explanation of Refraction.—Theory asserts that the index of refraction from one medium to another is the ratio of the velocities of light in the two media. Let v_1 be the velocity in the first, and v_2 in the second medium, and let ${}_1\mu_2$ denote the index of refraction from the first to the second, that is, $\sin\phi_1/\sin\phi_2$, ϕ_1 being the angle of incidence, and ϕ_2 the angle of refraction. According to the wave-theory (which is now universally accepted) ${}_1\mu_2$ is equal to v_1/v_2. According to the corpuscular theory (which was favoured by Newton) ${}_1\mu_2$ is equal to v_2/v_1. Both theories lead obviously to the results ${}_1\mu_2 \,.\, {}_2\mu_1 = 1$, ${}_1\mu_2 \,.\, {}_2\mu_3 \,.\, {}_3\mu_1 = 1$, and so on to any number of factors.

Using the subscript 0 for vacuum we have therefore—

$$ {}_0\mu_1 \cdot {}_1\mu_2 \cdot {}_2\mu_0 = 1, $$

or

$$ {}_1\mu_2 = {}_0\mu_2 / {}_0\mu_1, $$

that is, $\sin\phi_1/\sin\phi_2 = \mu_2/\mu_1$, or $\mu_1 \sin\phi_1 = \mu_2 \sin\phi_2$, where μ_1 μ_2 are the absolute indices.

We have accordingly the following rules:—

1. The *relative index* from one medium to another is equal to the *absolute index of the second divided by that of the first.*

2. *The value of μ sin ϕ is the same for the incident as for the refracted ray,* μ denoting the absolute index of either medium, and ϕ the angle which the ray in this medium makes with the normal at the point of incidence.

131. Refraction through Superposed Plates.—A *plate* in optics means a transparent body with two parallel faces. Let any number of plates of different materials be superposed; then the surfaces of contact are parallel planes, and all the normals are parallel lines. Let a ray be refracted through them all, and let ϕ_1 ϕ_2 ϕ_3 &c., be the angles which the rays in the successive plates make with the common direction of the normal. We have for the first refraction $\mu_1 \sin \phi_1 = \mu_2 \sin \phi_2$; for the second $\mu_2 \sin \phi_2 = \mu_3 \sin \phi_3$, and so on. Thus the expression $\mu \sin \phi$ has the same value at all points of a ray which traverses a pile of plates.

132. Application to Astronomical Refraction.—The portion of the atmosphere over any portion of the earth's surface, small enough for the curvature of the earth to be negligible, may be regarded as consisting of an indefinitely large number of superposed plates, their densities and indices of refraction diminishing as we ascend till we reach vacuum, where $\mu = 1$. Accordingly the value of $\mu \sin \phi$ for a ray will be the same at the point where the ray meets an observer's eye as at the point where it first meets the earth's atmosphere.

Suppose the ray to come from a star, then ϕ is the apparent zenith distance of the star at any point in the course of the ray, and $\mu \sin \phi$ will be its true zenith distance, the difference being due to the refraction of the earth's atmosphere. The direction of the ray at the earth's surface is the same as if it had passed directly out of vacuum into the lowest plate of air.

If z be the apparent and $z+h$ the true zenith distance, we shall have (since h is small)—

$$\begin{aligned} \mu \sin z &= \sin (z+h) \\ &= \sin z \cos h + \cos z \sin h \\ &= \sin z + h \cos z, \text{ nearly} \\ \text{whence } h &= (\mu - 1) \tan z. \end{aligned}$$

This is practically correct so long as z does not exceed 20° or 30°. When z is large, the upper plates which are traversed are no longer

parallel to the lower plates, and h is less than the above formula would make it.

133. Displacement produced by Plate.—When a ray is transmitted through a plate, its incident and emergent portions are parallel, as

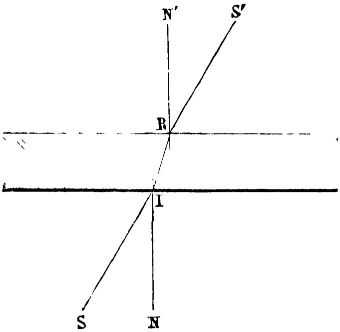


Fig 109 —Parallel Plate

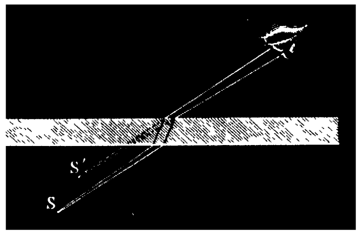


Fig 110 —Vision through Plate.

in Fig 109, the distance between them being proportional to the thickness of the plate, and increasing with the obliquity of incidence, as well as with the index of refraction. Objects seen obliquely through a plate are accordingly displaced laterally, as shown in Fig. 110.

134. Multiple Images produced by a Plate.—Let S (Fig. 111) be a luminous point in front of a transparent plate with parallel faces. Of the rays which it sends to the plate, some will be reflected from the front, thus giving rise to an image S′. Another portion will enter the plate, undergo reflection at the back, and emerge with refraction at the front, giving rise to a second image S^0. Another portion will undergo internal reflection at the front, then again at the back, and by emerging in front will form a third image S_1. The same process may be repeated several times; and if the luminous object be a candle, or a piece of bright metal, a number of images, one behind another, will be visible to an eye properly placed

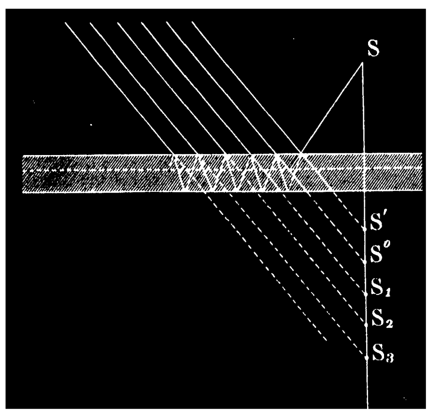


Fig 111.—Multiple Images in Plate

in front (Fig. 112). All the successive images, after the first two, continually diminish in brightness. If the glass be silvered at the back, the second image is much brighter than the first when the incidence is nearly normal, but as the angle of incidence increases, the first image gains upon the second, and ultimately surpasses it. This is due to the fact that the reflecting power of a surface of glass increases with the angle of incidence.

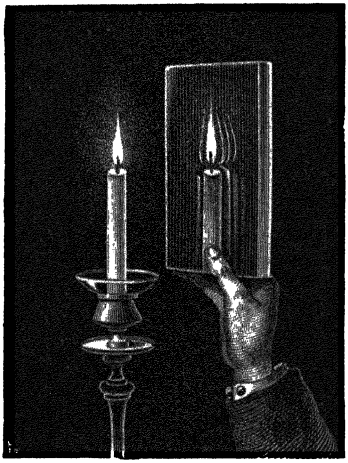


Fig 112.
Images of Candle in Looking-glass.

If the luminous body is at a distance which may be regarded as infinite,—if it is a star, for example,—all the images should coincide, and form only a single image, occupying a position which does not vary with the position of the observer, provided that the plate is perfectly homogeneous, and its faces perfectly plane and parallel. A severe test is thus furnished of the fulfilment of these conditions.

Plates are sometimes tested, for parallelism and uniformity, by supporting them in a horizontal position on three points, viewing the image of a star in them with a telescope furnished with cross wires, and observing whether the image is displaced on the wires when the plate is shifted into a different position, still resting on the same three points.

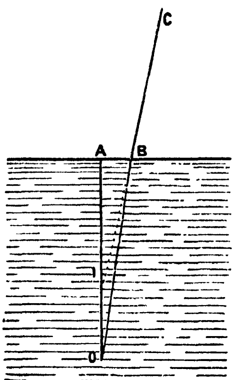


Fig. 113
Image by Refraction.

135. Images by Refraction at a Plane Surface.—Let O (Fig. 113) be a small object in the interior of a solid or liquid bounded by a plane surface AB. Let OBC be the path of a nearly normal ray, and let BC (the portion in air) be produced backwards to meet the normal in I. Then, since AIB and AOB are the inclinations of the two portions of the ray to the normal, we have (if μ be the index of refraction from air into the substance)—

$$\mu = \frac{\sin AIB}{\sin AOB} = \frac{OB}{IB}.$$

But OB is ultimately equal to OA, and IB to IA. Hence, if we make AI equal to $\frac{AO}{\mu}$, all the emergent rays of a small and nearly normal pencil emitted by O will, if produced backwards, intersect OA at points indefinitely near to the point I thus determined. If the eye of an observer be situated on the production of the normal OA, the rays by which he sees the object O constitute such a pencil. He accordingly sees the image at I. As the value of μ is $\frac{4}{3}$ for water, and about $\frac{3}{2}$ for glass, it follows that the apparent depth of a pool of clear water when viewed vertically is $\frac{3}{4}$ of the true depth, and that the apparent thickness of a piece of plate-glass when viewed normally is only $\frac{2}{3}$ of the true thickness.

136.—When the incident pencil (Fig 114) is not small, but includes rays of all obliquities, those of them which make angles with the

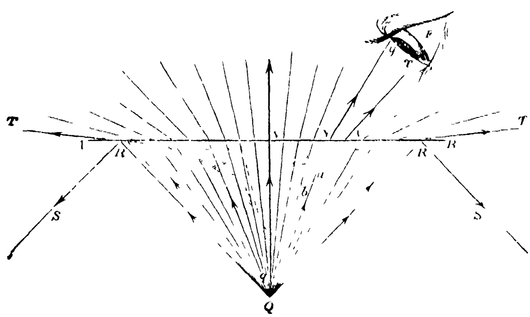


Fig 114 —Caustic by Refraction

normal less than the critical angle NQR will emerge into air, and the emergent rays, if produced backwards, will all touch a certain caustic surface, which has the normal QN for its axis of revolution, and touches the surface at all points of a circle of which NR is the radius. Wherever the eye may be situated, a tangent drawn from it to the caustic will be the direction of the visible image If the observer sees the image with both eyes, both being equidistant from the surface and also equidistant from the normal, the two lines of sight thus determined (one for each eye) will meet at a point on the normal, which will accordingly be the apparent position of the image. If, on the other hand, both eyes are in the same plane containing the normal, the two lines of sight will intersect at a point between the normal and the observer.

The image, whether seen with one eye or two, approaches nearer to the surface as the direction of vision becomes more oblique, and ultimately coincides with it. The apparent depth of water, which is only $\frac{3}{4}$ of the real depth when seen vertically, is accordingly less than $\frac{3}{4}$ when seen obliquely, and becomes a vanishing quantity as the direction of vision approaches to parallelism with the surface. The focus I determined in the preceding section is at the cusp of the caustic.

137. Refraction through a Prism.—For optical purposes, any portion of a transparent body lying between two plane faces which are not parallel may be regarded as a prism.[1] The line in which these faces meet, or would meet if produced, is called the edge of the prism, and a

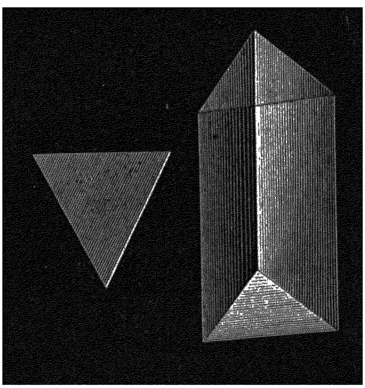


Fig. 115.—Equilateral Prism.

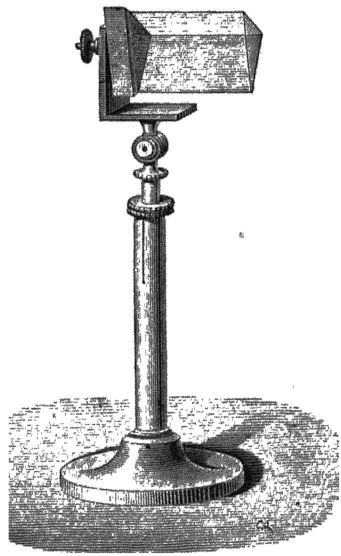


Fig. 116.—Prism mounted on Stand.

section made by a plane perpendicular to them both is called a *principal section* or a *principal plane*. The prisms chiefly employed are really prisms in the geometrical sense of the word. Their principal sections are usually triangular, and are very frequently equilateral, as in Fig. 115. The stand usually employed for prisms when mounted separately is represented in Fig. 116. It contains several joints. The uppermost is for rotating the prism about its own axis. The second is for turning the prism so that its edges shall make any required angle with the vertical. The third gives

[1] This amounts to saying that the word *prism* in optics means *wedge*.

motion about a vertical axis, and also furnishes the means of raising and lowering the prism through a range of several inches.

Let S I (Fig. 117) be an incident ray in a principal plane of the prism. If the external medium be air, or any other substance of less refractive power than the prism, the ray in entering the prism will be bent nearer to the normal, taking such a course as I E, and in leaving the prism will be bent away from the normal, taking the course E B. The effect of these two refractions is, therefore, to turn the ray away from the edge (or refracting angle) of the prism. In practice, the prism is usually so placed that I E, the path of the ray through the prism, makes equal angles with the two faces at which refraction occurs. If the prism is turned very far from this position, the course of the ray may be altogether different from that represented in the figure; it may, for example, enter at one face, be internally reflected at another, and come out at the third; but we at present exclude such cases from consideration.

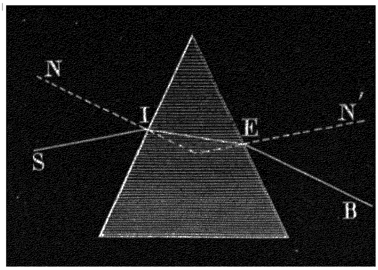


Fig. 117.—Refraction through Prism.

The direction of deviation is easily shown experimentally, by admitting a narrow beam of sunlight into a dark room, and introducing a prism in its course. It will be found that the refracted beam, in the circumstances represented in Fig. 117, is turned aside some 40° or 50° from its original course.[1]

Since the rays which traverse a prism are bent away from the edge, the object from which they proceed will appear, to an observer looking through the prism, to be more nearly in the direction of the edge than it really is. If, for example, he looks at the flame of a candle through a prism placed so that the edge which corresponds to the refracting angle is at the top (Fig. 118), the apparent place of the flame will be above its true place.

138. Formulæ for Refraction through Prisms. Minimum Deviation.—Let S I (Fig. 119) be an incident ray in the plane of a principal

[1] The phenomena here described are complicated in practice by the unequal refrangibility of rays of different colours (Chap. xii.). The complication may be avoided by employing homogeneous light, of which a spirit-lamp, with common salt sprinkled on the wick, affords a nearly perfect example.

section A B C of a prism. Let i be the angle of incidence S I N, and

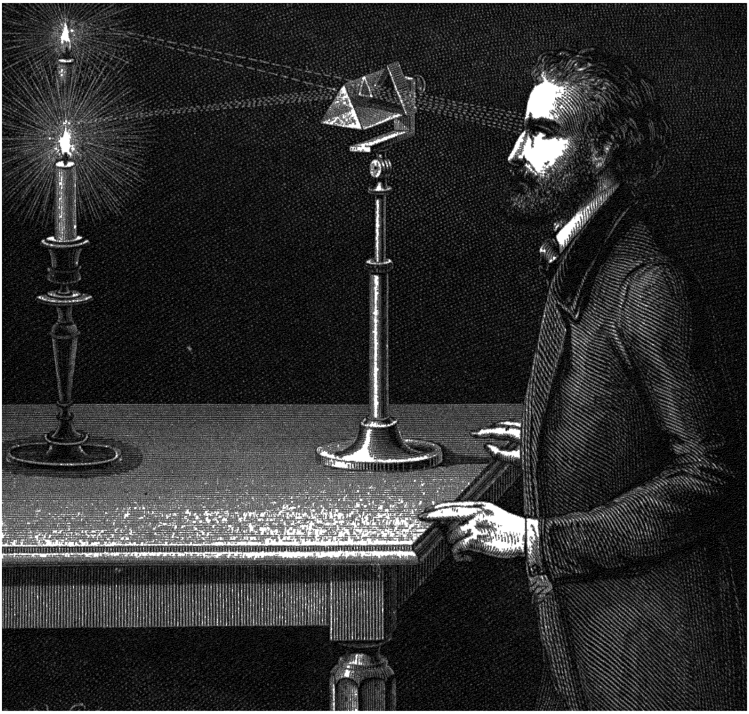


Fig. 118 —Vision through Prism.

r the angle of refraction M I I′. Then, denoting the index of refrac-

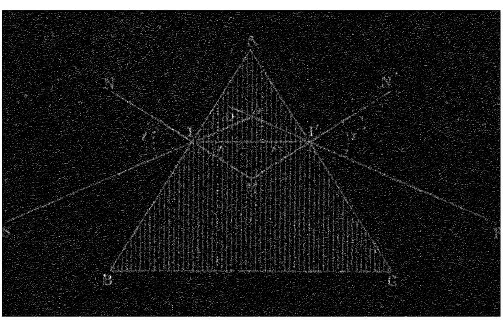


Fig. 119.—Refraction through Prism.

tion by μ, we have $sin\ i = \mu\ sin\ r$. In like manner, putting r' for

the angle of internal incidence on the second face I I′M, and i' for the angle of external refraction N′I′R, we have $\sin i' = \mu \sin r'$.

The deviation produced at I is $i - r$, and that at I′ is $i' - r'$, so that the total deviation, which is the acute angle D contained between the rays S I, R I′, when produced to meet at o, is

$$D = i - r + i' - r'. \quad (1)$$

But if we drop a perpendicular from the angular point A on the ray I I′, it will divide the refracting angle B A C into two parts, of which that on the left will be equal to r, and that on the right to r', since the angle contained between two lines is equal to that contained between their perpendiculars. We have therefore $A = r + r'$, and by substitution in the above equation

$$D = i + i' - A. \quad (2)$$

When the path of the ray through the prism I I′ makes equal angles with the two faces, the whole course of the ray is symmetrical with respect to a plane bisecting the refracting angle, so that we have

$$i = i'; \; r = r' = \frac{A}{2}.$$

Equation (2) thus becomes

$$D = 2i - A, \text{ whence } i = \frac{A + D}{2}, \quad (3)$$

$$\text{and } \mu = \frac{\sin i}{\sin r} = \frac{\sin \frac{A+D}{2}}{\sin \frac{A}{2}}. \quad (4)$$

This last result is of great practical importance, as it enables us to calculate the index of refraction μ from measurements of the refracting angle A of the prism, and of the deviation D which occurs when the ray passes symmetrically.

When a beam of sunlight in a dark room is transmitted through a prism, it will be found, on rotating the prism about its axis, that there is a certain mean position which gives smaller deviation of the transmitted light than positions on either side of it, and that, when the prism is in this position, a small rotation of it has no sensible effect on the amount of deviation. The position determined experimentally by these conditions, and known as the *position of minimum deviation*, is the position in which the ray passes symmetrically.

139. Construction for Deviation in a Single Refraction.—When a ray is refracted at a plane surface, the deviation increases with the angle of incidence; as may be proved by the following construction.

Describe (Fig. 120) two concentric circles, the ratio of their radii being the index of refraction. Draw any line N A B cutting the circles in A and B, and draw the radii O A, O B. Then by Snell's rule, if O A N be the angle of incidence in the rarer medium, O B N will be the corresponding angle of refraction, and their difference A O B will be the deviation.

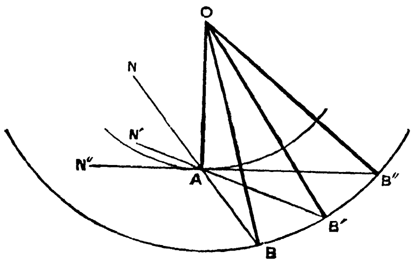


Fig. 120.—General Construction for Deviation.

As the angle of incidence increases from OAN to OAN′, the deviation increases from AOB to AOB′. The deviation vanishes as A N moves into coincidence with A O, and attains its greatest value AO B″ when A N″ is a tangent to the smaller circle, the angle of incidence OA N″ being then a right angle, and the sine of the angle of refraction O B″ N″ being OA/O B″, or the reciprocal of the index of refraction.

140. Construction for Refraction through a Prism.—To apply Snell's construction to the deviation of a ray refracted through a prism in a principal plane, let F E F′ (Fig. 121) be the refracting angle of the prism. Draw E B parallel to the ray in the prism, and from any point B in it drop normals B N, B N′ on the two faces or their planes produced. Find points A A′ on these normals such that $EA = EA' = EB/\mu$ Then E A, E A′ are parallel to the two rays in air, and the whole course $a\,b\,b'\,a'$ of the ray can be traced by making its three portions parallel to A E, B E, A′E respectively AEA′ is the total deviation, and ABA′, being the angle between the normals to the two faces, is equal to the angle of the prism.

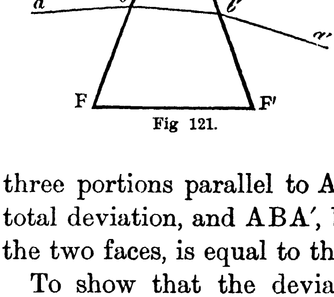


Fig 121.

To show that the deviation is least when the course is symmetrical, describe a circle round E as centre, passing through A and

A′, as in Fig 122, where O takes the place of E, the other letters being unchanged. The deviation A O A′ is proportional to the arc A A′. Let O B be fixed and O A, O A′ movable, keeping the angle A B A′ constant (= angle of prism). Let A B A′, a B a' (Fig. 123), be two consecutive positions, B A′ and B a' being greater than B A and B a. Then, since the small angles A B a, A′ B a' are equal, it is obvious, for a double reason,—greater distance from B and greater obliquity,—that the small arc A′ a' is greater than A a, and hence

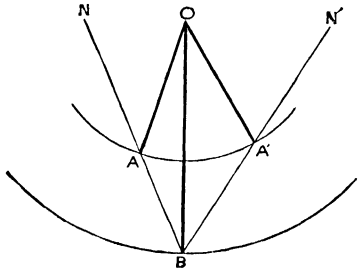


Fig 122 —Application to Prism.

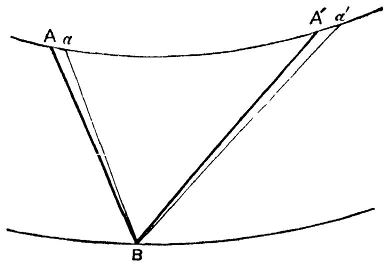


Fig 123 —Proof of Minimum Deviation.

the whole arc $a\,a'$ is greater than A A′. The deviation is therefore increased by altering the position in such a way as to make B A and B A′ depart further from equality, and is a minimum when they are equal.

141. Conjugate Foci for Minimum Deviation.—When the angle of incidence is nearly that corresponding to minimum deviation, a small change in this angle has no sensible effect on the amount of deviation.

Hence a small pencil of rays coming from a luminous point, and incident near the refracting edge at this angle, will emerge with their divergence sensibly unaltered, so that if produced backwards they would meet in a virtual focus at the same distance (but of course not in the same direction) as the point from which they came.

In like manner, if a small pencil of rays converging towards a point, are turned aside by interposing the edge of a prism in the position of minimum deviation, they will on emergence converge to another point at the same distance. We may therefore assert that, neglecting the thickness of a prism, *conjugate foci are at the same distance from it, and on the same side, when the deviation is a minimum.*

142. Spectrometer.—The instrument commonly employed for

measuring the minimum deviation of a prism is called a *spectrometer*, and is represented in plan in Fig. 124. It consists of a circular brass table on which are mounted two tubes, one fixed and the other movable, and a small revolving table in its centre. The fixed tube $s\,c$ is called the *collimator*, and has a slit at s in the focus of a convex lens at c. $c\,o$ is the observing telescope, with a cross of threads or similar mark in the common focus of its object-glass o and its eyepiece e. It can be moved round the graduated circle A A, and carries a vernier on which its exact position can be read. Its line of collimation is always directed to the centre of the circle. The small circular revolving table T is raised a little above the larger table to serve as a support for the prism p, and the angles through which it revolves are indicated either on the circle A A or on a smaller circle provided for the purpose.

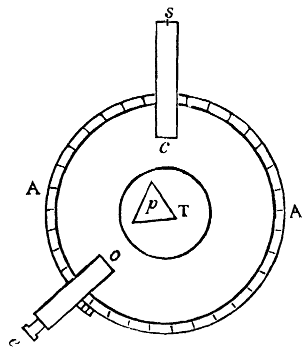


Fig. 124 —Spectrometer.

For measuring minimum deviation, the telescope is first placed directly opposite the collimator so that the slit is seen on its cross-wires, and the reading then taken for the position of the telescope is the zero from which deviations are to be measured. The revolving table with the prism on it is then turned into the position of minimum deviation, as shown in the figure, the slit being illuminated by a sodium flame or other source of monochromatic light; and the telescope is moved along the circle A A till the slit is again seen on its cross-wires. The difference between the reading in this position and the zero reading is the minimum deviation.

The same instrument serves for measuring the angle of the prism by one of the two methods described in § 101, and the index of refraction can then be computed by formula (4) of § 138.

143. Double Refraction.—Thus far we have been treating of what is called *single refraction*. We have assumed that to each given incident ray there corresponds only one refracted ray. This is true when the refraction is into a liquid, or into well-annealed glass, or into a crystal belonging to the cubic system.

On the other hand, when an incident ray is refracted into a crystal of any other than the cubic system, or into glass which

is unequally stretched or compressed in different directions—for example, into unannealed glass—it gives rise in general to two refracted rays which take different paths; and this phenomenon is called *double refraction*. Attention was first called to it in 1670 by Bartholin, who observed it in the case of Iceland-spar, and its laws for this substance were accurately determined by Huygens.

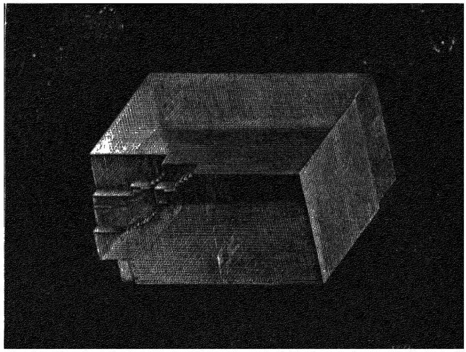


Fig. 125.—Iceland-spar.

144. Phenomena of Double Refraction in Iceland-spar.—Iceland-spar or calc-spar is a form of crystallized carbonate of lime, and is found in large quantity in the country from which it derives its name. It is usually found in rhombohedral form, as represented in Figs. 125, 126.

To observe the phenomenon of double refraction, a piece of the spar may be laid on a page of a printed book. All the letters seen through it will appear double, as in Fig. 126; and the depth of their blackness is considerably less than that of the originals, except where the two images overlap.

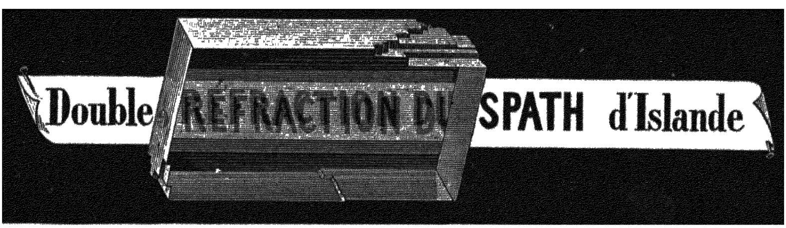


Fig. 126.—Double Refraction of Iceland-spar.

In order to state the laws of the phenomena with precision, it is necessary to attend to the crystalline form of Iceland-spar.

At the corner which is represented as next us in Fig. 125 three equal obtuse angles meet; and this is also the case at the opposite corner which is out of sight. If a line be drawn through one of these corners, making equal angles with the three edges which meet

there, it or any line parallel to it is called the *axis* of the crystal; the axis being properly speaking not a definite *line* but a definite *direction*.

The angles of the crystal are the same in all specimens; but the lengths of the three edges (which may be called the oblique length,

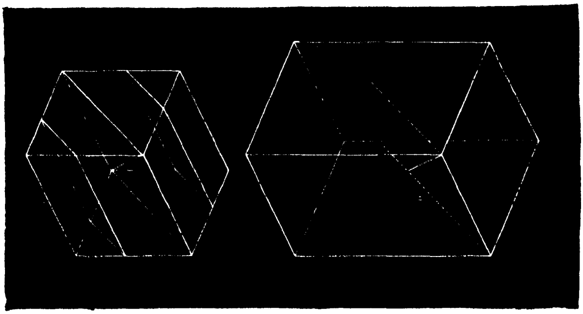


Fig. 127.—Axis of the Crystal.

breadth, and thickness) may have any ratios whatever. If the crystal is of such proportions that these three edges are equal, as in the first part of Fig. 127, the axis is the direction of one of its diagonals, which is represented in the figure.

Any plane containing (or parallel to) the axis is called a *principal plane* of the crystal.

If the crystal is laid over a dot on a sheet of paper, and is made to rotate while remaining always in contact with the paper, it will be observed that, of the two images of the dot, one remains unmoved, and the other revolves round it. The former is called the *ordinary*, and the latter the *extraordinary* image. It will also be observed that the former appears nearer than the latter, being more lifted up by refraction.

The rays which form the ordinary image follow the ordinary law of sines (§ 124). They are called the ordinary rays. Those which form the extraordinary image (called the extraordinary rays) do not follow the law of sines, except when the refracting surface is parallel to the axis, and the plane of incidence perpendicular to the axis; and in this case their index of refraction (called the extraordinary index) is different from that of the ordinary rays. The ordinary index is 1·65, and the extraordinary 1·48.

When the plane of incidence is parallel to the axis, the extraordinary ray always lies in this plane, whatever be the direction of

the refracting surface; but the ratio of the sines of the angles of incidence and refraction is variable.

When the plane of incidence is oblique to the axis, the extraordinary ray generally lies in a different plane.

We shall recur to the subject of double refraction in the concluding chapter of this volume.

145. Curved Image of Straight Slit.—The image of a straight slit when obtained by means of a prism which gives large deviation exhibits marked curvature, the rays from the ends of the slit being more deviated than those from the centre. This depends on the fact that the rays from the ends go through the prism in planes which are not principal planes; a principal plane being defined as a plane which is perpendicular to the edges of the prism.

Let x, y, z be rectangular co-ordinates of any point of a ray incident on the first face of the prism, measured from the point of incidence, z being parallel to the slit and to the edges of the prism, y normal to the face, and therefore x parallel to the face in a principal plane; and let x' y' z' be the co-ordinates of a point on the corresponding refracted ray produced backwards as in Snell's construction.

Since the plane of incidence and refraction intersects the face in a straight line, the ratio of x to z will be the same as the ratio of x' to z', and in Snell's construction we shall have $x=x'$, $z=z'$. The square of the index of refraction will be

$$\mu^2 = \frac{x'^2+y'^2+z'^2}{x^2+y^2+z^2} = \frac{x^2+y'^2+z^2}{x^2+y^2+z^2}.$$

Projecting the path of the ray upon the plane of x, y (that is upon a principal plane through the origin), the normal will coincide with its own projection, the point x, y, z will project into the point x, y, 0, and the angle of incidence will project into an angle whose sine is $x/\sqrt{(x^2+y^2)}$. The angle of refraction will in like manner project into an angle whose sine is $x'/(\sqrt{x'^2+y'^2})$, that is $x/\sqrt{(x^2+y'^2)}$, and the square of their ratio is $\frac{x^2+y'^2}{x^2+y^2}$. If μ^2 is greater than unity, this ratio is obviously greater still, since the numerators and denominators have the same difference z^2. Hence the projected ray makes a greater bend than an actual ray at the same inclination to the normal would make. The bending thus measured by its projection on a principal plane will evidently be greatest for the rays which have the largest values of z^2 for given values of x^2 and y^2,

that is, for the rays which come from the ends of the slit. Similar reasoning will apply to the refraction at the second face, and thus the ends of the image will exhibit more deviation in a principal plane than the central parts of the image.

146. Angles made with Normal Plane.—It is worth while to note in passing that the sine of the angle which the ray incident on the first face makes with the principal plane is $z/\sqrt{x^2+y^2+z^2}$, and the sine of the angle which the refracted ray makes with the same plane is $z'/\sqrt{x'^2+y'^2+z'^2}$, whereas the sines of the angles of incidence and refraction are $\sqrt{x^2+z^2}/\sqrt{x^2+y^2+z^2}$ and $\sqrt{x'^2+z'^2}/\sqrt{x'^2+y'^2+z'^2}$. The two former are in the same ratio as the two latter, since $z/z'=x/x'$, and their ratio is therefore equal to the index of refraction.

All that we have proved for refraction at the first face of a prism holds for refraction at any plane surface, the plane of x, y denoting any plane which contains the normal. Hence in every case of refraction at a plane surface *the sines of the angles which the incident and the refracted ray make with any plane perpendicular to the surface are in a constant ratio, which is equal to the index of refraction.*

CHAPTER IX.

LENSES.

147. Forms of Lenses.—A lens is usually a piece of glass bounded by two surfaces which are portions of spheres. There are two principal classes of lenses.

1. *Converging* lenses or *convex* lenses, which have one or other of the three forms represented in Fig 128. The first of these is called double convex, the second plano-convex, and the third concavo-convex. This last is also called a converging meniscus. All three

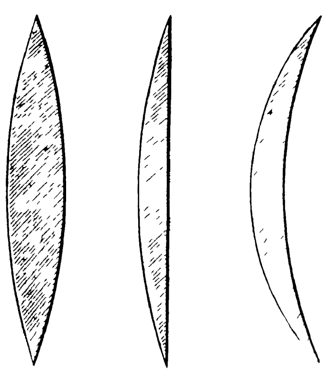


Fig 128 —Converging Lenses

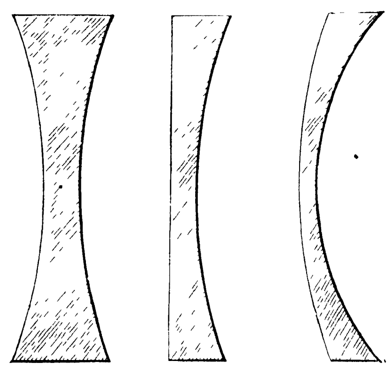


Fig 129 —Diverging Lenses

are thicker in the middle than at the edges. They are called converging, because rays are always more convergent or less divergent after passing through them than before.

2. *Diverging* lenses or *concave* lenses (Fig 129) produce the opposite effect, and are characterized by being thinner in the middle than at the edges. Of the three forms represented, the first is double concave, the second plano-concave, and the third convexo-concave (also called a diverging meniscus).

From the immense importance of lenses, especially convex lenses, in practical optics, it will be necessary to explain their properties at some length.

148. **Principal Focus.**—A lens is usually a solid of revolution, and the axis of revolution is called the *axis* of the lens, or sometimes the *principal axis.* When the surfaces are spherical, it is the line joining their centres of curvature.

When rays which were originally parallel to the principal axis pass through a convex lens (Fig. 130), the effect of the two refractions which they undergo, one on entering and the other on leaving the lens, is to make them all converge approximately to one point F, which is called the *principal focus.* The distance A F of the principal focus from the lens is called the *principal focal distance,* or more briefly and usually, the *focal length* of the lens. There is another principal focus at the same distance on the other side of the lens, corresponding to an incident beam coming in the opposite direction. The focal length depends on the convexity of the surfaces of the lens, and also on the refractive power of the material of which it is composed, being shortened either by an increase of refractive power or by a diminution of the radii of curvature of the faces.

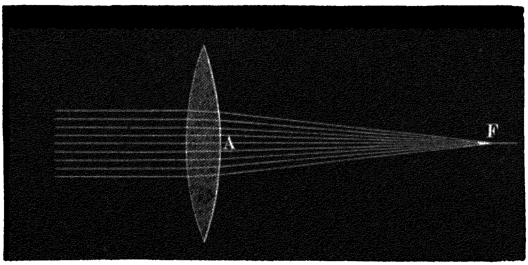


Fig 130.—Principal Focus of Convex Lens.

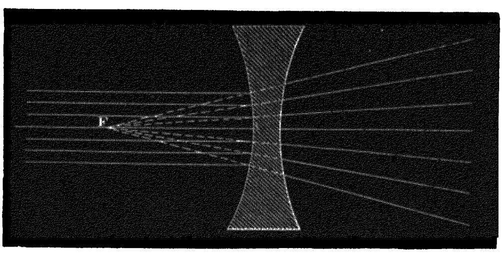


Fig. 131.—Principal Focus of Concave Lens.

In the case of a concave lens, rays incident parallel to the principal axis diverge after passing through; and their directions, if produced backwards, would approximately meet in a point F (Fig. 131), which is still called the principal focus. It is only a virtual focus, inasmuch as the emergent rays do not actually pass through it, whereas the principal focus of a converging lens is real.

149. Optical Centre of a Lens. Secondary Axes.—Let O and O′ (Fig. 132) be the centres of the two spherical surfaces of a lens. Draw any two parallel radii O I, O′ E to meet these surfaces, and let the joining line I E represent a ray passing through the lens. This ray makes equal angles with the normals at I and E, since these latter are parallel by construction; hence the incident and emergent rays S I, E R also make equal angles with the normals, and are therefore parallel. In fact, if tangent planes (indicated by the dotted lines in the figure) are drawn at I and E, the whole course of the ray SIER will be the same as if it had passed through a plate bounded by these planes.

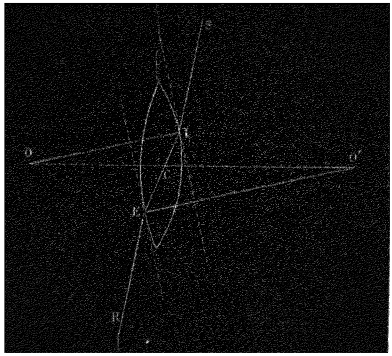


Fig 132.—Centre of Lens.

Let C be the point in which the line I E cuts the principal axis, and let R, R′ denote the radii of the two spherical surfaces. Then, from the similarity of the triangles O C I, O′ C E, we have

$$\frac{OC}{CO'} = \frac{R}{R'}; \qquad (1)$$

which shows that the point C divides the line of centres O O′ in a definite ratio depending only on the radii. Every ray whose direction on emergence is parallel to its direction before entering the lens, must pass through the point C in traversing the lens; and conversely, every ray which, in its course through the lens, traverses the point C, has parallel directions at incidence and emergence. The point C which possesses this remarkable property is called the *centre*, or *optical centre*, of the lens.

In the case of a double convex or double concave lens, the optical centre lies in the interior, its distances from the two surfaces being directly as their radii. In plano-convex and plano-concave lenses it is situated on the convex or concave surface. In a meniscus of either kind it lies outside the lens altogether, its distances from the surfaces being still in the direct ratio of their radii of curvature.[1]

[1] These consequences follow at once from equation (1); for the distances of C from the

In elementary optics it is usual to neglect the thickness of the lens. The incident and emergent rays S I, E R may then be regarded as lying in one straight line which passes through C, and we may lay down the proposition that *rays which pass through the centre of a lens undergo no deviation.* Any straight line through the centre of a lens is called a *secondary axis.*

The approximate convergence of the refracted rays to a point, when the incident rays are parallel, is true not only for rays parallel to the axis, but for rays making any small angle with it, and the point to which the emergent rays approximately converge (*f*, Fig. 133) is always situated on the secondary axis (*a c f*) parallel to the incident rays. The focal distance is sensibly the same as for rays parallel to the principal axis, unless the obliquity is considerable.

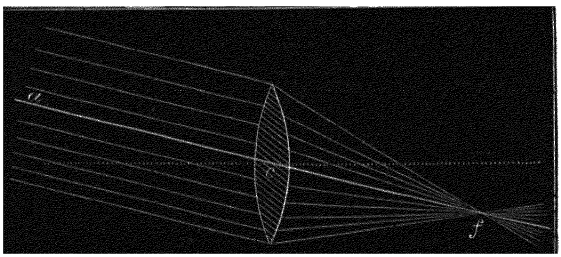


Fig. 133.—Principal Focus on Secondary Axis.

When the incidence is very oblique the refracted rays no longer pass approximately through one point, but through two focal lines, as in the case of mirrors.

150. Conjugate Foci.—When a luminous point S on or near the axis sends rays to a lens (Fig. 134), the emergent rays converge (approximately) to one point S′; whence it follows that rays sent from S′ to the lens would converge (approximately) to S. Two points thus related are called *conjugate foci* of the lens, and the line joining them always passes through the centre of the lens; in other words, they must either be both on the principal axis, or both on the same secondary axis.

The fact that rays which come from one point go to one point is

two faces are respectively the difference between R and O C, and the difference between R′ and O′ C, and we have

$$\frac{R}{R'} = \frac{OC}{O'C} = \frac{R - OC}{R' - O'C}.$$

the foundation of the theory of images, as we have already explained in connection with mirrors (§ 106).

The diameters of object and image are directly as their distances from the centre of the lens, and the image will be erect or inverted

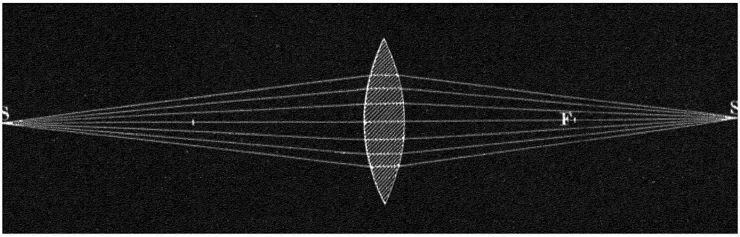


Fig. 134 —Conjugate Foci, both Real.

according as the object and image lie on the same side or on opposite sides of this centre (§ 112). There is also, in the case of lenses, the same difference between an image seen in mid-air and an image thrown on a screen, which we have pointed out in § 119.

It is to be remarked that the distinction between principal and secondary axes has much more significance in the case of lenses than of mirrors, and images produced by a lens are more distinct in the neighbourhood of the principal axis than at a distance from it.

151. Formulæ relating to Lenses.—The deviation produced in a ray by transmission through a lens will not be altered by substituting

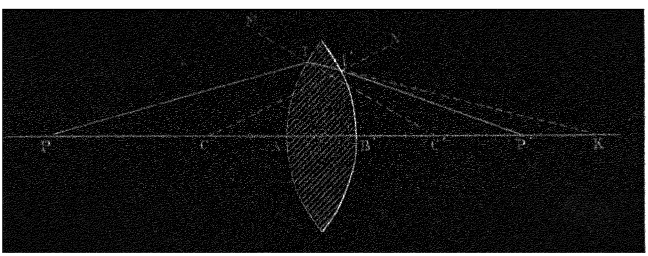


Fig 135.—Diagram showing Path of Ray, and Normals

for the lens a prism bounded by planes which touch the lens at the points of incidence and emergence; and in the actual use of lenses, the direction of the rays with respect to the supposed prism is such as to give a deviation not differing much from the minimum. The expression for the minimum deviation (§ 138) is $2i-2r$ or $2i-A$; and when the angle of the prism is small, as it is in the case of

ordinary lenses, we may assume $\frac{i}{r} = \frac{\sin i}{\sin r} = \mu$; so that $2\,i$ becomes $2\,\mu\,r$ or μ A, and the expression for the deviation becomes

$$(\mu - 1)\ \mathrm{A}, \tag{1}$$

A being the angle between the tangent planes (or between the normals) at the points of entrance and emergence.

Let x_1 and x_2 denote the distances of these points respectively from the principal axis, and r_1, r_2 the radii of curvature of the faces on which they lie. Then $\frac{x_1}{r_1}, \frac{x_2}{r_2}$ are the sines of the angles which the normals make with the axis, and the angle A is the sum or difference of these two angles, according to the shape of the lens. In the case of a double convex lens it is their sum, and if we identify the sines of these small angles with the angles themselves, we have

$$\mathrm{A} = \frac{x_1}{r_1} + \frac{x_2}{r_2}. \tag{2}$$

But if p_1, p_2 denote the distances from the faces of the lens to the points where the incident and emergent rays cut the principal axis, $\frac{x_1}{p_1}, \frac{x_2}{p_2}$ are the sines of the angles which these rays make with the axis, and the deviation is the sum or difference of these two angles, according as the conjugate foci are on opposite sides or on the same side of the lens. In the former case, identifying the angles with their sines, the deviation is $\frac{x_1}{p_1} + \frac{x_2}{p_2}$, and this, by formula (1), is to be equal to $(\mu - 1)$ A, that is, to $(\mu - 1)\left(\frac{x_1}{r_1} + \frac{x_2}{r_2}\right)$.

If the thickness of the lens is negligible in comparison with p_1, p_2, we may regard x_1 and x_2 as equal, and the equation

$$\frac{x_1}{p_1} + \frac{x_2}{p_2} = (\mu - 1)\left(\frac{x_1}{r_1} + \frac{x_1}{r_2}\right) \tag{3}$$

will reduce to

$$\frac{1}{p_1} + \frac{1}{p_2} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right). \tag{4}$$

If p_1 is infinite, the incident rays are parallel, and p_2 is the principal focal length, which we shall denote by f. We have therefore

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \tag{5}$$

and

$$\frac{1}{p_1} + \frac{1}{p_2} = \frac{1}{f}. \tag{6}$$

152. Conjugate Foci on Secondary Axis.—Let M (Fig. 136) be a luminous point on the secondary axis M O M′, O being the centre of the lens, and let M′ be the point in which an emergent ray corresponding to the incident ray M I cuts this axis. Let x denote x_1 or

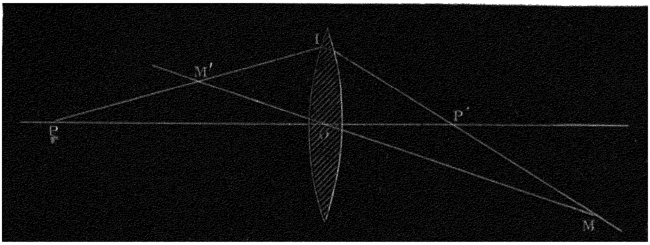


Fig 136 —Conjugate Foci on Secondary Axis.

x_2, the distances of the points of incidence and emergence from the principal axis, and θ the obliquity of the secondary axis; then $x \cos\theta$ is the length of the perpendicular from I upon M M′, and $\frac{x\cos\theta}{\mathrm{MI}}$, $\frac{x\cos\theta}{\mathrm{M'I}}$, are the sines of the angles O M I, O M′ I respectively. But the deviation is the sum of these angles; hence, proceeding as in last section, we have

$$\frac{x\cos\theta}{\mathrm{MI}}+\frac{x\cos\theta}{\mathrm{M'I}} = (\mu-1)\left(\frac{x}{r_1}+\frac{x}{r_2}\right) = \frac{x}{f} \tag{7}$$

$$\frac{1}{\mathrm{MI}}+\frac{1}{\mathrm{M'I}} = \frac{1}{f\cos\theta}. \tag{8}$$

The fact that x does not appear in equations (6) and (8) shows that, for every position of a luminous point, there is a conjugate focus, lying on the same axis as the luminous point itself. Equation (8) shows that the effective focal length becomes shorter as the obliquity becomes greater, its value being $f\cos\theta$, where θ is the obliquity.

If we take account of the fact that the rays of an oblique pencil make the angles of incidence and emergence more unequal than the rays of a direct pencil, and thus (by the laws of prisms) undergo larger deviation, we obtain a still further shortening of the effective focal length for oblique pencils.

When the obliquity is small, $\cos\theta$ may be regarded as unity, and we may employ the formula

$$\frac{1}{p_1}+\frac{1}{p_2} = \frac{1}{f} \tag{6}$$

for oblique as well as for direct pencils.

153. Discussion of the Formula for Convex Lenses.—For convex lenses f is to be regarded as positive; p will be positive when measured from the lens towards the incident light, and p' when measured in the direction of the emergent light.

Formula (6), being identical with equation (2) of § 105, leads to results analogous to those already deduced for concave mirrors.

As one focus advances from infinite distance to a principal focus, its conjugate moves away from the other principal focus to infinite distance on the other side. The more distant focus is always moving more rapidly than the nearer, and the least distance between them is accordingly attained when they are equidistant from the lens; in which case the distance of each of them from the lens is $2f$, and their distance from each other $4f$.

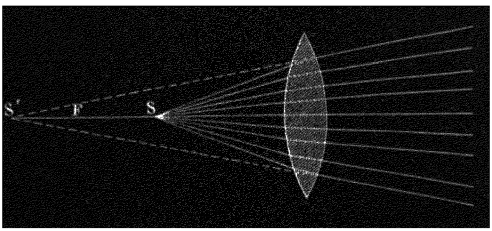


Fig. 137.—Conjugate Foci, one Real, one Virtual.

If either of the distances, as p, is less than f, the formula shows that the other distance p' is negative. The meaning is that the two foci are on the same side of the lens, and in this case one of them (the more distant of the two) must be virtual. For example, in Fig. 137, if S, S′ are a pair of conjugate foci, one of them S being between the principal focus F and the lens, rays sent to the lens by a luminous point at S, will, after emergence, diverge as if from S′; and rays coming from the other side of the lens, if they converge to S′ before incidence, will in reality be made to meet in S. As S moves towards the lens, S′ moves in the same direction more rapidly; and they become coincident at the surface of the lens. The formula in fact shows that if $\frac{1}{p}$ is very great in comparison with $\frac{1}{f}$, and positive, $\frac{1}{p'}$ must be very great and negative; that is to say, if p is a very small positive quantity, p' is a very small negative quantity.

154. Formation of Real Images.—Let AB (Fig. 138) be an object in front of a lens, at a distance exceeding the principal focal length. It will have a real image on the other side of the lens. To determine the position of the image by construction, draw through any point A of the object a line parallel to the principal axis, meeting

the lens in A′. The ray represented by this line will, after refraction, pass through the principal focus F; and its intersection with

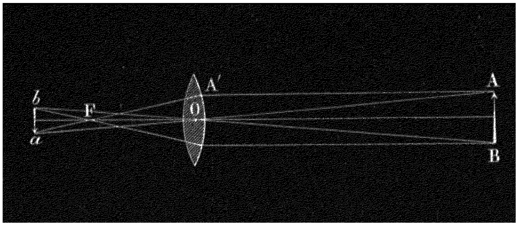


Fig 138 —Real and Diminished Image

the secondary axis AO determines the position of a, the focus conjugate to A. We can in like manner determine the position of b, the focus conjugate to B, another point of the object; and the joining line ab will then be the image of the line AB. It is evident that if $a\,b$ were the object, A B would be the image.

Figs. 138, 139 represent the cases in which the distance of the

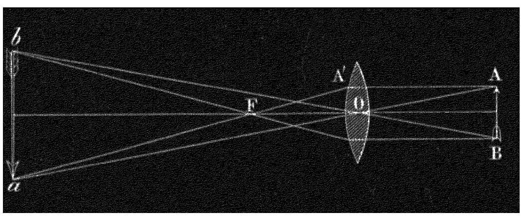


Fig 139 —Real and Magnified Image.

object is respectively greater and less than twice the focal length of the lens.

155. Size of Image.—In each case it is evident that $\frac{AB}{ab}=\frac{OA}{Oa}=\frac{p}{p'}$, or *the linear dimensions of object and image are directly as their distances from the centre of the lens.*

Again, as in § 113, we can deduce,

$$\frac{AB}{ab}=\frac{p}{p'}=\frac{p-f}{f}=\frac{f}{p'-f}; \tag{9}$$

$$(p-f)\,(p'-f)=f^2, \tag{10}$$

where $p-f$ is the distance of AB in Figs. 138, 139 to the right of the right-hand principal focus, and $p'-f$ is the distance of ab to the left of the left-hand principal focus.

Example.—A straight line $25^{mm.}$ long is placed perpendicularly on the axis, at a distance of 35 centimetres from a lens of 15 centimetres' focal length; what are the position and magnitude of the image?

To determine the distance p' we have

$$\frac{1}{35}+\frac{1}{p'}=\frac{1}{15}; \text{ whence } p'=\frac{35\times 15}{35-15}=26\tfrac{1}{4}^{\text{cm.}}$$

For the length of the image we have

$$25\,\frac{f}{p-f}=25\,\frac{15}{35-15}=18\tfrac{3}{4}^{\text{mm}}$$

156. Image on Cross-wires.—The position of a real image seen in mid-air can be tested by means of a cross of threads, or other convenient mark, so arranged that it can be fixed at any required point. The observer must fix this cross so that it appears approximately to coincide with a selected point of the image. He must then try whether any relative displacement of the two occurs on shifting his eye to one side. If so, the cross must be pushed further from him, or drawn nearer, according to the nature of the observed displacement, which follows the general rule of parallactic displacement, that the more distant object is displaced in the same direction as the observer's eye. The cross may thus be brought into exact coincidence with the selected point of the image, so as to remain in apparent coincidence with it from all possible points of view. When this coincidence has been attained, the cross is at the focus conjugate to that which is occupied by the selected point of the object.

By employing two crosses of threads, one to serve as object, and the other to mark the position of the image, it is easy to verify the fact that when the second cross coincides with the image of the first, the first also coincides with the image of the second.

157. Aberration of Lenses.—In the investigations of §§ 150–152, we made several assumptions which were only approximately true. The rays which proceed from a luminous point to a lens are in fact not accurately refracted to one point, but touch a curved surface called a caustic. The cusp of this caustic is the conjugate focus, and is the point at which the greatest concentration of light occurs. It is accordingly the place where a screen must be set to obtain the brightest and most distinct image. Rays from the central parts of the lens pass very nearly through it; but rays from the circumferential portions fall short of it. This departure from exact concurrence

is called *aberration*. The distinctness of an image on a screen is improved by employing an annular diaphragm to cut off all except the central rays; but the brightness is of course diminished.

By holding a convex lens in a position very oblique to the incident light, a primary and secondary focal line can be exhibited on a screen perpendicular to the beam, just as in the case of concave mirrors (§ 120). The experiment, however, is rather more difficult of performance.

158. Virtual Images.—Let an object AB be placed between a convex lens and its principal focus. Then the foci conjugate to the points A, B are virtual, and their positions can be found by construction from the consideration that rays through A, B, parallel to the principal axis, will be refracted to F, the principal focus on the other side. These refracted rays, if produced backward, must meet the secondary axes OA, OB in the required points. An eye placed on the other side of the lens will accordingly see a virtual image, erect, magnified, and at a greater distance from the lens than the object. This is the principle of the simple microscope. The formula for the distances D, d of object and image from the lens, when both are on the same side, is

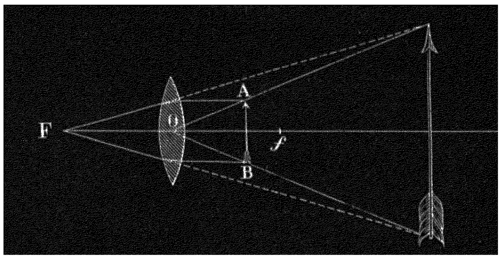


Fig 140 —Virtual Image formed by Convex Lens

$$\frac{1}{D} - \frac{1}{d} = \frac{1}{f}, \qquad (11)$$

f denoting the principal focal length.

159. Concave Lens.—For a concave lens, if the focal length be still regarded as positive, and denoted by f, and if the distances D, d be on the same side of the lens, the formula becomes

$$\frac{1}{d} - \frac{1}{D} = \frac{1}{f}, \qquad (12)$$

which shows that d is always less than D; that is, the image is nearer to the lens than the object.

In Fig. 141, AB is the object, and ab the image. Rays incident from A and B parallel to the principal axis will emerge as if they

came from the principal focus F. Hence the points *ab* are determined by the intersections of the dotted lines in the figure with the secondary axes OA, OB. An eye on the other side of the lens sees the image *ab*, which is always virtual, erect and diminished.

160. Action of Lenses on Converging Rays.—Illustrations of the interchangeableness of object and image—a principle which holds good for lenses as well as for mirrors—are furnished by reversing the course of the rays in any of the diagrams in which we have

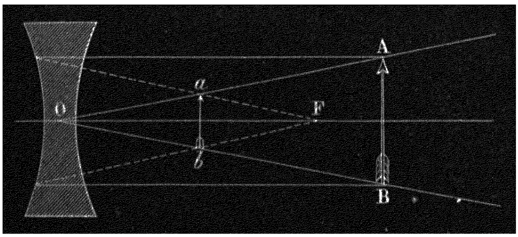


Fig. 141.—Virtual Image formed by Concave Lens.

traced the positions of points of the image. For example, to explain the effect of a convex lens on a converging pencil, we may take Fig. 137 as showing how rays on their way to S′ are made by the lens to converge to the nearer point S. In Fig. 140 the large arrow may represent a real image formed by rays coming from the left; and when the lens is interposed the nearer and smaller image A B will be formed instead.

So again, as regards concave lenses, Fig. 131 may be interpreted as illustrating the fact that rays converging from the right hand to F will by interposing the lens be made to emerge parallel.

In Fig. 141 rays converging from the left to form the real image *a b* will by interposing the lens in the position shown be made to form a new and larger image A B at a greater distance. To produce this effect, the lens must be at a distance less than its own focal length from the first image. If it is interposed at a distance greater than its own focal length it will form a virtual image on the side from which the rays proceed, which will be seen by looking through the lens from the other side.

161. Measurement of Focal Length of Concave Lens.—Fig. 141 interpreted as above illustrates the readiest method of measuring the focal length of a concave lens. Throw on a screen, by means of a convex lens, a real image *a b*; then interpose the concave lens and

move the screen further away till the image again becomes sharp. Measure the distance P of this image from the concave lens. Then remove the concave lens after marking its place on the table. Move the screen back to its original place so that the first image is formed upon it again, and measure the distance p of this image from the place where the concave lens stood. The focal length f can then be calculated by the formula

$$\frac{1}{p}-\frac{1}{P}=\frac{1}{f}.$$

162. Focometers.—Silbermann's focometer (Fig. 142) is an instrument for measuring the focal lengths of convex lenses, and is based

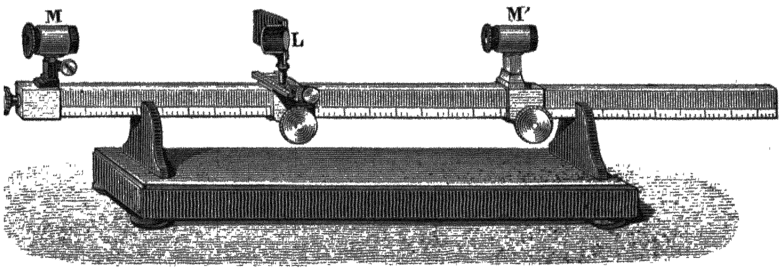


Fig 142 —Silbermann's Focometer.

on the principle (§ 153) that, when the object and its image are equidistant from the lens, their distance from each other is four times the focal length. It consists of a graduated rule carrying three runners M, L, M′. The middle one L is the support for the lens which is to be examined; the other two, M M′, contain two thin plates of horn or other translucent material, ruled with lines, which are at the same distance apart in both. The sliders must be adjusted until the image of one of these plates is thrown upon the other

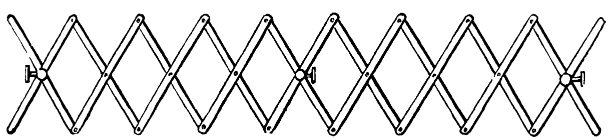


Fig. 143 —Everett's Focometer

plate, without enlargement or diminution, as tested by the coincidence of the ruled lines of the image with those of the plate on which it is cast. The distance between M and M′ is then read off, and divided by 4.

Everett's Focometer consists of a jointed frame, shown in Fig. 143,

with arrangements for mounting a lens at its centre and an object and screen at its ends as shown in Fig. 144. The distance between the ends can be altered between very wide limits, but the lens always remains midway between them, so that the observer has only one

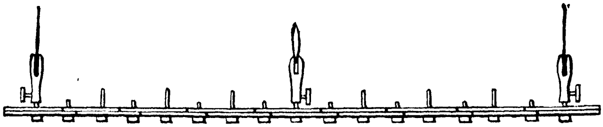


Fig. 144.

adjustment of distance to make. A lamp is placed in line with the object and lens, and as soon as the image is sharply depicted on the screen the distance between the object and image is to be measured and divided by 4. The screen may conveniently be a white card and the object an upright cross of coarse threads stretched across a hole in another card.

Another mode of operating is to turn the end at which the object is mounted towards a window, and, standing at the other end of the apparatus, to adjust the length of the frame till the image falls without parallax on a piece of wire-gauze which takes the place of the screen. Or the observer may stand with his back to the window and adjust till the image of the gauze falls without parallax on the cross of threads.

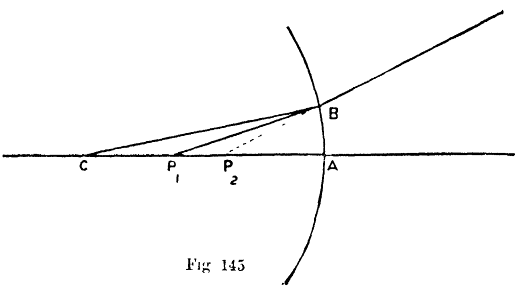


Fig 145

If the cross is upright, the wires of the gauze should be slanting to obtain the best visibility.

163. Refraction at a Single Spherical Surface.—Suppose a small pencil of rays to be incident nearly normally upon a spherical surface which forms the boundary between two media in which the indices are μ_1 and μ_2 respectively. Let C (Fig. 145) be the centre of curvature, and CA the axis. Let P_1 be the focus of the incident, and P_2 of the refracted rays. Then for any ray P_1B, CBP_1 is the angle of incidence and CBP_2 the angle of refraction. Hence by the law of sines we have (§ 130)

$$\mu_1 \sin CBP_1 = \mu_2 \sin CBP_2.$$

Dividing by sin BCA, and observing that

$$\frac{\sin CBP_1}{\sin BCA} = \frac{CP_1}{BP_1} = \frac{CP_1}{AP_1} \text{ ultimately;}$$

$$\frac{\sin CBP_2}{\sin BCA} = \frac{CP_2}{BP_2} = \frac{CP_2}{AP_2} \text{ ultimately;}$$

we obtain the equation

$$\mu_1 \frac{CP_1}{AP_1} = \mu_2 \frac{CP_2}{AP_2}, \tag{13}$$

which expresses the fundamental relation between the positions of the conjugate foci.

Let $AC=r$, $AP_1=p_1$, $AP_2=p_2$, then equation (13) becomes

$$\mu_1 \frac{r-p_1}{p_1} = \mu_2 \frac{r-p_2}{p_2}, \tag{14}$$

or, dividing by r,

$$\mu_1\left(\frac{1}{p_1} - \frac{1}{r}\right) = \mu_2\left(\frac{1}{p_2} - \frac{1}{r}\right),$$

which may be written

$$\frac{\mu_2}{p_2} - \frac{\mu_1}{p_1} = \frac{\mu_2 - \mu_1}{r}. \tag{15}$$

Again, let $CA=\rho$, $CP_1=q_1$, $CP_2=q_2$, then equation (13) gives

$$\mu_1 \frac{q_1}{\rho - q_1} = \mu_2 \frac{q_2}{\rho - q_2},$$

or

$$\frac{1}{\mu_1} \frac{\rho - q_1}{q_1} = \frac{1}{\mu_2} \frac{\rho - q_2}{q_2}, \tag{16}$$

an equation closely analogous to (14) and leading to the result (analogous to (15))

$$\frac{1}{\mu_2 q_2} - \frac{1}{\mu_1 q_1} = \left(\frac{1}{\mu_2} - \frac{1}{\mu_1}\right)\frac{1}{\rho} \tag{17}$$

The signs of p_1, p_2, r, in (14) and (15) are to be determined by the rule that, if one of the three points P_1, P_2, C lies on the opposite side of A from the other two, its distance from A is to be reckoned opposite in sign to theirs.

In like manner the signs of q_1, q_2, ρ, in (16) and (17) are to be determined by the rule that, if one of the three points P_1, P_2, A lies on the opposite side of C from the other two, its distance from C is to be reckoned opposite in sign to theirs

It is usual to reckon distances positive when measured *towards the incident light;* but the formulæ will remain correct if the opposite convention be adopted.

If f denote the principal focal length, measured from A, we have, by (15), writing f for p_2 and making p_1 infinite,

$$\frac{1}{f} = \frac{\mu_2 - \mu_1}{\mu_2} \frac{1}{r},$$

and (15) may now be written

$$\frac{\mu_2}{p_2} - \frac{\mu_1}{p_1} = \frac{\mu_2}{f},$$

it being understood that the positive direction for f is the same as for p_1, p_2, and r.

The application of these formulæ to lenses in cases where the thickness of the lens cannot be neglected, may be illustrated by the following example.

164. To find the position of the image formed by a spherical lens.

Let distances be measured from the centre of the sphere, and be reckoned positive on the side next the incident light.

Then, if x denote the distance of the object, y the distance of the image formed by the first refraction, z the distance of the image formed by the second refraction, a the radius of the sphere, and μ its index of refraction; we have, at the first surface,

$$\rho = a \qquad \mu_1 = 1 \qquad \mu_2 = \mu,$$

and at the second surface

$$\rho = -a \qquad \mu_1 = \mu \qquad \mu_2 = 1.$$

Hence equation (17) gives, for the first refraction,

$$\frac{1}{\mu y} - \frac{1}{x} = \left(\frac{1}{\mu} - 1\right)\frac{1}{a},$$

and for the second refraction,

$$\frac{1}{z} - \frac{1}{\mu y} = -\left(1 - \frac{1}{\mu}\right)\frac{1}{a} = \left(\frac{1}{\mu} - 1\right)\frac{1}{a}.$$

By adding these two equations, we obtain

$$\frac{1}{z} - \frac{1}{x} = \left(\frac{1}{\mu} - 1\right)\frac{2}{a} = -\frac{\mu - 1}{\mu} \cdot \frac{2}{a}.$$

If the incident rays are parallel, we have x infinite and $z = -\frac{\mu}{\mu - 1}\frac{a}{2}$; that is to say, the principal focus is at a distance $\frac{\mu}{\mu-1}\frac{a}{2}$ from the centre, on the side remote from the incident light.

165. Camera Obscura.—The images obtained by means of a hole in the shutter of a dark room (§ 73) become sharper as the size of the hole is diminished; but this diminution involves loss of light, so that it is impossible by this method to obtain an image at once bright and sharp. This difficulty can be overcome by employing a lens. If the

objects in the external landscape depicted are all at distances many times greater than the focal length of the lens, their images will all be formed at sensibly the same distance from the lens, and may be received upon a screen placed at this distance. The images thus obtained are inverted, and are of the same size as if a simple aperture were employed instead of a lens. This is the principle on which the *camera obscura* is constructed.

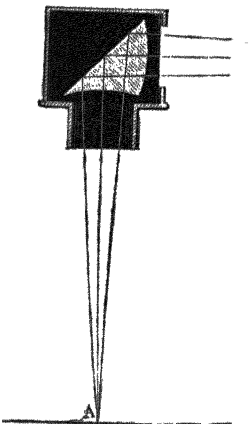


Fig 146.—Objective of Camera.

It is a kind of tent surrounded by opaque curtains, and having at its top a revolving lantern, containing a lens with its axis horizontal, and a mirror placed behind it at a slope of 45°, to reflect the transmitted light downwards on to a sheet of white paper lying on the top of a table. Images of external objects are thus depicted on the paper, and their outlines can be traced with a pencil if desired. It is still better to combine lens and mirror in one, by the arrangement represented in section in Fig. 146. Rays from external objects are first refracted at a convex surface, then totally reflected at the back of the lens, which is plane, and finally emerge through the bottom of the lens, which is concave, but with a larger radius of curvature than the first surface. The

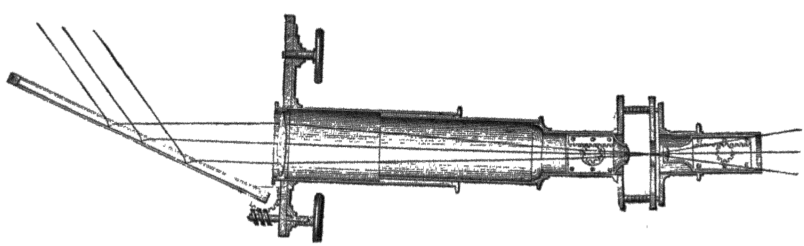


Fig 147.—Solar Microscope.

two refractions produce the effect of a converging meniscus. The instrument is now only employed for purposes of amusement.

166. Solar and Electric Microscope.—Lenses are extensively employed in the lecture-room, for rendering experiments visible to a whole audience at once, by projecting them on a screen.

In the solar microscope, a convex lens of short focal length is employed to throw upon a screen a highly-magnified image of a small

object placed a little beyond the principal focus. As the image is always much less bright than the object, and the more so as the magnification is greater, it is necessary that the object should be very highly illuminated. For this purpose the rays of the sun are directed upon it by means of a mirror and large lens; the latter serving to increase the solid angle of the cone of rays which fall upon the object, and thus to enable a larger portion of the magnifying lens to be utilized. The objects magnified are always transparent; and the images are formed by rays which have been transmitted through them.

The electric light or the lime light is more frequently employed instead of the sun. The mirror is then no longer necessary, but the other arrangements are the same as for the solar microscope. A trough with glass sides containing a solution of alum is frequently inserted between the light and the object to prevent the latter from being overheated, alum having great absorbing power for the non-luminous heat-rays.

The lanterns employed for ordinary lime-light views act in the same way but with less magnification. The image in all these cases is inverted.

CHAPTER X.

VISION AND OPTICAL INSTRUMENTS.

167. Description of the Eye.—The human eye (Fig. 148) is a nearly spherical ball, capable of turning in any direction in its socket. Its outermost coat is thick and horny, and is opaque except in its

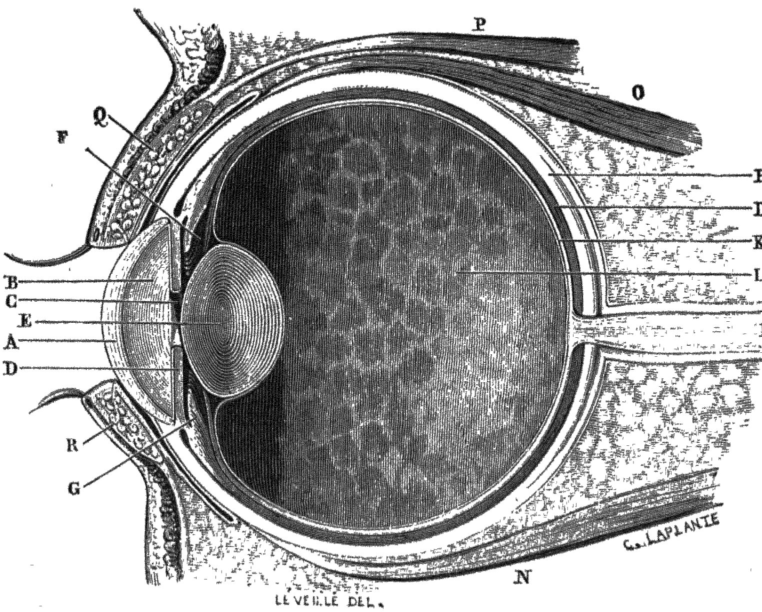


Fig. 148.—Human Eye

anterior portion. Its opaque portion H is called the *sclerotica*, or in common language the white of the eye. Its transparent portion A is called the *cornea*, and has the shape of a very convex watch-glass Behind the cornea is a diaphragm D, of annular form, called the *iris*. It is coloured and opaque, and the circular aperture C in its centre

is called the *pupil*. By the action of the involuntary muscles of the iris, this aperture is enlarged or contracted on exposure to darkness or light. The colour of the iris is what is referred to when we speak of the colour of a person's eyes. Behind the pupil is the *crystalline lens* E, which has greater convexity at back than in front. It is built up of layers or shells, increasing in density inwards, the outermost shell having nearly the same index of refraction as the media in contact with it; an arrangement which tends to prevent the loss of light by reflection. The cavity B between the cornea and the crystalline is called the anterior chamber, and is filled with a watery liquid called the *aqueous humour*. The much larger cavity L, behind the crystalline, is called the posterior chamber, and is filled with a transparent jelly called the *vitreous humour*, inclosed in a very thin transparent membrane (the hyaloid membrane). The posterior chamber is inclosed, except in front, by the *choroid coat* or *uvea* I, which is saturated with an intensely black and opaque mucus, called the *pigmentum nigrum*. The choroid is lined, except in its anterior portion, with another membrane K, called the retina, which is traversed by a ramified system of nerve filaments diverging from the optic nerve M. Light incident on the retina gives rise to the sensation of vision; and there is no other part of the eye which possesses this property.

168. The Eye as an Optical Instrument.—It is clear, from the above description, that a pencil of rays entering the eye from an external point will undergo a series of refractions, first at the anterior surface of the cornea, and afterwards in the successive layers of the crystalline lens, all tending to render them convergent (see table of indices, § 127). A real and inverted image is thus formed of any external object to which the eye is directed. If this image falls on the retina, the object is seen; and if the image thus formed on the retina is sharp and sufficiently luminous, the object is seen distinctly.

169. Adaptation to Different Distances.—As the distance of an image from a lens varies with the distance of the object, it would only be possible to see objects distinctly at one particular distance, were there not special means of adaptation in the eye. Persons whose sight is not defective can see objects in good definition at all distances exceeding a certain limit. When we wish to examine the minute details of an object to the greatest advantage, we hold it at a particular distance, which varies in different individuals, and averages about eight inches. As we move it further away, we

experience rather more ease in looking at it, though the diminution of its apparent size, as measured by the visual angle, renders its minuter features less visible On the other hand, when we bring it nearer to the eye than the distance which gives the best view, we cannot see it distinctly without more or less effort and sense of strain; and when we have brought it nearer than a certain lower limit (averaging about six inches), we find distinct vision no longer possible. In looking at very distant objects, if our vision is not defective, we have very little sense of effort. These phenomena are in accordance with the theory of lenses, which shows that when the distance of an object is a large multiple of the focal length of the lens, any further increase, even up to infinity, scarcely alters the distance of the image, but that, when the object is comparatively near, the effect of any change of its distance is considerable. There has been much discussion among physiologists as to the precise nature of the changes by which we adapt our eyes to distinct vision at different distances Such adaptation might consist either in a change of focal length, or in a change of distance of the retina. Observations in which the eye of the patient is made to serve as a mirror, giving images by reflection at the front of the cornea, and at the front and back of the crystalline, have shown that the convexity of the front of the crystalline is materially changed as the patient adapts his eye to near or remote vision, the convexity being greatest for near vision. This increase of convexity corresponds to a shortening of focal length, and is thus consistent with theory.

170. Binocular Vision.—The difficulty which some persons have felt in reconciling the fact of an inverted image on the retina with the perception of an object in its true position, is altogether fanciful, and arises from confused notions as to the nature of perception.

The question as to how it is that we see objects single with two eyes, rests upon a different footing, and is not to be altogether explained by habit and association.[1] To each point in the retina of one eye there is a *corresponding point,* similarly situated, in the other. An impression produced on one of these points is, in ordinary circumstances, undistinguishable from a similar impression produced on the other, and when both at once are similarly impressed, the effect is simply more intense than if one were impressed alone; or, to describe the same phenomena subjectively, we have only one field

[1] Binocular vision is a subject which has been much debated. For the account here given of it, the Editor is responsible

of view for our two eyes, and in any part of this field of view we see either one image, brighter than we should see it by one alone, or else we see two overlapping images. This latter phenomenon can be readily illustrated by holding up a finger between one's eyes and a wall, and looking at the wall. We shall see, as it were, two transparent fingers projected on the wall. One of these transparent fingers is in fact seen by the right eye, and the other by the left, but our visual sensations do not directly inform us which of them is seen by the right eye, and which by the left.

The principal advantage of having two eyes is in the estimation of distance, and the perception of relief. In order to see a point as single by two eyes, we must make its two images fall on corresponding points of the retinæ; and this implies a greater or less convergence of the optic axes according as the object is nearer or more remote. We are thus furnished with a direct indication of the distance of the object from our eyes; and this indication is much more precise than that derived from the adjustment of their focal length.

In judging of the comparative distances of two points which lie nearly in the same direction, we are greatly aided by the parallactic displacement which occurs when we change our own position.

We can also form an estimate of the nearness of an object, from the amount of change in its apparent size, contour, and bearing, produced by shifting our position. This would seem to be the readiest means by which very young animals can distinguish near from remote objects.

171. Stereoscope.—The perception of relief is closely connected with the doubleness of vision which occurs when the images on corresponding portions of the two retinæ are not similar. In surveying an object we run our eyes rapidly over its surface, in such a way as always to attain single vision of the particular point to which our attention is for the instant directed. We at the same time receive a somewhat indistinct impression of all the points within our field of view; an impression which, when carefully analysed, is found to involve a large amount of doubleness. These various impressions combine to give us the perception of relief; that is to say, of *form in three dimensions.*

The perception of relief in binocular vision is admirably illustrated by the *stereoscope,* an instrument which was invented by Wheatstone, and reduced to its present more convenient form by Brewster. Two figures are drawn, as in Fig. 149, being perspective representations of

the same object from two neighbouring points of view, such as might be occupied by the two eyes in looking at the object. Thus if the object be a cube, the right eye will have a fuller view of the right

Fig. 149.—Stereoscopic Pictures.

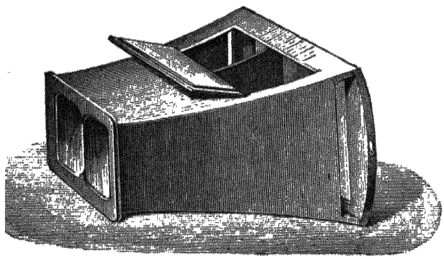


Fig. 150.—Stereoscope

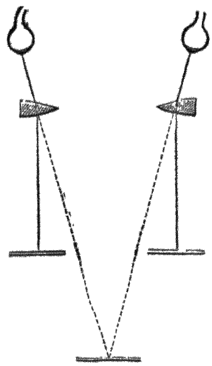


Fig 151 —Path of Rays in Stereoscope

face, and the left eye of the left face. The two pictures are placed in the right and left compartments of a box, which has a partition down the centre serving to insure that each eye shall see only the picture intended for it; and over each of the compartments a half-lens is fixed, serving, as in Fig. 151, not only to magnify the picture, but at the same time to displace it, so that the two virtual images are brought into approximate coincidence. Stereoscopic pictures are usually photographs obtained by means of a double camera, having two objectives, one beside the other, which play the part of two eyes.

When matters are properly arranged, the observer seems to see the object in relief. He finds himself able to obtain single view of any one point of the solid image which is before him; and the adjustments of the optic axes which he finds it necessary to make, in shifting his view from one point of it to another, are exactly such as would be required in looking at a solid object.

When one compartment of the stereoscope is empty, and the other contains an object, an observer, of normal vision, looking in in the ordinary way, is unable to say which eye sees the object. If two pictures are combined, consisting of two equal circles, one of them

having a cross in its centre, and the other not, he is unable to decide whether he sees the cross with one eye or both.

When two entirely dissimilar pictures are placed in the two compartments, they compete for mastery, each of them in turn becoming more conspicuous than the other, in spite of any efforts which the observer may make to the contrary. A similar fluctuation will be observed on looking steadily at a real object which is partially hidden from one eye by an intervening object. This tendency to alternate preponderance renders it well nigh impossible to combine two colours by placing one under each eye in the stereoscope.

The immediate visual impression, when we look either at a real solid object, or at the apparently solid object formed by properly combining a pair of stereoscopic views, is a single picture formed of two slightly different pictures superimposed upon each other. The coincidence becomes exact at any point to which attention is directed, and to which the optic axes are accordingly made to converge, but in the greater part of the combined picture there is a want of coincidence, which can easily be detected by a collateral exercise of attention. The fluctuation above described to some extent tends to conceal this doubleness; and in looking at a real solid object, the concealment is further assisted by the blurring of parts which are out of focus.

172. Visual Angle. Magnifying Power.—The angle which a given straight line subtends at the eye is called its *visual angle*, or the *angle under which it is seen*. This angle is the measure of the length of the image of the straight line on the retina. Two discs at different distances from the eye, are said to have the same apparent size, if their diameters are seen under equal angles. This is the condition that the nearer disc, if interposed between the eye and the remoter disc, should be just large enough to conceal it from view.

The angle under which a given line is seen, evidently depends not only on its real length, and the direction in which it points, but also on its distance from the eye; and varies, in the case of small visual angles, in the inverse ratio of this distance. The *apparent length* of a straight line may be regarded as measured by the visual angle which it subtends.

By the *magnifying power* of an optical instrument, is usually meant the ratio in which it increases *apparent lengths* in this sense. In the case of telescopes, the comparison is between an object as

seen in the telescope, and the same object as seen with the naked eye at its actual distance. In the case of microscopes, the comparison is between the object as seen in the instrument, and the same object as seen by the naked eye at the least distance of distinct vision, which is usually assumed as 10 inches.

But two discs, whose diameters subtend the same angle at the eye, may be said to have the same *apparent area;* and since the areas of similar figures are as the squares of their linear dimensions, it is evident that the apparent area of an object varies as the square of the visual angle subtended by its diameter. The number expressing *magnification of apparent area* is therefore the square of the magnifying power as above defined. Frequently, in order to show that the comparison is not between apparent areas, but between apparent lengths, an instrument is said to magnify so many *diameters.* If the diameter of a sphere subtends 1° as seen by the naked eye, and 10° as seen in a telescope, the telescope is said to have a magnifying power of 10 diameters. The superficial magnification in this case is evidently 100.

The apparent length and apparent area of an object are respectively proportional to the length and area of its image on the retina.

Apparent length is measured by the plane angle, and apparent area by the solid angle, which an object subtends at the eye.

173. Spectacles.—Spectacles are of two kinds, intended to remedy two opposite defects of vision. Short-sighted persons can see objects distinctly at a smaller distance than persons whose vision is normal, but always see distant objects confused. On the other hand, persons whose vision is normal in their youth, usually become over-sighted with advancing years, so that, while they can still adjust their eyes correctly for distant vision, objects as near as 10 or 12 inches always appear blurred. Spectacles for over-sighted persons are convex, and should be of such focal length, that, when an object is held at about 10 inches distance, its virtual image is formed at the nearest distance of distinct vision for the person who is to use them. This latter distance must be ascertained by trial. Call it p inches; then, by § 158, the formula for computing the required focal length x (in inches) is

$$\frac{1}{10} - \frac{1}{p} = \frac{1}{x}.$$

shows that, fc

For example varies inversely a e nearest distance at which the person

can conveniently read without spectacles, the focal length required is 30 inches.

In Fig. 152, A represents the position of a small object, and A′ that of its virtual image as seen with spectacles of this kind.

Over-sight is not the only defect which the eye is liable to acquire

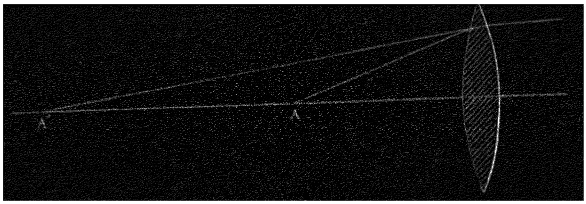


Fig. 152.—Spectacle-glass for Over-sighted Eye.

by age, but it is the defect which ordinary spectacles are designed to remedy.

Spectacles for short-sighted persons are concave, and the focal

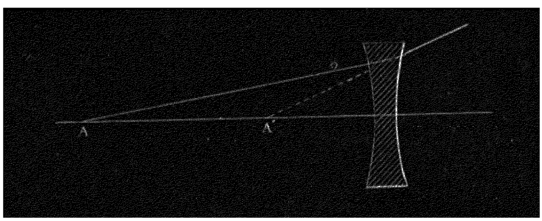


Fig. 153.—Spectacle-glass for Short-sighted Eye.

length which they ought to have, if designed for reading, may be computed by the formula

$$\frac{1}{p} - \frac{1}{10} = \frac{1}{x},$$

p denoting the nearest distance at which the person can read, and x the focal length, both in inches. If his *greatest* distance of distinct vision exceeds the focal length, he will be able, by means of the spectacles, to obtain distinct vision of ob[illegible]at all distances, from 10 inches upwards. *lengths* in [illegible]

174. Simple Magnifier.—A *magnif* between an objec[illegible]vex lens, of

shorter focal length than the human eye, and is placed at a distance somewhat less than its focal length from the object to be viewed. In Fig. 154, ab is the object, and A B the virtual image which is seen by the eye K. The construction which we have employed for drawing the image is one which we have several times used before. Through the point a, the line a M is drawn parallel to the principal axis. F M is then drawn from the principal focus F; O a is drawn from the optical centre O; and these two lines are produced till they meet in A.

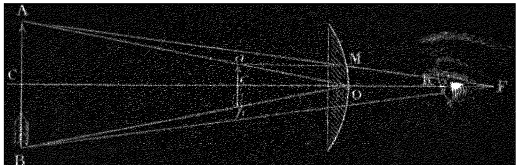


Fig. 154.—Magnifying Glass.

Distance of lens from object. In order that the image may be properly seen, its distance from the eye must fall between the limits of distinct vision; and in order that it may be seen under the largest possible visual angle, the eye must be close to the lens, and the object must be as near as is compatible with distinct vision. This and other interesting properties are established by the following investigation:—

Let θ denote the visual angle under which the observer sees the image of the portion $a\,c$ of the object. Also let x denote the distance c O of the object from the lens, and y the distance O K of the lens from the eye. Then we have

$$\tan\theta = \frac{\mathrm{A\,C}}{\mathrm{C\,K}} = \frac{\mathrm{A\,C}}{\mathrm{C\,O}+y};$$

but, by formulæ (10) and (11) of last chapter, we have

$$\mathrm{A\,C} = a\,c . \frac{f}{f-x}, \quad \mathrm{C\,O} = x . \frac{f}{f-x}.$$

Substituting these values for A C and C O, and reducing, we have

$$\tan\theta = a\,c . \frac{f}{(x+y)\,f-xy}. \qquad \text{(A)}$$

This equation shows that, for a given lens and a given object, the visual angle varies inversely as the quantity $(x+y)\,f-xy$.

The following practical consequences are easily drawn:—

(1) If the distance $x+y$ of the eye from the object is given, the visual angle increases as the two distances x, y approach equality, and is not altered by interchanging them.

(2) If one of the two distances x, y be given, and be less than f, the other must be made as small as possible, if we wish to obtain the largest possible visual angle.

To obtain the absolute maximum of visual angle, we must select, from the various positions which make C K equal to the nearest distance of distinct vision, that which gives the largest value of A C, since the quotient of A C by C K is the tangent of the visual angle. Now A C increases as the image moves further from the lens, and hence the absolute maximum is obtained by making its distance from the lens equal to the nearest distance of distinct vision, and making the eye come up close to the lens. In this case the distance p of the object from the lens is given by the equation $\frac{1}{p} - \frac{1}{D} = \frac{1}{f}$, where D denotes the nearest distance of distinct vision; and tan θ is $\frac{ac}{p}$ or $ac\left(\frac{1}{f}+\frac{1}{D}\right)$. But the greatest angle under which the body could be seen by the naked eye is the angle whose tangent is $\frac{ac}{D}$; hence the visual angle (or its tangent) is increased by the lens in the ratio $1+\frac{D}{f}$, which is called the *magnifying power*. If the object were in the principal focus, and the eye close to the lens, the magnifying power would be $\frac{D}{f}$. In either case, the thickness of the lens being neglected, the visual angle is the angle which the object subtends at the centre of the lens, and therefore varies inversely as the distance of this centre from the object. For lenses of small focal length, the reciprocal of the focal length may be regarded as proportional to the magnifying power.

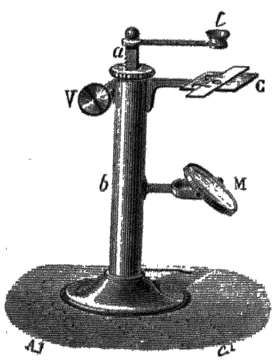


Fig. 155.—Simple Microscope.

Simple Microscope.—By a *simple microscope* is usually understood a lens of short focal length mounted in a manner convenient for the examination of small objects. Fig. 155 represents an instrument of this kind. The lens l is mounted in brass, and carried at the end of an arm. It is raised and lowered by turning the milled head V,

which acts on the rack *a*. C is the platform on which the object is laid, and M is a concave mirror, which can be employed for increasing the illumination of the object.

175. Compound Microscope.—In the compound microscope, there is one lens which forms a real and greatly enlarged image of the object; and this image is itself magnified by viewing it through another lens.

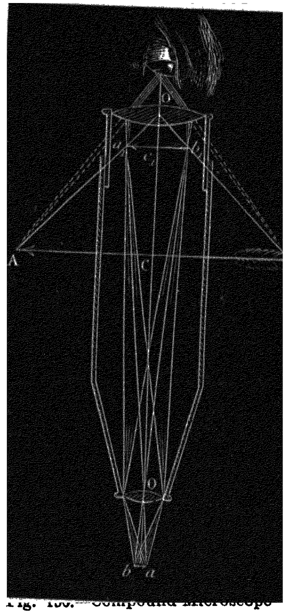


Fig. 156.—Compound Microscope.

In Fig. 156, $a\,b$ is the object, O is the first lens, called the *objective*, and is placed at a distance only slightly exceeding its focal length from the object; an inverted image $a_1\,b_1$ is thus formed at a much greater distance on the other side of the lens, and proportionally larger. O′ is the second lens, called the *ocular* or *eye-piece*, which is placed at a distance a little less than its focal length from the first image $a_1\,b_1$, and thus forms an enlarged virtual image of it A B, at a convenient distance for distinct vision.

If we suppose the final image A B to be at the least distance of distinct vision from the eye placed at O′ (this being the arrangement which gives the largest visual angle), the magnifying power will be simply the ratio of the length of this image to that of the object $a\,b$, and will be the product of the two factors $\frac{A\,B}{a_1\,b_1}$ and $\frac{a_1\,b_1}{a\,b}$. The former is the magnification produced by the eye-piece, and is, as we have just shown (§ 174), $1+\frac{D}{f}$. The other factor $\frac{a_1\,b_1}{a\,b}$ is the magnification produced by the objective, and is equal to the ratio of the distances $\frac{O\,a_1}{O\,a}$. If the objective is taken out, and replaced by another of different focal length, the readjustment will consist in altering the distance O a, leaving the distance O a_1 unchanged. The total magnification therefore varies inversely as O a, that is, nearly in the inverse ratio of the focal length of the objective. Compound microscopes are usually provided with several objectives, of various focal lengths, from which the observer makes a selection according to the magnifying power which he requires for the object to be examined. The powers most used range from 50 to 350 diameters.

The magnifying power of a microscope can be determined by direct observation, in the following way. A plane reflector pierced with a hole in its centre, is placed directly over the eye-piece (Fig. 157), at an inclination of 45°, and another plane reflector, or still better, a totally reflecting prism, as in the figure, is placed parallel to it at the distance of an inch or two, so that the eye, looking down upon the first mirror, sees, by means of two successive reflections, the image of a divided scale placed at a distance of 8 or 10 inches below the second reflector. In taking an observation, a micrometer scale engraved on glass, its divisions being at a known distance apart (say $\frac{1}{100}$ of a millimetre), is placed in the microscope as the object to be magnified; and the observer holds his eye in such a position that, by means of different parts of his pupil, he sees at once the magnified image of the micrometer scale in the microscope, and the reflected and unmagnified image of the other scale. The two images will be superimposed in the same field of view; and it is easy to observe how many divisions of the one coincide with a given number of divisions of the other. Let the divisions on the large scale be millimetres, and those on the micrometer scale hundredths of a millimetre. Then the magnifying power is 100, if one of the magnified covers one of the unmagnified divisions; and is $\frac{100\,N}{n}$, if n of the former cover N of the latter. This is on the assumption that the large scale is placed at the nearest distance of convenient vision. In stating the magnifying power of a microscope, this distance is usually reckoned as 10 inches.

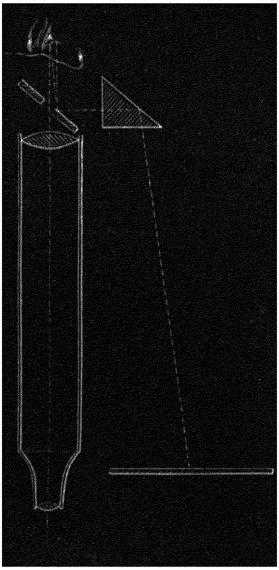


Fig. 157.
Measurement of Magnifying Power.

A short-sighted person sees an image in a microscope (whether simple or compound) under a larger visual angle than a person of normal sight; but the inequality is not so great as in the case of objects seen by the naked eye. In fact, if f be the focal length of the eye-piece in a compound microscope, or of the microscope itself if simple, and D the nearest distance of distinct vision for the

observer, the visual angle under which the image is seen in the microscope is proportional to $\frac{1}{f} + \frac{1}{D}$, the greatest visual angle for the naked eye being represented by $\frac{1}{D}$. Both these angles increase as D diminishes, but the latter increases in a greater ratio than the former. When f is as small as $\frac{1}{10}$ of an inch, the visual angle in the microscope is sensibly the same for short as for normal sight.

Before reading off the divisions in the observation above described, care should be taken to focus the microscope in such a way, that the image of the micrometer scale is at the same distance from the eye as the image of the large scale with which it is compared. When this is done, a slight motion of the eye does not displace one image with respect to the other.

Instead of a single eye-lens, it is usual to employ two lenses separated by an interval, that which is next the eye being called the *eye-glass*, and the other the *field-glass*. This combination is equivalent to the Huygenian or negative eye-piece employed in telescopes (§ 237).

176. Astronomical Telescope.—The astronomical refracting telescope consists essentially (like the compound microscope) of two lenses, one of which forms a real and inverted image of the object, which is looked at through the other.

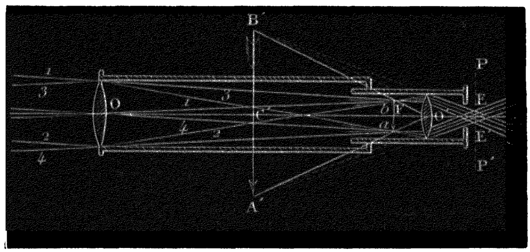


Fig. 158.—Astronomical Telescope.

In Fig. 158, O is the object-glass, which is sometimes a foot or more in diameter, and is always of much greater focal length than the eye-piece O'. The inverted image of a distant object is formed at the principal focus F. This image is represented at $a\,b$. The parallel rays marked 1, 2 come from the upper extremity of the object, and meet at a; and the parallel rays 3, 4, from the other extremity, meet at b. A' B' is the virtual image of $a\,b$ formed by the eye-piece. Its distance from the eye can be changed by pulling out or pushing in the eye-tube; and may in practice have any value intermediate between the least distance of distinct vision and infinity, the visual angle under which it is seen being but slightly affected by

this adjustment. The rays from the highest point of the object emerge from the eye-piece as a pencil diverging from A′; and the rays from the lowest point of the object form a pencil diverging from B′.

Magnification.—The angle under which the object would be seen by the naked eye is $a\,\mathrm{O}\,b$; for the rays $a\,\mathrm{O}$, $b\,\mathrm{O}$, if produced, would pass through its extremities. The angle under which it is seen in the telescope, if the eye be close to the eye-lens, is A′ O′ B′ or $a\,\mathrm{O}'\,b$. The magnification is therefore $\frac{a\,\mathrm{O}'\,b}{a\,\mathrm{O}\,b}$, which is approximately the same as the ratio of the distances of the image ab from the two lenses $\frac{\mathrm{OF}}{\mathrm{O'F}}$. If the eye-tube is so adjusted as to throw the image A′ B′ to infinite distance, F will be the principal focus of both lenses, and the magnification is the ratio of the focal length of the object-glass to that of the eye-piece. If the eye-tube be pushed in as far as is compatible with distinct vision (the eye being close to the lens), the magnification is greater than this in the ratio $\frac{\mathrm{D}+f}{\mathrm{D}}$, D denoting the nearest distance of distinct vision, and f the focal length of the eye-piece.

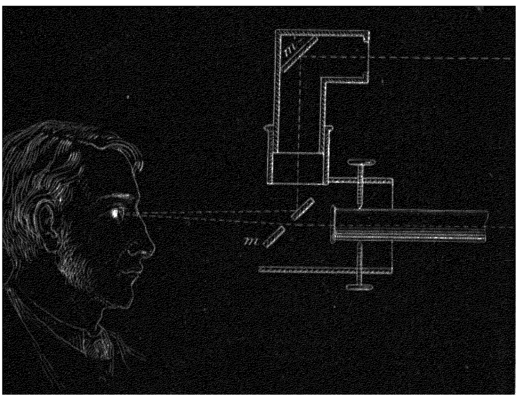


Fig. 159.—Measurement of Magnifying Power.

The magnification can be directly observed by looking with one eye through the telescope at a brick wall, while the other eye is kept open. The image will thus be superimposed on the actual wall, and we have only to observe how many courses of the latter coincide with a single course of the magnified image.

If the telescope is large, its tube may prevent the second eye from seeing the wall, and it may be necessary to employ a reflecting arrangement, as in Fig. 159, analogous to that described in connection with the microscope.

Telescopes without stands seldom magnify more than about 10 diameters. Powers of from 20 to 60 are common in telescopes with

stands, intended for terrestrial purposes. The powers chiefly employed in astronomical observation are from 100 to 500.

Mechanical Arrangements.—The achromatic object-glass O is set in a mounting which is screwed into one end of a strong brass tube A A (Fig. 160). In the other end slides a smaller tube F containing the eye-piece O′; and by turning the milled head V in one direction or the other, the eye-piece is moved forwards or backwards.

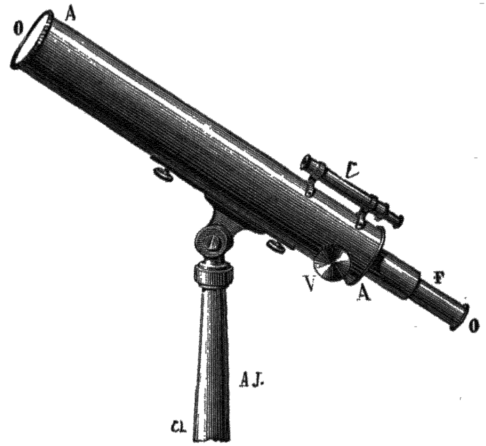


Fig 160 —Astronomical Telescope

Finder. — The small telescope *l*, which is attached to the principal telescope, is called a *finder*. This appendage is indispensable when the principal telescope has a high magnifying power; for a high magnifying power involves a small field of view, and consequent difficulty in directing the telescope so as to include a selected object within its range. The finder is a telescope of large field; and as it is set parallel to the principal telescope, objects will be visible in the latter if they are seen in the centre of the field of view of the former.

177. Best Position for the Eye.—The eye-piece forms a real and inverted image of the object-glass[1] at E E′ (Fig. 158), through which all rays transmitted by the telescope must of necessity pass. If the telescope be directed to a bright sky, and a piece of white paper held behind the eye-piece to serve as a screen, a circular spot of light will be formed upon it, which will become sharply defined (and at the same time attain its smallest size) when the screen is held in the correct position. This image (which we shall call the *bright spot*) may be regarded as marking the proper place for the pupil of the observer's eye. Every ray which traverses the centre of the object-glass traverses the centre of this spot; every ray which traverses the upper edge of the object-glass traverses the lower edge of the

[1] Or it may be called an image of *the aperture which the object-glass fills.* It remains sensibly unchanged on removing the object-glass so as to leave the end of the telescope open.

spot; and any selected point of the spot receives all the rays which have been transmitted by one particular point of the object-glass. An eye with its pupil anywhere within the limits of the bright spot, will therefore see the whole field of view of the telescope. If the spot and pupil are of exactly the same size, they must be made to coincide with one another, as the necessary condition of seeing the whole field of view with the brightest possible illumination. Usually in practice the spot is much smaller than the pupil, so that these advantages can be obtained without any nicety of adjustment; but to obtain the most distinct vision, the centre of the pupil should coincide as closely as possible with the centre of the spot. To facilitate this adjustment, a brass diaphragm, with a hole in its centre, is screwed into the eye-end of the telescope, the proper place for the eye being close to this hole.

One method of determining the magnifying power of a telescope consists in measuring the diameter of the bright spot, and comparing it with the effective aperture of the object-glass. In fact, let F and f denote the focal lengths of object-glass and eye-piece, and a the distance of the spot from the centre of the eye-piece; then $F+f$ is approximately the distance of the object-glass from the same centre, and, by the formula for conjugate focal distances, we have $\frac{1}{F+f}+\frac{1}{a}=\frac{1}{f}$. Multiplying both sides of this equation by $F+f$, and then subtracting unity, we have $\frac{F+f}{a}=\frac{F}{f}$. But the ratio of the diameter of the object-glass to that of its image is $\frac{F+f}{a}$; and $\frac{F}{f}$ is the usual formula for the magnifying power. Hence, *the linear magnifying power of a telescope is the ratio of the diameter of the object-glass to that of the bright spot.*

178. **Terrestrial Telescope.**—The astronomical telescope just described gives inverted images. This is no drawback in astronomical observation, but would be inconvenient in viewing terrestrial objects. In order to re-invert the image, and thus make it erect, two additional lenses O″ O‴ (Fig. 161) are introduced between the real image $a\,b$ and the eye-lens O′. If the first of these O″ is at the distance of its principal focal length from $a\,b$, the pencils which fall upon the second will be parallel, and an erect image $a'\,b'$ will thus be formed in the principal focus of O‴. This image is viewed through the eye-lens O′, and the virtual image A′ B′ which is perceived by the eye will therefore be erect. The two lenses O″, O‴, are usually

made precisely alike, in which case the two images $a\,b$, $a'\,b'$ will be equal. In the better class of terrestrial telescopes, a different ar-

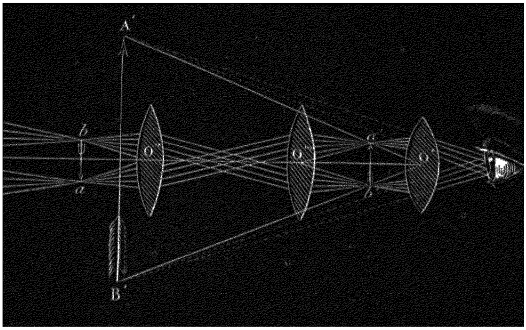


Fig. 161.—Terrestrial Eye-piece

rangement is adopted, requiring one more lens; but whatever system be employed, the reinversion of the image always involves some loss both of light and of distinctness.

179. Galilean Telescope.—Besides the disadvantages just mentioned, the erecting eye-piece involves a considerable addition to the length of the instrument. The telescope invented by Galileo, and the earliest of all telescopes, gives erect images with only two lenses, and with shorter length than even the astronomical telescope. O (Fig. 162) is the object-glass, which is convex as in the astronomical telescope, and would form a real and inverted image $a\,b$ at its principal focus; but the eye-glass O′, which is a concave lens, is interposed at a distance equal to or slightly exceeding its own focal length from the place of this image, and forms an erect virtual image A′ B′, which the observer sees.

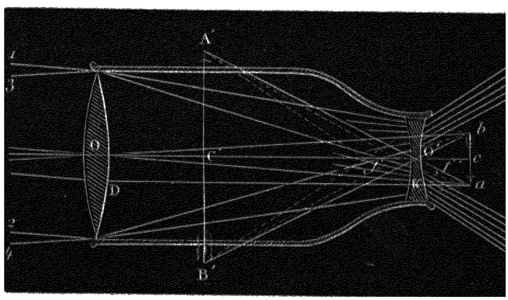


Fig. 162.—Galilean Telescope

Neglecting the distance of his eye from the lens, the angle under which he sees the image is A′ O′ B′, which is equal to a O′ b, whereas

the visual angle to the naked eye would be $a\,O\,b$. The magnification is therefore $\frac{a\,O'\,b}{a\,O\,b}$, which is approximately equal to $\frac{O\,c}{O'\,c}$, c being the principal focus of the object-glass. If the instrument is focussed in such a way that the image A′ B′ is thrown to infinite distance, c is also the principal focus of the eye-lens, and the magnification is simply the ratio of the focal lengths of the two lenses. This is the same rule which we deduced for the astronomical telescope; but the Galilean telescope, if of the same power, is shorter by twice the focal length of the eye-lens, since the distance between the two lenses is the difference instead of the sum of their focal lengths.

This telescope has the disadvantage of not admitting of the employment of cross wires; for these, in order to serve their purpose, must coincide with the real image; and no such image exists in this telescope.

Fig. 163.—Opera-glass

There is another peculiarity in the absence of the *bright spot* above described, the image of the object-glass formed by the eye-glass being virtual. In other telescopes, if half the object-glass be covered, half the bright spot will be obliterated; but the remaining half suffices for giving the whole field of view, though with diminished brightness. In the Galilean telescope, on the contrary, if half the object-glass be covered, half the field of view will be cut off, and the remaining half will be unaffected.

The *opera-glass*, single or binocular, is a Galilean telescope, or a pair of Galilean telescopes. In the best instruments, both object-glass and eye-glass are achromatic combinations of three pieces, as shown in section in the figure (Fig. 163); the middle piece in each case being flint, and the other two crown (§ 231).

180. Reflecting Telescopes.—In reflecting telescopes, the place of an object-glass is supplied by a concave mirror called a *speculum*, usually composed of solid metal. The real and inverted image which it forms of distant objects is, in the Herschelian telescope, viewed directly through an eye-piece, the back of the observer being towards the object, and his face towards the speculum. This construction is only applicable to very large specula; as in instruments of ordinary

size the interposition of the observer's head would occasion too serious a loss of light.

An arrangement more frequently adopted is that devised by Sir Isaac Newton, and employed by him in the first reflecting telescope ever constructed. It is represented in Fig. 164. The speculum is at the bottom of a tube whose open end is directed towards the distant object to be examined. The rays 1 and 2 from one extremity of the object are reflected towards a, and the rays 3, 4 from the other extremity are reflected towards b. A real inverted image $a\,b$ would thus be formed at the principal focus of the concave speculum; but a small plane mirror M is interposed obliquely, and causes the real image to be formed at $a'\,b'$ in a symmetrical position with respect to the mirror M. The eye-lens O′ transforms this into the enlarged and virtual image A′ B′.

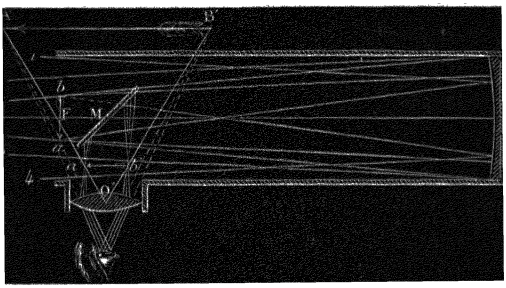


Fig. 164.—Newtonian Telescope.

Magnifying Power.—The approximate formula for the magnifying power is the same as in the case of the refracting telescopes already described. In fact the first image $a\,b$ subtends, at the optical centre O (not shown in the figure) of the large speculum, an angle $a\,\mathrm{O}\,b$ equal to the visual angle for the naked eye; and the second image $a'\,b'$ (which is equal to the former) subtends, at the centre of the eye-piece, an angle $a'\,\mathrm{O}'\,b'$ equal to the angle under which the image is seen in the telescope. The magnifying power is therefore $\frac{a'\,\mathrm{O}'\,b'}{a\,\mathrm{O}\,b}$, or, what is the same thing, is the ratio of the distance of $a\,b$ from O to the distance of $a'\,b'$ from O′, or the ratio of the focal length of the speculum to that of the eye-piece.

In the Gregorian telescope, which was invented before that of Newton, but not manufactured till a later date, there are two concave specula. The large one, which receives the direct rays from the object, forms a real and inverted image. The smaller speculum which is suspended in the centre of the tube, with its back to the object, receives the rays reflected from the first speculum, and forms a second real image, which is the enlarged and inverted image of the

first, and is therefore erect as compared with the object. This real and erect image is then magnified by means of an eye-piece, as in the instruments previously described, the eye-piece being contained in a tube which slides in a hole pierced in the middle of the large speculum.

As this arrangement gives an erect image, and enables the observer to look directly towards the object, it is specially convenient for terrestrial observation. It is the construction almost universally adopted in reflecting telescopes of small size.

The Cassegranian telescope resembles the Gregorian, except that the second speculum is convex, and the image which the observer sees is inverted.

181. Silvered Specula.—Achromatic refracting telescopes give much better results, both as regards light and definition, than reflectors of the same size or weight; but it has been found practicable to make specula of much larger size than object-glasses. The aperture of Lord Rosse's largest telescope is 6 feet, whereas the aperture of the largest achromatic telescopes yet constructed is less than two feet, and increase of size involves increased thickness of glass, and consequent absorption of light.

The massiveness which is found necessary in the speculum in order to prevent flexure, is a serious inconvenience, as is also the necessity for frequent repolishing—an operation of great delicacy, as the slightest change in the form of the surface impairs the definition of the images. Both these defects have been to a certain extent remedied by the introduction of glass specula, covered in front with a thin coating of silver. Glass is much more easily worked than speculum-metal (which is remarkable for its brittleness in casting), and has only one-third of its specific gravity. Silver is also much superior to speculum-metal in reflecting power; and as often as it becomes tarnished it can be removed and renewed, without liability to change of form in the reflecting surface.[1]

182. Measure of Brightness.—The apparent brightness of a surface is most naturally measured by the amount of light per unit area of its image on the retina; and therefore varies *directly as the amount of light which the surface sends to the pupil, and inversely as the apparent area of the surface.*

To avoid complications arising from the varying condition of the

[1] The merits of silvered specula are fully set forth in a brochure published by Mr. Browning, the optician, entitled *A Plea for Reflectors.*

observer, we shall leave dilatation and contraction of the pupil out of account.

When a body is looked at through a small pinhole in a card held close to the eye, it appears much darker than when viewed in the ordinary way; and in like manner images formed by optical instruments often furnish beams of light too narrow to fill the pupil. In all such cases it becomes necessary to distinguish between *effective brightness* and *intrinsic brightness*, the former being less than the latter in the same ratio in which the cross section of the beam which enters the pupil is less than the area of the pupil. We may correctly speak of the *intrinsic brightness* of a surface *for a particular point* of the pupil; and the effective brightness will in every case be the average value of the intrinsic brightness taken over the whole pupil.

In the case of natural bodies viewed in the ordinary way, the distinction may be neglected, since they usually send light in sensibly equal amounts to all parts of the pupil.

To obtain a numerical measure of intrinsic brightness, let us denote by s a small area on a surface directly facing towards the eye, or the foreshortened projection of a small area oblique to the line of vision, and by ω the solid angle which the pupil of the eye subtends at any point of s. Then the quantity of light q which s sends to the pupil per unit time, varies jointly as the solid angle ω, the area s, and the intrinsic brightness of s, which we will denote by I. We may therefore write

$$q = \mathrm{I}\, s\, \omega, \text{ and } \mathrm{I} = \frac{q}{s\,\omega}.$$

The intrinsic brightness of a small area s is therefore measured by $\frac{q}{s\,\omega}$, where q denotes the quantity of light which s emits per unit time in directions limited by the small angle of divergence ω

183. **Applications.**—One of the most obvious consequences is that *surfaces appear equally bright at all distances* in the same direction, provided that no light is stopped by the air or other intervening medium; for q and ω both vary inversely as the square of the distance. The area of the image formed on the retina in fact varies directly as the amount of light by which it is formed.

Images formed by Lenses.—Let A B (Fig. 165) be an object, and $a\,b$ its real image formed by the lens C D, whose centre is O. Let S denote a small area at A, and Q the quantity of light which it sends to the lens; also let s denote the corresponding area of the

image, and q the quantity of light which traverses it. Then q would be identical with Q if no light were stopped by the lens; the areas, S, s, are directly as the squares of the conjugate focal distances O A, O a; and the solid angles of divergence Ω and ω, for Q and q, being the solid angles subtended by the lens at A and a (for the plane angle $c\,a\,d$ in the figure is equal to the vertical angle CaD), are inversely as the squares of the conjugate focal distances. We have accordingly $S\,\Omega = s\,\omega$. The intrinsic brightness $\frac{q}{s\,\omega}$ of the image is therefore equal to the intrinsic brightness $\frac{Q}{S\,\Omega}$ of the object except in so far as light is stopped by the lens. Precisely similar reasoning applies to virtual images formed by lenses.[1]

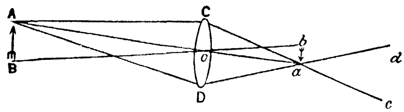


Fig 165.—Brightness of Image.

In the case of images formed by mirrors, Ω and ω are the solid angles subtended by the mirror at the conjugate foci, and are inversely as the squares of the distances from the mirror; while S and s are directly as the squares of the distances from the centre of curvature; but these four distances are proportional (§ 105), so that the same reasoning is still applicable. If the mirror only reflects half the incident light, the image will have only half the intrinsic brightness of the object.

If the pupil is filled with light from the image, the effective brightness will be the same as the intrinsic brightness thus computed. If this condition is not fulfilled, the former will be less than the latter. When the image is greatly magnified as compared with the object, the angle of divergence is greatly diminished in comparison with the angle which the lens or mirror subtends at the object, and often becomes so small that only a small part of the pupil is utilized. This is the explanation of the great falling off of light which is observed in the use of high magnifying powers, both in microscopes and telescopes.

[1] For refraction out of a medium of index μ_1 into another of index μ_2, we have by § 163, equation (13), $\mu_1 : \mu_2 : : \frac{A\,P_1}{C\,P_1} : \frac{A\,P_2}{C\,P_2}$. But since s_1, s_2 are the areas of corresponding parts of object and image, we have $s_1 : s_2 : : C\,P_1{}^2 : C\,P_2{}^2$, and since ω_1, ω_2 are the solid angles subtended at P_1, P_2 by one and the same portion of the bounding surface, we have $\omega_1 : \omega_2 : : A\,P_2{}^2 : A\,P_1{}^2$. Therefore $\frac{q}{s_1\,\omega_1} : \frac{q}{s_2\,\omega_2} : : \mu_1{}^2 : \mu_2{}^2$. The intrinsic brightnesses of a succession of images in different media are therefore directly as the squares of the absolute indices. On this point see *Phil. Mag.*, March 1888, p. 216.

184. Brightness of Image in a Telescope.—It has been already pointed out (§ 177) that in most forms of telescope (the Galilean being an exception), there is a certain position, a little behind the eye-piece, at which a well-defined bright spot is formed upon a screen held there while the telescope is directed to any distant source of light. It has also been pointed out that this spot is the image, formed by the eye-piece, of the opening which is filled by the object-glass, and that the magnifying power of the instrument is the ratio of the size of the object-glass to the size of this bright spot.

Let s denote the diameter of the bright spot, o the diameter of the object-glass, e the diameter of the pupil of the eye; then $\frac{o}{s}$ is the linear magnifying power.

We shall first consider the case in which the spot exactly covers the pupil of the observer's eye, so that $s=e$. Then the whole light which traverses the telescope from a distant object enters the eye; and if we neglect the light stopped in the telescope, this is the whole light sent by the object to the object-glass, and is $\left(\frac{o}{e}\right)^2$ times that which would be received by the naked eye. The magnification of apparent area is $\left(\frac{o}{s}\right)^2$, which, from the equality of s and e, is the same as the increase of total light. The brightness is therefore the same as to the naked eye.

Next, let s be greater than e, and let the pupil occupy the central part of the spot. Then, since the spot is the image of the object-glass, we may divide the object-glass into two parts—a central part whose image coincides with the pupil, and a circumferential part whose image surrounds the pupil. All rays from the object which traverse the central part, traverse its image, and therefore enter the pupil; whereas rays traversing the circumferential part of the object-glass, traverse the circumferential part of the image, and so are wasted. The area of the central part (whether of the object-glass or of its image) is to the whole area as $e^2 \cdot s^2$; and the light which the object sends to the central portion, instead of being $\left(\frac{o}{e}\right)^2$ times that which would be received by the naked eye, is only $\left(\frac{o}{s}\right)^2$ times. But $\left(\frac{o}{s}\right)^2$ is the magnification of apparent area. Hence the brightness is the same as to the naked eye. In these two cases, effective and intrinsic brightness are the same.

Lastly (and this is by far the most common case in practice), let s be less than e. Then no light is wasted, but the pupil is not filled. The light received is $\left(\frac{o}{e}\right)^2$ times that which the naked eye would receive; and the magnification of apparent area is $\left(\frac{o}{s}\right)^2$. The effective brightness of the image, is to the brightness of the object to the naked eye, as $\left(\frac{o}{e}\right)^2 : \left(\frac{o}{s}\right)^2$; that is, as $s^2 : e^2$; that is, as the area of the bright spot to the whole area of the pupil.

To correct for the light stopped by reflection and imperfect transparency, we have simply to multiply the result in each case by a proper fraction, expressing the ratio of the transmitted to the incident light. This ratio, for the central parts of the field of view, is about 0·85 in the best achromatic telescopes. In such telescopes, therefore, the brightness of the image cannot exceed 0·85 of the brightness of the object to the naked eye. It will have this precise value, when the magnifying power is equal to or less than $\frac{o}{e}$; and from this point upwards will vary inversely as the square of the linear magnification.

The same formulæ apply to reflecting telescopes, o denoting now the diameter of the large speculum which serves as objective; but the constant factor is usually considerably less than 0·85.

It may be accepted as a general principle in optics, that while it is possible, by bad focussing or instrumental imperfections, to obtain a confused image whose brightness shall be intermediate between the brightest and the darkest parts of the object, *it is impossible, by any optical arrangement whatever, to obtain an image whose brightest part shall surpass the brightest part of the object.*

185. Brightness of Stars.—There is one important case in which the foregoing rules regarding the brightness of images become nugatory. The fixed stars are bodies which subtend at the earth angles smaller than the *minimum visibile*, but which, on account of their excessive brightness, *appear* to have a sensible angular diameter. This is an instance of *irradiation*, a phenomenon manifested by all bodies of excessive brightness, and consisting in an extension of their apparent beyond their actual boundary. What is called, in popular language, a bright star, is a star which sends a large total amount of light to the eye.

Denoting by a the ratio of the transmitted to the whole incident light, a ratio which, as we have seen, is about 0·85 in the most

favourable cases, and calling the light which a star sends to the naked eye unity, the light perceived in its image will be $a\left(\frac{o}{s}\right)^2$, or $a \times$ square of linear magnification, if the bright spot is as large as the pupil. When the eye-piece is changed, increase of power diminishes the size of the spot, and increases the light received by the eye, until the spot is reduced to the size of the pupil. After this, any further magnification has no effect on the quantity of light received, its constant value being $a\left(\frac{o}{e}\right)^2$.

The value of this last expression, or rather the value of $a\,o^2$, is the measure of what is called the *space-penetrating power* of a telescope; that is to say, the power of rendering very faint stars visible, and it is in this respect that telescopes of very large aperture, notably the great reflector of Lord Rosse, are able to display their great superiority over instruments of moderate dimensions

We have seen that the total light in the visible image of a star remains unaltered, by increase of power in the eye-piece above a certain limit. But the visibility of faint stars in a telescope is promoted by darkening the back-ground of sky on which they are seen. Now the brightness of this back-ground varies directly as s^2, or inversely as the square of the linear magnification (s being supposed less than e). Hence it is advantageous, in examining very faint stars, to employ eye-pieces of sufficient power to render the bright spot much smaller than the pupil of the eye

186. Images on a Screen.—Thus far we have been speaking of the brightness of images as viewed directly. Images cast upon a screen are, as a matter of fact, much less brilliant; partly because the screen sends out light in all directions, and therefore through a much larger solid angle than that formed by the beam incident on the screen, and partly because some of the incident light is absorbed.

Let A be the area of the object, which we suppose to face directly towards the lens by which the image is thrown upon the screen, a the area of the image, and D, d their respective distances from the lens. Then if I denote the intrinsic brightness of the object, the light sent from A to the lens will be the product of I A by the solid angle which the lens subtends at the object. This solid angle will be $\frac{L}{D^2}$, if L denote the area of the lens. $I A \frac{L}{D^2}$ is therefore the light sent by the object to the lens, and if we neglect reflection and absorption all this light falls upon the image. *The light which falls*

on unit area of the image is therefore $I\frac{A}{a}\frac{L}{D^2}$, that is $I\frac{L}{d^2}$; it *is* therefore *the same as if the lens were a source of light of brightness I.* Accordingly, if the image of a lamp flame be thrown upon the pupil of an observer's eye, and be large enough to cover the pupil, he will see the lens filled with light of a brightness equal to that of the flame seen directly.

187. **Field of View in Astronomical Telescope.**—Let $p\,m\,n\,q$ (Fig. 166) be the common focal plane of the object-glass and eye-glass. Draw Ba, Ab joining the highest points of both, and the lowest points of both; also Aa, Bb joining the highest point of each with the lowest point of the other. Evidently Ba, Ab will be the boundaries of the beam of light transmitted through the telescope, and therefore the points p and q in which these lines intersect the focal plane, will be the extremities of that part of the real image which sends rays to the eye. The angle subtended by $p\,q$ at the centre of the object-glass will therefore be the angular diameter of the complete field of view. But the outer portions of this field will be less bright than the centre, and the full amount of brightness, as calculated in § 184 for the case in which the "bright spot" is smaller than the pupil, will belong only to the portion $m\,n$ bounded by the cross-lines Aa, Bb; for all the rays sent by the object-glass through the part $m\,n$ traverse the eye-glass, and therefore the bright spot, whereas some of the rays sent by the object-glass to any point between m and p, or between n and q pass wide of the eye-glass and therefore do not reach the bright spot. The complete field of view, as seen by an eye whose pupil includes the bright spot, accordingly consists of a central disc $m\,n$ of full brightness, surrounded by a ring extending to p and q whose brightness gradually diminishes from full brightness at its junction with the disc to zero at its outer boundary. This ring is called the "ragged edge," and is put out of sight in actual telescopes by an opaque stop of annular form in the focal plane. The angular diameter of the field of view, excluding the ragged edge, will be equal to the angle which $m\,n$ subtends at the centre of the object-glass.

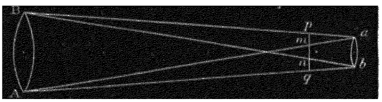


Fig. 166.—Field of View.

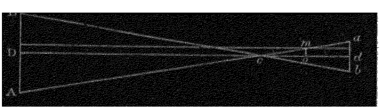


Fig. 167.—Calculation of Field.

To calculate the length of $m n$, join D, d, the centres of the object-glass and eye-glass (Fig. 167). The joining line will obviously pass through the intersection of Aa, Bb, and also through the middle point of $m n$. Draw a parallel to this line through m. Then, by comparing the similar triangles of which $a m$, Am are the hypotenuses, we have

$$ad - mo \ : \ od \ :: \ \mathrm{AD} + mo \ : \ \mathrm{D}o.$$

Hence, multiplying extremes and means, and denoting the focal lengths Do, $o d$ by F, f, we have

$$\mathrm{F}\,(ad - mo) = f\,(\mathrm{AD} + mo),$$

whence

$$mo = \frac{\mathrm{F}\,.\,ad - f\,.\,\mathrm{AD}}{\mathrm{F}+f}.$$

This is the radius of the real image, excluding the ragged edge; and the angular radius of the field of view will be

$$\frac{mo}{\mathrm{F}} = \frac{\mathrm{F}\,.\,ad - f\,.\,\mathrm{AD}}{\mathrm{F}\,(\mathrm{F}+f)}$$

$$= \frac{ad}{\mathrm{F}+f} - \frac{f\,.\,\mathrm{AD}}{\mathrm{F}\,(\mathrm{F}+f)}.$$

The first term $\frac{ad}{\mathrm{F}+f}$ is the angle which the radius of the eye-glass subtends at the object-glass. But, it is obvious from Fig. 166 that the line aD would bisect $m p$. Hence the second term represents half the breadth of the ragged edge, and the whole field of view, including the ragged edge, has an angular radius

$$\frac{ad}{\mathrm{F}+f} + \frac{f\,.\,\mathrm{AD}}{\mathrm{F}\,(\mathrm{F}+f)}.$$

188. Cross-wires of Telescopes.—We have described in § 156 a mode of marking the place of a real image by means of a cross of threads. When telescopes are employed to assist in the measurement of angles, a contrivance of this kind is almost always introduced. A cross of silkworm threads, in instruments of low power, or of spider threads in instruments of higher power, is stretched across a metallic frame just in front of the eye-piece. The observer must first adjust the eye-piece for distinct vision of this cross, and must then (in the case of theodolites and other surveying instruments) adjust the distance of the object-glass until the object which is to be observed is also seen distinctly in the telescope. The image of the object will then be very nearly in the plane of the cross. If it is not exactly in the plane, parallactic displacement will be observed when the eye is shifted, and this must be cured by slightly

altering the distance of the object-glass. When the adjustment has been completed, the cross always marks one definite point of the object, however the eye be shifted. This coincidence will not be disturbed by pushing in or pulling out the eye-piece; for the frame which carries the cross is attached to the body of the telescope, and the coincidence of the cross with a point of the image is real, so that it could be observed by the naked eye, if the eye-piece were removed. The adjustment of the eye-piece merely serves to give distinct vision, and this will be obtained simultaneously for both the cross and the object.

189. **Line of Collimation.**—The employment of *cross-wires* (as these crossing threads are called) enormously increases our power of making accurate observations of direction, and constitutes one of the greatest advantages of modern over ancient instruments.

The line which is regarded as the line of sight, or as the direction in which the telescope is pointed, is called the *line of collimation.* If we neglect the curvature of rays due to atmospheric refraction, we may define it as the *line joining the cross to the object whose image falls on it.* More rigorously, the line of collimation is the *line joining the cross to the optical centre of the object-glass.* When it is desired to adjust the line of collimation,—for example, to make it truly perpendicular to the horizontal axis on which the telescope is mounted, the adjustment is performed by shifting the frame which carries the wires, slow-motion screws being provided for this purpose. Telescopes for astronomical observation are often furnished with a number of parallel wires, crossed by one or two in the transverse direction; and the line of collimation is then defined by reference to an imaginary cross, which is the centre of mean position of all the actual crosses.

190. **Micrometers.**—Astronomical micrometers are of various kinds, some of them serving for measuring the angular distance between two points in the same field of view, and others for measuring their apparent direction from one another. They often consist of spider threads placed in the principal focus of the object-glass, so as to be in the same plane as the images of celestial objects, one or more of the threads being movable by means of slow-motion screws, furnished with graduated circles, on which parts of a turn can be read off.

One of the commonest kinds consists of two parallel threads, which can thus be moved to any distance apart, and can also be turned round in their own plane.

CHAPTER XI.

SYSTEMS OF LENSES.

191. Homographic Relation. — Two variable quantities x and y are said to be *homographic* when they are connected by an equation of the form—

$$xy + bx + ay + c = 0, \tag{1}$$

$$\text{or} \quad 1 + \frac{a}{x} + \frac{b}{y} + \frac{c}{xy} = 0; \tag{2}$$

a, b, and c being constants, which may be either positive, negative, or zero

Two variables which are homographic to the same third variable are homographic to each other.

For the simultaneous equations $\begin{cases} xz + b_1 x + a_1 z + c_1 = 0 \\ yz + b_2 y + a_2 z + c_2 = 0 \end{cases}$

give, by eliminating z,

$$(b_1 - b_2)xy + (b_1 a_2 - c_2)x + (c_1 - a_1 b_2)y + c_1 a_2 - a_1 c_2 = 0$$

which can evidently be reduced to the form (1).

The distances of a small object and its image from a spherical mirror, or from a thin lens, are homographic. For, denoting them by x and y, and the focal length by f, we have

$$\frac{1}{x} + \frac{1}{y} - \frac{1}{f} = 0, \quad \text{or } xy - fx - fy = 0;$$

so that here we have $a = -f$, $b = -f$, $c = 0$.

The distances of a small object and its image from a spherical refracting surface are also homographic; for they are connected by an equation of the form—

$$\frac{\mu_1}{x} - \frac{\mu_2}{y} = \frac{\mu_1 - \mu_2}{r}, \quad \text{or } xy - \frac{r\mu_2}{\mu_2 - \mu_1} x + \frac{r\mu_1}{\mu_2 - \mu_1} y = 0.$$

If two homographic variables are increased or diminished by constants, they will still be homographic

For, putting $x = x' + m$, $y = y' + n$,
(1) becomes $x'y' + bx' + ay' + an + bm + c$,

which is of the same form, with a new constant in place of c.

192. Conjugate Points. — Let P and Q be points which travel along two fixed straight lines, and let their respective distances O P, O′ Q from two fixed

points O, O′ in these lines be expressed algebraically by x and y. The proposition last proved shows that, if x and y are homographic, the distances of P and Q from any other fixed points in their respective lines of motion will also be homographic. Two points P and Q thus connected are said to be homographic, and simultaneous positions of P and Q are called *conjugate points.* It appears from above that the positions of a small object and its image by direct refraction or reflection at a spherical surface are thus connected.

Let a pencil of rays, diverging from a point P on the common axis of any number of refracting or reflecting surfaces, be successively refracted or reflected at these surfaces. Let the successive images of P thus formed on the axis be at points Q, R, S, &c. Then, since P is homographic to Q, and Q is homographic to R, therefore P is homographic to R; and by continuing this reasoning we can show that P is homographic to the final image formed by the system. This result leads to some important general properties of such an optical arrangement, to which the remainder of this chapter will be devoted.

193. Foci of a System. — Take an arbitrary origin on the common axis, and let x and y denote the distances of a small object and its final image from this origin, with the same convention as to sign for both.

The standard equation when put in the form—

$$1+\frac{a}{x}+\frac{b}{y}+\frac{c}{xy}=0 \qquad (2)$$

shows that—

$$\text{when } x=\infty,\ y=-b;$$
$$\text{when } y=\infty,\ x=-a.$$

Thus the system has two principal foci. The focus $x=-a$ is called the *first*, and the focus $y=-b$ the *second* principal focus. We shall denote them by F and F′ respectively.

In some special cases, parallel rays incident on the system give rise to parallel emergent rays; a and b are then infinite. We shall reserve these cases for subsequent consideration, and proceed in the meantime to deduce some properties of systems in which a and b are finite.

The general equation may evidently be put in the form—

$$(x+a)(y+b) = ab-c. \qquad (1)$$

But $x+a$ is the distance of P on the positive side of the first principal focus F; and $y+b$ is the distance of Q on the positive side of the second principal focus F′. We have, therefore,

$$\mathrm{FP} \, . \, \mathrm{F'Q} = \text{constant}. \qquad (3)$$

This proposition will be referred to hereafter as the *constancy of the product of focal distances.*

194. — Three known numerical values of x, with the corresponding values of y, give, by substitution in the general equation, three numerical equations of the

first degree for determining the three constants a, b, c. The relation between x and y will then be completely known, so that the position of the image for any position of a small object on the axis can at once be calculated.

195. Projective Construction for Conjugate Points. — The general equation is simplified if the two origins O, O′ are conjugate, for in this case the values $x=0$, $y=0$ must be simultaneous. Hence the constant c must be zero, and the equation becomes

$$xy+ay+bx=0, \tag{4}$$

or
$$1+\frac{a}{x}+\frac{b}{y}=0; \tag{5}$$

that is,
$$\frac{\mathrm{OF}}{\mathrm{OP}}+\frac{\mathrm{O'F'}}{\mathrm{O'Q}}=1. \tag{6}$$

This gives the following construction for conjugate points, when one pair O, O′ and the two foci F, F′ are given.

Let the two lines O F, O′ F′, on which the two sets of conjugate points lie, be crossed, with O lying upon O′. Draw F M (Fig. 168) parallel to O′ F′, and F′ M

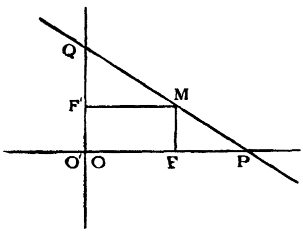


Fig 168

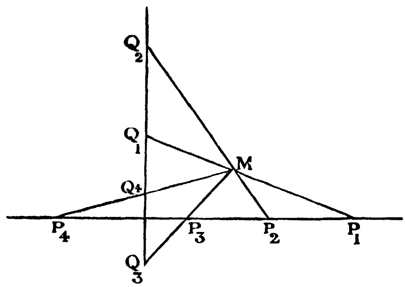


Fig 169

parallel to O F. Every line (such as P M Q) drawn through the point M will be the join of a pair of conjugate points; for we have in the figure

$$\frac{\mathrm{OF}}{\mathrm{OP}}=\frac{\mathrm{QM}}{\mathrm{QP}},\quad \frac{\mathrm{O'F'}}{\mathrm{O'Q}}=\frac{\mathrm{MP}}{\mathrm{QP}},\quad \mathrm{QM}+\mathrm{MP}=\mathrm{QP};$$

hence the values of O P, O′Q thus found satisfy equation (6). Four pairs of conjugate points thus found are shown in Fig 169.

196. Relation of Magnification to Focal Distance. — We now proceed to some inferences respecting magnification

The magnification in a given optical system varies directly as the distance of the image from the second principal focus

Let 1, 1 and 2, 2 (Fig 170) be two rays incident parallel to the axis of the system. They will emerge as two rays crossing in the second principal focus. Consider an object whose length is the distance between the two parallels, and let it move along them with one end in each The images of its ends will travel along the two intersecting lines, and the length of the image will vary directly as the distance of the image from the point of intersection.

The image will evidently be erect on one side of the principal focus, and inverted on the other side. Magnification is reckoned positive when the image is erect,

and negative when inverted. Thus when the distance from the focus changes sign, the magnification changes sign also.

On the other hand, two incident rays intersecting in the first principal focus will emerge as two rays parallel to the axis. If we regard the image as of con-

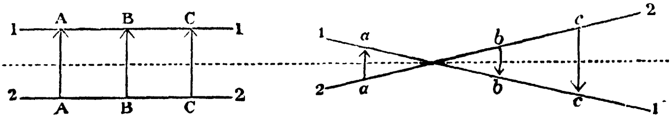


Fig 170.

stant size, and bounded by these two parallel lines, the size of the object must vary directly as the distance of the object from the point of intersection.

Hence the magnification varies inversely as the distance of the object from the first principal focus.

Combining these two propositions respecting magnification, we deduce that the distance of the object from the first principal focus varies inversely as the distance of the image from the second principal focus; or the product of these two distances is constant, a conclusion which we have already obtained by a different line of reasoning.

197. Principal Planes. — Since the size of the image changes continuously from zero at the second principal focus up to any magnitude we please, and is erect when on one side of the second focus and inverted when on the other, it is in general possible to choose one position at which the image will be erect and equal to the object. In this case the magnification is said to be unity, and the positions of the object and image on the axis are called the *Principal Points* of the optical system. The position of the object is called the *first*, and the position of the image the *second* principal point. They are usually denoted by H and H Since they are conjugate points, we have, from the constancy of the product of focal distances,

$$FP \cdot F'Q = FH \cdot F'H', \tag{6}$$

or

$$\frac{FP}{FH} = \frac{F'H'}{F'Q}.$$

Since both members of this last equation must have the same sign, Q and H′ will be on the same or opposite sides of F′, according as P and H are on the same or opposite sides of F. F H and F′ H′ are called the *principal focal distances*, and planes through H and H′ perpendicular to the axis are called the *Principal Planes*. An object lying in one principal plane gives an equal and erect image in the other principal plane. Hence if we take a pair of directly opposite points in these two planes (that is, points whose join is parallel to the axis), every ray that passes through one of these points passes also through the other.

198. Conditions for determining Magnification. — We have seen that the relation between x and y (that is, between the positions of conjugate points) is completely determined by three pairs of conjugate points. This relation however does not suffice for determining magnifications. It fixes the positions of the two principal foci; and the magnification will vary as the distance of the image from the second principal focus, or inversely as the distance of the object from

the first; but the constant factor implied in this statement is undetermined unless the magnification for some one pair of conjugate points is also given

Three given pairs of conjugate points, together with the magnification corresponding to one of them,[1] suffice for completely determining the relation between objects and their images As a particular case of this rule, if we know the position and magnification of the image for one given position of the object, and also know the positions of the two principal foci, we have just sufficient data for predicting all other positions and magnifications. The calculation with these data is specially simple; for, assigning any other position to the object, the position of the image will be at once determined by the constancy of the product of focal distances; and the magnification will have a known ratio to the given magnification

The following construction (Fig 171) is equivalent to this calculation

Given an object A B, its image A′B′ (vertical heights being exaggerated for convenience of drawing), and the principal foci F, F′, to find the position and size of the image of an object placed at any other point C on the axis Join F B and let the vertical C D (meeting it in D) represent the object. Draw D m parallel to the axis, meeting A B in m

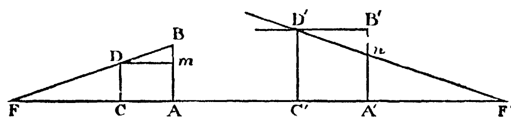


Fig 171

The image of m will be the point n which divides A′B′ in the same ratio in which m divides A B The incident ray D m will emerge as n F′, and the incident ray D B will emerge as a parallel to the axis through B′ The meeting point D′ of these two emergent rays must be the image of D, and the vertical D′C′ will be the image of D C.

199. Relation between Magnification and Divergence.

—We now proceed to an investigation which will lead to a simple relation between the two principal focal distances.

Let C (Fig 172) be the centre of curvature of a spherical surface A B, at which rays issuing from P_1 are refracted, and let P_2 be the focus after refraction, so that P_2 is the image of P_1 Let $P_1 L_1$ be a small object perpendicular to the axis C A, and draw the radius C L_1 B. Since the ray L_1 B is incident normally it undergoes no deviation, hence L_2 the image of L_1 lies in the line C L_1 B, and to first approximations $P_2 L_2$ drawn perpendicular to the axis is the image of $P_1 L_1$ We have evidently $P_1 L_1 . P_2 L_2 : : C P_1 \quad C P_2$; that is, the linear dimensions are directly as the distances from the centre of curvature.

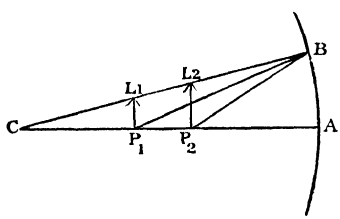


Fig 172

Next compare the angles of divergence of the incident and refracted pencils. The incident pencil A P_1 B gives rise to the refracted pencil A P_2 B But the plane angles A P_1 B, A P_2 B are in the inverse ratio of A P_1 and A P_2. Hence if

[1] Instead of the magnification corresponding to one of them, it is sufficient to know the *sign* of this magnification together with the index of refraction from the first medium to the last. This will be proved hereafter

θ_1 θ_2 denote the plane angles of divergence (or convergence) of the incident and refracted pencils, we have $\theta_1 : \theta_2 : : A P_2 : A P_1$. Accordingly denoting $P_1 L_1$, $P_2 L_2$ by l_1, l_2, equation (13) of § 163,

namely
$$\mu_1 \frac{C P_1}{A P_1} = \mu_2 \frac{C P_2}{A P_2},$$

will become
$$\mu_1 l_1 \theta_1 = \mu_2 l_2 \theta_2. \qquad (7)$$

In reflection at a spherical surface we have the still simpler relation

$$\frac{C P_1}{A P_1} = -\frac{C P_2}{A P_2} \text{ leading to } l_1 \theta_1 = -l_2 \theta_2.$$

It thus appears that the product $\mu l \theta$ retains its value unchanged through all the successive changes which a pencil undergoes in traversing such an optical system as we have been considering, l denoting the length of any one of the successive images, and θ the plane angle of the pencils which form this image. It should be borne in mind that each point of an image is the vertex of a solid cone of rays, and that if it is a *real* image formed in mid-air the cone is a double cone, the angle of divergence after passing the image being equal to the previous angle of convergence. Also that, to first approximations, the angle of the cone has the same value for all the points of an image; for example, in the above figure, the arc A B subtends sensibly equal angles at P_2 and L_2.

We have spoken of the *plane* angle of divergence in contradistinction from the solid angle of divergence, which is the solid angle of one of the cones in question and is proportional to θ^2. In like manner the areas of object and image are proportional to l_1^2 and l_2^2. Accordingly if we denote these areas by a_1, a_2, &c., and the solid angles of the pencils by ω_1, ω_2, &c., we have $\mu_1^2 a_1 \omega_1 = \mu_2^2 a_2 \omega_2 =$ &c., or the product $\mu^2 a \omega$ remains unchanged in passing through the optical system. At present we are concerned only with linear dimensions and plane angles.

200. Ratio of two Focal Lengths.—We know that, to first approximations, parallel rays at a small inclination to the axis of a lens meet, after refraction, at a point which lies in a plane drawn through the principal focus perpendicular to the axis; and conversely that rays incident from such a point will emerge parallel. We shall assume this property to be general for such optical systems as we are discussing.

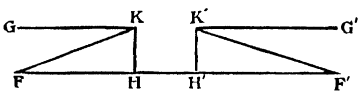


Fig. 173

Applying the formula $\mu_1 l_1 \theta_1 = \mu_2 l_2 \theta_2$ to an object H K (Fig. 173) and its image H′ K′ in the two principal planes, we have $l_1 = l_2$, hence $\mu_1 \theta_1 = \mu_2 \theta_2$.

The incident ray F K emerges as K′G′, and G K emerges as K′ F′. Thus the incident pencil bounded by F K and G K gives rise to the emergent pencil bounded by K′G′ and K′F′. Taking the plane angles of these pencils as the values of θ_1 and θ_2, we have

$$\theta_1/\theta_2 = H' F'/F H = -F' H'/F H.$$

Putting f for F H, and f' for $-F'H'$, we have $f\theta_1 = f'\theta_2$; but from above $\mu_1\theta_1 = \mu_2\theta_2$; hence $f/f' = \mu_1/\mu_2$; that is, *the focal lengths are directly as the indices of refraction of the first and last media.* When the last medium is the same as

the first, this gives $f=f'$, or $FH = -F'H'$, the minus sign indicating that if H lies on the right of F, H′ lies on the left of F′, as in the figure.

201. **Ambiguous determination of Principal Points.**—We are now in a position to determine the "principal points" and the magnifications of a system from the positions of the two foci and of one pair of conjugate points, combined with a knowledge of the index of refraction from the first to the last medium. Call this index μ, and the given conjugate points P and Q, P being for the incident and Q for the emergent rays. We have

$$F'H' = -\mu \, . \, FH, \text{ and } FP \, . \, F'Q = FH \, . \, F'H' = -\mu \, (FH)^2,$$
$$(FH)^2 = -FP \, . \, F'Q/\mu. \tag{8}$$

This determination of FH is ambiguous as regards sign, and two different optical systems satisfy the given conditions, the two systems being identical as regards the positions of conjugate points, but differing in sign of magnification. The ambiguity will be removed by knowing whether the image at Q of an object at P is erect or inverted. If it is erect, H and P lie on the same side of F; if inverted, on opposite sides.

If the first and last media are the same, as is usually the case in practice, we have simply

$$FH^2 = -FP \, . \, F'Q. \tag{9}$$

202. **Nodal Points.**—Take FN (Fig. 174) equal to H′F′, and F′N′ equal to HF, so that HN = H′N′ = difference of focal lengths. The points N and N′ possess a remarkable property, and are called the first and second *Nodal Points*.

Let GN be any incident ray through N. Let it meet the principal plane in S, and the vertical at FG in G. Make HK and H′K′ equal to FG, and H′S′ equal

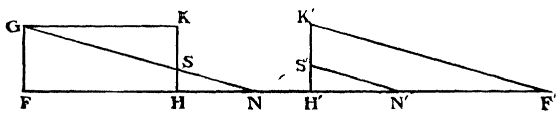


Fig. 174

to HS. Triangles SHN, S′H′N′ are equal in all respects, as are also triangles GFN, K′H′F′.

The ray GK will emerge as K′F′; and since G is in a focal plane, GN will emerge as S′N′ parallel to K′F′ or to GN.

Thus *every incident ray through the first nodal point emerges as a parallel ray through the second nodal point.* An obvious consequence of this property is that the angle subtended by an object at the first nodal point is equal to the angle subtended by its image at the second nodal point.[1] In the human eye the second nodal point is within the crystalline lens about ·4 of a millimetre from its back, and the image of a distant object formed on the retina subtends the same angle at this point which the object subtends at the eye. The "line of collimation" of a telescope, which we have defined on page 198 as "the line joining the cross to

[1] Hence we can prove that no other pair of conjugate points can possess the property which we have proved to belong to the nodal points, for the image of a distant object cannot subtend equal angles at two different points on the axis.

the optical centre of the object-glass," would be still more accurately defined as the line joining the cross to the second nodal point of the object-glass.

When the last medium is the same as the first, the nodal points coincide with the principal points, since the focal lengths are equal and H N is their difference. This is on the supposition that the emergent rays have the same general direction as the incident rays; if in consequence of reflection they have the opposite direction, the ratio μ_1/μ_2 must be regarded as being -1.

203. Simple Applications.—We shall now apply the foregoing principles to a few practical cases.

Taking first the case of refraction at a single spherical surface, and availing ourselves of the results established in § 163; a small area on the surface coincides with its own image, hence the two principal planes coincide with one another and are at the intersection of the axis with the surface. Again, every incident ray through the centre of curvature continues its course unchanged. Hence the two nodal points coincide at the centre of curvature.

With the usual conventions as to sign, the first focal length f is the value of p_1 in equation (15) when p_2 is infinite, and the second focal length f' is *minus* the value of p_2 when p_1 is infinite. We have thus $f = \mu_1 r / (\mu_1 - \mu_2)$, $f' = \mu_2 r / (\mu_1 - \mu_2)$. The ratio of these distances is μ_1 / μ_2 and their difference is r, which is the distance between the principal points and the nodal points.

Secondly, take the case of refraction through a sphere, which has been worked out in § 164.

Since every incident ray through the centre continues its course unchanged, the nodal points coincide at the centre. The principal points are identical with the nodal points, since the first and last media are the same. The focal length is $\frac{1}{2}\mu a / (\mu - 1)$. This is the distance of F from the centre on the side next the incident light, and the distance of F′ from the centre on the opposite side.

Thirdly, take the case of reflection at a spherical surface.

The principal planes coincide at the intersection of the axis with the surface. The two nodal points coincide at the centre of curvature. Since the emergent rays are opposite in general direction to the incident rays, the equal distances H F, H′ F′ are in the same direction, thus F and F′ coincide. The distance of the nodal points from the principal points will for the same reason be the sum instead of the difference of the focal lengths, and as they are equal the focus is half-way between them.

204. Application to a Thick Lens.

CASE 4.—A double-convex lens of equal curvatures. Let a denote the radius of curvature of either face, and $2t$ the thickness.

Referring to Fig. 132, § 149, the nodal points will be the ultimate intersections of the lines S I and R E with the axis; in other words, they will be the images formed by rays from the centre of the lens refracted into air at the two surfaces. By the aid of formula (15) of § 163 the distance of the first nodal point from the first surface, or of the second nodal point from the second surface, is found to be

$$\frac{at}{\mu(a-t)+t}.$$

CASE 5.—A double-concave lens with equal radii of curvature b, the thickness at the centre being $2t$.

The first nodal point is found by the method of case 4 to be between the first surface and the centre of the lens, at a distance $\frac{bt}{\mu(b+t)-t}$ from the first surface; and the second nodal point is symmetrically placed on the other side of the centre.

In both the cases 4 and 5 it is easy to show, either from the formulæ here obtained, or directly, that the distance of the first nodal point from the first surface is less than t, the half-thickness of the lens. In both cases the principal points are the same as the nodal points.

CASE 6.—For a plano-convex or plano-concave lens the "centre" is on the convex or concave face, and if t now denote the whole thickness (not the half-thickness as before), the law of refraction at a plane surface gives t/μ as the distance of the image of this centre from the plane face. The image formed by refraction at the curved face coincides with the "centre" itself. These images are the two principal points and nodal points of the lens, and their distance apart is $(1-1/\mu)t$.

In all cases, for a lens in air, the focal length, neglecting sign, is to be defined as the distance between either focus and the corresponding principal plane.

H F is opposite in direction to H′F′, and if we put $f = \text{H F} = -\text{H}'\text{F}'$, we have $\text{F P}.\text{F}'\text{Q} = \text{F H}.\text{F}'\text{H}' = -f^2$, from which can be deduced by substitution

$$\frac{\text{H F}}{\text{H P}} + \frac{\text{H}'\text{F}'}{\text{H}'\text{Q}} = 1, \text{ or } \frac{1}{\text{H P}} - \frac{1}{\text{H}'\text{Q}} = \frac{1}{f}. \tag{10}$$

° 205. Application to Eye-pieces.

CASE 7.—Ramsden's eye-piece. Two thin convex lenses, each of focal length $3a$, at distance $2a$.

Rays from infinity incident on either lens are first collected to a focus at distance a beyond the other lens, and then still further converged by it to a principal focus F or F′ at distance $\frac{3}{4}a$. F and F′ are thus at distances $\frac{3}{4}a$ outside the system.

An object at distance $3a$ from the system will first give an image at infinity, and then a final image, which will be inverted, and at distance $3a$ beyond the further lens. Symmetry shows that the magnitudes of the object and image will be equal. The distance of this object from F is $2\frac{1}{4}a$, and H will be $2\frac{1}{4}a$ on the other side of F, or $\frac{1}{2}a$ short of the further lens; H′ will by symmetry be $\frac{1}{2}a$ from the near lens. The order of succession of the four points will be F H′ H F′, with an interval a between H′ and H.

CASE 8.—Huygens' eye-piece. Two thin convex lenses, the smaller having focal length a, and the larger $3a$, at distance $2a$; the larger being next the object.

Parallel rays incident first on the small lens will be collected by it to a focus midway between the two lenses, and will finally diverge from a point distant $3a/2$ from the large, or $a/2$ from the small lens. This point is F.

Parallel rays incident first on the large lens are collected by it to a focus at distance a beyond the small lens, and are finally brought to the principal focus F′ at a distance $a/2$ outside the small lens.

Rays from a point P, at distance $3a$ beyond the large lens, will fall as a parallel pencil on the small lens, and be brought by it to a focus Q at distance a beyond it, and the image at Q will be inverted. The distances F P and F′Q, neglecting

sign, are $4\frac{1}{2}a$ and $\frac{1}{2}a$; hence f^2 is $\frac{9}{4}a^2$, and f is $\pm 3a/2$. This is the distance of H from F, and since the image at Q is inverted, F H will be opposite in direction to F P. H will therefore be at distance a beyond the small lens. F′ H′ must be opposite in direction to F′ Q; hence H′ is midway between the two lenses.

Definition of Equivalent Lens.—A single lens is said to be *equivalent* to a given system of lenses when the value of H F or its equal F′ H′ is the same for the lens as for the given system. The student should verify that for a system consisting of two thin convex lenses of focal lengths f_1, f_2, at distance D, the focal length f of the equivalent lens is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{D}{f_1 f_2}. \tag{11}$$

206. Principal Foci at Infinite Distance.—We have seen that the magnification varies as the distance of the image from F′, the second principal focus. If this focus is at a practically infinite distance, the magnification will be practically the same for all positions of the image within moderate distance of the instrument. In like manner, as the first principal focus F is then at infinite distance, the magnification is independent of the position of the object. These results are applicable to a telescope focussed for long sight. The diameters of the images which it forms bear a constant ratio to the diameters of their objects, and this is true even if the object comes close up to the telescope. We are speaking here of diameters in linear, not in angular measure. Taking the simple case of two convex lenses, at a distance equal to the sum of their focal lengths, it is easy to see that a parallel incident pencil gives a parallel emergent pencil, and that the breadths of these pencils are in the direct ratio of the focal lengths of the two lenses. An object whose breadth is that of the incident pencil will give an image whose breadth is that of the emergent pencil whatever its distance may be. In an ordinary telescope this image is smaller than the object, so that the magnification (in the sense in which we have thus far used the word) is a proper fraction We shall denote this magnification by m, and call it for distinction the *transverse* magnification.

207. Adaptation of the General Equation.—In the general equation $xy+ay+bx+c=0$ we have seen that $-a$ is the value of x corresponding to $y=\infty$, and that $-b$ is the value of y for $x=\infty$. If the values $x=\infty$ and $y=\infty$ correspond to each other, both a and b must be infinite, and this is the case which we are now discussing.

Dividing the equation by a, the first term will vanish, and we shall have

$$y=-bx/a-c/a,$$

which may be more conveniently written

$$y=kx+h, \tag{12}$$

k denoting $-b/a$ and h denoting $-c/a$.

If x_2-x_1 be the length of an object measured parallel to the axis, the length of its image similarly measured will be y_2-y_1, or $k(x_2-x_1)$; hence the axial magnification is constant and equal to k or $-b/a$. These results are true whether x and y are measured from the same origin, or from different origins.

Let the origin be the same for both, and let a new origin be taken at any distance e in the positive direction. When x and y are measured from this new origin the equation will become

$$y+e = k(x+e)+h,$$

which will reduce to the simple form

$$y=kx, \tag{13}$$

if $e=ke+h$, that is if

$$e = h/(1-k). \tag{14}$$

The new origin thus determined is called the *centre* of the system. The object will meet its image at the centre, and as it moves away to any distance, its image will move k times as fast.

When $k=1$, the centre goes off to infinity unless $h=0$.

208. Relation between the Three Magnifications.—The angular magnification of a telescope, which is what is commonly understood by its "magnifying power," is equal to m/k, that is, to the transverse magnification divided by the axial magnification.

For, if A B (Fig. 175) is a line which just covers a more distant line C D as seen by the observer (both being perpendicular to the line of sight), the angle subtended by either is the inclination of B D to A C, that is, the angle $(\text{C D}-\text{A B})/\text{A C}$.

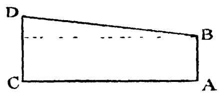


Fig 175

In the image this is changed to $(m.\text{C D}-m.\text{A B})/k.\text{A C}$, and is therefore multiplied by m/k. In order that the image of A B may exactly cover the image of C D the eye must be at the "centre" of the telescope

209. As a first example we will take a telescope composed of two thin convex lenses of focal lengths 1 and n (the latter being the object-glass), at a distance equal to the sum of their focal lengths. Taking the common focus as origin, and considering the positive direction to be that in which the light from the object travels, we can determine k and h as follows.

An object at distance n beyond the object-glass will send parallel pencils to the eye-glass and give an image at distance 1 beyond the eye-glass, thus the value $x=-2n$ gives $y=2$.

Again, an object in contact with the object-glass will coincide with its first image and give a final image at distance $1+\frac{1}{n}$ beyond the eye-glass Thus the value $x=-n$ gives $y=2+\frac{1}{n}$. Substituting these two pairs of values in the equation $y=kx+h$ we obtain $k=1/n^2$, $h=2+2/n$; and for the centre of the telescope $e=h/(1-k)=2n/(n-1)$ Subtracting 1 we get the distance of the "centre" beyond the eye-glass, which is $(n+1)/(n-1)$.

A ray incident parallel to the axis crosses the axis at the common focus, and emerges parallel to the axis on the other side at a distance $1/n$ of its original distance. Hence $m=-1/n$

The angular magnification is m/k and is therefore $-n$, which agrees with the rule deduced in § 176.

The image is only $1/n$ of the size of the object, but is n^2 times as near, and hence subtends n times as large an angle.

When the two glasses have the same focal length, n is 1, and the "centre" is at infinity. We have then $y=x+4$, and as x is negative for a real object, the image is 4 nearer. The angular magnification will therefore be $D/(D-4)$, D denoting the actual distance of the object from the observer's eye.

210. Second example. Three thin convex lenses, of focal lengths 4, 1, 4, the second midway between the first and third, at a distance 4 from each.

A ray parallel to the axis incident on the first lens will pass through the centre of the middle lens without deviation, and emerge from the third parallel to the axis. Its final distance from the axis will be equal and opposite to its initial distance. Hence $m=-1$.

Rays from an object at distance 4 beyond the first lens will fall as a parallel pencil on the second lens, which will bring them to a focus at distance 3 from the third lens; and the final image will be an inverted virtual image at distance 12 from the third lens, or 4 from the first lens. It will therefore coincide with the object. By symmetry it can be shown that incident rays converging to a point at distance 4 beyond the third lens will, after emergence, still converge to this point.

Measuring x and y from the middle lens we have thus $x=-8$ when $y=-8$, and $x=8$ when $y=8$. Substituting in $y=kx+h$ we deduce $k=1$, $h=0$.

The expression $h/(1-k)$ for the distance of the "centre" takes the form $0/0$, and every point on the axis has the properties of a centre.

The transverse and angular magnifications are each -1.

211.—Third example. Keeping the distances as above, let the focal lengths of the first and third lenses be each 2.

A ray parallel to the axis incident on the first lens will cross the axis midway between the first and second lens. It will then cross back again midway between the second and third, and finally emerge from the third parallel to the axis at the same side of it and at the same distance as the original ray. Thus $m=1$.

An object at distance 2 outside the first lens sends parallel rays to the second lens, which brings them to a focus at distance 3 from the third, which forms a real image at distance 6 beyond the system.

An object at distance 6 outside the first would by symmetry give an image at distance 2 beyond the third.

An object at distance 4 from the first gives an equal and inverted image at the centre of the second; and an equal and reinverted image of this at distance 4 beyond the third.

We have here one datum more than we require. The equations are

$$\left.\begin{aligned} 10 &= -6k+h \\ 6 &= -10k+h \\ 8 &= -8k+h \end{aligned}\right\} \text{from any two of which we find } k=1,\ h=16,\ y=x+16.$$

Since k is 1, the "centre" is at infinite distance. When the combination is used as a telescope, the angular magnification will be $x/(x-16)$, x denoting the distance of the object.

CHAPTER XII.

DISPERSION. STUDY OF SPECTRA.

212. Newton's Experiment.—The index of refraction of a substance is not the same for light of different colours. This is the explanation of the following experiment, due to Sir Isaac Newton.

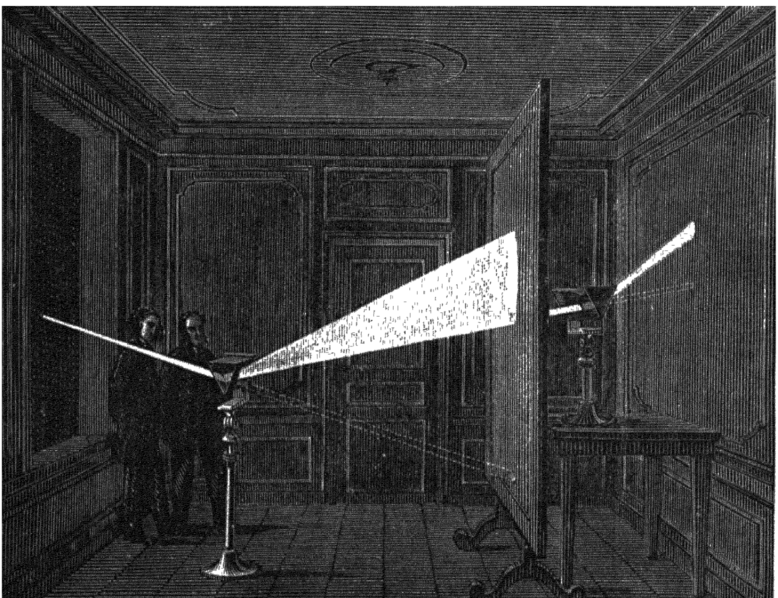


Fig 176 —Solar Spectrum by Newton's Method.

Let a beam of sunlight be admitted through a small opening into a dark room (Fig. 176). If allowed to fall normally on a white screen not too near the opening it produces a round white spot

which is an image of the sun. Now let a prism be placed in its path edge downwards as in the figure. The beam will thus be deflected upwards and at the same time resolved into a number of different colours, forming what is called the *solar spectrum*. Red will be the lowest and violet the highest, showing that red light is the least and violet light the most refrangible. The image depicted on the screen will be a rectangle rounded off at the ends and having the same width as the original white image. It is formed by the overlapping of a number of round images of different colours. The order of succession is red, orange, yellow, green, blue, violet, each passing into the next by insensible gradations. If a small opening is made in the screen so as to allow some of the coloured rays to pass through it and fall upon a second prism which deflects them still further upwards, they will be still further separated, so that there will be less overlapping of the images; in other words, the transmitted portion of the spectrum will be rendered less impure.

213. Pure Spectrum. Virtual.—In order to obtain the purest possible spectrum we must, in the first place, employ as the object for yielding the images a very narrow slit, and in the second place we must take care that the images which we obtain of this slit are not blurred, but have the greatest possible sharpness. A spectrum possessing these characteristics is called *pure*.

The simplest mode of obtaining a pure spectrum is to look through a prism at a fine slit in the shutter of a dark room, the edges of the prism being parallel to the slit, and the eye being close to the prism. The observer should rotate the prism on its own axis till he sees the spectrum in its greatest perfection, and it is not necessary that the position should be that of minimum deviation. In Fig. 177, E is the position of the eye, S that of the slit. The extreme red and violet images of the slit will be seen at R, V, and the other images, which compose the remainder of the spectrum, will occupy positions between R and V. The spectrum, in this mode of operating, is virtual.

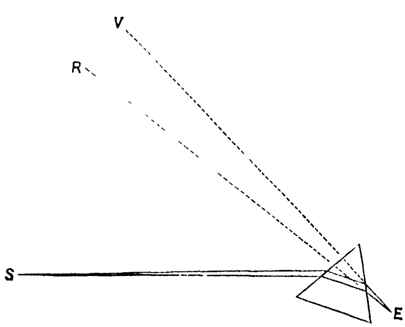


Fig. 177 —Arrangement for seeing a Pure Spectrum.

If the prism is some yards from the slit, a telescope may be

employed to assist the eye, and the position of minimum deviation will then be necessary in order to obtain sharp definition, as in any other position the deviations are not sufficiently uniform. It was in this manner that Fraunhofer's observations, which were the first accurate observations of the spectrum, were made, the distance of the slit from the prism being 24 feet. A long distance combined with minimum deviation are essential in order that rays may fall on different parts of the first face of the prism at sensibly the same angle and may undergo sensibly equal deviation. The telescope must be focussed for viewing objects at the distance of the slit.

214. Collimator.—To obviate the necessity of a long distance between the slit and prism, Professor Swan[1] introduced a collimating lens, that is to say, an achromatic lens fixed at its own focal length from the slit so that the virtual image which it forms of the slit is at a practically infinite distance. To this image the observing telescope is directed, focussed as if for viewing very distant objects This arrangement is now universally employed in spectroscopes.

215. Pure Spectrum. Real.—To throw a pure spectrum on a screen, the first operation is to throw an image of a strongly illuminated

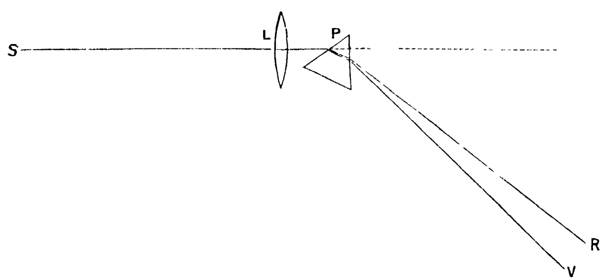


Fig 178 —Arrangement for Pure Spectrum on Screen

slit on another screen at about the same distance from the lens. In Fig. 178, S is the slit, L the lens, and I this first image. The prism P is then introduced in the position shown, and rotated slightly on its axis till minimum deviation is obtained, in other words, till the spectrum R V is brought as near to the first image I as possible. The spectrum will then be approximately in focus, and a final adjustment can be made by moving the screen a little nearer or further off.

For exhibiting the solar spectrum in this way, the light of the sky

[1] *Trans. Roy. Soc. Edin.*, 1847 and 1856.

is not sufficiently powerful. Sunshine is necessary, and the sun's rays are usually thrown through the slit upon the lens by means of a movable mirror called a *heliostat*. Sometimes the movements of the mirror are produced by hand, sometimes by an ingenious clockwork arrangement which causes the reflected beam to keep its direction unchanged notwithstanding the progress of the sun through the heavens.

216. Dark Lines in the Solar Spectrum.—When a pure spectrum of solar light is examined by any of these methods, it is seen to be traversed by numerous dark lines, constituting, if we may so say, dark images of the slit. Each of these is an indication that a particular kind of elementary ray is wanting[1] in solar light. Every elementary ray that is present gives its own image of the slit in its own peculiar colour; and these images are arranged in strict contiguity, so as to form a continuous band of light passing by perfectly gradual transitions through the whole range of simple colour, except at the narrow intervals occupied by the dark lines. Fig. 1, Plate III., is a rough representation of the appearance thus presented. If the slit is illuminated by a gas flame, or by any ordinary lamp, instead of by solar light, no such lines are seen, but a perfectly continuous spectrum is obtained. The dark lines are therefore not characteristic of light in general, but only of solar light.

Wollaston saw and described some of the more conspicuous of them. Fraunhofer counted about 600, and marked the places of 354 upon a map of the spectrum, distinguishing some of the more conspicuous by the names of letters of the alphabet, as indicated in fig. 1. These lines are constantly referred to as reference marks for the accurate specification of different portions of the spectrum. They always occur in precisely the same places as regards colour, but do not retain exactly the same relative distances one from another when prisms of different materials are employed, different parts of the spectrum being unequally expanded by different refracting substances.[2] The inequality, however, is not so great as to introduce any difficulty in the identification of the lines.

The dark lines in the solar spectrum are often called Fraunhofer's lines. Fraunhofer himself called them the "fixed lines."

217. Invisible Portion of the Spectrum.—The brightness of the solar spectrum, however obtained, is by no means equal throughout, but

[1] Probably not absolutely wanting, but so feeble as to appear black by contrast.

[2] This property is called the *irrationality of dispersion.*

is greatest between the dark lines D and E; that is to say, in the yellow and the neighbouring colours orange and light green; and falls off gradually on both sides.

The heating effect upon a small thermometer or thermopile increases in going from the violet to the red, and still continues to increase for a certain distance beyond the visible spectrum at the red end. Prisms and lenses of rock-salt should be employed for this investigation, as glass largely absorbs the invisible rays which lie beyond the red.

When the spectrum is thrown upon the sensitized surfaces employed in photography, the action is usually insensible in the red, strong in the blue and violet, and sensible to a great distance beyond the violet end. When proper precautions are taken to insure a very pure spectrum, the photograph reveals the existence of dark lines, like those of Fraunhofer, in the invisible ultra-violet portion of the spectrum. The strongest of these have been named L, M, N, O, P

218. Phosphorescence and Fluorescence.—There are some substances which, after being exposed in the sun, are found for a long time to appear self-luminous when viewed in the dark, and this without any signs of combustion or sensible elevation of temperature Such substances are called *phosphorescent.* Sulphuret of calcium and sulphuret of barium have long been noted for this property, and have hence been called respectively *Canton's phosphorus* and *Bologna phosphorus.* The phenomenon is chiefly due to the action of the violet and ultra-violet portion of the sun's rays.

More recent investigations have shown that the same property exists, in a much lower degree, in an immense number of bodies, their phosphorescence continuing, in most cases, only for a fraction of a second after their withdrawal from the sun's rays. E. Becquerel contrived an instrument, called the *phosphoroscope,* which is extremely appropriate for the observation of this phenomenon. It is represented in Fig. 179. Its most characteristic feature is a pair of rigidly connected discs (Fig. 180), each pierced with four openings, those of the one being not opposite but midway between those of the other. This pair of discs can be set in very rapid rotation by means of a series of wheels and pinions. The body to be examined is attached to a fixed stand between the two discs, so that it is alternately exposed on opposite sides as the discs rotate. One side is turned towards the sun and the other towards the observer, who accordingly only sees the body when it is not exposed to the sun's

rays. The cylindrical case within which the discs revolve is fitted into a hole in the shutter of a dark room, and is pierced with an opening on each side exactly opposite the position in which the body is fixed. The body, if not phosphorescent, will never be seen by the observer, as it is always in darkness except when it is hidden by the intervening disc. If its phosphorescence lasts as long as an

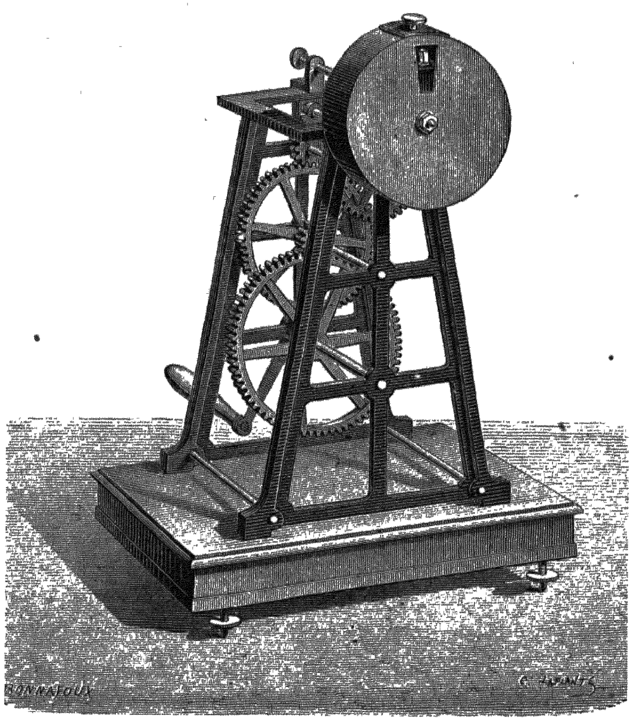


Fig. 179 —Becquerel's Phosphoroscope.

eighth part of the time of one rotation it will become visible in the darkness.

Nearly all bodies, when thus examined, show traces of phosphorescence, lasting, however, in some cases, only for a ten-thousandth of a second.

The phenomenon of *fluorescence*, which is illustrated in Plate II. at the beginning of Part III., appears to be essentially identical with phosphorescence. The former name is applied to the phenomenon if it is observed while the body is actually exposed to the source of light, the latter to the effect of the same kind, but usually

less intense, which is observed after the light from the source is cut off. Both forms of the phenomenon occur in a strongly-marked degree in the same bodies. Canary-glass, which is coloured with oxide of uranium, is a very convenient material for the exhibition of fluorescence. A thick piece of it, held in the violet or ultra-violet portion of the solar spectrum is filled to the depth of from $\frac{1}{8}$ to $\frac{1}{4}$ of an inch with a faint nebulous light. A solution of sulphate of quinine is also frequently employed for exhibiting the same effect, the luminosity in this case being bluish. If the solar spectrum be thrown upon a screen freshly washed with sulphate of quinine, the ultra-violet portion will become visible by fluorescence; and if the spectrum be very pure the presence of dark lines in this portion will be detected.

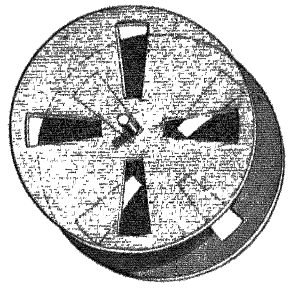


Fig 180.
Discs of Phosphoroscope.

The light of the electric lamp is particularly rich in ultra-violet rays, this portion of its spectrum being much longer than in the case of solar light, and about twice as long as the spectrum of luminous rays. Prisms and lenses of quartz should be employed for this purpose, as this material is specially transparent to the highly-refrangible rays. Flint-glass prisms, however, if of good quality, answer well in operating on solar light. The luminosity produced by fluorescence has sensibly the same tint in all parts of the spectrum in which it occurs, and depends upon the fluorescent substance employed. Prismatic analysis is not necessary to the exhibition of fluorescence. The phenomenon is very conspicuous when the electric discharge of a Holtz's machine or a Ruhmkorff's coil is passed near fluorescent substances, and it is faintly visible when these substances are examined in bright sunshine. The light emitted by a fluorescent substance is found by analysis not to be homogeneous, but to consist of rays having a wide range of refrangibility. It is less refrangible than the incident light which gives rise to it.

The ultra-violet rays, though usually styled invisible, are not altogether deserving of this title. By keeping all the rest of the spectrum out of sight, and carefully excluding all extraneous light, the eye is enabled to perceive these highly refrangible rays. Their colour is described as lavender-gray or bluish white, and has been attributed, with much appearance of probability, to fluorescence of

the retina. The ultra-red rays, on the other hand, are never seen; but this may be owing to the fact, which has been established by experiment, that they are largely, if not entirely, absorbed before they can reach the retina.

219. Recomposition of White Light.—The composite nature of white light can be established by actual synthesis. This can be done in several ways.

1. If a second prism, precisely similar to the first, but with its refracting edge turned in exactly the opposite direction, is interposed in the path of the coloured beam, very near its place of emergence from the first prism, the deviation produced by the second prism will be equal and opposite to that produced by the first, the two prisms will produce the effect of a parallel plate, and the image on the screen will be a white spot, nearly in the same position as if the prisms were removed.

2. Let a convex lens (Fig. 181) be interposed in the path of the

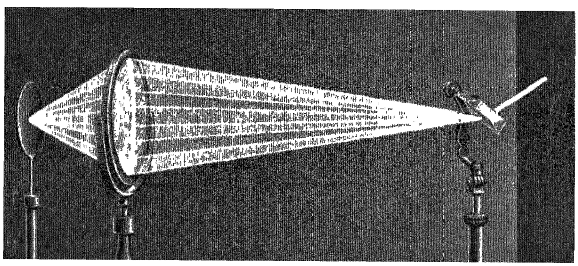


Fig 181.—Recomposition by Lens.

coloured beam, in such a manner that it receives all the rays, and that the screen and the prism are at conjugate focal distances. The image thus obtained on the screen will be white, at least in its central portions.

3. Let a number of plane mirrors be placed so as to receive the successive coloured rays, and to reflect them all to one point of a screen, as in Fig. 182. The bright spot thus formed will be white or approximately white.

More complete information respecting the mixture of colours will be given in the next chapter.

220. Spectroscope.—When we have obtained a pure spectrum by any of the methods above indicated, we have in fact effected an analysis of the light with which the slit is illuminated. In recent

years, many forms of apparatus have been constructed for this purpose, under the name of *spectroscopes*.

A spectroscope contains, besides a slit, a prism, and a telescope, a convex lens called a *collimator*, which is fixed between the prism

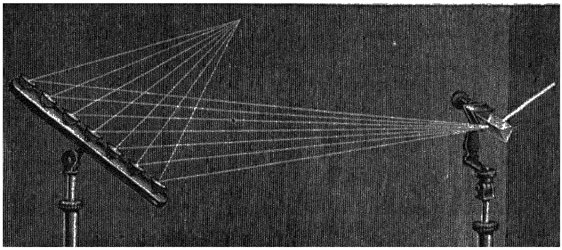


Fig. 182.—Recomposition by Mirrors.

and the slit, at the distance of its principal focal length from the latter. The effect of this arrangement is, that rays from any point of the slit emerge parallel, as if they came from a much larger slit (the virtual image of the real slit) at a much greater distance. The prism (set at minimum deviation) forms a virtual image of this image, at the same distance, but in a different direction, on the principle of Fig. 177. To this second virtual image the telescope is directed, being focussed as if for a very distant object.

Fig. 183 represents a spectroscope thus constructed. The tube of the collimator is the further tube in the figure, the lens being at the end of the tube next the prism, while at the far end, close to the lamp flame, there is a slit (not visible in the figure) consisting of an opening between two parallel knife-edges, one of which can be moved to or from the other by turning a screw. The knife-edges must be very true, both as regards straightness and parallelism, as it is often necessary to make the slit exceedingly narrow. The tube on the left hand is the telescope, furnished with a broad guard to screen the eye from extraneous light. The near tube, with a candle opposite its end, is for purposes of measurement. It contains, at the end next the candle, a scale of equal parts, engraved or photographed on glass. At the other end of the tube is a collimating lens, at the distance of its own focal length from the scale; and the collimator is set so that its axis and the axis of the telescope make equal angles with the near face of the prism. The observer thus sees in the telescope, by reflection from the surface of the prism, a magnified image of the scale, serving as a standard of reference for assigning the positions

of the lines in any spectrum which may be under examination. This arrangement affords great facilities for rapid observation.

Another plan is, for the arm which carries the telescope to be

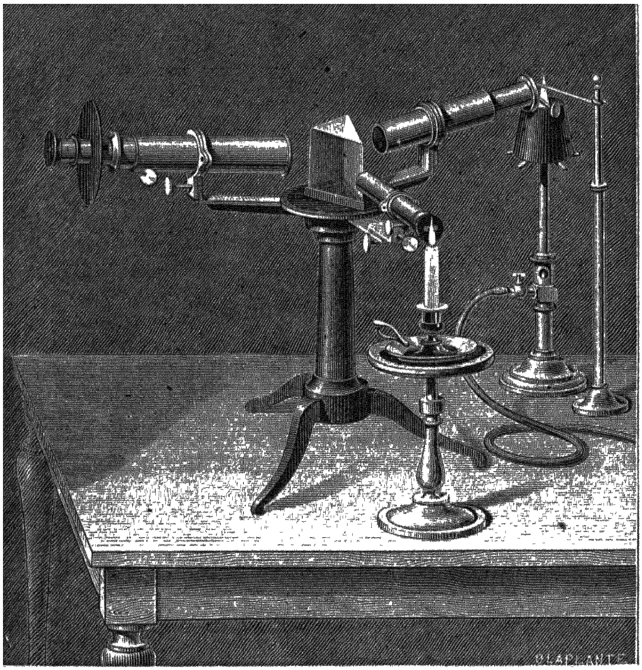


Fig. 183.—Spectroscope.

movable round a graduated circle, the telescope being furnished with cross-wires, which the observer must bring into coincidence with any line whose position he desires to measure.

Arrangements are frequently made for seeing the spectra of two different sources of light in the same field of view, one half of the length of the slit being illuminated by the direct rays of one of the sources, while a reflector, placed opposite the other half of the slit, supplies it with reflected light derived from the other source. This method should always be employed when there is a question as to the exact coincidence of lines in the two spectra. The reflector is usually an equilateral prism. The light enters normally at one of its faces, is totally reflected at another, and emerges normally at the third, as in the annexed sketch (Fig. 184), where the dotted line represents the path of a ray.

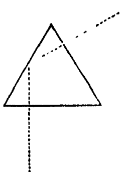


Fig. 184. Reflecting Prism

A one-prism spectroscope is amply sufficient for the ordinary purposes of chemistry. For some astronomical applications a much greater dispersion is required. This is attained by making the light pass through a number of prisms in succession, each being set in the proper position for giving minimum deviation to the rays which have passed through its predecessor. Fig. 185 represents the ground plan of such a battery of prisms, and shows the gradually increasing width of a pencil as it passes round the series of nine prisms on its way from the collimator to the telescope. The prisms are usually connected by a special arrangement, which enables the observer, by a single movement, to bring all the prisms at once into the proper position for giving minimum deviation to the particular ray under examination.

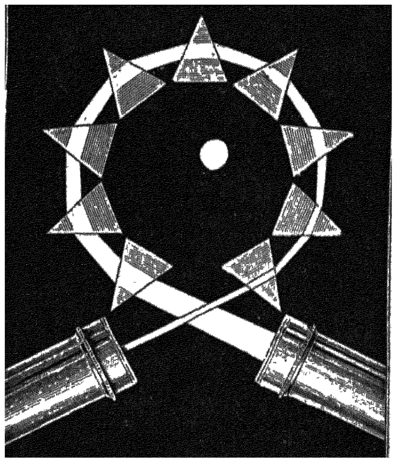


Fig 185 —Train of Prisms.

221. Direct-Vision Spectroscopes.—Direct-vision spectroscopes, having the form of a short tube, which is pointed towards the source of light to be examined, are much more convenient for rapid work, and are very largely employed. They are sufficiently powerful for ordinary chemical purposes, even when small enough to be easily carried in the pocket. They consist of the following parts:—

First. A slit in the principal focus of a collimating lens of short focal length.

Secondly. A compound prism built up of prisms of two kinds of glass—flint and crown—the refracting angles of the two kinds being turned in opposite directions, and so proportioned that their deviations exactly counteract each other for the mean rays. The dispersion of the flint is then only partially counteracted by that of the crown, so that a spectrum is produced extending on both sides of the direct line of sight.

Thirdly. A very short telescope which magnifies the spectrum. The observer has usually to focus the telescope, regulate the width of the slit, and turn the slit so that it is parallel to the edges of the prisms.

222. Different Kinds of Spectra.—The examination of a great variety of sources of light has shown that spectra may be divided into the following classes:—

1. The solar spectrum is characterized, as already observed, by a definite system of dark lines interrupting an otherwise continuous succession of colours. The same system of dark lines is found in spectra of the moon and planets, this being merely a consequence of the fact that they shine by the reflected light of the sun. The spectra of the fixed stars also contain systems of dark lines, which are different for different stars.

2. The spectra of incandescent solids and liquids are completely continuous, containing light of all refrangibilities from the extreme red to a higher limit depending on the temperature.

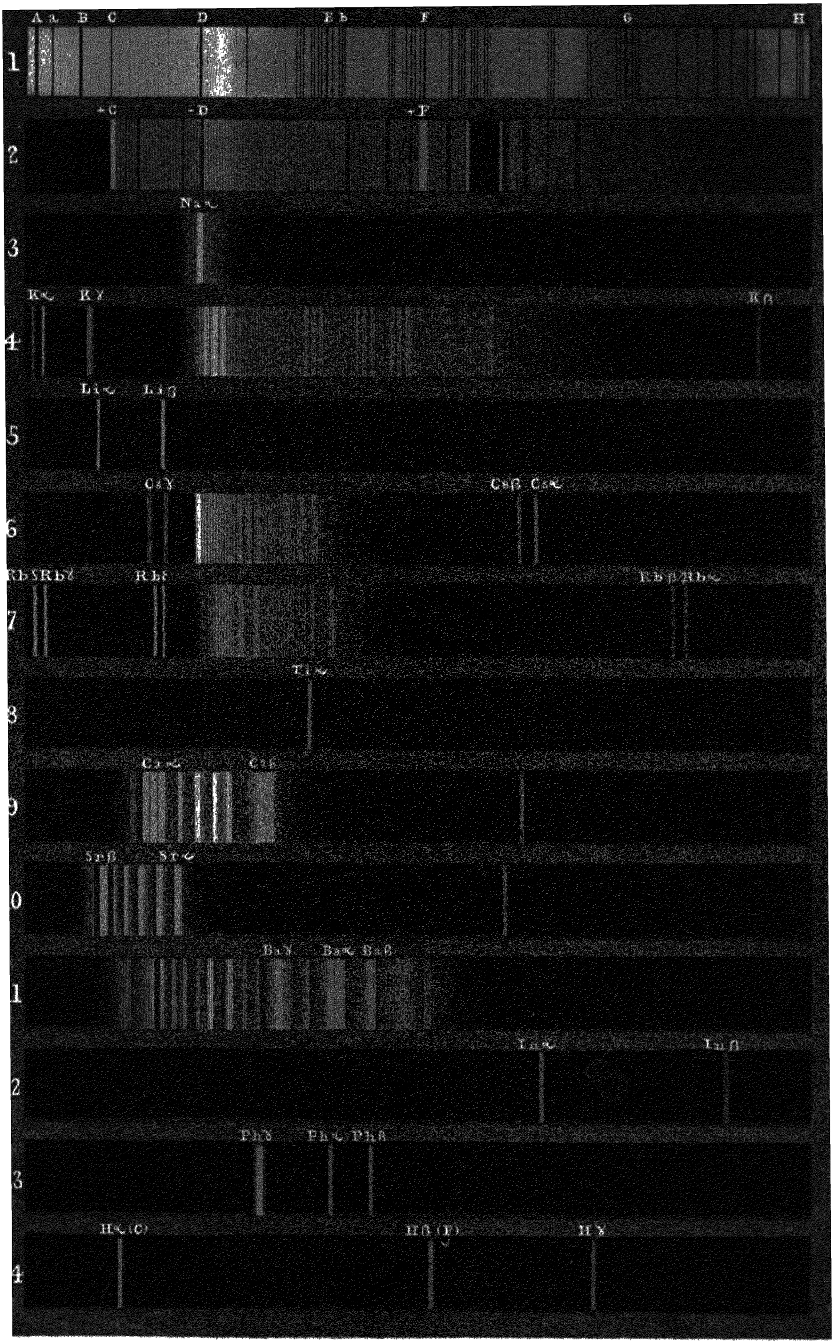
3. Flames not containing solid particles in suspension, but merely emitting the light of incandescent gases, give a discontinuous spectrum, consisting of a finite number of bright lines. The continuity of the spectrum of a gas or candle flame, arises from the fact that nearly all the light of the flame is emitted by incandescent particles of solid carbon—particles which we can easily collect in the form of soot. When a gas-flame is fed with an excessive quantity of air, as in Bunsen's burner, the separation of the solid particles of carbon from the hydrogen with which they were combined no longer takes place: the combustion is purely gaseous, and the spectrum of the flame is found to consist of bright lines. When the electric light is produced between metallic terminals, its spectrum contains bright lines due to the incandescent vapour of these metals, together with other bright lines due to the incandescence of the oxygen and nitrogen of the air. When it is taken between charcoal terminals, its spectrum is continuous; but if metallic particles be present, the bright lines due to their vapours can be seen as well.

The spectrum of the electric discharge in a Geissler's tube consists of bright lines characteristic of the gas contained in the tube.

223. Spectrum Analysis.—As the spectrum exhibited by a compound substance when subjected to the action of heat, is frequently found to be identical with the spectrum of one of its constituents, or to consist of the spectra of its constituents superimposed,[1] the spectroscope affords an exceedingly ready method of performing qualitative analysis.

[1] These appear to be merely examples of the dissociation of the elements of a chemical compound at high temperatures.

SPECTRA OF VARIOUS SOURCES OF LIGHT



If a salt of a metal which is easily volatilized is introduced into a Bunsen lamp flame, by means of a loop of platinum wire, the bright lines which form the spectrum of the metal will at once be seen in a spectroscope directed to the flame; and the spectrum of the Bunsen flame itself is too faint to introduce any confusion. For those metals which require a higher temperature to volatilize them, electric discharge is usually employed. Geissler's tubes are commonly used for gases.

Plate III. contains representations of the spectra of several of the more easily volatilized metals, as well as of phosphorus and hydrogen; and the solar spectrum is given at the top for comparison. The bright lines of some of these substances are precisely coincident with some of the dark lines in the solar spectrum.

The fact that certain substances when incandescent give definite bright lines, had been known for many years, from the researches of Brewster, Herschel, Talbot, and others, but it was for a long time thought that the same line might be produced by different substances, more especially as the bright yellow line of sodium was often seen in flames in which that metal was not supposed to be present Professor Swan, having ascertained that the presence of the 2,500,000th part of a grain of sodium in a flame was sufficient to produce it, considered himself justified in asserting, in 1856, that this line was always to be taken as an indication of the presence of sodium in larger or smaller quantity.

But the greatest advance in spectrum analysis was made by Bunsen and Kirchhoff, who, by means of a four-prism spectroscope, obtained accurate observations of the positions of the bright lines in the spectra of a great number of substances, as well as of the dark lines in the solar spectrum, and called attention to the identity of several of the latter with several of the former. Since the publication of their researches, the spectroscope has come into general use among chemists, and speedily led to the discovery of four new metals, cæsium, rubidium, thallium, and indium.

224. Reversal of Bright Lines. Analysis of the Sun's Atmosphere.—It may seem surprising that, while incandescent solids and liquids are found to give continuous spectra containing rays of all refrangibilities, the solar spectrum is interrupted by dark lines indicating the absence or relative feebleness of certain elementary rays. It seems natural to suppose that the deficient rays have been removed by selective absorption, and this conjecture was thrown out long

since. But where and how is this absorption produced? These questions have now received an answer which appears completely satisfactory.

According to the theory of exchanges, which has been explained in Part II. in connection with the radiation of heat, every substance which emits certain kinds of rays to the exclusion of others, absorbs the same kind which it emits; and when its temperature is the same in the two cases compared, its emissive and absorbing power are precisely equal for any one elementary ray.

When an incandescent vapour, emitting only rays of certain definite refrangibilities, and therefore having a spectrum of bright lines, is interposed between the observer and a very bright source of light, giving a continuous spectrum, the vapour allows no rays of its own peculiar kinds to pass; so that the light which actually comes to the observer consists of transmitted rays in which these particular kinds are wanting, together with the rays emitted by the vapour itself, these latter being of precisely the same kind as those which it has refused to transmit. It depends on the relative brightness of the two sources whether these particular rays shall be on the whole in excess or defect as compared with the rest. If the two sources are at all comparable in brightness, these rays will be greatly in excess, inasmuch as they constitute the whole light of the one, and only a minute fraction of the light of the other; but the light of the electric lamp, or of the lime-light, is usually found sufficiently powerful to produce the contrary effect; so that if, for example, a spirit-lamp with salted wick is interposed between the slit of a spectroscope and the electric light, the bright yellow line due to the sodium appears black by contrast with the much brighter back-ground which belongs to the continuous spectrum of the charcoal points. By employing only some 10 or 15 cells, a light may be obtained, the yellow portion of which, as seen in a one-prism spectroscope, is sensibly equal in brightness to the yellow line of the sodium flame, so that this line can no longer be separately detected, and the appearance is the same whether the sodium flame be interposed or removed.

The dark lines in the solar spectrum would therefore be accounted for by supposing that the principal portion of the sun's light comes from an inner stratum which gives a continuous spectrum, and that a layer external to this contains vapours which absorb particular rays, and thus produce the dark lines. The stratum which gives the continuous spectrum might be solid, liquid, or even gaseous, for

the experiments of Frankland and Lockyer have shown that, as the pressure of a gas is increased, its bright lines broaden out into bands, and that the bands at length become so wide as to join each other and form a continuous spectrum.[1]

Hydrogen, sodium, calcium, barium, magnesium, zinc, iron, chromium, cobalt, nickel, copper, and manganese have all been proved to exist in the sun by the accurate identity of position of their bright lines with certain dark lines in the sun's spectrum.

The strong line D, which in a good instrument is seen to consist of two lines near together, is due to sodium, and the lines C and F are due to hydrogen. No less than 450 of the solar dark lines have been identified with bright lines of iron.

225. Telespectroscope. Solar Prominences.—For astronomical investigations, the spectroscope is usually fitted to a telescope, and takes the place of the eye-piece, the plane of the slit being placed in the principal focus of the object-glass, so that the image is thrown upon it, and the light which enters the slit is the light which forms one strip (so to speak) of the image, and which therefore comes from one strip of the object A telescope thus equipped is called a telespectroscope Extremely interesting results have been obtained by thus subjecting to examination a strip of the sun's edge, the strip being sometimes tangential to the sun's disc, and sometimes radial When the former arrangement is adopted, the appearance presented is that depicted in No. 2, Plate III, consisting of a few bright lines scattered through a back-ground of the ordinary solar spectrum The bright lines are due to an outer layer called the *chromosphere,* which is thus proved to be vaporous. The ordinary solar spectrum which accompanies it, is due to that part of the sun from which most of our light is derived. This part is called the *photosphere*, and if not solid or liquid, it must consist of vapour so highly compressed that its properties approximate to those of a liquid.

When the slit is placed radially, in such a position that only a small portion of its length receives light from the body of the sun, the spectra of the photosphere and chromosphere are seen in immediate contiguity, and the bright lines in the latter (notably those of hydrogen, No. 14, Plate III.) are observed to form continuations of some of the dark lines of the former.

[1] The gradual transition from a spectrum of bright lines to a continuous spectrum may be held to be an illustration of the continuous transition which can be effected from the condition of ordinary gas to that of ordinary liquid (see Part II.).

The chromosphere is so much less bright than the photosphere, that, previous to the application of spectroscopy to astronomy, its existence was never revealed except during total eclipses of the sun, when projecting portions of it were seen extending beyond the dark body of the moon. The spectrum of these projecting portions, which have been variously called "prominences," "red flames," and "rose-coloured protuberances," was first observed during the "Indian eclipse" of 1868, and was found to consist of bright lines, including those of hydrogen. From their excessive brightness, M. Janssen, who was one of the observers, expressed confidence that he should be able to see them in full sunshine; and the same idea had been already conceived and published by Mr. Lockyer. The expectation was shortly afterwards realized by both these observers, and the chromosphere has ever since been an object of frequent observation. The visibility of the chromosphere lines in full sunshine, depends upon the principle that, while a continuous spectrum is extended, and therefore made fainter, by increased dispersion, a bright line in a spectrum is not sensibly broadened, and therefore loses very little of its intrinsic brightness (§ 228). Very high dispersion is necessary for this purpose.

Still more recently, by opening the slit to about the average width of the prominence-region, as measured on the image of the sun which is thrown on the slit, it has been found possible to see the whole of an averaged-sized prominence at one view. This will be understood by remembering that a bright line as seen in a spectrum is a monochromatic image of the illuminated portion of the slit, or when a tele-spectroscope is used, as in the present case, it is a monochromatic image of one strip of the image formed by the object-glass, namely, that strip which coincides with the slit. If this strip then contains a prominence in which the elementary rays C and F (No. 2, Plate III.) are much stronger than in the rest of the strip, a red image of the prominence will be seen in the part of the spectrum corresponding to the line C, and a blue image in the place corresponding to the line F. This method of observation requires greater dispersion than is necessary for the mere detection of the chromosphere lines; the dispersion required for enabling a bright-line spectrum to predominate over a continuous spectrum being always nearly proportional to the width of the slit (§ 228).

Of the nebulæ, it is well known that some have been resolved by powerful telescopes into clusters of stars, while others have as yet

proved irresolvable. Huggins has found that the former class of nebulæ give spectra of the same general character as the sun and the fixed stars, but that some of the latter class give spectra of bright lines, indicating that their constitution is gaseous.

226. Displacement of Lines consequent on Celestial Motions.— According to the undulatory theory of light, which is now universally accepted, the fundamental difference between the different rays which compose the complete spectrum, is a difference of wave-frequency, and, as connected with this, a difference of wave-length in any given medium, the rays of greatest wave-frequency or shortest wave-length being the most refrangible.

Doppler first called attention to the change of refrangibility which must be expected to ensue from the mutual approach or recess of the observer and the source of light, the expectation being grounded on reasoning which we have explained in connection with acoustics (§ 33).

Doppler adduced this principle to explain the colours of the fixed stars, a purpose to which it is quite inadequate, but it has rendered very important service in connection with spectroscopic research. Displacement of a line towards the more refrangible end of the spectrum indicates approach, displacement in the opposite direction indicates recess, and the velocity of approach or recess admits of being calculated from the observed displacement.

When the slit of the spectroscope crosses a spot on the sun's disc, the dark lines lose their straightness in this part, and are bent, sometimes to one side, sometimes to the other. These appearances clearly indicate uprush and downrush of gases in the sun's atmosphere in the region occupied by the spot.

Huggins detected a displacement of the F line towards the red end, in the spectrum of Sirius, as compared with the spectrum of the sun or of hydrogen. The displacement is so small as only to admit of measurement by very powerful instrumental appliances; but, small as it is, calculation shows that it indicates a motion of recess at the rate of about 30 miles per second.[1]

[1] The observed displacement corresponded to recess at the rate of 41·4 miles per second; but 12·0 of this must be deducted for the motion of the earth in its orbit at the season of the year when the observation was made. The remainder, 29·4, was therefore the rate at which the distance between the sun and Sirius was increasing.

In a more recent paper Dr. Huggins gave the results of observations with more powerful instrumental appliances. The recess of Sirius was found to be only 20 miles per second Arcturus was found to be approaching at the rate of 50 miles per second. Community

227. Spectra of Artificial Lights.—The spectra of the artificial lights in ordinary use (including gas, oil-lamps, and candles) differ from the solar spectrum in the relative brightness of the different colours, as well as in the entire absence of dark lines. They are comparatively strong in red and green, but weak in blue; hence all colours which contain much blue in their composition appear to disadvantage by gas-light.

It is possible to find artificial lights whose spectra are of a completely different character. The salts of strontium, for example, give red light, composed of the ingredients represented in spectrum No. 10, Plate III., and those of sodium yellow light (No. 3, Plate III.). If a room is illuminated by a sodium flame (for example, by a spirit-lamp with salt sprinkled on the wick), all objects in the room will appear of a uniform colour (that of the flame itself), differing only in brightness, those which contain no yellow in their spectrum as seen by day-light being changed to black. The human countenance and hands assume a ghastly hue, and the lips are no longer red.

A similar phenomenon is observed when a coloured body is held in different parts of the solar spectrum in a dark room, so as to be illuminated by different kinds of monochromatic light. The object either appears of the same colour as the light which falls upon it, or else it refuses to reflect this light and appears black. Hence a screen for exhibiting the spectrum should be white.

228. Brightness and Purity.—The laws which determine the brightness of images generally, and which have been expounded at some length in the preceding chapter, may be applied to the spectroscope. We shall, in the first instance, neglect the loss of light by reflection and imperfect transmission.

Let Δ denote the *prismatic dispersion,* as measured by the angular separation of two specified monochromatic images when the naked eye is applied to the last prism, the observing telescope being removed. Then, putting m for the linear magnifying power of the telescope, $m\Delta$ is the angular separation observed when the eye is applied to the telescope. We shall call $m\Delta$ the *total dispersion.*

Let θ denote the angle which the breadth of the slit subtends at the centre of the collimating lens, and which is measured by $\frac{\text{breadth of slit}}{\text{focal length of lens}}$. Then θ is also the apparent breadth of any absolutely

of motion was established in certain sets of stars; and the belief previously held by astronomers, as to the direction in which the solar system is moving with respect to the stars as a whole, was fully confirmed

monochromatic image of the slit, formed by rays of minimum deviation, as seen by an eye applied either to the first prism, the last prism, or any one of the train of prisms. The change produced in a pencil of monochromatic rays by transmission through a prism at minimum deviation, is in fact simply a change of direction, without any change of mutual inclination, and thus neither brightness nor apparent size is at all affected. In ordinary cases, the bright lines of a spectrum may be regarded as monochromatic, and their apparent breadth, as seen without the telescope, is sensibly equal to θ. Strictly speaking, the effect of prismatic dispersion in actual cases, is to increase the apparent breadth by a small quantity, which, if all the prisms are alike, is proportional to the number of prisms, but the increase is usually too small to be sensible.

Let I denote the intrinsic brightness of the source as regards any one of its (approximately) monochromatic constituents, in other words, the brightness which the source would have if deprived of all its light except that which goes to form a particular bright line. Then, still neglecting the light stopped by the instrument, the brightness of this line as seen without the aid of the telescope will be I, and as seen in the telescope it will either be equal to or less than this, according to the magnifying power of the telescope and the effective aperture of the object-glass (§ 184). If the breadth of the slit be halved, the breadth of the bright line will be halved, and its brightness will be unchanged. These conclusions remain true so long as the bright line can be regarded as practically monochromatic

The brightness of any part of a *continuous* spectrum follows a very different law. It varies directly as the width of the slit, and inversely as the prismatic dispersion Its value without the observing telescope, or its maximum value with a telescope, is $\frac{\theta}{\Delta} i$, where i is a coefficient depending only on the source

The *purity* of any part of a continuous spectrum is properly measured by the ratio of the *distance between two specified monochromatic images* to the *breadth of either*, the distance in question being measured from the centre of one to the centre of the other. This ratio is unaffected by the employment of an observing telescope, and is $\frac{\Delta}{\theta}$.

The ratio of the brightness of a bright line to that of the adjacent portion of a continuous spectrum forming its back-ground, is $\frac{\Delta \mathrm{I}}{\theta i}$,

assuming the line to be so nearly monochromatic that the increase of its breadth produced by the dispersion of the prisms is an insignificant fraction of its whole breadth. As we widen the slit, and so increase θ, we must increase Δ in the same ratio, if we wish to preserve the same ratio of brightness. As $\frac{\Delta}{\theta}$ is increased indefinitely, the predominance of the bright lines does not increase indefinitely, but tends to a definite limit, namely, to the predominance which they would have in a perfectly pure spectrum of the given source.

The loss of light by reflection and imperfect transmission, increases with the number of surfaces of glass which are to be traversed; so that, with a long train of prisms and an observing telescope, the actual brightness will always be much less than the theoretical brightness as above computed.

The actual purity is always less than the theoretical purity, being greatly dependent on freedom from optical imperfections; and these can be much more completely avoided in lenses than in prisms. It is said that a single good prism, with a first-class collimator and telescope (as originally employed by Swan), gives a spectrum much more free from blurring than the modern multiprism spectroscopes, when the total dispersion $m\Delta$ is the same in both the cases compared.

229. Chromatic Aberration.—The unequal refrangibility of the different elementary rays is a source of grave inconvenience in connection with lenses. The focal length of a lens depends upon its index of refraction, which of course increases with refrangibility, the focal length being shortest for the most refrangible rays. Thus a lens of uniform material will not form a single white image of a white object, but a series of images, of all the colours of the spectrum, arranged at different distances, the violet images being nearest, and the red most remote. If we place a screen anywhere in the series of images, it can only be in the right position for one colour. Every other colour will give a blurred image, and the superposition of them all produces the image actually formed on the screen. If the object be a uniform white spot on a black ground, its image on the screen will consist of white in its central parts, gradually merging into a coloured fringe at its edge. Sharpness of outline is thus rendered impossible, and nothing better can be done than to place the screen at the focal distance corresponding to the brightest part of the spectrum. Similar indistinctness will attach to images observed in

mid-air, whether directly or by means of another lens. This source of confusion is called *chromatic aberration.*

230. Possibility of Achromatism.—In order to ascertain whether it was possible to remedy this evil by combining lenses of two different materials, Newton made some trials with a compound prism composed of glass and water (the latter containing a little sugar of lead), and he found that it was not possible, by any arrangement of these two substances, to produce deviation of the transmitted light without separation into its component colours. Unfortunately he did not extend his trials to other substances, but concluded at once that an *achromatic* prism (and hence also an achromatic lens) was an impossibility, and this conclusion was for a long time accepted as indisputable Mr. Hall, a gentleman of Worcestershire, was the first to show that it was erroneous, and is said to have constructed some achromatic telescopes, but the important fact thus discovered did not become generally known till it was rediscovered by Dolland, an eminent London optician, in whose hands the manufacture of achromatic instruments attained great perfection.

231. Conditions of Achromatism—The conditions necessary for achromatism are easily explained. The angular separation between the brightest red and the brightest violet ray transmitted through a prism is called the *dispersion* of the prism, and is evidently the difference of the deviations of these rays. These deviations, for the position of minimum deviation of a prism of small refracting angle A, are $(\mu'-1)$ A and $(\mu''-1)$ A, μ' and μ'' denoting the indices of refraction for the two rays considered (§ 151, equation (1)) and their difference is $(\mu''-\mu')$ A. This difference is always small in comparison with either of the deviations whose difference it is, and its ratio to either of them, or more accurately its ratio to the value of $(\mu-1)$ A for the brightest part of the spectrum, is called the *dispersive power* of the substance. As the common factor A may be omitted, the formula for the dispersive power is evidently $\frac{\mu''-\mu'}{\mu-1}$.

If this ratio were the same for all substances, as Newton supposed, achromatism would be impossible; but in fact its value varies greatly, and is greater for flint than for crown glass. If two prisms of these substances, of small refracting angles, be combined into one, with their edges turned opposite ways, they will achromatize one another if $(\mu''-\mu')$ A, or the product of deviation by dispersive power, is the same for both. As the deviations can be made to have any ratio we

please by altering the angles of the prisms, the condition is evidently possible.

The deviation which a simple ray undergoes in traversing a lens, at a distance x from the axis, is $\frac{x}{f}$, f denoting the focal length of the lens (§ 151), and the separation of the red and violet constituents of a compound ray is the product of this deviation by the dispersive power of the material. If a convex and concave lens are combined, fitting closely together, the deviations which they produce in a ray traversing both, are in opposite directions, and so also are the dispersions. If we may regard x as having the same value for both (a supposition which amounts to neglecting the thicknesses of the lenses in comparison with their focal lengths) the condition of no resultant dispersion is that

$$\text{dispersive power} \times \frac{1}{f}$$

has the same value for both lenses. *Their focal lengths* must therefore be *directly as the dispersive powers of their materials.* These latter are about ·033 for crown and ·052 for flint glass. A converging achromatic lens usually consists of a double convex lens of crown fitted to a diverging meniscus of flint. In every achromatic combination of two pieces, the direction of resultant *deviation* is that due to the piece of smaller dispersive power.

The definition above given of dispersive power is rather loose. To make it accurate, we must specify, by reference to the "fixed lines," the precise positions of the two rays whose separation we consider.

Since the distances between the fixed lines have different proportions for crown and flint glass, achromatism of the whole spectrum is impossible. With two pieces it is possible to unite any two selected rays, with three pieces any three selected rays, and so on. It is considered a sign of good achromatism when no colours can be brought into view by bad focussing except purple and green.

232. Achromatic Eye-pieces.—The eye-pieces of microscopes and astronomical telescopes, usually consist of two lenses of the same kind of glass, so arranged as to counteract, to some extent, the spherical and chromatic aberrations of the object-glass. The *positive* eye-piece, invented by Ramsden, is suited for observation with cross-wires or micrometers; the *negative* eye-piece, invented by Huygens, is not so well adapted for purposes of measurement, but is preferred when distinct vision is the sole requisite. These eye-pieces are commonly

called achromatic, but their achromatism is in a manner spurious. It consists not in bringing the red and violet images into true coincidence, but merely in causing one to cover the other as seen from the position occupied by the observer's eye.

In the best opera-glasses (§ 179), the eye-piece, as well as the object-glass, is composed of lenses of flint and crown so combined as to be achromatic in the more proper sense of the word.

233. Rainbow.—The unequal refrangibility of the different elementary rays furnishes a complete explanation of the ordinary phenomena of rainbows. The explanation was first given by Newton, who confirmed it by actual measurement.

It is well known that rainbows are seen when the sun is shining on drops of water. Sometimes one bow is seen, sometimes two, each of them presenting colours resembling those of the solar spectrum. When there is only one bow, the red arch is above and the violet below. When there is a second bow, it is at some distance outside of this, has the colours in reverse order, and is usually less bright.

Rainbows are often observed in the spray of cascades and fountains, when the sun is shining.

In every case, a line joining the observer to the sun is the axis of the bow or bows; that is to say, all parts of the length of the bow are at the same angular distance from the sun.

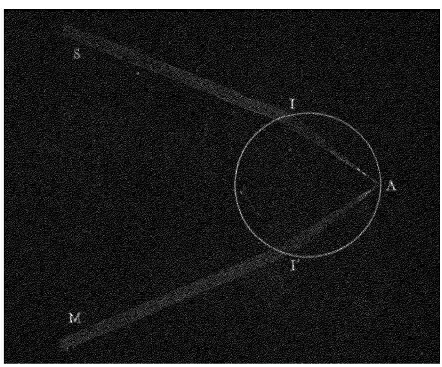


Fig 186 —Production of Primary Bow

The formation of the primary bow is illustrated by Fig. 186. A ray of solar light, falling on a spherical drop of water, in the direction S I, is refracted at I, then reflected internally from the back of the drop, and again refracted into the air in the direction I′M. If we take different points of incidence, we shall obtain different directions of emergence, so that the whole light which emerges from the drop after undergoing, as in the figure, two refractions and one reflection, forms a widely-divergent pencil. Some portions of this

pencil, however, contain very little light. This is especially the case with those rays which, having been incident nearly normally, are returned almost directly back, and also with those which were almost tangential at incidence. The greatest condensation, as regards any particular species of elementary ray, occurs at that part of the emergent pencil which has undergone *minimum deviation*. It is by means of rays which have undergone this minimum deviation, that the observer sees the corresponding colour in the bow; and the deviation which they have undergone is evidently equal to the angular distance of this part of the bow from the sun.

The minimum deviation will be greatest for those rays which are most refrangible. If the figure, for example, be supposed to represent the circumstances of minimum deviation for violet, we shall obtain smaller deviation in the case of red, even by giving the angle I A I′ the same value which it has in the case of minimum deviation for violet, and still more when we give it the value which corresponds to the minimum deviation of red. The most refrangible colours are accordingly seen furthest from the sun. The effect of the rays which undergo other than minimum deviation, is to produce a border of white light on the side remote from the sun; that is to say, on the inner edge of the bow.[1]

The condensation which accompanies minimum deviation is merely a particular case of the general mathematical law that magnitudes remain nearly constant in the neighbourhood of a maximum or minimum value. The rays which compose a small parallel pencil S I

[1] When the drops are very uniform in size, a series of faint *supernumerary bows*, alternately purple and green, is sometimes seen beneath the primary bow. These bows are produced by the mutual interference of rays which have undergone other than minimum deviation, and the interference arises in the following way. Any two parallel directions of emergence, for rays of a given refrangibility, correspond in general to two different points of incidence on any given drop, one of the two incident rays being more nearly normal, and the other more nearly tangential to the drop than the ray of minimum deviation. These two rays have pursued dissimilar paths in the drop, and are in different phases when they reach the observer's eye. The difference of phase may amount to one, two, three, or more exact wave-lengths, and thus one, two, three, or more supernumerary bows may be formed. The distances between the supernumerary bows will be greater as the drops of water are smaller. This explanation is due to Dr. Thomas Young.

A more complete theory, in which diffraction is taken into account, is given by Airy in the *Cambridge Transactions* for 1838; and the volume for the following year contains an experimental verification by Miller. It appears from this theory that the maximum of intensity is less sharply marked than the ordinary theory would indicate, and does not correspond to the geometrical minimum of deviation, but to a deviation sensibly greater. Also that the region of sensible illumination extends beyond this geometrical minimum and shades off gradually.

incident at and around the precise point which corresponds to minimum deviation, will thus have deviations which may be regarded as equal, and will accordingly remain sensibly parallel at emergence. A parallel pencil incident on any other part of the drop, will be divergent at emergence.

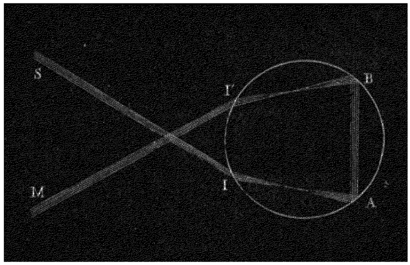


Fig. 187 —Production of Secondary Bow

The indices of refraction for red and violet rays from air into water are respectively $\frac{108}{81}$ and $\frac{109}{81}$, and calculation shows that the distances from the centre of the sun to the parts of the bow in which these colours are strongest should be the supplements of 42° 2′ and 40° 17′ respectively. These results agree with observation. The angles 42° 2′ and 40° 17′ are the distances from the *antisolar point*, which is always the centre of the bow.

The rays which form the secondary bow have undergone two internal reflections, as represented in Fig. 187, and here again a special concentration occurs in the direction of minimum deviation. This deviation is greater than 180° and is greatest for the most refrangible rays. The distance of the arc thus formed from the sun's centre, is 360° *minus* the deviation, and is accordingly least for the most refrangible rays. Thus the violet arc is nearest the sun, and the red furthest from it, in the secondary bow.

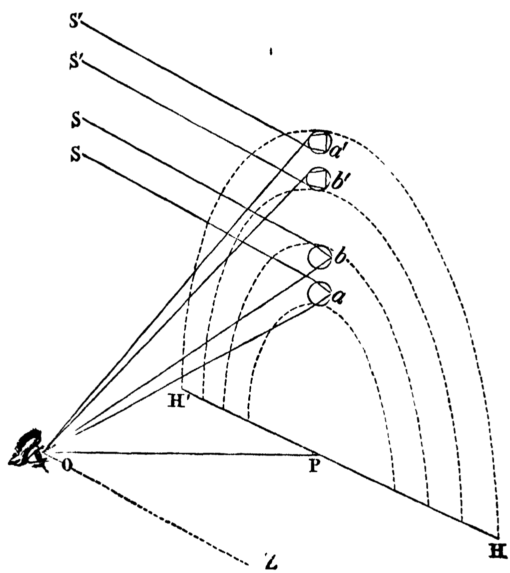


Fig. 188.—Relative Positions.

Some idea of the relative situations of the eye, the sun, and the drops of water, may be obtained from an inspection of Fig. 188.

CHAPTER XIII.

COLOUR.

234. Colour as a Property of Opaque Bodies.—A body which reflects (by irregular reflection) all the rays of the spectrum in equal proportion, will appear of the same colour as the light which falls upon it; that is to say, in ordinary cases, white or gray. But the majority of bodies reflect some rays in larger proportion than others, and are therefore coloured, their colour being that which arises from the mixture of the rays which they reflect. A body reflecting no light would be perfectly black. Practically, white, gray, and black differ only in brightness. A piece of white paper in shadow appears gray, and in stronger shadow black.

235. Colour of Transparent Bodies.—A transparent body, seen by transmitted light, is coloured, if it is more transparent to some rays than to others, its colour being that which results from mixing the transmitted rays. No new ingredient is added by transmission, but certain ingredients are more or less completely stopped out.

Some transparent substances appear of very different colours according to their thickness. A solution of chloride of chromium, for example, appears green when a thin layer of it is examined, while a greater thickness of it presents the appearance of reddish brown. In such cases, different kinds of rays successively disappear by selective absorption, and the transmitted light, being always the sum of the rays which remain unabsorbed, is accordingly of different composition according to the thickness.

When two pieces of coloured glass are placed one behind the other, the light which passes through both has undergone a double process of selective absorption, and therefore consists mainly of those rays which are abundantly transmitted by both glasses; or to speak broadly, the colour which we see in looking through the combination

is not the sum of the colours of the two glasses, but their common part. Accordingly, if we combine a piece of ordinary red glass, transmitting light which consists almost entirely of red rays, with a piece of ordinary green glass, which transmits hardly any red, the combination will be almost black. The light transmitted through two glasses of different colour and of the same depth of tint, is always less than would be transmitted by a double thickness of either; and the colour of the transmitted light is in most cases a colour which occupies in the spectrum an intermediate place between the two given colours. Thus, if the two glasses are yellow and blue, the transmitted light will, in most cases, be green, since most natural yellows and blues when analysed by a prism show a large quantity of green in their composition. Similar effects are obtained by mixing coloured liquids.

236. Colours of Mixed Powders.—"In a coloured powder, each particle is to be regarded as a small transparent body which colours light by selective absorption. It is true that powdered pigments when taken in bulk are extremely opaque. Nevertheless, whenever we have the opportunity of seeing these substances in compact and homogeneous pieces before they have been reduced to powder, we find them transparent, at least when in thin slices Cinnabar chromate of lead, verdigris, and cobalt glass are examples in point

"When light falls on a powder thus composed of transparent particles, a small part is reflected at the upper surface, the rest penetrates, and undergoes partial reflection at some of the surfaces of separation between the particles. A single plate of uncoloured glass reflects $\frac{1}{25}$ of normally incident light, two plates $\frac{1}{13}$, and a large number nearly the whole. In the powder of such glass, we must accordingly conclude that only about $\frac{1}{25}$ of normally incident light is reflected from the first surface, and that all the rest of the light which gives the powder its whiteness comes from deeper layers. It must be the same with the light reflected from blue glass; and in coloured powders generally only a very small part of the light which they reflect comes from the first surface, it nearly all comes from beneath. The light reflected from the first surface is white, except when the reflection is metallic. That which comes from below is coloured, and so much the more deeply the further it has penetrated. This is the reason why coarse powder of a given material is more deeply coloured than fine, for the quantity of light returned at each successive reflection depends only on the number of reflections and not on the

thickness of the particles. If these are large, the light must penetrate so much the deeper in order to undergo a given number of reflections; and will therefore be the more deeply coloured.

"The reflection at the surfaces of the particles is weakened if we interpose between them, in the place of air, a fluid whose index of refraction more nearly approaches their own. Thus powders and pigments are usually rendered darker by wetting them with water, and still more with the more highly refracting liquid, oil.

"If the colours of powders depended only on light reflected from their first surfaces, the light reflected from a mixed powder would be the sum of the lights reflected from the surfaces of both. But most of the light, in fact, comes from deeper layers, and having had to traverse particles of both powders, must consist of those rays which are able to traverse both. The resultant colour therefore, as in the case of superposed glass plates, depends not on addition but rather on subtraction. Hence it is that a mixture of two pigments is usually much more sombre than the pigments themselves, if these are very unlike in the average refrangibility of the light which they reflect. Vermilion and ultramarine, for example, give a black-gray (showing scarcely a trace of purple, which would be the colour obtained by a true mixture of lights), each of these pigments being in fact nearly opaque to the light of the other." [1]

237. **Mixtures of Colours.**—By the colour resulting from the mixture of two lights, we mean the colour which is seen when they both fall on the same part of the retina. Propositions regarding mixtures of colours are merely subjective. The only objective differences of colour are differences of refrangibility, or if traced to their source, differences of wave-frequency. All the colours in a pure spectrum are objectively simple, each having its own definite period of vibration by which it is distinguished from all others. But whereas, in acoustics, the quality of a sound as it affects the ear varies with every change in its composition, in colour, on the other hand, very different compositions may produce precisely the same visual impression. Every colour that we see in nature can be exactly imitated by an infinite variety of different combinations of elementary rays.

To take, for example, the case of white. Ordinary white light consists of all the colours of the spectrum combined; but any one of the elementary colours, from the extreme red to a certain point in yellowish green, can be combined with another elementary colour

[1] Translated from Helmholtz's *Physiological Optics*, § 20.

on the other side of green in such proportion as to yield a perfect imitation of ordinary white. The prism would instantly reveal the differences, but to the naked eye all these whites are completely undistinguishable one from another.

238.. **Methods of Mixing Colours.**—The following are some of the best methods of mixing colours (that is coloured lights):—

1. By combining reflected and transmitted light; for example, by looking at one colour through a piece of glass, while another colour is seen by reflection from the near side of the glass. The lower sash of a window, when opened far enough to allow an arm to be put through, answers well for this purpose. The brighter of the two coloured objects employed should be held inside the window, and seen by reflection; the second object should then be held outside in such a position as to be seen in coincidence with the image of the first. As the quantity of reflected light increases with the angle of incidence, the two colours may be mixed in various proportions by shifting the position of the eye. This method is not however adapted to quantitative comparison, and can scarcely be employed for combining more than two colours.

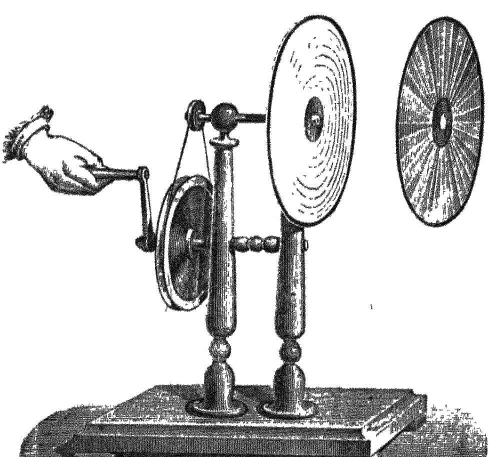


Fig. 189.—Rotating Disc

2. By employing a rotating disc (Fig 189) composed of differently coloured sectors. If the disc be made to revolve rapidly, the sectors will not be separately visible, but their colours will appear blended into one on account of the persistence of visual impressions. The proportions can be varied by varying the sizes of the sectors. Coloured discs of paper, each having a radial slit, are very convenient for this purpose, as any moderate number of such discs can be combined, and the sizes of the sectors exhibited can be varied at pleasure.

The mixed colour obtained by a rotating disc is to be regarded as

a *mean* of the colours of the several sectors—a mean in which each of these colours is assigned a weight proportional to the size of its sector. Thus, if the 360 degrees which compose the entire disc consist of 100° of red paper, 100° of green, and 160° of blue, the intensity of the light received from the red when the disc is rotating will be only $\frac{10}{36}$ of that which would be received from the red sector when seen at rest; and the total effect on the retina is represented by $\frac{10}{36}$ of the intensity of the red, *plus* $\frac{10}{36}$ of the intensity of the green, *plus* $\frac{16}{36}$ of the intensity of the blue; or if we denote the colours of the sectors by their initial letters, the effect may be symbolized by the formula $\frac{10\,R+10\,G+16\,B}{36}$. Denoting the resultant colour by C, we have the symbolic equation

$$10\,R+10\,G+16\,B=36\,C;$$

and the resultant colour may be called the mean of 10 parts of red, 10 of green, and 16 of blue. Colour-equations, such as the above, are frequently employed, and may be combined by the same rules as ordinary equations.

3. By causing two or more spectra to overlap. We thus obtain mixtures which are the *sums* of the overlapping colours.

If, in the experiment of § 213, we employ, instead of a single straight slit, a pair of slits meeting at an angle, so as to form either an X or a V, we shall obtain mixtures of all the simple colours two and two, since the coloured images of one of the slits will cross those of the other. The display of colours thus obtained upon a screen is exquisitely beautiful, and if the eye is placed at any point of the image (for example, by looking through a hole in the screen), the prism will be seen filled with the colour which falls on this point.

239. Experiments of Helmholtz and Maxwell.—Helmholtz, in an excellent series of observations of mixtures of simple colours, employed a spectroscope with a V-shaped slit, the two strokes of the V being at right angles to one another; and by rotating the V he was able to diminish the breadth and increase the intensity of one of the two spectra, while producing an inverse change in the other. To isolate any part of the compound image formed by the two overlapping spectra, he drew his eye back from the eye-piece, so as to limit his view to a small portion of the field.

But the most effective apparatus for observing mixtures of simple colours is one devised by Professor Clerk Maxwell, by means of which any two or three colours of the spectrum can be combined in

any required proportions. In principle, this method is nearly equivalent to looking through the hole in the screen in the experiment above described.

Let P (Fig. 190) be a prism, in the position of minimum deviation L a lens; E and R conjugate foci for rays of a particular refrangibility, say red; E and V conjugate foci for rays of another given refrangibility, say violet. If a slit is opened at R, an eye at E will receive only red rays, and will see the lens filled with red light. If this slit be closed, and a slit opened at V, the eye, still placed at E, will see the lens filled with violet light. If both slits be opened, it will see the lens filled with a uniform mixture of the two lights; and if a third slit be opened, between R and V, the lens will be seen filled with a mixture of three lights.

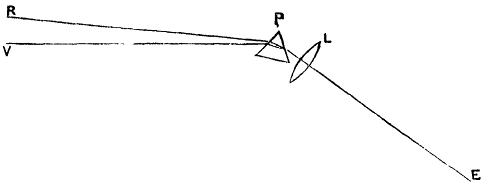


Fig 190 —Principle of Maxwell's Colour-box

Again, from the properties of conjugate foci, if a slit is opened at E, its spectral image will be formed at R V, the red part of it being at R, and the violet part at V.

The apparatus was inclosed in a box painted black within. There was a slit fixed in position at E, and a frame with three movable slits at R V. When it was desired to combine colours from three given parts of the spectrum, specified by reference to Fraunhofer's lines, the slit E was first turned towards the light, giving a real spectrum in the plane R V, in which Fraunhofer's lines were visible, and the three movable slits were set at the three specified parts of the spectrum. The box was then turned end for end, so that light was admitted (reflected from a large white screen placed in sunshine) at the movable slits, and the observer, looking in at the slit E, saw the resultant colour.

240. **Results of Experiment.**—The following are some of the principal results of experiments on the mixture of coloured lights:—

1. Lights which appear precisely alike to the naked eye yield identical results in mixtures; or employing the term *similar* to express apparent identity as judged by the naked eye, *the sums of similar lights are themselves similar.* It is by reason of this physical fact, that colour-equations yield true results when combined according to the ordinary rules of elimination.

In the strict application of this rule, the same observer must be the judge of similarity in the different cases considered. For

2. Colours may be similar as seen by one observer, and dissimilar as seen by another; and in like manner, colours may be similar as seen through one coloured glass, and dissimilar as seen through another. The reason, in both cases, is that selective absorption depends upon real composition, which may be very different for two merely similar lights. Most eyes are found to exhibit selective absorption of a certain kind of elementary blue, which is accordingly weakened before reaching the retina.

3. Between any four colours, given in intensity as well as in kind, one colour-equation subsists; expressing the fact that, when we have the power of varying their intensities at pleasure, there is one definite way of making them yield a *match*, that is to say, a pair of similar colours. Any colour can therefore be completely specified by three numbers, expressing its relation to three arbitrarily selected colours. This is analogous to the theorem in statics that a force acting at a given point can be specified by three numbers denoting its components in three arbitrarily selected directions.

4. Between any five colours, given in intensity as well as in kind, a match can be made in one definite way by taking means;[1] for example, by mounting the colours on two rotating discs. If we had the power of illuminating one disc more strongly than the other in any required ratio, four colours would be theoretically sufficient; and we can, in fact, do what is nearly equivalent to this, by employing black as one of our five colours. Taking means of colours is analogous to finding centres of gravity. In following out the analogy, a colour given in kind merely must be represented by a material point given in position merely, and the intensity of the colour must be represented by the mass of the material point. The means of two given colours will be represented by points in the line joining two given points. The means of three given colours will be represented by points lying within the triangle formed by joining three given points, and the means of four given colours will be represented by points within a tetrahedron whose four corners are given. When we have five colours given, we have five points given, and of these generally no four will lie in one plane. Call them A, B, C, D, E.

[1] Propositions 4 and 5 are not really independent, but represent different aspects of one physical (or rather physiological) law.

Then if E lies within the tetrahedron A B C D, we can make the centre of gravity of A, B, C, and D coincide with E, and the colour E can be matched by a mean of the other four colours.

If E lies outside the tetrahedron, it must be situated at a point from which either one, two, or three faces are visible (the tetrahedron being regarded as opaque).

If only one face is visible, let it be B C D; then the point where the straight line E A cuts B C D is the match; for it is a mean of E and A, and is also a mean of B, C, and D.

If two faces are visible, let them be A C D and B C D; then the intersection of the edge C D with the plane E A B is the match.

If three faces are visible, let them be the three which meet at A; then A is the match, for it lies within the tetrahedron E B C D.

With six given colours, combined five at a time, six different matches can be made, and six colour-equations will thus be obtained, the consistency of which among themselves will be a test of the accuracy both of theory and observation, as only three of the six can be really independent. Experiments which have been conducted on this plan have given very consistent results.

241. **Cone of Colour.**—All the results of mixing colours can be represented geometrically by means of a cone or pyramid within which all possible colours will have their definite places. The vertex will represent total blackness, or the complete absence of light; and colours situated on the same line passing through the vertex will differ only in intensity of light. Any cross-section of the cone will contain all colours, except so far as intensity is concerned, and the colours residing on its perimeter will be the colours of the spectrum ranged in order, with purple to fill up the interval between violet and red. It appears from Maxwell's experiments, that the true form of the cross-section is approximately triangular;[1] with red, green, and violet at the three corners. When all the colours have been assigned their proper places in the cone, a straight line joining any two of them passes through colours which are means of these two; and if two lines are drawn from the vertex to any two colours, the parallelogram constructed on these two lines will have at its further corner the colour which is the sum of these two colours. A certain axial line of the cone will contain

[1] The shape of the triangle is a mere matter of convenience, not involving any question of fact.

white or gray at all points of its length, and may be called the *line of white.*

It is convenient to distinguish three qualities of colour which may be called *hue, depth,* and *brightness. Brightness* or *intensity* of light is represented by distance from the vertex of the cone. *Depth* depends upon angular distance from the line of white, and is the same for all points on the same line through the vertex. *Paleness* or *lightness* is the opposite of depth, and is measured by angular nearness to the line of white. *Hue* or *tint* is that which is often *par excellence* termed colour. If we suppose a plane, containing the line of white, to revolve about this line as axis, it will pass successively through different tints; and in any one position it contains only two tints, which are separated from each other by the line of white, and are complementary.

Red is complementary to	Bluish green.
Orange " "	Sky blue.
Yellow " "	Violet blue.
Greenish yellow "	Violet.
Green " "	Pink.

Any two colours of complementary tint give white, when mixed in proper proportions; and any three colours can be mixed in such proportions as to yield white, if the triangle formed by joining them is pierced by the line of white.

Every colour in nature, except purple, is similar to a colour of the spectrum either pure or diluted with gray; and all purples are similar to mixtures of red and blue with or without dilution. Brown can be imitated by diluting orange with dark gray. The orange and yellow of the spectrum can themselves be imitated by adding together red and green.

242. Three Primary Colour-sensations.—All authorities are now agreed in accepting the doctrine, first propounded by Dr. Thomas Young, that there are three elements of colour-sensation; or, in other words, three distinct physiological actions, which, by their various combinations, produce our various sensations of colour. Each is excitable by light of various wave-lengths lying within a wide range, but has a maximum of excitability for a particular wave-length, and is affected only to a slight degree by light of wave-length very different from this. The cone of colour is theoretically a triangular pyramid, having for its three edges the colours which correspond to these three wave-lengths; but it is probable that we cannot obtain

one of the three elementary colour-sensations quite free from admixture of the other two, and the edges of the pyramid are thus practically rounded off. One of these sensations is excited in its greatest purity by the green near Fraunhofer's line *b*, another by the extreme red, and the third by the extreme violet.

Helmholtz ascribes these three actions to three distinct sets of nerves, having their terminations in different parts of the thickness of the retina—a supposition which aids in accounting for the approximate achromatism of the eye, for the three sets of nerve-terminations may thus be at the proper distances for receiving distinct images of red, green, and violet respectively, the focal length of a lens being shorter for violet than for red.

Light of great intensity, whatever its composition, seems to produce a considerable excitement of all three elements of colour-sensation. If a spectroscope, for example, be directed first to the clouds and then to the sun, all parts of the spectrum appear much paler in the latter case than in the former.

The popular idea that red, yellow, and blue are the three primaries, is quite wrong as regards mixtures of lights or combinations of colour-sensations. The idea has arisen from facts observed in connection with the mixture of pigments and the transmission of light through coloured glasses. We have already pointed out the true interpretation of observations of this nature, and have only now to add that in attempting to construct a theory of the colours obtained by mixtures of pigments, the law of substitution of *similars* cannot be employed. Two pigments of *similar* colour will not in general give the same result in mixtures.

243. **After-images.**—If we look steadily at a bright stained-glass window, and then turn our eyes to a white wall, we see an image of the window with the colours changed into their complementaries. The explanation is that the nerves which have been strongly exercised in the perception of the bright colours have had their sensibility diminished, so that the balance of action which is necessary to the sensation of white no longer exists, but those elements of sensation which have not been weakened preponderate. The subjective appearances arising from this cause are called *negative after-images.* Many well-known effects of contrast are similarly explained. White paper, when seen upon a background of any one colour, often appears tinged with the complementary colour, and stray beams of sunlight entering a room, shaded with yellow holland

blinds, produce blue streaks when they fall upon a white tablecloth.

In some cases, especially when the object looked at is painfully bright, there is a *positive* after-image; that is, one of the same colour as the object; and this is frequently followed by a negative image. A positive after-image may be regarded as an extreme instance of the persistence of impressions.

244. Colour-blindness.—What is called colour-blindness has been found, in most cases which have been carefully investigated, to consist in the absence of the elementary sensation corresponding to red. To persons thus affected the solar spectrum appears to consist of two decidedly distinct colours, with white or gray at their place of junction, which is a little way on the less refrangible side of the line F. One of these two colours is doubtless nearly identical with the normal sensation of blue or violet. It attains its maximum about midway between F and G, and extends beyond G as far as the normally visible spectrum. The other colour extends a considerable distance into what to normal eyes is the red portion of the spectrum, attaining its maximum about midway between D and E, and becoming deeper and more faint till it vanishes at about the place where to normal eyes crimson begins. The scarlet of the spectrum is thus visible to the colour-blind, not as scarlet but as a deep dark colour, perhaps a kind of dark green, orange and yellow as brighter shades of the same colour, while bluish-green appears nearly white.

It is obvious from this account that what is called "colour-blindness" should rather be called *dichroic vision*, normal vision being distinctively designated as *trichroic*. To the dichroic eye any colour can be matched by a mixture of yellow and blue, and a match can be made between any three (instead of four) given colours. Objects which have the same colour to the trichroic eye have also the same colour to the dichroic eye.

245. Colour and Musical Pitch.—As it is completely established that the difference between the colours of the spectrum is a difference of vibration-frequency, there is an obvious analogy between colour and musical pitch; but in almost all details the relations between colours are strikingly different from the relations between sounds.

The compass of visible colour, including the lavender rays which lie beyond the violet, and are perhaps visible not in themselves,

but by the fluorescence which they produce on the retina, is, according to Helmholtz, about an octave and a fourth; but if we exclude the lavender, it is almost exactly an octave. Attempts have been made to compare the successive colours of the spectrum with the notes of the gamut; but much forcing is necessary to bring out any trace of identity, and the gradual transitions which characterize the spectrum, and constitute a feature of its beauty, are in marked contrast to the transitions *per saltum* which are required in music.

CHAPTER XIV.

WAVE THEORY OF LIGHT.

246. Principle of Huygens.[1]—The propagation of waves, whether of sound or light, is a propagation of energy. Each small portion of the medium experiences successive changes of state, involving changes in the forces which it exerts upon neighbouring portions. These changes of force produce changes of state in these neighbouring portions, or in such of them as lie on the forward side of the wave, and thus a disturbance existing at any one part is propagated onwards.

Let us denote by the name *wave-front* a continuous surface drawn through particles which have the same phase; then each wave-front advances with the velocity of light, and each of its points may be regarded as a secondary centre from which disturbances are continually propagated. This mode of regarding the propagation of light is due to Huygens, who derived from it the following principle, which lies at the root of all practical applications of the undulatory theory: *The disturbance at any point of a wave-front is the resultant* (given by the parallelogram of motions) *of the separate disturbances which the different portions of the same wave-front in any one of its earlier positions, would have occasioned if acting singly.* This principle involves the physical fact that rays of light are not affected by crossing one another; and its truth, which has been experimentally tested by a variety of consequences, must be taken as an indication that the amplitudes of luminiferous vibrations are infinitesimal in comparison with the wave-lengths. A similar law applies to the resultant of small disturbances generally, and is called by writers on dynamics the law of "superposition of small motions." It is analogous to the arithmetical principle that, when a and b are very small fractions, the product of $1 + a$ and $1 + b$ may be identified with

[1] **For the spelling of this name see remarks by Lalande,** ***Mémoires de l'Académie,*** **1773.**

$1+a+b$; the term $a\,b$, which represents the mutual influence of two small changes, being negligible in comparison with the sum $a+b$ of the small changes themselves.

247. Explanation of Rectilinear Propagation.—In a medium in which light travels with the same velocity in all parts and in all directions, the waves propagated from any point will be concentric spheres, having this point for centre; and the lines of propagation, in other words the rays of light, will be the radii of these spheres. It can in fact be shown that the only part of one of these waves which needs to be considered, in computing the resultant disturbance of an external point, is the part which lies directly between this external point and the centre of the sphere. The remainder of the wave-front can be divided into small parts, each of which, by the mutual interference of its own subdivisions, gives a resultant effect of zero at the given point. We express these properties by saying that *in a homogeneous and isotropic medium the wave-surface is a sphere, and the rays are normal to the wave-fronts.* This class of media includes gases, liquids, crystals of the cubic system, and well-annealed glass.

If a medium be homogeneous but not isotropic, disturbances emanating from a point in it will be propagated in waves which will retain their form unchanged as they expand in receding from their source, but this form will not generally be spherical. The rays of light in such a medium will be straight, proceeding directly from the centre of disturbance, and any one ray will cut all the wave-fronts at the same angle; but this angle will generally be different for different rays. In this case, as in the last, the disturbance produced at any point may be computed by merely taking into account that small portion of a wave-front which lies directly between the given point and the source,—in other words, which lies on or very near to the ray which traverses the given point.

A disturbance in such a medium usually gives rise to two sets of waves, having two distinct forms, and these remarks apply to each set separately.

The tendency of the different parts of a wave-front to propagate disturbances in other directions besides the single one to which such propagation is usually confined, is manifested in certain phenomena which are included under the general name of *diffraction.*

The only wave-fronts with which it is necessary to concern ourselves are those which belong to waves emanating from a single

point,—that is to say, either from a surface really very small, or from a surface which, by reason of its distance, subtends a very small solid angle at the parts of space considered.

248. Application to Refraction.—When waves are propagated from one medium into another, the principle of Huygens leads to the following construction:—

Let A E (Fig 191) represent a portion of the surface of separation between two media, and A B a portion of a wave-front in the first medium; both portions being small enough to be regarded as plane.

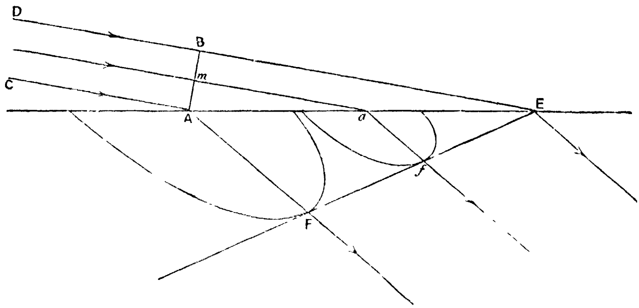


Fig. 191.—Huygens' Construction for Wave-front.

Then straight lines C A, D B E, normal to the wave-front, represent rays incident at A and E. From A as centre, describe a wave-surface, of such dimensions that light emanating from A would reach this surface in the same time in which light in air travels the distance B E, and draw a tangent plane (perpendicular to the plane of incidence) through E to this surface. Let F be the point of contact (which is not necessarily in the plane of incidence). Then the tangent plane E F is a wave-front in the second medium, and A F is a ray in the second medium; for it can be shown that disturbances propagated from all points in the wave-front A B will just have reached E F when the disturbance propagated from B has reached E. For example, a ray proceeding from m, the middle point of the line A B, will exhaust half the time in travelling to the middle point a of A E, and the remaining half in travelling through af, equal and parallel to half of A F.

When the wave-surfaces in both media are spherical, the planes of incidence and refraction A B E, A F E coincide, the angle B A E (Fig. 192) between the first wave-front and the surface of separation is the same as the angle between the normals to these surfaces, that

is to say, is the angle of incidence; and the angle A E F between the surface of separation and the second wave-front is the angle of refraction. The sine of the former is $\frac{BE}{EA}$, and the sine of the latter is $\frac{AF}{EA}$. The ratio $\frac{\sin i}{\sin r}$ is therefore $\frac{BE}{AF}$. But B E and A F are the distances travelled in the same time in the two media Hence the sines of the angles of incidence and refraction are directly as the velocities of propagation of the incident and refracted light. The *relative index* of refraction from one medium into another is therefore the *ratio of the velocity of light in the first medium to its velocity in the second;* and *the absolute index of refraction of any medium is inversely as the velocity of light in that medium.*

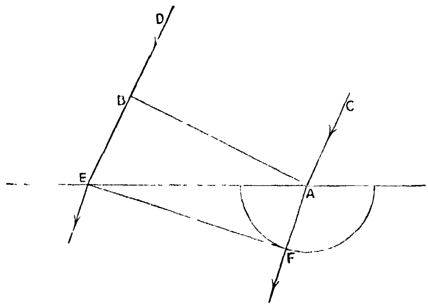


Fig 192 —Wave-front in Ordinary Refraction

249. **Application to Reflection.**—The explanation of reflection is precisely similar. Let C A, D E (Fig. 193) be parallel rays incident at A and E; A B the wave-front. As the successive points of the wave-front arrive at the reflecting surface, hemispherical waves diverge from the points of incidence; and by the time that B reaches E, the wave from A will have diverged in all directions to a distance equal to B E. If then we describe in the plane of incidence a semicircle, with centre A and radius equal to B E, the tangent E F to this semicircle will be the wave-front of the reflected light, and A F will be the reflected ray corresponding to the incident ray C A. From the equality of the right-angled triangles A B E, E F A, it is evident that the angles of incidence and reflection are equal.

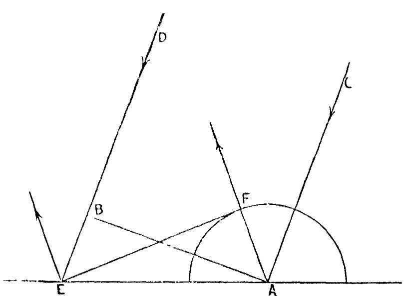


Fig 193 —Wave-front in Reflection.

250. **Newtonian Explanation of Refraction.**—In the Newtonian theory, the change of direction which a ray experiences at the bound-

ing surface of two media, is attributed to the preponderance of the attraction of the denser medium upon the particles of light. As the resultant force of this attraction is normal to the surface, the tangential component of velocity remains unchanged, and the normal component is increased or diminished according as the incidence is from rare to dense or from dense to rare. Let μ denote the relative index of refraction from rare to dense. Let v, v' be the velocities of light in the rarer and denser medium respectively, and i, i' the angles which the rays in the two media make with the normal. Then the tangential components of velocity in the two media are $v \sin i, v' \sin i'$ respectively, and these by the Newtonian theory are equal; whence $\frac{v'}{v}=\frac{\sin i}{\sin i'}=\mu$; whereas according to the undulatory theory $\frac{v'}{v}=\frac{1}{\mu}$. In the Newtonian theory, the velocity of light in any medium is directly as the absolute index of refraction of the medium; whereas, in the undulatory theory, the reverse rule holds.

The main design of Foucault's experiment with the rotating mirror (§ 77), in its original form, was to put these opposite conclusions to the test of direct experiment. For this purpose it was not necessary to determine the velocity of the rotating mirror, since it affected both the observed displacements alike. The two images were seen in the same field of view, and were easily distinguished by the greenness of the water-image. In every trial the water-image was more displaced than the air-image, indicating longer time and slower velocity; and the measurements taken were in complete accordance with the undulatory theory, while the Newtonian theory was conclusively disproved.

251. **Principle of Least Time.**—The path by which light travels from one point to another is in the generality of cases that which occupies least time. For example, in ordinary cases of reflection (except from very concave[1] surfaces), if we select any two points, one on the incident and the other on the reflected ray, the sum of their distances from the point of incidence is less than the sum of their distances from any neighbouring point on the reflecting surface. In this case, since only one medium is concerned, distance is proportional to time. When a ray in air is refracted into water, if we select any two points,

[1] Suppose an ellipse described, having the two selected points for foci, and passing through the point of incidence. If the curvature of the reflecting surface in the plane of incidence is greater than the curvature of this ellipse, the length of the path is a maximum, if less, a minimum. This follows at once from the constancy of the sum of the focal distances in an ellipse.

one on the incident and the other on the refracted ray, and call their distances from any point of the refracting surface s, s' respectively, and the velocities of propagation in the two media v, v', then the sum of $\frac{s}{v}$ and $\frac{s'}{v'}$ is generally less when s and s' are measured to the point of incidence than when they are measured to any neighbouring point on the surface. $\frac{s}{v}$ is evidently the time of going from the first point to the refracting surface, and $\frac{s'}{v'}$ the time from the refracting surface to the second point.

The proposition as above enunciated admits of certain exceptions, the time being sometimes a maximum instead of a minimum. The really essential condition (which is fulfilled in both these opposite cases) is that all points on a small area surrounding the point of incidence give sensibly *the same time*. The component waves sent from all parts of this small area will be in the same phase, and will propagate a ray of light by their combined action.

When the two points considered are conjugate foci, and there is no aberration, this condition must be fulfilled by all the rays which pass through both; and the *time of travelling from one focus to the other is the same for all the rays.* Spherical waves diverging from one focus will, after incidence, become spherical waves converging to or diverging from its conjugate focus. An effect of this kind can be beautifully exhibited to the eye by means of an elliptic dish containing mercury. If agitation is produced at one focus of the ellipse by dipping a small rod into the liquid at this point, circular waves will be seen to converge towards the other focus. A circular dish exhibits a similar result somewhat imperfectly; waves diverging from a point near the centre will be seen to converge to a point symmetrically situated on the other side of the centre.

When the second point lies on a caustic surface formed by the reflection or refraction of rays emanating from the first point, all points on an area of sensible magnitude in the neighbourhood of the point of incidence would give sensibly the same time of travelling as the actual point of incidence, so that the light which traverses a point on a caustic may be regarded as coming from an area of sensible magnitude instead of (as in the case of points not on the caustic) an excessively small area. An eye placed at a point on a caustic will see this portion of the surface filled with light.

As the velocity of light is inversely proportional to the index of

refraction μ, the time of travelling a distance s with constant velocity may be represented by μs, and if a ray of light passes from one point to another by a crooked path, made up of straight lines $s_1, s_2, s_3, \ldots$ lying in media whose absolute indices are $\mu_1, \mu_2, \mu_3, \ldots$, the expression $\mu_1 s_1 + \mu_2 s_2 + \mu_3 s_3 + \ldots$ represents the time of passage. This expression, which may be called *the sum of such terms as* μs, must therefore fulfil the above condition; that is to say, the points of incidence on the surfaces of separation must be so situated that this sum either remains absolutely constant when small changes are supposed to be made in the positions of these points, or else retains that approximate constancy which is characteristic of maxima and minima. Conversely, all lines from a luminous point which fulfil this condition, will be paths of actual rays.

252. **Terrestrial Refraction.**[1]—The atmosphere may be regarded as homogeneous when we confine our attention to small portions of it, and hence it is sensibly true, in ordinary experiments where no great distances are concerned, that rays of light in air are straight, just as it is true in the same limited sense that the surface of a liquid at rest is a horizontal plane. The surface of an ocean is not plane, but approximately spherical, its curvature being quite sensible in ordinary nautical observations, where the distance concerned is merely that of the visible sea-horizon; and a correction for curvature is in like manner required in observing levels on land. If the observer is standing on a perfectly level plain, and observing a distant object at precisely the same height as his eye above the plain, it will appear to be below his eye, for a horizontal *plane* through his eye will pass above it, since a perfectly level *plain* is not *plane*, but shares in the general curvature of the earth. It is easily proved that the apparent depression due to this cause is half the angle between the verticals at the positions of the observer and of the object observed. But experience has shown that this apparent depression is to a considerable extent modified by an opposite disturbing cause, called *terrestrial refraction.* When the atmosphere is in its normal condition, a ray of light from the object to the observer is not straight, but is slightly concave downwards.

This curvature of a nearly horizontal ray is not due to the curvature of the earth and of the layers of equal density in the earth's atmosphere, as is often erroneously supposed, but would still exist,

[1] For the leading idea which is developed in §§ 252, 253, the Editor is indebted to suggestions from Professor James Thomson.

and with no sensible change in its amount, if the earth's surface were plane, and the directions of gravity everywhere parallel. It is due to the fact that light travels faster in the rarer air above than in the denser air below, so that time is saved by deviating slightly to the upper side of a straight course. The actual amount of curvature (as determined by surveying) is from $\frac{1}{2}$ to $\frac{1}{10}$ of the curvature of the earth; that is to say, the radius of curvature of the ray is from 2 to 10 times the earth's radius.

253. Calculation of Curvature of Ray.—In order to calculate the radius of curvature from physical data, it is better to approach the subject from a somewhat different point of view.

The wave-fronts of a ray in air are perpendicular to the ray; and if the ray is nearly horizontal, its wave-fronts will be nearly vertical. If two of these wave-fronts are produced downwards until they meet, the distance of their intersection from the ray will be the radius of curvature. Let us consider two points on the same wave-front, one of them a foot above the other; then the upper one being in rarer air will be advancing faster than the lower one, and it is easily shown that the difference of their velocities is to the velocity of either, as 1 foot is to the radius of curvature.

Put ρ for the radius of curvature in feet, v and $v+\delta v$ for the two velocities, μ and $\mu+\delta\mu$ for the indices of refraction of the air at the two points. Then we have

$$\frac{1}{\rho} = \frac{\delta v}{v} = \frac{-\delta\mu}{\mu}. \tag{1}$$

Now it has been ascertained, by direct experiment, that the value of $\mu-1$ for air, within ordinary limits of density, is sensibly proportional to the density (even when the temperature varies), and is ·0002943 or 1/3400 at the density corresponding to the pressure 760^{mm} (at Paris) and temperature 0° C. Hence, for the value of ρ in feet, we have

$$\frac{1}{\rho} = -\frac{\delta\mu}{\mu} = -\frac{\delta(\mu-1)}{1} = -(\mu-1)\frac{\delta(\mu-1)}{\mu-1} = \frac{1}{3400}\text{(expansion per foot).}$$

The expansion per foot, that is to say, the *difference of density for one foot of height, divided by the density*, would, if the temperature were uniform, be 1/H, H denoting the height of the homogeneous atmosphere in feet.

When the temperature diminishes upwards by $1/n$ of a degree

Centigrade per foot, there will be a contraction of about $1/273n$ to set against the above expansion, and we shall have

$$\frac{1}{\rho} = \frac{1}{3400}\left(\frac{1}{\mathrm{H}} - \frac{1}{273\,n}\right). \tag{2}$$

H is about 26000, and the average value of n is generally taken as 540. This will make the negative term about one-sixth of the positive, giving roughly

$$\frac{1}{\rho} = \frac{1}{3400} \cdot \frac{5}{6} \cdot \frac{1}{\mathrm{H}}, \text{ or } \frac{\rho}{\mathrm{H}} = 4080.$$

Since H is about 1/800 of the earth's radius, this makes ρ about five times the earth's radius; or the curvature of a horizontal ray about 1/5 of the curvature of the earth's surface, which is a close approximation to its average value as deduced from surveying. The variations in its value depend chiefly on the variations of the temperature-gradient $1/n$.

For a ray inclined at an angle θ to the horizon, the curvature $1/\rho$ will be less than for a horizontal ray, being proportional to $\cos\theta$; as appears by considering two points a foot apart lying in the same wave-front and in the vertical plane of the ray.

254. **Mirage.**—An appearance, as of water, is frequently seen in sandy deserts, where the soil is highly heated by the sun. The observer sees in the distance the reflection of the sky and of terrestrial objects, as in the surface of a calm lake. This phenomenon, which is called *mirage*, is explained by the heating and consequent rarefaction of the air in contact with the hot soil. The density in the lowest stratum of air (a foot or so in thickness) increases upwards, and rays entering this stratum at a small inclination are bent upwards in a manner resembling "total reflection," but with the corner rounded off.

A kind of inverted mirage is often seen across masses of calm water, and is called *looming;* images of distant objects, such as ships or hills, being seen in an inverted position immediately over the objects themselves. The explanation just given of the mirage of the desert will apply to this phenomenon also, if we suppose at a certain height, greater than that of the observer's eye, a layer of rapid transition from colder and denser air below to warmer and rarer air above.

An appearance similar to mirage may be obtained by gently depositing alcohol or methylated spirit upon water in a vessel with plate-glass sides. The spirit, though lighter, has a higher index of

refraction than the water, and rays traversing the layer of transition are bent upwards. This layer accordingly behaves like a mirror when looked at very obliquely by an eye above it.[1]

255. Curved Rays of Sound.—The reasoning of §§ 251, 253 can be applied, with a slight modification, to the propagation of sound.

Sound travels faster in warm than in cold air. On calm sunny afternoons, when the ground has become highly heated by the sun's rays, the temperature of the air is much higher near the ground than at moderate heights; hence sound bends upwards, and may thus become inaudible to observers at a distance by passing over their heads. On the other hand, on clear calm nights the ground is cooled by radiation to the sky, and the layers of air near the ground are colder than those above them, hence sound bends downwards, and may thus, by arching over intervening obstacles, become audible at distant points, which it could not reach by rectilinear propagation. This influence of temperature, which was first pointed out by Professor Osborne Reynolds, is one reason why sound from distant sources is better heard by night than by day.

A similar effect of wind had been previously pointed out by Professor Stokes. It is well known that sound is better heard with the wind than against it. This difference is due to the circumstance that wind is checked by friction against the earth, and therefore increases in velocity upwards. Sound travelling with the wind, therefore, travels fastest above, and sound travelling against the wind travels fastest below, its actual velocity being in the former case the sum, and in the latter the difference, of its velocity in still air and the velocity of the wind. The velocity of the wind is so much less than that of sound, that if uniform at all heights its influence on audibility would scarcely be appreciable.

To calculate the curvature of a ray of sound due to variation of temperature with height, we may employ, as in § 253, the formula $\frac{1}{\rho} = \frac{\delta v}{v}$, where δv denotes the difference of velocity for a difference of 1 foot in height. The value of v varies as $\sqrt{(1+\alpha t)}$, or approximately as $1+\frac{1}{2}\alpha t$, t denoting temperature, and α the coefficient of expansion, which is $\frac{1}{273}$. Hence if the velocity at 0° be denoted by 1, the value at $t°$ will be denoted by $1+\frac{1}{2}\alpha t$, and if the tempera-

[1] A more complete discussion of the optics of mirage will be found in two papers by the editor of this work in the *Philosophical Magazine* for March and April, 1873, and in *Nature* for Nov. 19 and 26, 1874.

ture varies by $\frac{1}{n}$ of a degree per foot, the value of $\frac{\delta v}{v}$ at temperatures near zero will be $\frac{a}{2n}$, that this, $\frac{1}{546n}$, and the radius of curvature will be $546n$ feet. This calculation shows that the bending is much more considerable for rays of sound than for rays of light.

256. Interference.—In §§ 21-23 we pointed out that the superposition of two nearly similar sets of sonorous waves produces alternate extinction and augmentation of sound. Similar results occur in optics, and are called by the same name—*interference.* But there is one notable difference. Two distinct sources of sound—for instance, two organ pipes nearly in unison—can interfere; but two distinct sources of light—for instance, two flames or two different portions of the same flame—never exhibit the phenomena of interference. The explanation is that the vibrations of light, being transverse, are not parallel to one line, like those of sound, but to one plane, and are in general elliptic, and that variations in the ellipses prevent the necessary uniformity.

257. Colours of Thin Films.—The simplest example of interference of light is furnished by the colours of thin films. If two pieces of glass, with their surfaces clean, are brought into close contact, coloured fringes are seen surrounding the point where the contact is closest. They are best seen when light is obliquely reflected to the eye from the surfaces of the glass; and fringes of the complementary colours may be seen by transmitted light. A drop of oil placed on the surface of clean water spreads out into a thin film, which exhibits similar fringes of colour; and in general, a very thin film of any transparent substance, separating media whose indices of refraction are different from its own, exhibits colour, especially when viewed by obliquely reflected light. In the first experiment above-mentioned, the thin film is an air-film separating the pieces of glass. In soap-bubbles or films of soapy water stretched on rings, a similar effect is produced by a small thickness of water separating two portions of air.

The colours, in all these cases, when seen by reflected light, are produced by the mutual interference of the light reflected from the two surfaces of the thin film. An incident ray undergoes, as explained in § 134, a series of reflections and refractions; and we may thus distinguish, for light of any given refrangibility, several systems of waves, all of which originally came from the same source. These systems give by their interference a series of alternately bright and

dark fringes, and when ordinary white light is employed, the fringes are broadest for the colours of greatest wave-length. Their superposition thus produces the observed colours. The colours seen by transmitted light may be similarly explained. The following investigation applies to the colours seen by reflection.

Let DCBA (fig. 194) be the upper face of a thin transparent plate on which parallel rays fall. An eye at E receives not only the ray directly reflected at A, but also a ray which has been refracted at B, reflected internally at G, and again refracted at A; besides a third ray which has taken the course CKBGA, a fourth which has taken the course DLCKBGA, and so on.

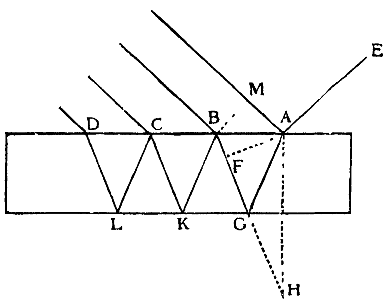


Fig 194

BM perpendicular to the incident rays will be a wave-front in the first medium, and AF perpendicular to BG is the position which this wave-front will have reached in the second medium when it has reached A in the first. The light which takes the course BGA will have to travel over the additional distance FG + GA before it arrives at A.

Producing BG to meet a perpendicular to the plate from A in H, AH will be twice the thickness of the plate, say $2t$, and GH will equal GA. Thus the retardation FG + GA is equal to FH or to $2t \cos \beta$, β denoting the angle which the refracted rays in the plate make with the normal. If μ denote the absolute index for the plate, the time of travelling the distance $2t \cos \beta$ in the plate is the same as the time of travelling a distance $2t\mu \cos \beta$ in vacuo. If the first medium is vacuum, the angle of incidence α will be such that $\sin \alpha = \mu \sin \beta$; and if the first medium be in the form of a thick plate, of any index, with vacuum above it, the angle of incidence on the thick plate will be the same. Hence we have

$$\mu^2 \cos^2 \beta = \mu^2 - \mu^2 \sin^2 \beta = \mu^2 - \sin^2 \alpha.$$

The retardation reduced to vacuum may therefore be expressed as $2t \sqrt{(\mu^2 - \sin^2 \alpha)}$.

Since μ for air is sensibly unity, the retardation for a thin layer of air of thickness t is $2t \cos \alpha$. The ray which has undergone three internal reflections will have a retardation double of this; the ray

which has been five times internally reflected, triple, and so on, these retardations being relative to the ray which has been directly reflected at the first surface. All this is on the assumption that no change of phase is produced by reflection.

The directly reflected light is much more intense than the light which has been once internally reflected; this again is much more intense than the light which has been thrice internally reflected, and so on. The observed effect mainly depends on the interference of the two strongest portions.

If no change of phase were produced by either of the reflections the difference of phase between these two portions would be $2t \cos \alpha$, and would vanish with t. We should then have complete accordance of phase, and a maximum of brightness for an air-film of vanishing thickness. This, however, is directly contrary to the fact; when the two pieces of glass are pressed as close together as possible a black spot is obtained. The explanation is that—just as in the case of sound waves—the direction of motion of a particle close to the point of incidence is reversed when a pulse is reflected from a denser medium, but retains its original direction when a pulse in a denser medium is reflected from a rarer. Of the two principal portions of light, one has been reflected at incidence on the denser and the other at incidence on the rarer medium; and this circumstance, if it were the only point of difference between them, would make their phases opposite. The actual difference of phase, expressed in wave-lengths, is accordingly $\frac{2t \cos \alpha}{\lambda} \pm \frac{1}{2}$. It is immaterial whether we take $+\frac{1}{2}$ or $-\frac{1}{2}$, since a whole wave-length makes no difference in phase. For a thin plate of any substance, the retardation, expressed in wave-lengths in vacuo, is $\frac{2t \sqrt{(\mu^2 - \sin^2 \alpha)}}{\lambda} + \frac{1}{2}$.

Let n be any integer. The most complete extinctions of light of wave-length λ will be obtained when

$$2t \sqrt{(\mu^2 - \sin^2 \alpha)} = 0, \text{ or } n\lambda,$$

and the greatest brightness when

$$2t \sqrt{(\mu^2 - \sin^2 \alpha)} = \tfrac{1}{2}\lambda, \text{ or } (n + \tfrac{1}{2})\lambda.$$

258. Newton's Rings.—Newton's rings, which are a good example of the colours of thin films, are usually obtained by pressing together two pieces of glass, one of them plane and the other slightly convex. Let ρ denote its radius of curvature. At distance x from the point of contact the thickness t of the intervening film is $x^2/2\rho$.

Hence if x is the radius of one of the bright rings, the successive values of x^2 will be as the odd integers 1, 3, 5, &c.; and the squares of the radii of the dark rings will be represented on the same scale by 2, 4, 6, &c., besides the dark spot, at the centre of which $x^2=0$

For the radius x of the first bright ring we have $x^2 = 2\rho t$ $= \frac{\frac{1}{2}\rho\lambda}{\surd(\mu^2 - \sin^2\alpha)}$, which for air is $\frac{1}{2}\rho\lambda \sec\alpha$. This shows that the size of the rings increases with the obliquity of the incidence—this obliquity being α. Comparing one medium with another, for the same obliquity of incidence, x^2 varies inversely as $\surd(\mu^2 - \sin^2\alpha)$, and therefore diminishes as μ increases, so that a film of liquid will give smaller rings than a film of air. Since x^2 is proportional to λ, the red rings will be the largest and the violet rings the smallest.

These results have been deduced from the consideration of the two most important components only. The action of the other components modifies the distribution of intensity, but does not alter the positions of maximum and minimum—in other words, does not affect the sizes of the rings.

Since the scale of the phenomenon (for an air film) is proportional to sec α, the broadest rings or bands will be obtained by making α nearly 90°. If the upper piece of glass is a plate, the directly reflected light will then overpower the other portions and obliterate the rings, but by employing an ordinary equilateral prism instead, this difficulty is obviated, and very broad bands of colour are seen in the neighbourhood of the part of the base where total reflection begins.

259. Diffraction Fringes.—When a beam of direct sunlight is admitted into a dark room through a narrow slit, a screen placed at any distance to receive it will show a line of white light, bordered with coloured fringes which become wider as the slit is narrowed. They also increase in width as the screen is removed further off If they are viewed through a piece of red glass which allows only red rays to pass, they will appear as a succession of bands alternately bright and dark.

To explain their origin, we shall suppose the sun's rays (which may be reflected from an external mirror) to be perpendicular to the plane of the slit,[1] so that the wave-fronts are parallel to this

[1] That is, to the plane of the two knife-edges by which the slit is bounded. This condition can only be strictly fulfilled for a single point on the sun's disc. Every point on the sun's surface sends out its own waves as an independent source; and waves from one point

plane, and we shall, in the first instance, confine our attention to light of a particular wave-length; for example, that of the light transmitted by the red glass. Then, if the slit be uniform through its whole length, the positions of the bright and dark bands will be governed by the following laws:—

1. The darkest parts will be at points whose distances from the two edges of the slit differ by an exact number of wave-lengths. If the difference be one wave-length, the light which arrives at any instant from different parts of the width of the slit is in all possible phases, and the resultant of the whole is zero. In fact, the disturbance produced by the nearer half of the slit cancels that produced by the remoter half. If the difference be n wave-lengths, we can divide the slit into n parts, such that the effect due to each part is thus *nil*.

2. The brightest parts will be at points whose distances from the two edges of the slit differ by an exact number of wave-lengths *plus* a half. Let the difference be $n+\frac{1}{2}$; then we can divide the slit into n inefficient parts and one efficient part, this latter having only half the width of one of the others.[1]

Each colour of light has its own alternate bands of brightness and darkness, the distance from band to band being greatest for red and least for violet. The superposition of all the bands constitutes the coloured fringes which are seen.

This experiment furnishes the simplest answer to the objection formerly raised to the undulatory theory, that light is not able, like sound, to pass round an obstacle, but can only travel in straight lines. In this experiment light does pass round an obstacle, and turns more and more away from a straight line as the slit is narrowed.

When the slit is not exceedingly narrow, the light sent in oblique directions is quite insensible in comparison with the direct light, and no fringes are visible. "We have reason to think that when *sound* passes through a very large aperture, or when it is reflected from a

cannot interfere with waves from another. In the experiment as described in the text the fringes due to different parts of the sun's surface are all produced at once on the screen, and overlap each other.

[1] Each element of the length of the slit tends to produce a system of circular rings (the screen being supposed parallel to the plane of the slit). If the width of the slit is uniform, these systems will be precisely alike, and will have for their resultant a system of straight bands, parallel to the slit and touching the rings. These are the bands described in the text. Hence, to determine the illumination of any point of the screen, it is only necessary to attend, as in the text, to the nearest points of the two edges of the slit.

large surface (which amounts nearly to the same thing), it is hardly sensible except in front of the opening, or in the direction of reflection."[1]

260. Fresnel's Fringes.—Very definite and regular fringes can be obtained by a method devised by Fresnel (Fig. 195).

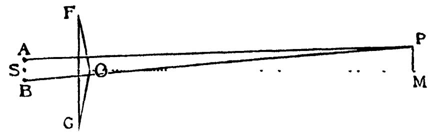


Fig 195

A prism (called *Fresnel's biprism*), whose section FOG is an isosceles triangle with a vertical angle of nearly 180°, is placed at a distance of a few inches in front of a strongly illuminated slit S. The edge which corresponds to the vertical angle O must be very accurately parallel to the slit, and either turned directly towards or directly away from the slit. An eye in the line SO produced will see two images A and B of the slit, one in each half of the biprism; and as the refracting angles of the prism are very small, the two images are very near together. The fringes are produced by the interference of the two refracted beams of light which may be regarded as coming from these two images. They can be thrown upon a screen a few feet in front of the prism, if the room is sufficiently dark, or they can be seen in mid air even in daylight with the aid of an eye-piece or a short focussed lens. As the eye-piece is moved further from the prism, the fringes become both broader and more numerous. The region in which they are formed is that in which the beams from the two images overlap, and would be shown in the diagram as the space lying between the productions of AO and BO.

To explain their formation, let M be a point on the axis of symmetry SOM, and P a point on the perpendicular to the axis at M, at such a distance that AP and BP lie on opposite sides of O. The point P will receive light from both A and B, and the nature of the interference will depend on the retardation—that is on the difference of the distances AP, BP.

Denoting AB by $2c$, SM by a, and MP by x,

we have $AP^2 = a^2 + (c \pm x)^2$,

$BP^2 = a^2 + (c \mp x)^2$,

hence $AP^2 \sim BP^2 = 4cx = (AP + BP)\,(AP \sim BP)$.

That is, $4cx = 2a\,(AP \sim BP)$, nearly.

[1] Airy, *Undulatory Theory*. Art. 28.

If the difference of distances $AP \sim BP$ is an odd number of wave-lengths, the two beams of light (which, it is to be noted, came from the same source, namely the slit) will be in opposite phases, and will give darkness; if an even number, they will be in the same phase and will give a maximum of light. The adjustments should be so made that the two images are of precisely equal brightness, otherwise the extinctions will not be complete.

Putting $n\lambda/2$ for $AP \sim BP$, we shall have $4cx = na\lambda$,

as the relation determining the values of x for the dark bands when n is odd, and for the bright bands when n is even.

In the apparatus usually provided for this experiment, there is a movable red glass, which can be placed behind the eye-piece and stops practically all light except that belonging to the middle red of the spectrum. It shows a succession of sharply defined strong and equidistant black lines, with bright intervals between. When the red glass is put aside, a gorgeous display of coloured stripes is seen, produced by the overlapping of the bands of the different spectral colours. The above equation shows that the distance from band to band is proportional to the wave-length.

The eye-piece is attached to a micrometer screw, by means of which it can be carried across the fringes through measured distances. Its cross-wires are thus brought into coincidence with the different black bands in succession, and the distances between the bands are read off on the micrometer. This gives x. A direct measurement is also made of a. To determine $2c$, the distance $OS = b$ between the prism and slit is directly measured, and the angular separation given by the prism is measured by means of a spectrometer, the prism being dismounted for this purpose after the other measurements have been made. This angle multiplied by the distance b is the distance $2c$ between the images. From these measurements the wave-length of the approximately monochromatic light employed can be computed by the above equation, $4cx = na\lambda$. For light of given wave-length, x is directly proportional to a, which is the distance from the slit to the focus of the eye-piece, and inversely proportional to $2c$, so that the pattern can be made coarser either by removing the eye-piece to a greater distance or by moving the prism nearer to the slit.

261. Fresnel's Mirrors. — In another arrangement, also devised by Fresnel, two plane mirrors, set at an angle of nearly 180°, are

employed instead of the biprism. They are usually plates of black glass. Light falls obliquely upon them from the slit, and the two images of the slit, which are formed one by each mirror, give the same effect as the two images formed by the biprism.

In another form of the experiment, devised by Lloyd, only one mirror is employed. The slit itself, and the image of it formed by this mirror, take the place of the two images A, B in the above investigation; but the effects differ in the following respects.

1. The reversal of phase which takes place in the reflection causes the places of the dark and bright bands to be interchanged.

2. The bands obtained lie on one side of the axis only.

3. The image by reflection being less bright than the slit, the extinctions are not complete.

The geometrical relations in these two arrangements are shown in Figs. 196, 197. In Fig. 196, S is the slit, giving images A, B in the two mirrors OF, OG. The interference takes place in the region lying between the prolongations OA′, OB′ of AO and BO, and the fringes are symmetrically disposed with reference to the bisector OM.

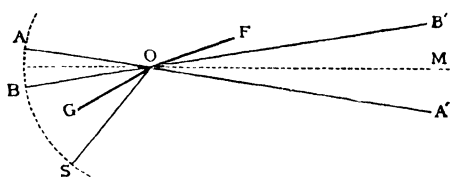


Fig 196

In Fig 197, S is the source, OF the mirror, and FM the prolongation of its plane. A is the image formed by the mirror, and the region of interference lies between the prolongations of AO and AF. The axis from which x is measured is OFM.

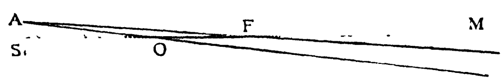


Fig 197

The true Fresnel fringes, to which the above calculations apply, are bordered by coarser and less regular fringes, which are similar in appearance and in origin to those given by a single slit, and which are not dependent on the interference of the two images.

The most convenient means of illumination for the slit in ordinary laboratory work is a naked flame of flat form placed edge-on to the slit, at a distance sufficient to prevent danger of injury to the slit There should be no lens between it and the slit. The paraffin lamp known as Hitchcock's is specially convenient, as it gives a very flat and steady flame without a chimney.

262. Young's Fringes.—Fringes similar to Fresnel's and similarly explained can be obtained by means of two very close pinholes in a sheet of tinfoil, illuminated by direct sunlight (or sunlight reflected from a mirror) which has first passed through a rather larger hole in another sheet of foil, or through a slit whose length is perpendicular to the line joining the holes. The slit or large hole must be sufficiently distant to ensure that the two small holes receive their light from practically the same part of the sun's surface. A short focussed lens, forming a very small image of the sun, may be used instead of the first hole.

This experiment was devised by Thomas Young, and was earlier in date than Fresnel's.

There are several other modes of producing diffraction fringes, which our limits do not permit us to notice. We proceed to describe the mode of obtaining a *pure spectrum* by diffraction.

263. Diffraction by a Grating.—If a piece of glass is ruled with parallel equidistant scratches (by means of a dividing engine and diamond point) at the rate of some hundreds or thousands to the inch, we shall find, on looking through it at a slit or other bright line (the glass being held so that the scratches are parallel to the slit), that a number of spectra are presented to view, ranged at nearly equal distances, on both sides of the slit. If the experiment is made under favourable circumstances, the spectra will be so pure as to show a number of Fraunhofer's lines.

Instead of viewing the spectra with the naked eye, we may with advantage employ a telescope, focussed on the plane of the slit; or we may project the spectra on a screen, by first placing a convex lens so as to form an image of the slit (which must be very strongly illuminated) on the screen, and then interposing the ruled glass in the path of the beam.

A piece of glass thus ruled is called a *grating*.[1] A grating for diffraction experiments consists essentially of a number of parallel strips alternately transparent and opaque.

The distances between the "fixed lines" of the spectra, and the distance from one spectrum to the next, are found to depend on the distance of the strips measured from centre to centre, in other words,

[1] Engraved glass gratings of sufficient size for spectroscopic purposes (say an inch square) are extremely expensive and difficult to procure. Lord Rayleigh has made numerous photographic copies of such gratings, and the copies appear to be equally effective with the originals.

on the number of scratches to the inch, but not at all on the relative breadths of the transparent and opaque strips. This latter circumstance only affects the brightness of the spectra.

Diffraction spectra are of great practical importance—

1. As furnishing a uniform standard of reference in the comparison of spectra.

2. As affording the most accurate method of determining the wave-lengths of the different elementary rays of light.

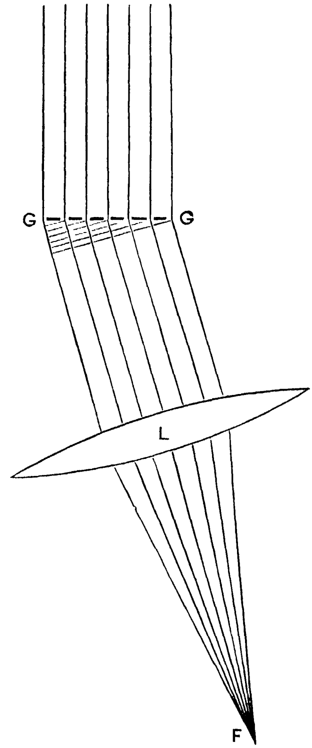


Fig 198
Principle of Diffraction Spectrum

264. Principle of Diffraction Spectrum.—Let GG (Fig. 198) be a grating, receiving light from an infinitely[1] distant point lying in a direction perpendicular to the plane of the grating, so that the wave-fronts of the incident light are parallel to this plane. Let a convex lens L be placed on the other side of the grating, and let its axis make an acute angle θ with the rays incident on the grating Then the light collected at its principal focus F consists of all the light incident upon the lens parallel to its axis. Let s denote the distance between the rulings measured from centre to centre, so that if, for example, there are 1000 lines to the inch, s will be $\frac{1}{1000}$ of an inch, and suppose first that $s \sin\theta$ is exactly equal to the wave-length λ of one of the elementary kinds of light. Then, of all the light which falls upon the lens parallel to its axis, the left-hand portion in the figure is most retarded (having travelled farthest), and the right-hand portion least, the retardation, in comparing each transparent interval with the next, being constant, and equal to $s \sin\theta$, as is evident from an inspection of the figure. Now, for the particular kind of light for which $\lambda = s \sin\theta$, this retardation is exactly a wave-length, and all the transparent intervals send light of the same phase to the focus F, so that, if there are 1000 such

[1] It is not *necessary* that the source should be infinitely distant (or the incident rays parallel); but this is the simplest case, and the most usual case in practice.

intervals, the resultant amplitude of vibration at F is 1000 times the amplitude due to one interval alone. For light of any other wave-length this coincidence of phase will not exist. For example, if the difference between λ and $s \sin\theta$ is $\frac{1}{1000}\lambda$, the difference of phase between the lights received from the 1st and 2d intervals will be $\frac{1}{1000}\lambda$, between the 1st and 3d $\frac{2}{1000}\lambda$, between the 1st and 501st $\frac{500}{1000}\lambda$, or just half a wave-length, and so on. The 1st and 501st are thus in complete discordance, as are also the 2d and 502d, &c. Light of every wave-length except one is thus almost completely destroyed by interference, and the light collected at F consists almost entirely of the particular kind defined by the condition

$$\lambda = s \sin \theta. \tag{1}$$

The purity of the diffraction spectrum is thus explained.

If a screen be held at F, with its plane perpendicular to the principal axis, any point on this screen a little to one side of F will receive light of another definite wave-length, corresponding to another direction of incidence on the lens, and a pure spectrum will thus be depicted on the screen.

265. Practical Application.—In the arrangement actually employed for accurate observation, the lens L L is the object-glass of a telescope with a cross of spider-lines at its principal focus F. The telescope is first pointed directly towards the source of light, and is then turned to one side through a measured angle θ. Any fixed line of the spectrum can thus be brought into apparent coincidence with the cross of spider-lines, and its wave-length can be computed by the formula (1).

The spectrum to which formula (1) relates is called the *spectrum of the first order.*

There is also a spectrum of the second order, corresponding to values of θ nearly twice as great, and for which the equation is

$$2\lambda = s \sin \theta. \tag{2}$$

For the spectrum of the third order the equation is

$$3\lambda = s \sin \theta; \tag{3}$$

and so on, the explanation of their formation being almost precisely the same as that above given There are two spectra of each order, one to the right, and the other at the same distance to the left of the direction of the source. In Ångström's observations,[1] which

[1] Ångström, *Recherches sur la Spectre solaire.* Upsal, 1868.

were for many years the standard authority, all the spectra, up to the sixth inclusive, were observed, and numerous independent determinations of wave-length were thus obtained for several hundred of the dark lines of the solar spectrum.

The source of light was the infinitely distant image of an illuminated slit, the slit being placed at the principal focus of a collimator, and illuminated by a beam of the sun's rays reflected from a mirror.

The purity of a diffraction spectrum increases with the number of lines on the grating which come into play, provided that they are exactly equidistant, and may therefore be increased either by increasing the size of the grating, or by ruling its lines closer together. The gratings employed by Ångstrom were about $\frac{3}{4}$ of an inch square, the closest ruled having about 4500 lines, and the widest 1500.

As regards brightness, diffraction spectra are far inferior to those obtained by prisms. To give a maximum of light, the opaque intervals should be perfectly opaque, and the transparent intervals perfectly transparent, but even under the most favourable conditions, the whole light of any one of the spectra cannot exceed about $\frac{1}{10}$ of the light which would be received by directing the telescope to the slit The greatest attainable intrinsic brightness in any part of a diffraction spectrum is thus not more than $\frac{1}{10}$ of the intrinsic brightness in the same part of a prismatic spectrum, obtained with the same slit, collimator, and observing telescope, and with the same angular separation of fixed lines. The brightness of the spectra partly depends upon the ratio of the breadths of the transparent and opaque intervals. In the case of the spectra of the first order, the best ratio is that of equality, and equal departures from equality in opposite directions give identical results; for example, if the breadth of the transparent intervals is to the breadth of the opaque either as 1 : 5 or as 5 : 1, it can be shown that the quantity of light in the first spectrum is just a quarter of what it would be with the breadths equal. (See § 273.)

When a diffraction spectrum is seen with the naked eye, the cornea and crystalline of the eye take the place of the lens L L, and form a real image on the retina at F.

266. Retardation Gratings—If, instead of supposing the bars of the grating to be opaque, we suppose them to be transparent, but to produce a definite change of phase either by acceleration or retardation, the spectra produced will be the same as in the case above discussed, except as regards brightness. We may regard the effect

as consisting of the superposition of two exactly coincident sets of spectra, one due to the spaces and the other to the bars. Any one of the resultant spectra may be either brighter or less bright than either of its components, according to the difference of phase between them. If the bars and spaces are equally transparent, the two superimposed spectra will be equally bright, and their resultant at any part may have any brightness intermediate between zero and four times that of either component.

267. Reflection Gratings.—Transmission gratings (such as we have hitherto been describing) are now superseded, for high-class work, by reflection gratings ruled with a diamond point on speculum metal, which give much brighter spectra. The first great advance in their construction was made by Rutherfurd of New York, and their manufacture has been brought to a very high degree of perfection by Rowland. They usually contain above 14,000 lines, ruled within a space of one inch.

If ϕ is the angle of incidence, the difference of path from the slit to two consecutive lines of the grating is $s \sin\phi$; and if θ has the same meaning as in §§ 264, 265, the difference of path from these two lines to the focus of the telescope is $s \sin\theta$, as there shown. If the two distances are on the same side of the normal, the two differences will have the same sign, and the condition for identity of phase is

$$s \sin\phi + s \sin\theta = n\lambda. \qquad (4)$$

If they are on opposite sides of the normal, the difference must be taken instead of the sum. The same formula is applicable to transmission gratings, and the case discussed in § 265 is included by putting $\phi =$ zero.

If the grating and slit are fixed, and the telescope is sensibly normal to the grating, ϕ is constant and θ is small. Hence it can be shown from (4) that the variations of λ are proportional to the corresponding variations of θ, so that the *distances between the "fixed lines" of the spectrum* will be *proportional to the differences of their wave-lengths.* A spectrum thus characterized is called "NORMAL," and is by common consent adopted as the standard of reference.

268. To measure wave-lengths by means of a plane reflection grating, it should be mounted on a spectrometer (§ 142) in the place usually occupied by the prism. We shall suppose the circle round which the observing telescope travels to be graduated from 0° to 360°, commencing at the collimator. (See Fig. 124, § 142.)

Let β denote the angle which the plane of the grating makes with the direction of the rays from the collimator (the complement of the angle of incidence), and let α_n denote the reading at the observing telescope when a line formed by light of wave-length λ in a spectrum of the n_{th} order is on the cross-wires. Then $\beta+\alpha_n$ and $180-\beta-\alpha_n$ are the angles made by the diffracted rays with the plane of the grating, and we have

$$n\lambda = s\,\{\cos\beta + \cos(\beta+\alpha_n)\}. \qquad (5)$$

Let α_0 be the reading when the image formed by ordinary reflection is on the wires; then

$$180 - \beta - \alpha_0 = \beta, \text{ whence } \beta = 90 - \tfrac{1}{2}\alpha_0. \qquad (6)$$

This value of β is to be substituted in the equation

$$\frac{n\lambda}{s} = \cos\beta + \cos(\beta+\alpha_n); \qquad (7)$$

thus $n\lambda/s$ is determined. Several independent determinations of it should be made, first by shifting the observing telescope so as to obtain spectra of different orders on both sides of the incident rays; and secondly, by turning the grating into a new position and repeating the process. In the case of Rowland's gratings, the number of lines to the inch is recorded on the margin of the grating; hence s can be computed, and then λ.

269. Comparison with Prismatic Spectra.—The simplicity of the law connecting wave-length with position in a diffraction spectrum, whether "normal" or not, offers a remarkable contrast to the lawlessness or "irrationality" of the dispersion produced by prisms. The material of the grating has no influence on the law.

Diffraction spectra differ notably from prismatic spectra in the much greater relative extension of the red end Owing to this circumstance, the brightest part of the diffraction spectrum of solar light is nearly in its centre.

The first three columns of numbers in the subjoined table indicate the approximate distances between the fixed lines B, D, E, F, G in certain prismatic spectra, and in the "normal" diffraction spectrum, the distance from B to G being in each case taken as 1000:—

	Flint-glass Angle of 60°	Bisulphide of Carbon, Angle of 60°.	Diffraction, or Difference of Wave-length	Difference of Wave-frequency.
B to D, . .	220	194	381	278
D to E, . .	214	206	243	232
E to F, . .	192	190	160	184
F to G, . .	374	410	216	306

It has been suggested that a more convenient reference-spectrum would be constructed by assigning to each colour a deviation proportional to its wave-frequency (or to the reciprocal of its wave-length), so that the distance between two colours will represent the difference between their wave-frequencies. The result of thus disposing the fixed lines is shown in the last column of the above table. It differs from prismatic spectra in the same direction, but to a much less extent than the diffraction spectrum.

270. Wave-lengths.—Wave-lengths of light are commonly stated in terms of a unit of which 10^{10} make a metre,—hence called the *tenth-metre.* The following are the wave-lengths of some of the principal "fixed lines" as determined by Ångström:[1]—

WAVE-LENGTHS IN TENTH-METRES.

A	7604	E	5269
B	6867	F	4861
C	6562	G	4307
D_2	5895	H_1	3968
D_1	5889	H_2	3933

The velocity of light is 300 million metres per second, or 300×10^{16} tenth-metres per second. The number of waves per second for any colour is therefore 300×10^{16} divided by its wave-length as above expressed. Hence we find approximately:—

For A	395	millions of millions per second
„ D	510	„ „ „
„ H	760	„ „ „

271. Concave Gratings.—Besides his improvements in plane gratings, Professor Rowland has introduced the novelty of ruling gratings on concave spherical surfaces. He is thus enabled to dispense both with a collimating lens and with the object-glass of the observing telescope. As this invention has become very important, we shall explain it at some length.

The rule which determines the kind of light that will be thrown in a given direction by any part of a reflection grating is, that the total length of path from the slit to a point lying in this direction differs by one wave-length or an exact number of wave-lengths of this particular light, as we pass from one bar to the next. If the slit is at the same distance from all the bars, the difference of path

[1] The wave-lengths of the spectral lines of all elementary substances will be found in Dr. W. M. Watts' *Index of Spectra;* and the wave-lengths and wave-frequencies of the dark lines in the solar spectrum, with the names of the substances to which many of them are due, will be found in the *British Association Report* for 1878 (Dublin), pp. 40–91.

will be the difference of the two reflected rays, and will be the projection of the distance between the bars on one of these rays, so that, as in § 264, we shall have $s \sin \theta$ equal to λ or a multiple of λ, θ now denoting the angle of reflection.

In Fig. 199, O is the centre of the sphere of which the grating forms part, A the middle point of the grating, BAC a section of the grating perpendicular to the rulings.

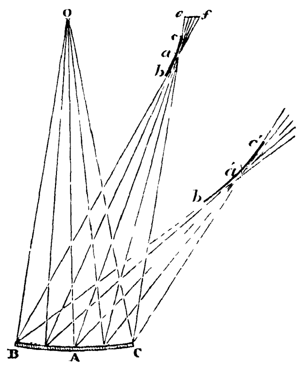


Fig 199 Concave Grating

First let the slit be at O. For the spectra of the first order the angle of reflection θ for a given kind of light is determined by the equation

$$s \sin \theta = \lambda,$$

and if, round O as centre, we describe a circle of radius OA $\sin \theta$, this circle will be touched by all the reflected rays An arc bac of this circle containing the same number of degrees as the grating BAC will be the caustic The narrowest part of the sheaf of rays will be at a the middle point of this arc, being the point where it is touched by the ray Aa from the middle point of the grating. Since Oa if joined is at right angles to Aa, the point a lies on the circle described on OA as diameter—the dotted circle in the figure. For the spectra of the second order, θ is determined by the equation

$$s \sin \theta = 2 \lambda,$$

and the caustic will be the circular arc $b'a'c'$ described about O as centre with a radius double of the radius of $b\,a\,c$, the construction being in other respects the same, and similar reasoning applies to the spectra of higher orders.

All the caustics of every order and for every value of λ will thus have their brightest points on the circle described on OA as diameter.

If the slit be at a, a spectrum of the first order will be formed at O and one of the third order at a'; for the rays incident from a on consecutive bars of the grating at A will have a common difference of λ, and the distances of these bars from a' have a common difference of 2λ.

Neglecting the breadth of the reflected beam at its narrowest part, we may regard a and a' as foci conjugate to O, and may regard the dotted circle as the locus of a point from which rays to all parts of the arc BAC have the same angle of incidence; whence it follows that, if the slit be anywhere on this circle, all the spectra will focus themselves along the circle. Hence the grating, the slit, and the screen for receiving the spectra, or the eye-piece for viewing them, may be fixed at the ends of three equal arms all pivoted at the centre of this circle.

A pencil of rays from a single point at O will not meet *in a single point* at a, even if we regard the breadth of the beam at a as negligible, but in a focal line perpendicular to the plane of the diagram, and they will meet again in a second focal line ef in the plane of the diagram. The lines of the spectrum due to a slit at O, perpendicular to the plane of the diagram, will therefore be focussed at a, while the transverse lines due to particles of dust in the slit will be focussed at ef. Similar remarks apply to the spectra of the second and higher orders. Hence the spectra as actually observed with a concave grating have the advantage of not showing dust lines.

272. In the arrangement adopted by Rowland for photographing the spectrum, the photographic plate was at O, being fixed to one end of a movable diameter O A, which carried the grating at the other end, as shown in Fig. 200, where the reference letters agree with those in Fig. 199. The slit was at a fixed point a on the circle.

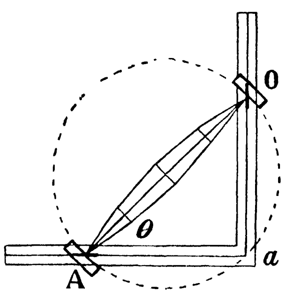


Fig 200.

Let R denote the diameter of this circle (which is the radius of curvature of the grating), and x the variable chord aO or R $\sin\theta$. The equation $s \sin\theta = n\lambda$ gives

$$x = \frac{nR}{s}\lambda, \qquad (1)$$

so that a scale of wave-lengths magnified nR/s times commencing from a will indicate the wave-length of the light which falls at O.

Further, if θ' denote the angle subtended at A by a small arc of the circle commencing at O, and x' its chord, the wave-length of the light which falls at the other end of this arc will by equation (4) be given by

$$n\lambda = s(\sin\theta \pm \sin\theta'), \tag{2}$$

that is,

$$x \pm x' = \frac{nR}{s}\lambda. \tag{3}$$

The distance between two lines of the spectrum as thrown on the plate is the difference between the two corresponding values of x' for the same value of x, and equation (3) shows that this difference is nR/s times the difference of their wave-lengths The scale of magnification is accordingly the same whether measured by distance along aO, or by the photograph, and is the *same for all wave-lengths* In virtue of this last fact it fulfils the condition of an absolute standard of reference mentioned in § 267, or to use the technical phrase it is a "normal spectrum" This condition would not be fulfilled if the plate were at any other part of the dotted circle, because equal differences in the chord x' drawn from O would not correspond to equal differences in the arc

The mechanical appliances employed are sketched in Fig 200 The slit is at the intersection of two lines of railway set at right angles, along which two carriages travel supporting the two ends of the tubular iron girder AO, the length of which is $21\frac{1}{2}$ feet The grating and photographic plate are rigidly attached (with proper adjustments) to the ends of this girder, which are supported on pivots at A and O. A micrometer eye-piece for measuring the distances between the lines can be substituted for the photographic plate when desired

273 Widths of Bars and Spaces.—The relation of the resultant amplitude in a diffraction spectrum (and hence of the brightness of the spectrum) to the widths of the bars and spaces is most simply shown by the following investigation. The bars are regarded as perfectly opaque, and the spaces as slits between them.

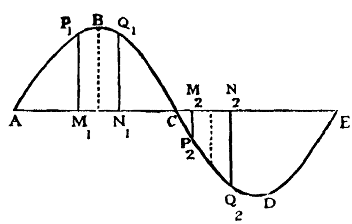


Fig 201.

Draw a curve of sines ABCDE (Fig. 201), and regard its wave-length AE as representing a width of slit which gives a retardation of one wave-length between its edges.

Take anywhere on the base AE a distance MN, representing, on the same scale, the width of the actual slit or space, and erect ordinates MP, NQ at its ends. The area PMNQ between these ordinates represents the resultant disturbance at a particular

instant; for the elementary strips which compose it represent the disturbances due to the elementary strips which compose the slit. As MN travels along the base line this area is sometimes positive, sometimes negative, and sometimes zero, and it can be shown to have a constant ratio to the ordinate at the middle point of MN. Its maximum value represents the resultant amplitude, and is clearly obtained by placing MN so as to be bisected by the maximum ordinate of the curve, as at M_1N_1 in the figure. The width of slit which gives the greatest amplitude is represented by AC or by any odd multiple of AC, for this gives the whole positive area ABC as representing the amplitude. When the width is represented by an even multiple of AC, the positive and negative areas will cancel each other, and the resultant amplitude will be nil.

In dealing with a diffraction spectrum of the first order, AE represents the width of a bar and space together, and AC represents half this width; hence the greatest brightness is obtained by making the bars equal to the spaces.

In dealing with a spectrum of the second order, if AI (Fig. 202) represent the joint width of a bar and space, the retardation between A and I is two wave-lengths. Reasoning similar to the foregoing shows that the brightness is a maximum when the width of a space is represented by AC or AG, that is, by one-fourth or three-fourths of the joint width, and that it is zero when the spaces are equal to the bars, since the area ABCDE is zero. In this case all the other spectra of even order will also vanish.

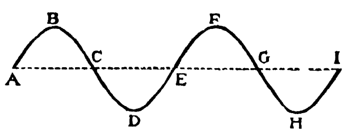


Fig. 202.

A spectrum of the third order will vanish when the width of a space is 1/3 or 2/3 of the joint width, and will have maximum brightness when the width is an odd number of sixths of the joint width.

Putting a for the amplitude of the curve of sines (that is for the ordinate at B, which, multiplied by AE, represents the amplitude of vibration of the direct light), and 2π for AE, also denoting MN by 2ϕ, we have area $M'PBQN = \int a \sin\theta\, d\theta$, from $\theta = \frac{\pi}{2} - \phi$ to $\theta = \frac{\pi}{2} + \phi$, which will be found to be $2a \sin\phi$.

In a spectrum of the first order, when the bars and spaces are

equal, ϕ is $\pi/2$, and when the spaces are 1/5 of the width of the bars, or 1/6 of the joint width, ϕ is $\pi/6$. The resulting amplitudes in the two cases are as $\sin \pi/2$ to $\sin \pi/6$, that is, as 1 to $\frac{1}{2}$, and the quantities of light, being as the squares of the amplitudes, are as 1 to $\frac{1}{4}$.

To compare the resulting amplitude with the amplitude of the direct light is to compare the area MPBQN with the area of the rectangle whose base is AE and height the ordinate at B. On the above scale the area of this rectangle is $2\pi a$. Hence the required ratio is the ratio of $2a \sin \phi$ to $2\pi a$, or of $\sin \phi$ to π. The maximum value of this is $1/\pi$, and the ratio of the quantities of light is therefore $1/\pi^2$.

274. Diffraction in Telescopes.—In consequence of diffraction, the image of a star, formed even by a theoretically perfect object-glass, is not a mere point, but consists of a central bright spot surrounded by bright rings.

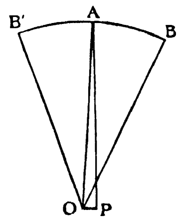


Fig 203

Let O (Fig. 203) be the focus. The wave-fronts which form the image are portions of spheres having O for their common centre and converging upon it as they advance, the solid angle subtended by any one of them at O being the same as that subtended by the object-glass. Let BAB′ be a section of one of these wave-fronts through its axis OA, and let $OP = x$ be a short line perpendicular to the axis. Put θ for one of the equal angles AOB, AOB′, also let

$R = OA = OB = OB'$, $\rho = PB$, $\rho' = PB'$ We have

$$\rho^2 = R^2 + x^2 - 2Rx \sin \theta.$$

$$\rho'^2 = R^2 + x^2 + 2Rx \sin \theta.$$

Hence $\rho'^2 - \rho^2 = 4Rx \sin \theta = (\rho' - \rho)(\rho' + \rho)$.

Putting $2R$ for $\rho' + \rho$, this gives $\rho' - \rho = 2x \sin \theta$.

For light of wave-length λ, as we make x increase from zero, there will not be complete opposition between any two portions of the wave-front as regards their effects at P until $\rho' - \rho$ attains the value $\lambda/2$, and the combined effect will not become nil till $\rho' - \rho$ has attained a certain value comparable to λ.

As $\rho' - \rho$ (which is proportional to x) continues to increase, till it amounts to several wave-lengths, the effect will vanish several times, and will have intervening maxima of continually decreasing intensity The values of x for the maxima are the radii of the brightest parts of the bright rings, and the values of x which make the effect vanish correspond to dark rings.

Assuming that each point of nil effect corresponds to a definite difference of phase between the effects of the nearest and furthest points of the wave-front, independent of the angular aperture 2θ, the radii of corresponding rings for different objectives will be inversely as $\sin \theta$; hence they will be inversely as the diameters of the objective when the focal length is given, and directly as the focal length when the diameter is given. Denoting the diameter of the objective by D, and the focal length by f, the radius of the first or any other specified ring will vary simply as f/D; but the distance between the centres of the images of two given stars is directly as f; hence the ratio of the distance between the centres of the images to the radius of the first ring varies directly as D. If this ratio is below a certain value, the central spots will overlap so much that the image, however much it may be magnified by the eye-piece, will not be distinguishable from that due to a single star. It accordingly appears from theory that the ability of a telescope to separate the components of a double star is proportional simply to the diameter of the objective. This inference is in accordance with observation.

According to Foucault's observations, it requires an objective of a diameter of 13 centimetres to resolve a double star whose components are 1″ asunder and of nearly equal brightness.

CHAPTER XV.

POLARIZATION AND DOUBLE REFRACTION.

275. Polarization.—When a piece of the semi-transparent mineral called tourmaline is cut into slices by sections parallel to its axis, it is found that two of these slices, if laid one upon the other in a particular relative position, as A, B (Fig. 204), form an opaque combination. Let one of them, in fact, be turned round upon the other through various angles (Fig. 204). It will be found that the combination is most transparent in two positions differing by 180°, one of them *a b* being the natural position which they originally occupied in the crystal, and that it is most opaque in the two positions at right angles to these. It is not necessary that the slices should be cut from the same crystal. Any two plates of tourmaline with their faces parallel to the axis of the crystals from which they were cut, will exhibit the same phenomenon. The experiment shows that light which has passed through one such plate is in a peculiar and so to speak unsymmetrical condition. It is said to be *plane-polarized.* According to the undulatory theory, a ray of common light contains vibrations in all planes passing through the ray, and a ray of plane-polarized light contains vibrations in one plane only. Polarized light cannot be distinguished from common light by the naked eye; and for all experiments in polarization two pieces of apparatus must be employed—one to produce polarization, and the other to show it. The former is called the *polarizer,* the latter the *analyser;* and every apparatus that serves for one of these purposes will also serve for the other. In the experiment above described,

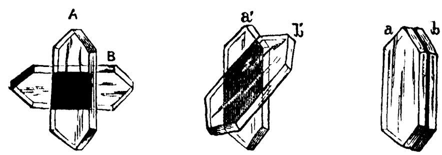


Fig 204 —Tourmaline Plates

the plate next the eye is the analyser. The usual process in examining light with a view to test whether it is polarized, consists in looking at it through an analyser, and observing whether any change of brightness occurs as the analyser is rotated. When the light of the blue sky is thus examined, a difference of brightness can always be detected according to the position of the analyser, especially at the distance of about 90° from the sun. In all such cases there are two positions differing by 180°, which give a minimum of light, and the two positions intermediate between these give a maximum of light.

The extent of the changes thus observed is a measure of the completeness of the polarization of the light.

276. Polarization by Reflection.—Transmission through tourmaline is only one of several ways in which light can be polarized. When a beam of light is reflected from a polished surface of glass, wood, ivory, leather, or any other non-metallic substance, at an angle of from 50° to 60° with the normal, it is more or less polarized, and in like manner a reflector composed of any of these substances may be employed as an analyser. In so using it, it should be rotated about an axis parallel to the incident rays which are to be tested, and the observation consists in noting whether this rotation produces changes in the amount of reflected light.

Malus' Polariscope (Fig. 205) consists of two reflectors A, B, one serving as polarizer and the other as analyser, each consisting of a pile of glass plates. Each of these reflectors can be turned about a horizontal axis; and the upper one (which is the analyser) can also be turned about a vertical axis, the amount of rotation being measured on the horizontal circle C C. To obtain the most powerful effects, each of the reflectors should be set at an angle of about 33° to the vertical, and a strong beam of common light should be allowed to fall upon the lower pile in such a direction as to be reflected vertically upwards. It will thus fall upon the centre of the upper pile, and the angles of incidence and reflection on both

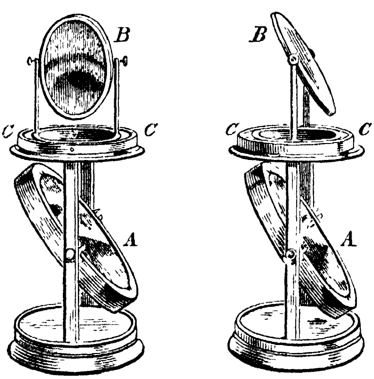


Fig 205.—Malus' Polariscope.

the piles will be about 57°. The observer looking into the upper pile in such a direction as to receive the reflected beam, will find that, as the upper pile is rotated about a vertical axis there are two positions (differing by 180°) in which he sees a black spot in the centre of the field of view, these being the positions in which the upper pile refuses to reflect the light reflected to it from the lower pile. They are 90° on either side of the position in which the two piles are parallel; this latter, and the position differing from it by 180°, being those which give a maximum of reflected light.

For every reflecting substance there is a particular angle of incidence which gives a maximum of polarization in the reflected light. It is called the *polarizing angle* for the substance, and its tangent is always equal to the index of refraction of the substance, or what amounts to the same thing, it is that particular angle of incidence which is the complement of the angle of refraction, so that the refracted and reflected rays are at right angles[1] This important law was discovered experimentally by Sir David Brewster.

The reflected ray under these circumstances is in a state of almost complete polarization, and the advantage of employing a *pile* of plates consists merely in the greater intensity of the reflected light thus furnished. The transmitted light is also polarized; it diminishes in intensity, but becomes more completely polarized, as the number of plates is increased. The reflected and the transmitted light are in fact mutually complementary, being the two parts into which common light has been decomposed; and their polarizations are accordingly opposite, so that, if both the transmitted and reflected beams are examined by a tourmaline, the maxima of obscuration will be obtained by placing the axis of the tourmaline in the one case parallel and in the other perpendicular to the plane of incidence

It is to be noted that what is lost in reflection is gained in transmission, and that polarization never favours reflection at the expense of transmission.

277. Plane of Polarization.—We have seen that there are two principal planes at right angles to each other associated with polarized light, one being the plane of greatest, and the other of least reflection. One of these, namely the plane of best reflection, is con-

[1] Adopting the indices of refraction given in the table § 127, we find the following values for the polarizing angle for the undermentioned substances:—

Diamond, . .	67° 43′ to 70° 3′	Crown-glass, . .	56° 51′ to 57° 23′
Flint-glass, . .	57° 36′ to 58° 40′	Pure Water, ...	53° 11′

ventionally styled the *plane of polarization.* Frequent perplexity arises from the circumstance that there is nothing in the name itself to indicate which of the two planes it denotes *Plane of best reflection* is shorter by one syllable, and leaves no room for doubt. When light is polarized by reflection, the plane of this reflection is the "plane of polarization."

278. Polarization by Double Refraction.—We have described in § 143 some of the principal phenomena of double refraction in uniaxal crystals. We have now to mention the important fact that the two rays furnished by double refraction are polarized, the polarization in this case being more complete than in any of the cases thus far discussed. On looking at the two images through a plate of tourmaline, or any other analyser, it will be found that they undergo great variations of brightness as the analyser is rotated, one of them becoming fainter whenever the other becomes brighter, and the maximum brightness of either being simultaneous with the absolute extinction of the other. If a second piece of Iceland-spar be used as the analyser, four images will be seen, of which one pair become dimmer as the other pair become brighter, and either of these pairs can be extinguished by giving the analyser a proper position.

Tourmaline, like Iceland-spar, is double refracting; and its use as a polarizer depends on the property which it possesses of absorbing the ordinary much more rapidly than the extraordinary ray, so that a thickness which is tolerably transparent to the latter is almost completely opaque to the former.

279. Theory of Double Refraction.—The existence of double refraction admits of a very natural explanation on the undulatory theory. In uniaxal crystals it is assumed that the elasticity of the luminiferous æther is the same for all vibrations executed in directions perpendicular to the axis; and that, for vibrations in other directions, the elasticity varies solely according to the inclination of the direction of vibration to the axis. There are two classes of doubly-refracting uniaxal crystals, called respectively *positive* and *negative.* In the former the elasticity for vibrations perpendicular to the axis is a maximum; in the latter it is a minimum. Iceland-spar and tourmaline belong to the latter class; and as small elasticity implies slow propagation, a ray propagated by vibrations perpendicular to the axis will, in these crystals, travel with minimum velocity; while the most rapid propagation will be attained by rays whose vibrations are parallel to the axis.

Consider any plane oblique to the axis. Through any point in this plane we can draw one line perpendicular to the axis; and the line at right angles to this will have smaller inclination to the axis than any other line in the plane. These two lines are the directions of least and greatest resistance to vibration, the former is the direction of vibration for an ordinary, and the latter for an extraordinary ray. The velocity of propagation is the same for the ordinary rays in all directions in the crystal, so that the wave-surface for these is spherical, but the velocity of propagation for the extraordinary rays differs according to their inclination to the axis, and their wave-surface is a spheroid whose polar diameter is equal to the diameter of the aforesaid sphere. The sphere and spheroid touch one another at the extremities of this diameter (which is parallel to the axis of the crystal), and the ordinary and extraordinary rays coincide both in direction and velocity along this common diameter. The general construction for the path of the extraordinary ray is due to Huygens, and has been described in § 248, Fig. 191, where CA is the incident and AF the refracted ray.

280. Huygens' Construction Extended.—Huygens' construction, as applied to any two media in which the velocities of rays in different directions are known, may be stated as follows:

Round the point of incidence C (Fig. 206) as centre describe the wave-surfaces for the two media, that is, surfaces whose radii vectores measured from C are proportional to the ray-velocities along them. Take any diameter AA′ of the wave-surface of the first medium, and at either end of this diameter draw a tangent plane AS or A′S′. Through the intersection S or S′ of this tangent plane with the plane surface which separates the two media draw, if possible, a tangent plane SB or S′B′ to the wave-surface of the second medium, and draw a diameter BB′ through the point of contact. This diameter is the direction of the refracted ray corresponding to the incident ray AA′.

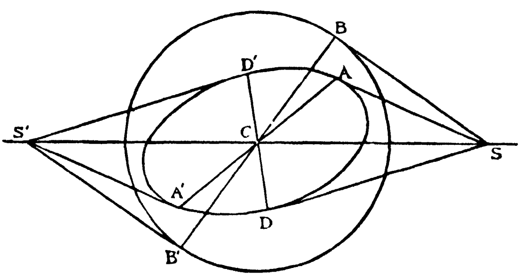


Fig 206

Again if through the intersection S or S′ we draw a second tangent

plane SD or S′D′ to the wave-surface of the first medium, the diameter DD′ will be the direction of the reflected ray in the first medium, whether the reflection be total or partial. It is partial if the line of intersection S or S′ lies outside the second wave-surface, as in the figure, and is total if it cuts this surface, as in this case a tangent plane cannot be drawn.

The plane of incidence does not in general coincide with either the plane of refraction or the plane of reflection. It does so, however, when the plane of incidence divides both wave-surfaces symmetrically, as is the case when one medium is air, and the other a uniaxal crystal with its axis either in or perpendicular to the plane of incidence.

281. When the axis is perpendicular to the plane of incidence, the construction takes the form shown in Fig. 207, where the largest semicircle is a section of the wave-surface for air, and the other two semicircles are sections of the sphere and spheroid for the crystal. In this case the law of sines is followed by the extraordinary as well

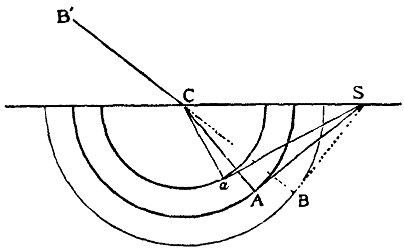


Fig 207

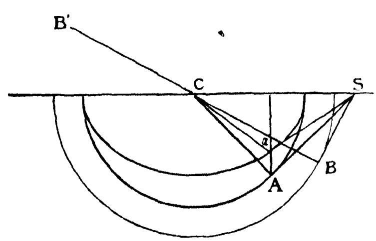


Fig 208

as by the ordinary ray. If Ca is the extraordinary ray, its index of refraction is C B / Ca, which may be described as the ratio of the velocity in air to that velocity in the crystal which differs most from the constant velocity of the ordinary ray.

When the axis is the intersection of the plane of incidence with the surface of the crystal, the construction takes the form shown in Fig. 208. The sections of the sphere and spheroid in the crystal are a circle and an ellipse, one of the axes of the ellipse being a diameter of the circle, as shown by the two smaller curves. The larger circle is the section of the air wave-surface. All three rays lie in the plane of the paper, and, by a property of the ellipse, the two points of contact a, A lie on the same ordinate.

282. Nicol's Prism.—One of the most convenient and effective

contrivances for polarizing light, or analysing it when polarized, is that known, from the name of its inventor, as Nicol's prism. It is made by slitting a rhomb of Iceland-spar along a diagonal plane *a c b d* (Fig 209), and cementing the two pieces together in their natural position by Canada balsam, a substance whose refractive index is intermediate between the ordinary and extraordinary indices of the crystal.[1] A ray of common light S I undergoes double refraction on entering the prism. Of the two rays thus formed, the ordinary ray is totally reflected on meeting the first surface of the balsam, and passes out at one side of the crystal, as *o* O; while the extraordinary ray is transmitted through the balsam as through a parallel plate, and finally emerges at the end of the prism, in the direction *e* E, parallel to the original direction S I. This apparatus has nearly all the convenience of a tourmaline plate, with the advantages of much greater transparency and of complete polarization.

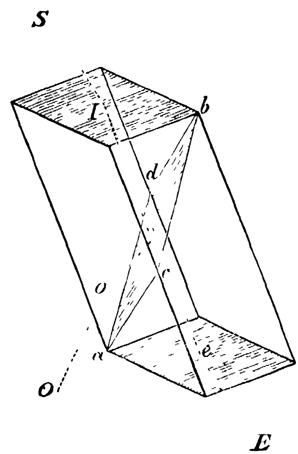


Fig 209 —Nicol's Prism

In Foucault's prism, which is extensively used instead of Nicol's, the Canada balsam is omitted, and there is nothing but air between the two pieces. This change has the advantage of shortening the prism (because the critical angle of total reflection depends on the relative index of refraction of the two media), but gives a smaller field of view, and rather more loss of light by reflection.

283. Colours produced by Elliptic Polarization.—Very beautiful colours may be produced by the peculiar action of polarized light For example, if a piece of selenite (crystallized gypsum) about the thickness of thin paper, is introduced between the polarizer and analyser of any polarizing arrangement, and turned about into different directions, it will in some positions appear brightly coloured, the colour being most decided when the analyser is in either of the two critical positions which give respectively the greatest light and the

[1] *a* and *b* are the corners at which three equal obtuse angles meet (§ 144). The ends of the rhomb which are shaded in the figure are rhombuses. Their diagonals drawn through *a* and *b* respectively will lie in one plane, which will contain the axis of the crystal, and will cut the plane of section *a c b d* at right angles. The length of the rhomb is about three and a half times its breadth.

greatest darkness. The colour is changed to its complementary by rotating the analyser through a right angle; but rotation of the piece of selenite, when the analyser is in either of the critical positions, merely alters the depth of the colour without changing its tint, and in certain critical positions of the selenite there is a complete absence of colour. Thicker plates of selenite restore the light when extinguished by the analyser, but do not show colour

284. Explanation.—The following is the explanation of these appearances. Let the analyser be turned into such a position as to produce complete extinction of the plane-polarized light which comes to it from the polarizer; and let the plane of polarization and the plane perpendicular thereto (and parallel to the polarized rays) be called the two *planes of reference.* Let the slice of selenite be laid so that the polarized rays pass through it normally. Then there are two directions, at right angles to each other, which are the directions of greatest and least elasticity in the plane of the slice. Unless the slice is laid so that these directions coincide with the two planes of reference, the plane-polarized light which is incident upon it will be broken up into two rays, one of which will traverse it more rapidly than the other. Referring to the diagram of Lissajous' figures (Fig. 43), let the sides of the rectangle be the directions of greatest and least elasticity, and let the diagonal line in the first figure be the direction of the vibrations of an incident ray,—this diagonal accordingly lies in one of the two planes of reference. In traversing the slice, the component vibrations in the directions of greatest and least elasticity will be propagated with unequal velocities, and if the incident ray be homogeneous, the emergent light will be elliptically polarized; that is to say, its vibrations, instead of being rectilinear, will be elliptic, precisely on the principle[1] of Blackburn's pendulum (§ 59). The shape of the ellipse depends, as in the case of Lissajous' figures, on the amount of retardation of one of the two component vibrations as compared with the other, and this is directly proportional to the thickness of the slice. The analyser resolves these elliptic vibrations into two rectilinear components parallel and perpendicular to the original direction of vibration, and sup-

[1] The principle is that, whereas displacement of a particle parallel to either of the sides of the rectangle calls out a restoring force directly opposite to the displacement, displacement in any other direction calls out a restoring force inclined to the direction of displacement, being in fact the resultant of the two restoring forces which its two components parallel to the sides of the rectangle would call out.

presses one of these components, so that only the other remains. Thus if the ellipse in the annexed figure (Fig 210) represent the vibrations of the light as it emerges from the selenite, and C D, E F be tangents parallel to the original direction of vibration, the perpendicular distance between these tangents, A B, is the component vibration which is not suppressed when the analyser is so turned that all the light would be suppressed if the selenite were removed By rotating the analyser, we shall obtain vibrations of various amplitudes, corresponding to the distances between parallel tangents drawn in various directions.

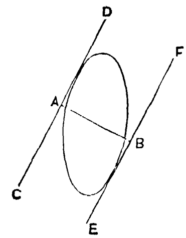


Fig. 210 — Colours of Selenite Plates

For a certain thickness of selenite the ellipse may become a circle, and we have thus what is called *circularly polarized* light, which is characterized by the property that rotation of the analyser produces no change of intensity. Circularly polarized light is not, however, identical with ordinary light, for the interposition of an additional thickness of selenite converts it into elliptically (or in a particular case into plane) polarized light (§ 292).

The above explanation applies to homogeneous light. When the incident light is of various refrangibilities, the retardation of one component upon the other is greatest for the rays of shortest wave-length The ellipses are accordingly different for the different elementary colours, and the analyser in any given position will produce unequal suppression of different colours. But since the component which is suppressed in any one position of the analyser, is the component which is not suppressed when the analyser is turned through a right angle, the light yielded in the former case *plus* the light yielded in the latter must be equal to the whole light which was incident on the selenite.[1] Hence the colours exhibited in these two positions must be complementary

It is necessary for the exhibition of colour in these experiments that the plate of selenite should be very thin, otherwise the retardation of one component vibration as compared with the other will be greater by several complete periods for violet than for red, so that

[1] We here neglect the light absorbed and scattered; but the loss of this does not sensibly affect the *colour* of the whole. It is to be borne in mind that the intensity of light is measured by the *square* of the amplitude, and is therefore the simple sum of the intensities of its two components when the resolution is rectangular.

the ellipses will be identical for several different colours, and the total non-suppressed light will be sensibly white in all positions of the analyser.

Two thick plates may, however, be so combined as to produce the effect of one thin plate. For example, two selenite plates, of nearly equal thickness, may be laid one upon the other, so that the direction of greatest elasticity in the one shall be parallel to that of least elasticity in the other. The resultant effect in this case will be that due to the difference of their thicknesses. Two plates so laid are said to be *crossed*.

285. Colours of Plates perpendicular to Axis.—A different class of appearances are presented when a plate cut from a uniaxal crystal by sections perpendicular to the axis is inserted between the polar-

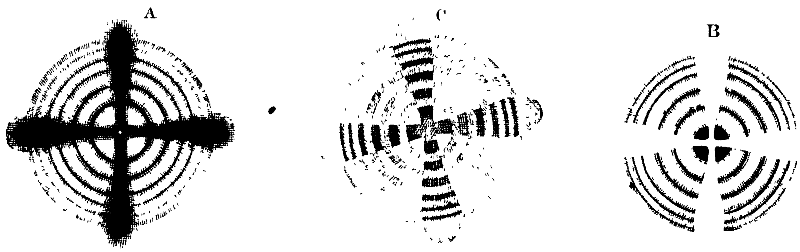


Fig. 211.—Rings and Cross.

izer and the analyser. Instead of a broad sheet of uniform colour, we have now a system of coloured rings, interrupted, when the analyser is in one of the two critical positions, by a black or white cross, as at A, B (Fig. 211).

286. Explanation.—The following is the explanation of these appearances. Suppose, for simplicity, that the analyser is a plate of tourmaline held close to the eye. Then the light which comes to the eye from the nearest point of the plate under examination (the foot of a perpendicular dropped upon it from the eye), has traversed the plate normally, and therefore parallel to its optic axis. It has therefore not been resolved into an ordinary and an extraordinary ray, but has emerged from the plate in the same condition in which it entered, and is therefore black, gray, or white according to the position of the analyser, just as it would be if the plate were removed. But the light which comes obliquely to the eye from any other part of the plate, has traversed the plate obliquely, and has undergone double refraction. Let E (Fig. 212) be the position of the eye, E O

a perpendicular on the plate, P a point on the circumference of a circle described about O as centre. Then, since E O is parallel to the axis of the plate, the direction of vibration for the ordinary ray at P is perpendicular to the plane E O P, and is tangential to the circle. The direction of vibration for the extraordinary ray lies in the plane E O P, is nearly perpendicular to E O (or to the axis), if the angle O E P is small, and deviates more from perpendicularity to the axis as the angle O E P increases. Both for this reason, and also on account of the greater thickness traversed, the retardation of one ray upon the other is greater as P is taken further from O, and, from the symmetry of the circumstances, it must be the same at the same distance from O all round. In consequence of this retardation, the light which emerges at P in the direction P E is elliptically polarized; and by the agency of the analyser it is accordingly resolved into two components one of which is suppressed. With homogeneous light, rings alternately dark and bright would thus be formed at distances from O corresponding to retardations of 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, . . . complete periods, and it can be shown that the radii of these rings would be proportional to the numbers 0, $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$, . . . The rings are larger for light of long than of short wave-length, and the coloured rings actually exhibited when white light is employed, are produced by the superposition of all the systems of monochromatic rings. The monochromatic rings for red light are easily seen by looking at the actual rings through a piece of red glass.

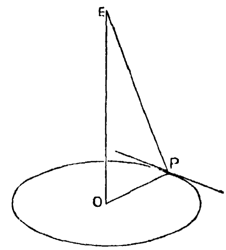


Fig 212
Theory of Rings and Cross

Let O, P, Fig 213, be the same points which were denoted by these letters in Fig. 212, and let A B be the direction of vibration of the light incident on the crystal at P. Draw A C, D B parallel to O P, and complete the rectangle A C B D. Then the length and breadth of this rectangle are approximately the directions of vibration of the two components one of which loses upon the other in traversing the crystal. The vibration of the emergent ray is represented by an ellipse inscribed in the rectangle ACBD (§ 57,

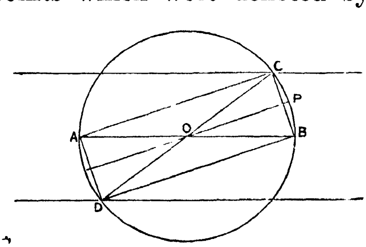


Fig 213 —Theory of Rings and Cross

note 2); and when the loss is half a period, this ellipse shrinks into a straight line, namely, the diagonal C D. Through C and D draw lines parallel to A B; then the distance between these parallels represents the double amplitude of the vibration which is transmitted when there has been a retardation of half a period, and is greater than the distance between the tangents in the same direction to any of the inscribed ellipses. A retardation of another half period will again reduce the inscribed ellipse to the straight line A B, as at first. The position D C corresponds to the brightest and A B to the darkest part of any one of the series of rings for a given wave-length of light, the analyser being in the position for suppressing all the light if the crystal were removed. When the analyser is turned into the position at right angles to this, A B corresponds to the brightest, and D C to the darkest parts of the rings. It is to be remembered that amount of retardation depends upon distance from the centre of the rings, and is the same all round. The two diagonals of our rectangle therefore correspond to different sizes of rings.

If the analyser is in such a position, with respect to the point P considered, that the suppressed vibration is parallel to one of the sides of the rectangle, (in other words, if O P, or a line perpendicular to O P, is the direction of suppression,) the retardation of one component upon the other has no influence, inasmuch as one of the two components is completely suppressed and the other is completely transmitted. There are, accordingly, in all positions of the analyser, a pair of diameters, coinciding with the directions of suppression and non-suppression, which are alike along their whole length and free from colour.

Again if P is situated at B or at 90° from B, the corner C of the rectangle coincides with B or with A, and the rectangle, with all its inscribed ellipses, shrinks into the straight line A B. The two diameters coincident with and perpendicular to A B are therefore alike along their whole length and uncoloured.

The two colourless crosses which we have thus accounted for, one of them turning with the analyser and the other remaining fixed with the polarizer, are easily observed when the analyser is not near the critical positions. In the critical positions, the two crosses come into coincidence; and these are also the positions of maximum blackness or maximum whiteness for the two crosses considered separately. Hence the conspicuous character of the cross in either

of these positions, as represented at A, B, Fig. 211. As the analyser is turned away from these positions, the cross at first turns after it with half its angular velocity, but soon breaks up into rings, somewhat in the manner represented at C, which corresponds to a position not differing much from A.

287. Biaxal Crystals.—Crystals may be divided optically into three classes:—

1. Those in which there is no distinction of different directions, as regards optical properties. Such crystals are said to be optically *isotropic.*

2. Those in which the optical properties are the same for all directions equally inclined to one particular direction called the optic axis, but vary according to this inclination. Such crystals are called *uniaxal.*

3. All remaining crystals (excluding compound and irregular formations) belong to the class called *biaxal.* In any homogeneous elastic solid, there are three cardinal directions called *axes of elasticity,* possessing the same distinctive properties which belong to the two principal planes of vibration in Blackburn's pendulum (§ 59), that is to say, if any small portion of the solid be distorted by forcibly displacing one of its particles in one of these cardinal directions, the forces of elasticity thus evoked tend to urge the particle *directly* back; whereas displacement in any other direction calls out forces whose resultant is generally oblique to the direction of displacement, so that when the particle is released it does not fly back through the position of equilibrium, but passes on one side of it, just as the bob of Blackburn's pendulum generally passes beside and not through the lowest point which it can reach

In biaxal crystals, the resistances to displacement in the three cardinal directions are all unequal, and this is true not only for the crystalline substance itself, but also for the luminiferous æther which pervades it, and is influenced by it.[1] The construction given by Fresnel for the wave-surface in any crystal is as follows:—First take an ellipsoid, having its axis parallel to the three cardinal directions, and of lengths depending on the particular crystalline substance considered. Then let any plane sections (which will of course be ellipses) be made through the centre of this ellipsoid, let normals to them be drawn through the centre, and on each normal let points

[1] The cardinal directions are however believed not to be the same for the æther as for the material of the crystal.

be taken at distances from the centre equal to the greatest and least radii of the corresponding section. The locus of these points is the complete wave-surface, which consists of two sheets cutting one another at four points. These four points of intersection are situated upon the normals to the two *circular sections* of the ellipsoid, and the two *optic axes*, from which *biaxal* crystals derive their name, are closely related to these two circular sections. The optic axes are the directions of *single wave-velocity*, and the normals to the two circular sections are the directions of *single ray-velocity*. The direction of advance of a wave is always regarded as normal to the front of the wave, whereas the direction of a ray (defined by the condition of traversing two apertures placed in its path) always passes through the centre of the wave-surface, and is not in general normal to the front. Both these pairs of directions of single velocity are in the plane which contains the greatest and least axes of the ellipsoid.

When two axes of the ellipsoid are equal, it becomes a spheroid, and the crystal is uniaxal. When all three axes are equal, it becomes a sphere, and the crystal is isotropic.

Experiment has shown that biaxal crystals expand with heat unequally in three cardinal directions, so that in fact a spherical piece of such a crystal is changed into an ellipsoid[1] when its temperature is raised or lowered. A spherical piece of a uniaxal crystal in the same circumstances changes into a spheroid; and a spherical piece of an isotropic crystal remains a sphere.

It is generally possible to determine to which of the three classes a crystal belongs, from a mere inspection of its shape as it occurs in nature. Isotropic crystals are sometimes said to be symmetrical about a point, uniaxal crystals about a line, biaxal crystals about neither. The following statement is rather more precise:—

If there is one and only one line about which if the crystal be rotated through 90° or else through 120° the crystalline form remains in its original position, the crystal is uniaxal, having that line for the axis. If there is more than one such line, the crystal is isotropic, while, if there is no such line, it is biaxal. In the last case, if there exist a plane of crystalline symmetry, such that one half of the crystal is the reflected image of the other half with respect to this

[1] This fact furnishes the best possible definition of an ellipsoid for persons unacquainted with solid geometry.

plane, it is also a plane of optical symmetry, and one of the three cardinal directions for the æther is perpendicular to it.[1]

Glass, when in a strained condition, ceases to be isotropic, and if inserted between a polarizer and an analyser, exhibits coloured streaks or spots, which afford an indication of the distribution of strain through its substance. The experiment is shown sometimes with anannealed glass, which is in a condition of permanent strain, sometimes with a piece of ordinary glass which can be subjected to force at pleasure by turning a screw. Any very small portion of a piece of strained glass has the optical properties of a crystal, but different portions have different properties, and hence the glass as a whole does not behave like one crystal.

The production of colour by interposition between a polarizer and an analyser, is by far the most delicate test of double refraction Many organic bodies (for example, grains of starch) are thus found to be doubly refracting; and microscopists often avail themselves of this means of detecting diversities of structure in the objects which they examine.

288. Conical Refraction.—Let a,b,c be the semiaxes of the Fresnel's ellipsoid mentioned in last section, a being the greatest and c the least. A little consideration will show that the three principal sections of the wave-surface, constructed as there directed, will each consist of a circle and an ellipse. One will be a circle of radius a surrounding an ellipse of semiaxes b,c. Another will be a circle of radius c lying within an ellipse of semiaxes a,b. The third will be a circle of radius b cutting in four points an ellipse of semiaxes a,c. This principal section of the wave-surface is shown in Fig. 214. The two diameters rr, rr of the wave-surface which join opposite points of section of the circle and ellipse are the directions

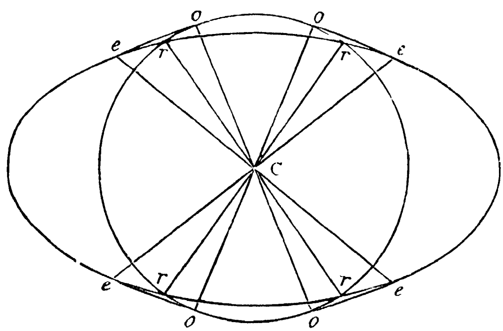


Fig 214.

[1] The optic axes either lie in the plane of symmetry, or lie in a perpendicular plane and are equally inclined to the plane of symmetry.

For the precise statement here given, the Editor is indebted to Professor Sir G. G. Stokes.

of single ray-velocity; and if the four common tangents *oe* to the two curves are drawn, the two diameters *oo,oo* of the circle which are normal to these tangents are the directions of single wave-velocity. These two diameters are defined to be the *optic axes.*

Sir Wm. R. Hamilton pointed out that through each of the four tangent lines *oe* a plane can be drawn touching the wave-surface along a circle. Each of the lines *oe* in the figure is a diameter of one of these circles of contact. He also pointed out that these geometrical relations involve the existence of two kinds of *conical refraction.* Applying the construction of § 280 to a ray passing out of air into the crystal, we are first to draw a plane perpendicular to the incident ray cutting the surface of the crystal in a line S, secondly to draw through this line a tangent plane to the wave-surface; and thirdly to join the point of contact to the centre of the wave-surface, this joining line being the course which the ray will take in the crystal. When the tangent plane has a circle of points of contact, a cone of refracted rays in the crystal will thus be obtained. This result is called *internal conical refraction.*

Again, in constructing for a ray refracted from the crystal into air, we are first to draw, at the point where the ray in the crystal meets the wave-surface, a tangent plane cutting the surface of the crystal in a line S; secondly, to draw through this line a tangent plane to a sphere of given radius described round the centre C; and thirdly, to join the point of contact on this sphere to C to obtain the direction of the refracted ray in air. When the incident ray in the crystal is in the direction of one of the common diameters of the circle and ellipse, the first tangent plane mentioned in these directions becomes indefinite, being any tangent plane to a certain cone. This will lead to an infinite number of refracted rays diverging in conical fashion from the point of emergence. This result is called *external conical refraction.*

It is of course to be understood that all the cones here mentioned are hollow cones or conical surfaces, not solid cones.

These two results, predicted by Hamilton, were experimentally verified by Lloyd with a plate of arragonite.[1]

The courses of the conically refracted rays in the two cases are roughly shown in Figs. 215, 216, the first representing external and the second internal conical refraction. The angle of the cone for

[1] See Lloyd's *Wave Theory,* pp. 176-179.

arragonite is about 3° for the former and 1° 55′ for the latter. The letters *r*, *o*, *e* have the same meanings as in Fig. 214.

If the ring-shaped image obtained by conical refraction of either kind is tested for polarization, it is found that points which are oppositely polarized are not 90° but 180° apart, and that the black

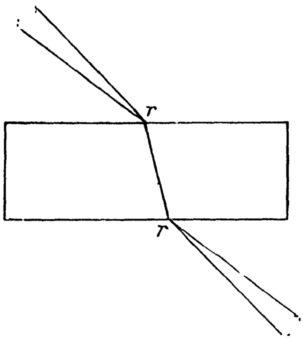


Fig 215

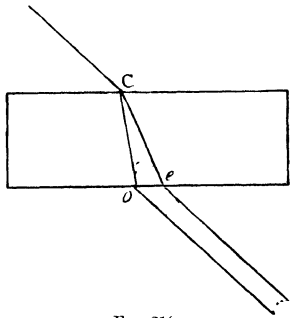


Fig 216

spot which indicates extinction by the analyser revolves twice as fast as the analyser. The plane of polarization for any ray of the internal cone passes through the diameter *ee*, and the plane which, in Fresnel's theory, contains the direction of vibration, passes through the optic axis *oo*.

289. Rotation of Plane of Polarization.—When a plate of quartz (rock-crystal), even of considerable thickness, cut perpendicular to the axis, is interposed between the polarizer and analyser, colour is exhibited, the tints changing as the analyser is rotated, and similar effects of colour are produced by employing, instead of quartz, a solution of sugar, inclosed in a tube with plane glass ends.

If homogeneous light is employed, it is found that if the analyser is first adjusted to produce extinction of the polarized light, and the quartz or saccharine solution is then introduced, there is a partial restoration of light. On rotating the analyser through a certain angle, there is again complete extinction of the light; and on comparing different plates of quartz, it will be found that the angle through which the analyser must be rotated is proportional to the thickness of the plate. In the case of solutions of sugar, the angle is proportional jointly to the length of the tube and the strength of the solution.

The action thus exerted by quartz or sugar is called *rotation of*

the plane of polarization, a name which precisely expresses the observed phenomena. In the case of ordinary quartz, and solutions of sugar-candy, it is necessary to rotate the analyser in the direction of watch-hands as seen by the observer, and the rotation of the plane of polarization is said to be *right-handed.* In the case of what is called *left-handed* quartz, and of solutions of non-crystallizable sugar, the rotation of the plane of polarization is in the opposite direction, and the observer must rotate the analyser against watch-hands.

The amount of rotation is different for the different elementary colours, and has been found to be inversely as the square of the wave-length. Hence the production of colour.

290. Biquartz Analyser.—The unequal rotation of the plane of polarization for different colours is utilized in the biquartz analyser.

Two semicircular plates of quartz ABA′, AB′A′ (Fig. 217), one giving right-handed and the other left-handed rotation, are placed side by side as in the figure, their common thickness being 3·75 mm., which is just sufficient for turning the plane of polarization of the brightest yellow of the spectrum through 90°. For the brightest red the rotation will be about 70°, and for the brightest blue about 120°. Accordingly, if AA′ be the direction of vibration of incident polarized light, the direction of vibration for the different spectral colours after transmission will be distributed in two fan-shaped arrangements as indicated in the figure, OR, OR′ being the directions for red in the two halves of the field, OY, OY′ for yellow, and OB, OB′ for blue. To the naked eye the colours will blend into white, the direction of vibration having no influence on the visual sensations. But if a Nicol is interposed, the component vibrations parallel to some one diameter of the circular field of view will be suppressed, while those perpendicular to it will be transmitted. When the Nicol is so placed that the direction of the suppressed vibrations coincides with YY′, which is perpendicular to AA′, the two halves of the field of view will be of the same colour and undistinguishable in appearance; but a small angular displacement either of the Nicol or of the plane of polarization of the incident light, will cause the suppression of more red and less blue in one semicircle, and of less red and more blue in the other. The duty of the observer is to

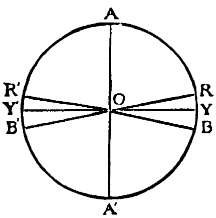


Fig. 217

obtain identity of colour in the two halves of the field, and the colour then obtained will be a pale purple called the *sensitive tint*, or *tint of passage*. The smallest displacement in either direction makes one half a reddish and the other a blueish purple. The apparatus thus affords a very delicate means of determining the plane of polarization of a given beam of light, and of measuring the rotation of this plane produced by transmission through a liquid.

291. **Magneto-optic Rotation.**—Faraday made the remarkable discovery that the plane of polarization can be rotated in certain circumstances by the action of magnetism. Let a long rectangular piece of "heavy-glass" (silico-borate of lead) be placed longitudinally between the poles of the powerful electro-magnet represented in Fig. 88, p. 126, Part III, which is for this purpose made hollow in its axis, so that an observer can see through it from end to end. Let a Nicol's prism be fitted into one end of the magnet, to serve as polarizer, and another into the other end to serve as analyser, and let one of them be turned till the light is extinguished. Then, as long as no current is passed round the electro-magnet, the interposition of the heavy-glass will produce no effect; but the passing of a current while the heavy-glass is in its place between the poles, produces rotation of the plane of polarization in the same direction as that in which the current circulates. Flint glass gives about half the effect of heavy-glass, and all transparent solids and liquids exhibit an effect of the same kind in a more or less marked degree.

A steel magnet, if extremely powerful, may be used instead of an electro-magnet; and in all cases, to give the strongest effect, the lines of magnetic force should coincide with the direction of the transmitted ray. The amount of rotation is directly as the distance traversed by the light in the substance, and directly as the longitudinal component of the intensity of the field; or, (combining these two independent variables,) it is simply proportional to the difference of magnetic potential between the points of incidence and emergence.

Faraday regarded these phenomena as proving the direct action of magnetism upon light; but it is now more commonly believed that the direct effect of the magnetism is to put the particles of the transparent body in a peculiar state of strain, to which the observed optical effect is due.

In every case tried by Faraday, the direction of the rotation was the same as the direction in which the current circulated; but cer-

tain substances[1] have since been found which give rotation against the current. The law for the relative amounts of rotation of different colours is approximately the same as in the case of quartz.

The direction of rotation is with watch-hands as seen from one end of the arrangement, and against watch-hands as seen from the other; so that the same piece of glass, in the same circumstances, behaves like right-handed quartz to light entering it at one end, and like left-handed quartz to light entering it at the other.

The rotatory power of quartz and sugar appears to depend upon a certain unsymmetrical arrangement of their molecules, an arrangement somewhat analogous to the thread of a screw; right-handed and left-handed screws representing the two opposite rotatory powers. It is worthy of note that the two kinds of quartz crystallize in different forms, each of which is unsymmetrical, one being like the image of the other as seen in a looking-glass. Pasteur has conducted extremely interesting researches into the relations existing between substances which, while in other respects identical or nearly identical, differ as regards their power of producing rotation. For the results we must refer to treatises on chemistry.

Dr. Kerr has obtained rotation of the plane of polarization by reflection from intensely magnetized iron. In some of the experiments the direction of magnetization was normal, and in others parallel to the reflecting surface.

292. Circular Polarization. Fresnel's Rhomb.—We have explained in § 284 the process by which elliptic polarization is brought about, when plane-polarized light is transmitted through a thin plate of selenite. To obtain circular polarization (which is merely a case of elliptic), the plate must be of such thickness as to retard one component more than the other by a *quarter of a wave-length,* and must be laid so that the directions of the two component vibrations make angles of 45° with the plane of polarization. Plates specially prepared for this purpose are in general use, and are called *quarter-wave plates.* They are usually of mica, which differs but little in its properties from selenite. It is impossible, however, in this way to obtain complete circular polarization of ordinary white light, since different thicknesses are required for light of different wave-lengths, the thickness which is appropriate for violet being too small for red.

[1] One such substance is a solution of Fe^2Cl^3 (old notation) in methylic (not methylated) alcohol.

Fresnel discovered that plane-polarized light is elliptically polarized by *total internal reflection* in glass, whenever the plane of polarization of the incident light is inclined to the plane of incidence. The rectilinear vibrations of the incident light are in fact resolved into two components, one of them in, and the other perpendicular to, the plane of incidence; and one of these is retarded with respect to the other in the act of reflection, by an amount depending on the angle of incidence. He determined the magnitude of this angle for which the retardation is precisely $\frac{1}{8}$ of a wave-length; and constructed a *rhomb*, or oblique parallelepiped of glass (Fig. 218), in which a ray, entering normally at one end, undergoes two successive reflections at this angle (about 55°), the plane of reflection being the same in both. The total retardation of one component on the other is thus $\frac{1}{4}$ of a wave-length; and if the rhomb is in such a position that the plane in which the two reflections take place is at an angle of 45° to the plane of polarization of the incident light, the emergent light is circularly polarized. The effect does not vary much with the wave-length, and sensibly white circularly polarized light can accordingly be obtained by this method.

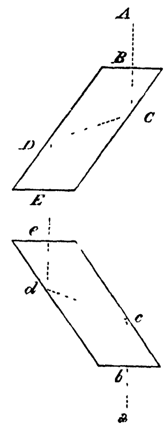


Fig 218
Two Fresnel's Rhombs.

When circularly polarized light is transmitted through a Fresnel's rhomb, or through a quarter-wave plate, it becomes plane-polarized,

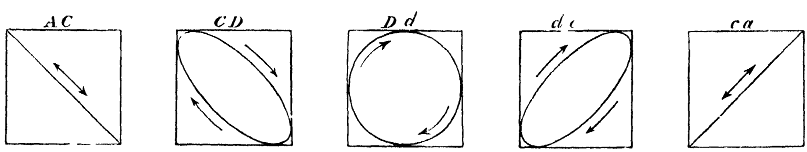


Fig 219 —Form of Vibration in traversing the Rhombs

and we have thus a simple mode of distinguishing circularly polarized light from common light; for the latter does not become polarized when thus treated. Two quarter-wave plates, or two Fresnel's rhombs, may be combined either so as to assist or to oppose one another. By the former arrangement, which is represented in Fig. 218, we can convert plane-polarized light into light polarized in a perpendicular plane, the final result being therefore the same as if the plane of polarization had been rotated through 90°. The

several steps of the process are illustrated by the five diagrams of Fig. 219, which represent the vibrations of the five portions AC, CD, D*d*, *dc*, *ca* of the ray which traverses the two rhombs in the preceding figure. The sides of the square are parallel to the directions of resolution; the initial direction of vibration is one diagonal of the square, and the final direction is the other diagonal; a gain or loss of half a complete vibration on the part of either component being just sufficient to effect this change.

293. Direction of Vibration of Plane-polarized Light.—When light is polarized by the double refraction of Iceland-spar, or of any other uniaxal crystal, it is found that the plane of polarization of the ordinary ray is the plane which contains the axis of the crystal. But the distinctive properties of the ordinary ray are most naturally explained by supposing that its vibrations are perpendicular to the axis. Hence we conclude that the direction of vibration in plane-polarized light is normal to the plane of best reflection (technically called the plane of polarization), and therefore that, in polarization by reflection, the vibrations of the reflected light are parallel to the reflecting surface.

This is Fresnel's doctrine. MacCullagh, however, reversed this hypothesis, and maintained that the direction of vibration is *in* the "plane of polarization." Both theories have been ably expounded; but Stokes contrived a crucial experiment in diffraction, which confirmed Fresnel's view;[1] and in his classical paper on "Change of Refrangibility," he has deduced the same conclusion from a consideration of the phenomena of the polarization of light by reflection from excessively fine particles of solid matter in suspension in a liquid.[2]

294. Vibrations of Ordinary Light.—Ordinary light agrees with circularly polarized light in always yielding two beams of equal intensity when subjected to double refraction; but it differs from circularly polarized light in not becoming plane-polarized by transmission through a Fresnel's rhomb or a quarter-wave plate. What, then, can be the form of vibration for common light? It is probably very irregular, consisting of ellipses of various sizes, positions, and forms (including circles and straight lines), rapidly succeeding one another. By this irregularity we can account for the fact that beams of light from different sources (even from different points of

[1] *Cambridge Transactions*, 1850.

[2] *Philosophical Transactions*, 1852; pp. 530, 531.

the same flame, or from different parts of the sun's disc), cannot, by any treatment whatever, be made to exhibit the phenomena of mutual interference; and for the additional fact that the two rectangular components into which a beam of common light is resolved by double refraction, cannot be made to interfere, even if their planes of polarization are brought into coincidence by one of the methods of rotation above described.

Certain phenomena of interference show that a few hundred consecutive vibrations of common light may be regarded as similar, but as the number of vibrations in a second is about 500 millions of millions, there is ample room for excessive diversity during the time that one impression remains upon the retina.

295. **Polarization of Radiant Heat.**—The fundamental identity of radiant heat and light is confirmed by thermal experiments on polarization. Such experiments were first successfully performed by Forbes in 1834, shortly after Melloni's invention of the thermo-multiplier. He first proved the polarization of heat by tourmaline; next by transmission through a bundle of very thin mica plates, inclined to the transmitted rays; and afterwards by reflection from the multiplied surfaces of a pile of thin mica plates placed at the polarizing angle. He next succeeded in showing that polarized heat, even when quite obscure, is subject to the same modifications which doubly refracting crystallized bodies impress upon light, by suffering a beam of heat, after being polarized by transmission, to pass through an interposed plate of mica, serving the purpose of the plate of selenite in the experiment of § 283, the heat traversing a second mica bundle before it was received on the thermo-pile. As the interposed plate was turned round in its own plane, the amount of heat shown by the galvanometer was found to fluctuate just as the amount of light received by the eye under similar circumstances would have done He also succeeded in producing circular polarization of heat by a Fresnel's rhomb of rock-salt. These results have since been fully confirmed by the experiments of other observers.

EXAMPLES IN ACOUSTICS.

Period, Wave-length, and Velocity.

1. If an undulation travels at the rate of 100 ft. per second, and the wave-length is 2 ft., find the period of vibration of a particle, and the number of vibrations which a particle makes per second.

2. It is observed that waves pass a given point once in every 5 seconds, and that the distance from crest to crest is 20 ft. Find the velocity of the waves in feet per second.

3. The lowest and highest notes of the normal human voice have about 80 and 800 vibrations respectively per second. Find their wave-lengths when the velocity of sound is 1100 ft. per second.

4. Find their wave-lengths in water in which the velocity of sound is 4900 feet per second.

5. Find the wave-length of a note of 500 vibrations per second in steel in which the velocity of propagation is 15,000 ft. per second.

Pitch and Musical Intervals.

6. Show that a "fifth" added to a "fourth" makes an octave.

7. Calling the successive notes of the gamut Do_1, Re_1, Mi_1, Fa_1, Sol_1, La_1, Si_1, Do_2, show that the interval from Sol_1 to Re_2 is a true "fifth."

8. Find the first 5 harmonics of Do_1.

9. A siren of 15 holes makes 2188 revolutions in a minute when in unison with a certain tuning-fork. Find the number of vibrations per second made by the fork.

10. A siren of 15 holes makes 440 revolutions in a quarter of a minute when in unison with a certain pipe. Find the note of the pipe (in vibrations per second).

Reflection of Sound, and Tones of Pipes.

11. Find the distance of an obstacle which sends back the echo of a sound to the source in $1\frac{1}{2}$ seconds, when the velocity of sound is 1100 ft. per second.

12. A well is 210 ft. deep to the surface of the water. What time will elapse between producing a sound at its mouth and hearing the echo?

13. What is the wave-length of the fundamental note of an open organ-pipe 16 ft. long; and what are the wave-lengths of its first two overtones? Find also their vibration-numbers per second.

14. What is the wave-length of the fundamental tone of a stopped organ pipe

5 ft. long; and what are the wave-lengths of its first two overtones? Find also their vibration-numbers per second.

15. What should be the length of a tube stopped at one end that it may resound to the note of a tuning-fork which makes 520 vibrations per second; and what should be the length of a tube open at both ends that it may resound to the same fork. [The tubes are supposed narrow, and the smallest length that will suffice is intended.]

16. Would tubes twice as long as those found in last question resound to the fork? Would tubes three times as long?

BEATS.

17. One fork makes 256 vibrations per second, and another makes 260. How many beats will they give in a second when sounding together?

18. Two sounds, each consisting of a fundamental tone with its first two harmonics, reach the ear together. One of the fundamental tones has 300 and the other 302 vibrations per second. How many beats per second are due to the fundamental tones, how many to the first harmonics, and how many to the second harmonics?

19. A note of 225 vibrations per second, and another of 336 vibrations per second, are sounded together. Each of the two notes contains the first two harmonics of the fundamental. Show that two of the harmonics will yield beats at the rate of 3 per second.

VELOCITY OF SOUND IN GASES.

20. If the velocity of sound in air at 0° C. is 33,240 cm. per second, find its velocity in air at 10° C., and in air at 100° C.

21. If the velocity of sound in air at 0° C. is 1090 ft. per second, what is the velocity in air at 10°?

22. Show that the difference of velocity for 1° of difference of temperature in the Fahrenheit scale is about 1 ft. per second.

23. If the wave-length of a certain note be 1 metre in air at 0°, what is it in air at 10°?

24. The density of hydrogen being ·06926 of that of air at the same pressure and temperature, find the velocity of sound in hydrogen at a temperature at which the velocity in air is 1100 ft. per second.

25. The quotient of pressure (in dynes per sq. cm.) by density (in gm. per cubic cm.) for nitrogen at 0° C. is 807 million. Compute (in cm. per second) the velocity of sound in nitrogen at this temperature.

26. If a pipe gives a note of 512 vibrations per second in air, what note will it give in hydrogen?

27. A pipe gives a note of 100 vibrations per second at the temperature 10° C. What must be the temperature of the air that the same pipe may yield a note higher by a major fifth?

VIBRATIONS OF STRINGS.

28. Find, in cm. per second, the velocity with which pulses travel along a string whose mass per cm. of length is ·005 gm., when stretched with a force of 7 million dynes.

29. If the length of the string in last question be 33 cm., find the number of vibrations that it makes per second when vibrating in its fundamental mode; also the numbers corresponding to its first two overtones.

30 The A string of a violin is 33 cm. long, has a mass of ·0065 gm. per cm., and makes 440 vibrations per second. Find the stretching force in dynes

31. The E string of a violin is 33 cm. long, has a mass of ·004 gm. per cm., and makes 660 vibrations per second. Find the stretching force in dynes.

32. Two strings of the same length and section are formed of materials whose specific gravities are respectively d and d'. Each of these strings is stretched with a weight equal to 1000 times its own weight. What is the musical interval between the notes which they will yield?

33. The specific gravity of platinum being taken as 22, and that of iron as 7·8, what must be the ratio of the lengths of two wires, one of platinum and the other of iron, both of the same section, that they may vibrate in unison when stretched with equal forces?

Longitudinal Vibrations of Rods.

34. If sound travels along fir in the direction of the fibres at the rate of 15,000 ft. per second, what must be the length of a fir rod that, when vibrating longitudinally in its fundamental mode, it may emit a note of 750 vibrations per second?

35. A rod 8 ft. long, vibrating longitudinally in its fundamental mode, gives a note of 800 vibrations per second. Find the velocity with which pulses are propagated along it

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EXAMPLES IN OPTICS.

Photometry, Shadows, and Plane Mirrors.

1. A lamp and a taper are at a distance of 4·15 m. from each other, and it is known that their illuminating powers are as 6 to 1. At what distance from the lamp, in the straight line joining the flames, must a screen be placed that it may be equally illuminated by them both?

2 Two parallel plane mirrors face each other at a distance of 3 ft., and a small object is placed between them at a distance of 1 ft. from the first mirror, and therefore of 2 ft. from the second. Calculate the distances, from the first mirror, of the three nearest images which are seen in it; and make a similar calculation for the second mirror.

3. Show that a person standing upright in front of a vertical plane mirror will just be able to see his feet in it, if the top of the mirror is on a level with his eyes, and its height from top to bottom is half the height of his eyes above his feet.

4. A square plane mirror hangs exactly in the centre of one of the walls of a cubical room. What must be the size of the mirror that an observer with his

eye exactly in the centre of the room may just be able to see the whole of the opposite wall reflected in it except the part concealed by his body?

5. Two plane mirrors contain an angle of 160°, and form images of a small object between them. Show that if the object be within 20° of either mirror there will be three images; and that if it be more than 20° from both, there will be only two.

6. Show that when the sun is shining obliquely on a plane mirror, an object directly in front of the mirror may give two shadows, besides the direct shadow.

7. A person standing beside a river near a bridge observes that the inverted image of the concavity of the arch receives his shadow exactly as a real inverted arch would do if it were in the place where the image appears to be. Explain this.

8. If a globe be placed upon a table, show that the breadth of the elliptic shadow cast by a candle (considered as a luminous point) will be independent of the position of the globe.

9. What is the length of the cone of the *umbra* thrown by the earth? and what is the diameter of a cross section of it made at a distance equal to that of the moon?

The radius of the sun is 112 radii of the earth; the distance of the moon from the earth is 60 radii of the earth; and the distance of the sun from the earth is 24,000 radii of the earth. Atmospheric refraction is to be neglected.

10. The stem of a siren carries a plane mirror, thin, polished on both sides, and parallel to the axis of the stem. The siren gives a note of 345 vibrations per second. The revolving plate has 15 holes. A fixed source of light sends to the mirror a horizontal pencil of parallel rays. What space is traversed in a second by a point of the reflected pencil at a distance of 4 metres from the axis of the siren? This axis is supposed vertical.

Spherical Mirrors.

11. Find the focal length of a concave mirror whose radius of curvature is 2 ft., and find the position of the image (*a*) of a point 15 in. in front of the mirror; (*b*) of a point 10 ft. in front of the mirror; (*c*) of a point 9 in. in front of the mirror; (*d*) of a point 1 in. in front of the mirror.

12. Calling the diameter of the object unity, find the diameters of the image in the four preceding cases.

13. The flame of a candle is placed on the axis of a concave spherical mirror at the distance of 154 cm., and its image is formed at the distance of 45 cm. What is the radius of curvature of the mirror?

14. On the axis of a concave spherical mirror of 1 m. radius, an object 9 cm high is placed at a distance of 2 m. Find the size and position of the image.

15. What is the size of the circular image of the sun which is formed at the principal focus of a mirror of 20 m. radius? The apparent diameter of the sun is 30′.

16. In front of a concave spherical mirror of 2 metres' radius is placed a concave luminous arrow, 1 decimetre long, perpendicular to the principal axis, and at the distance of 5 metres from the mirror. What are the position and size of the image? A small plane reflector is then placed at the principal focus of the spherical mirror, at an inclination of 45° to the principal axis, its polished side being next the mirror. What will be the new position of the image?

17. At what distance from a concave mirror of 1 ft. radius must an object be placed that a real image may be formed of twice the linear dimensions of the object?

18. A candle flame is 3 ft from a screen, and a concave mirror throws on the screen an image of it magnified $1\frac{2}{5}$ times. Find focal length of mirror.

19. A convex mirror is placed 6 in. in front of a screen on which an image has been thrown by a long-focussed lens, and reflects the rays so as to form a new image on a card 10 in. in front of the mirror. Find focal length of mirror.

REFRACTION

(The index of refraction of glass is to be taken as $\frac{3}{2}$, except where otherwise specified, and the index of refraction of water as $\frac{4}{3}$.)

20 The sine of 45° is $\sqrt{\frac{1}{2}}$, or ·707 nearly. Hence, determine whether a ray incident in water at an angle of 45° with the surface will emerge or will be reflected, and determine the same question for a ray in glass.

21. If the index of refraction from air into crown-glass be $1\frac{1}{2}$, and from air into flint-glass $1\frac{3}{5}$, find the index of refraction from crown-glass into flint-glass

22 The index of refraction from water into oil of turpentine is 1·11, find the index of refraction from air into oil of turpentine.

23. The index of refraction for a certain glass prism is 1·6, and the angle of the prism is 10°. Find approximately the deviation of a ray refracted through it nearly symmetrically

24 A ray of light falls perpendicularly on the surface of an equilateral prism of glass with a refracting angle of 60°. What will be the deviation produced by the prism? Index of refraction of glass 1 5

25. A speck in the interior of a piece of plate-glass appears to an observer looking normally into the glass to be 2 mm. from the near surface What is its real distance?

26 The rays of a vertical sun are brought to a focus by a lens at a distance of 1 ft from the lens. If the lens is held just above a smooth and deep pool of water, at what depth in the water will the rays come to a focus?

27 A mass of glass is bounded by a convex surface, and parallel rays incident nearly normally on this surface come to a focus in the interior of the glass at a distance a. Find the focal length of a plano-convex lens of the same convexity, supposing the rays to be incident on the convex side.

28 Show that the deviation of a ray going through an air-prism in water is towards the edge of the prism.

29. What is the greatest apparent zenith distance that a star can have as seen by an eye under water?

30. When the reflected and refracted rays into which an incident ray divides are at right angles, prove that the index of refraction is the tangent of the angle of incidence.

31. The sun shines on an isosceles prism whose base is horizontal. Show that the image of the sun seen by looking into the other face is in the same position as if formed by reflection in a trough of mercury.

32 Show that when a converging beam of light in air passes through a tank of water 2 ft. long, with parallel glass ends to which the axis of the beam is normal, its focus is shifted by 6 in.

33. Show that a ray incident on a solid refracting sphere will emerge parallel and

reversed after one internal reflection if the cosine of the angle of refraction at entrance is $\frac{1}{2}\mu$.

34. A ray passes through a prism in a principal plane, the deviation being equal to the angle of incidence, and each of them equal to twice the angle of the prism. Show that the cosine of the angle of the prism is the square root of $(\mu^2-1)/8$.

Lenses, &c.

35. Compare the focal lengths of two lenses of the same size and shape, one of glass and the other of diamond, their indices of refraction being respectively 1·6 and 2·6. Also compare their focal lengths in water.

36. If the index of refraction of glass be $\frac{3}{2}$, show that the focal length of an equi-convex glass lens is the same as the radius of curvature of either face.

37. What is the focal length of a double-convex lens of diamond, the radius of curvature of each of its faces being 4 millimetres? Index of refraction 2·5.

38. Prove that the focal length of a thin lens is equal to $a^2/2(\mu-1)t$, where t is the difference between the thickness at the rim and at the middle, μ the index of refraction, and a the radius of the rim.

39. The focal length of a convex lens is 1 ft. Find the positions of the image of a small object when the distances of the object from the lens are respectively 20 ft., 2 ft., and $1\frac{1}{2}$ ft. Are the images real or virtual?

40. When the distances of the object from the lens in last question are respectively 11 in., 10 in., and 1 in., find the distances of the image. Are the images real or virtual?

41. Calling the diameter of the object unity, find the diameter of the image in the six cases of questions 39, 40, taken in order.

42. Show that, when the distance of an object from a convex lens is double the focal length, the image is at the same distance on the other side.

43. The object is 6 ft. on one side of a lens, and the image is 1 ft. on the other side. What is the focal length of the lens?

44. The object is 3 in. from a lens, and its image is 18 in. from the lens on the same side. Is the lens convex or concave, and what is its focal length?

45. The object is 12 ft. from a lens, and the image 1 ft. from the lens on the same side. Find the focal length, and determine whether the lens is convex or concave.

46. A person who sees best at the distance of 3 ft., employs convex spectacles with a focal length of 1 ft. At what distance should he hold a book, to read it with the aid of these spectacles?

47 A person reads a book at the distance of 1 ft. with the aid of concave spectacles of 6 in. focal length. At what distance is the image which he sees?

48. An object 8 centimetres high is placed at 1 metre distance on the axis of an equi-convex lens of crown-glass of index 1·5, the radius of curvature of its faces being 0·4 m. Find the size and position of the image.

49. Show that, when an image is formed by a lens, the distance of the object from one principal focus multiplied by the distance of the image from the other is equal to the square of the focal length.

50. A sharp image of a fixed object is thrown on a fixed screen by a lens, and is found to measure 1 in. in diameter. The lens is then moved into a new position till another sharp image is thrown on the screen. This image measures 5 in. What is the diameter of the object?

51. Rays which form a real image on a screen are intercepted by a concave lens of 12 in. focal length, at a distance of 8 in. from the screen. How far must the screen be moved that it may receive the new image?

52. A converging lens placed at a distance of 5 in from an object throws on a screen a sharp image six times the size of the object. Find the focal length of the lens.

53 It is desired to throw upon a screen an image of a lamp-flame magnified threefold, by means of a lens of given focal length f What must be the distance of the screen from the flame?

54. A distinct image of an object is thrown on a screen at a distance d from it, by means of a convex lens, and when the lens is moved a distance a towards the screen, another distinct image is formed on it Show that the focal length of the lens is $(d^2-a^2)/4d$.

55. A source of light is placed at a distance a from a screen, and a thin convex lens of focal length f is moved from the source towards the screen till a distinct image is formed. Show that if the lens be now moved through a further distance $p=\sqrt{a(a-4f)}$, another image will be formed whose linear dimensions are to those of the former as $(a-p)^2$ to $(a+p)^2$.

56. A person who can see vertical lines most distinctly at the distance of 30 in, and horizontal lines most distinctly at the distance of 40 in, uses an eye-glass, the front of which is spherical and the back cylindrical, both being convex. Find their radii of curvature (taking μ as 1 5) that he may see an object at the distance of 10 in. with the eye-glass close to his eye.

57. P and Q are two small equal objects on opposite sides of a thin lens, between it and its principal foci Show that the angle subtended at Q by the image of P is equal to that subtended at P by the image of Q [This property belongs to all arrangements of lenses and mirrors with a common axis.]

58 A candle is at distance a from a screen on which a convex lens throws an image of the candle. On moving the lens towards the candle another image is formed, which is m times as long as the first. Prove that the focal length of the lens is

$$\frac{a\sqrt{m}}{(1+\sqrt{m})^2}.$$

59. A wafer 1 in. in diameter, 8 in. from the eye, is seen through a convex lens of 8 in. focal length placed half-way between What must be the diameter of the lens that the whole of the wafer may be visible through it at once?

60. At what distance from the eye must a concave lens be placed that the angular diameter of a distant object may be diminished one-half?

61. A concave lens is moved from contact with an object up to the eye. Show that the angular diameter is least when the lens is half-way between the eye and the object.

Combinations of Lenses or of Lens and Mirror.

62. The objective of a telescope has a focal length of 20 ft. What will be the magnifying power when an eye-piece of half-inch focus is used?

63. Two converging lenses, with a common focal length of 0·05 m., are at a distance of 0·05 m. apart, and their axes coincide. What image will this system give of a circle 0·01 m. in diameter, placed at a distance of ·1 m. on the prolongation of the common axis?

64. Show that if F denote the focal length of a combination of two lenses in contact, their thicknesses being neglected, we have

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

f_1 and f_2 denoting the focal lengths of the two lenses.

65. What is the focal length of a lens composed of a convex lens of 2 in. focal length, cemented to a concave lens of 9 in. focal length?

66. Apply the formulæ of § 163 to find the focal length of a lens, the thickness being neglected.

67. The flame of a candle is on the axis of a lens of 1 ft. focal length at a distance of 18 in., and a plane mirror at a distance of 1 ft. behind the lens reflects the rays to one side. At what distance from the mirror must a screen be held to receive the image?

68. A thin convex lens of focal length f has one face in contact with a plane mirror. Show that the combination behaves like a concave mirror of focal length $\frac{1}{2}f$.

69. A reflecting galvanometer has a plane mirror, and just in front of this is a fixed convex lens, not turning with the mirror. The illuminated slit is in the principal focus of this lens Show that the image will be formed at the same distance as the slit, and that its displacement will equal double the angular displacement of the mirror multiplied by the focal length of the lens

70. A flame is 2 ft. in front of a convex lens of 1 ft. focal length, and a plane mirror 6 in. behind the lens reflects the rays back through the lens. Show that they will form a real image 6 in. from the lens.

71. A concave mirror of radius of curvature r has its centre in the centre of a convex lens, their axes coinciding. If f be the focal length of the lens, and rays proceeding from a point a distance u from the lens emerge, after two transmissions and one reflection, as a pencil of parallel rays, prove that

$$\frac{1}{u}+\frac{2}{r}=\frac{2}{f}.$$

REFRACTION AT SPHERICAL SURFACES.

72. An eye is placed close to the surface of a large sphere of glass ($\mu=\frac{3}{2}$) which is silvered at the back. Show that the image which the eye sees of itself is three-fifths of the natural size.

73. A pencil of parallel rays fall upon a sphere of glass of 1 inch radius. Find the principal focus of rays near the axis, the index of refraction of glass being 1·5.

74. A sphere of glass of index 1·5 lying upon a horizontal plane receives the sun's rays. What must be the height of the sun above the horizon that the principal focus of the sphere may be in this horizontal plane?

75. A lens is made of two unequal hemispheres of the same kind of glass, of radii r and s, cemented together so as to be concentric. Prove that its focal length measured from the common centre is given by $\frac{1}{f}=\frac{\mu-1}{\mu}\left(\frac{1}{r}+\frac{1}{s}\right)$.

76. Prove that the principal foci of a spherical refracting shell of index μ are at the distance $\frac{1}{2}\frac{\mu}{\mu-1}\frac{Rr}{R-r}$ from its centre, R, r being the external and internal radii.

77. The radii of curvature of a thin double-convex lens are 10 in and 15 in. and its index of refraction 1·6. Find its focal lengths in air and in water. If it is laid on the surface of water with its 10 in face upwards and 15 in. face just immersed, find the positions of its two principal foci.

Systems of Lenses.

78. Show that, for refraction at a single spherical surface, the distances of the two principal foci from the surface are in the same ratio as the indices of refraction of the two media

79 In § 163, if f_1 denote the value of p_1 when p_2 is infinite, and f_2 the value of p_2 when p_1 is infinite, show that $f_1/p_1 + f_2/p_2 = 1$, and that $(p_1 - f_1)(p_2 - f_2) = f_1 f_2$.

80 Rays passing through a refracting sphere, parallel to the axis and near it, converge to an external point. Show that the distance of this point from the centre is twice the focal length of a thin lens "equivalent" to the sphere; and that a small object at this point gives an equal and inverted image.

81. When the two foci F, F′ of a system have been found by observation, and also that pair of conjugate points for which the magnification is -1, show how to deduce the positions of the two principal points

82 Show that, if the order of succession of the foci and principal points of an optical instrument is H F F′ H′, with H F = F′ H′ = $\frac{1}{2}$ F F′, all these distances being very great, and the instrument being in the centre, an object anywhere on the axis within moderate distance of the instrument will give an inverted image near and sensibly equal to the object itself.

83. The first and last media are the same, the two principal points are in or near the instrument, and the two foci are at great distances on opposite sides Show that, in the neighbourhood of the instrument, the distance between an object and its image is constant.

ANSWERS TO EXAMPLES IN ACOUSTICS.

Ex. 1. $\frac{1}{50}$ sec. 50. Ex. 2. 4. Ex. 3. $13\frac{3}{4}$ ft. $1\frac{3}{8}$ ft. Ex. 4. $61\frac{1}{4}$ ft., $6\frac{1}{8}$ ft Ex. 5. 30 ft.

Ex. 6. $\frac{3}{2} \times \frac{4}{3} = 2$. Ex. 7. $\frac{36}{54} = \frac{2}{3}$. Ex. 8. Do_2, Sol_2, Do_3, Mi_3, Sol_3.

Ex. 9. 547. Ex. 10. 440.

Ex. 11 825. Ex. 12. $\frac{21}{55} = {\cdot}382$ sec. Ex. 13. 32 ft, 16 ft., $10\frac{2}{3}$ ft.; $34\frac{3}{8}$, $68\frac{3}{4}$, $103\frac{1}{8}$. Ex. 14. 20 ft., $6\frac{2}{3}$ ft., 4 ft.; 55, 165, 275. Ex. 15. $\frac{55}{104}$ ft., $\frac{55}{52}$ ft. Ex. 16. An open tube twice or three times as long will resound, because one of its overtones will coincide with the note of the fork. A stopped tube three times as long will resound, but a stopped tube twice as long will not.

Ex. 17. 4. Ex. 18. 2, 4, 6. Ex. 19. $675 - 672 = 3$.

Ex. 20. 33843, 38850. Ex 21. 1110. Ex. 22. The velocity is 1090 at 32° and 1110 at 50°. Ex. 23. 1·018 metre. Ex. 24. 4180 ft. per second. Ex 25. 33732. Ex. 26. 1945 vibrations per second. Ex. 27. 364° C.

Ex. 28. 37417. Ex. 29. 567, 1134, 1701. Ex. 30. $v=29040$, $t=v^2m=5481600$. Ex. 31. $v=43560$, $t=7589900$. Ex. 32. Unison. Ex. 33. Length of iron = 1·68 times length of platinum.

Ex. 34. 10 ft. Ex. 35. 12800 ft. per second.

ANSWERS TO EXAMPLES IN OPTICS.

Ex. 1. 2·95 m. Ex. 2. 1, 5, and 7 ft. behind first mirror; 2, 4, and 8 ft. behind second. Ex. 4. Side of mirror must be $\frac{1}{3}$ of edge of cube.

Ex. 6. They are the shadows of the object and of its image, cast by the sun's image. The former is due to the intercepting of light after reflection; the latter to the intercepting of light before reflection. Ex. 7. The sun's image throws a shadow of the man's image on the real arch, owing to his intercepting rays on their way to the water. Ex. 8. First let the globe be vertically under the flame, and draw through the flame two equally inclined planes, touching the globe. Their intersections with the table will be parallel lines which will be tangents to the shadow, and will still remain tangents to it as the globe is rolled between the planes to any distance. Ex. 9. 216 radii of earth; $1\frac{4}{9}$ radii. Ex. 10. $368\pi=1156$ metres.

Ex. 11. Focal length 1 ft.; (*a*) 5 ft. in front of mirror; (*b*) $1\frac{1}{9}$ ft. in front; (*c*) 3 ft. behind mirror; (*d*) $1\frac{1}{11}$ in. behind. Ex. 12. 4, $\frac{1}{9}$, 4, $1\frac{1}{11}$.

Ex. 13 69·6 cm. Ex. 14. Distance $\frac{2}{3}$ m., height 3 cm. Ex. 15. 8·73 cm Ex. 16. Distance $1\frac{1}{4}$ m., length $\frac{1}{4}$ dec., new position $\frac{1}{4}$ m. laterally from focus. Ex. 17. 9 in. Ex. 18. 1 ft. 9 in. Ex. 19. 1 ft. 3 in.

Ex. 20. The ray in water will emerge, because $\frac{3}{4}$ is greater than ·707; the ray in glass will be totally reflected, because $\frac{2}{3}$ is less than ·707. Ex. 21. $\frac{16}{15}$. Ex. 22. 1·48. Ex. 23. 6°. Ex. 24. 60° (by total reflection).

Ex. 25. 3 mm. Ex. 26. 1 ft. 4 in. Ex. 27. $\frac{2}{3}a$.

Ex. 29. $\text{Sin}^{-1} \frac{3}{4} = 48° \ 35'$. Ex. 30. $\text{Sin}\ i = \mu \sin r$, with $i + r = 90°$. Ex. 31. The ray in the prism makes the angles of incidence and reflection at the base equal. Hence the other angles are equal in pairs.

Ex. 32. If original point of convergence is at distance p from first surface, it is $\frac{4}{3}p$ after first refraction. After second refraction, distance from second surface is $\frac{3}{4}(\frac{4}{3}p-24)=p-18$, instead of $p-24$ as at first.

Ex. 33. The ray must be reflected at the point where a radius parallel to incident ray meets surface of sphere. Hence angle of incidence is double angle of refraction, and $\mu=\sin 2r/\sin r=2\cos r$.

Ex. 34. We have $i=2\text{A}$, $i'=\text{A}$, $\sin r=\frac{1}{\mu}\sin 2\text{A}$, $\sin r'=\frac{1}{\mu}\sin \text{A}$. $\therefore \frac{1}{\mu}\sin 2\text{A} = \sin(\text{A}-r') = \sin \text{A}\cos r' - \cos \text{A}\sin r'$. Substitute for $\sin r'$ and $\cos r'$.

Ex. 35. Focal length of diamond is $\frac{3}{8}$ that of glass. In water it is $\frac{4}{19}$ that of glass. Ex. 37. $1\frac{1}{3}$ mm. Ex. 38. t = sum of sagittæ $=a^2/2r_1+a^2/2r_2$. $\therefore 1/r_1+1/r_2=2t/a^2$.

Ex. 39. $1\frac{1}{19}$ ft., 2 ft., 3 ft. on the other side of lens. All real. Ex. 40. 11 ft., 5 ft., $\frac{1}{11}$ ft. on same side of lens. All virtual. Ex. 41. $\frac{1}{19}$, 1, 2, 12, 6, $1\frac{1}{11}$.

Ex. 43. $\frac{6}{7}$ ft. Ex. 44. $3\frac{3}{5}$ in., convex. Ex. 45. $1\frac{1}{11}$ ft., concave. Ex. 46. 9 in. Ex. 47. 4 in.

Ex. 48. Distance $\frac{2}{3}$ m. on other side, height $5\frac{1}{3}$ cm. Ex. 49. See § 155. A direct proof can be obtained by tracing two rays from A to a, Fig 138, one of them parallel to the axis before incidence, and the other after emergence. They will pass through the two principal foci and will be cut proportionally by the three parallels. Hence $x:f::f:y$.

Ex. 50. Geometric mean, or $\sqrt{5}$ in. Ex. 51. Let x be distance of new image from lens. $\frac{1}{12}=\frac{1}{8}-1/x$ gives $x=24$ in. Screen must be moved 16 in. away. Ex. 52. Distance of lens from screen is 30 in.; $f=4\frac{2}{7}$ in. Ex. 53. $1/x+1/3x=1/f$ gives $x=4f/3$. Required distance is $4x=16f/3$. Ex. 54. The conjugate distances are $d/2+a/2$ and $d/2-a/2$.

Ex 55. The conjugate distances are $a/2 \pm p/2$, hence from above $f=(a^2-p^2)/4a$. $p^2=a^2-4af$. Magnification is $(a+p)/(a-p)$ or $(a-p)/(a+p)$. $\therefore$ ratio of lengths of images is $(a+p)^2/(a-p)^2$.

Ex. 56. Since horizontal lines are to be more affected than vertical, axis of cylinder must be horizontal. If r be radius of sphere, and s of cylinder, $\mu-1$ being 1/2, we have $1/2r=\frac{1}{10}-\frac{1}{30}$, $1/2r+1/2s=\frac{1}{10}-\frac{1}{40}$, whence $r=7\frac{1}{2}$ in., $s=60$ in.

Ex 57 The images will be virtual Call their distances from the lens p', q', the distances of the objects being p, q. Length of either object l Angle subtended by image of P at centre of lens is l/p, and angle subtended by it at Q is $\frac{l}{p}\frac{p'}{p'+q}$, which reduces to $\frac{lf}{f(p+q)-pq}$, and is not altered by interchanging p and q.

Ex. 58. The magnifications are $\sqrt{m}$ and its reciprocal. The conjugate distances are as 1 to $\sqrt{m}$, and their sum is a.

Ex. 59 Wafer being 4 in. from lens, image is 8 in. from lens, or 12 in. from eye Angle subtended at lens is $\frac{1}{4}$, and at eye $\frac{2}{3}$ of this, or $\frac{1}{6}$ Diameter of lens $=\frac{1}{6}$ of 4 in $=\frac{2}{3}$ in.

Ex. 60. If eye were close to lens, angular diameter would be unchanged and image would be at distance f. Eye must be at distance $2f$ from image or f from lens. Ex. 61. Let object be at distance a from eye and p from lens. Distance of image from lens is $p'=pf/(p+f)$. Angle subtended at lens by object l is l/p, and at eye $\frac{l}{p}\frac{p'}{p'+a-p}$, which reduces to $\frac{lf}{ap+af-p^2}$. This is least when $ap-p^2$ is greatest, that is, when $a-p=p$.

Ex. 62. 480. Ex. 63. A real image ·025 m. beyond second lens; diameter of image ·005 m

Ex. 64. Calling the distances of the object and of the two successive images x, y, z, we have $1/f_1=1/x \pm 1/y$, $1/f_2=\mp 1/y+1/z$. $\therefore 1/f_1+1/f_2=1/x+1/z=1/\mathrm{F}$. Ex. 65. $2\frac{1}{7}$ in.

Ex. 66. Let x, y, z be the distances of object and its two successive images from lens, positive if to left of lens. We have $\mu/y-1/x=(\mu-1)/-r_1$, $1/z-\mu/y=(1-\mu)/r_2$. $\therefore \frac{1}{z}-\frac{1}{x}=-(\mu-1)(1/r_1+1/r_2)$, and when x is infinite z is f for concave and $-f$ for convex lens. Ex. 67. First image is 3 ft. behind lens, or 2 ft beyond mirror. Second image is 2 ft. from mirror sideways. Ex. 68. An object at distance p in front gives image at distance p' behind. Mirror gives a second image at same distance in front. Lens gives a third image at distance q in front. $1/p+1/p'=1/f$, $1/q-1/p'=\frac{1}{f}$. $\therefore 1/p+1/q=2/f$.

Ex. 69. Slit at distance f gives parallel rays to mirror, which reflects them as a second parallel beam turned through 2θ when mirror has obliquity θ. Line through centre of lens in this direction, of length f, determines position of image. Ex. 70. First image is 2 ft. behind lens, or 1 ft. 6 in. behind mirror. Second image is 1 ft. 6 in. before mirror, or 1 ft. before lens. Third image is 6 in. before lens.

Ex. 71. Reversing the course of the rays, the lens first brings them to its own principal focus. The mirror then brings them to a conjugate focus at distance x such that $1/x + 1/(r-f) = 2/r$. The lens then brings them to distance u, giving $1/(r-x) + 1/u = 1/f$. Eliminating x, the required result is obtained.

Ex. 72. Sizes vary as distances from centre. First image is at distance r from centre on near side, erect and equal to eye. Second image is at distance $r/3$ beyond centre and inverted. Third image is at distance $3r/5$ beyond centre and inverted. Ex. 73. 1·5 in. from centre, or ·5 in. from sphere. Ex. 74. 1·5 is cosec. of alt., alt. is $41° 49'$. Ex. 75. Using formula (17) of § 163, we have

first, $\mu_1 = 1,\ \mu_2 = \mu,\ q_1 = \infty,\ q_2 = x,\ \rho = -r,$

second, $\mu_1 = \mu,\ \mu_2 = 1,\ q_1 = x,\ q_2 = f,\ \rho = s.$

First equation is $1/\mu x = (1 - 1/\mu)\,1/r$. Second is $1/f - 1/\mu x = (1 - 1/\mu)\,1/s$. By addition, $1/f = (1 - 1/\mu)(1/r + 1/s)$.

Ex. 76. Using the same formula, we have, first,

$\mu_1 = 1,\ \mu_2 = \mu,\ q_1 = \infty,\ q_2 = x,\ \rho = -\mathrm{R};$

second, $\mu_1 = \mu,\ \mu_2 = 1,\ q_1 = x,\ q_2 = y,\ \rho = -r;$

third, $\mu_1 = 1,\ \mu_2 = \mu,\ q_1 = y,\ q_2 = z,\ \rho = r;$

fourth, $\mu_1 = \mu,\ \mu_2 = 1,\ q_1 = z,\ q_2 = f,\ \rho = \mathrm{R}.$

The four equations are, $1/\mu x = (1 - 1/\mu)\,1/\mathrm{R}$, $1/y - 1/\mu x = (1 - 1/\mu)\,1/-r$, $1/\mu z - 1/y = (1/\mu - 1)\,1/r$, $1/f - 1/\mu z = (1 - 1/\mu)\,1/\mathrm{R}$.

By addition, $1/f = (1 - 1/\mu)\,(2/\mathrm{R} - 2/r)$. This is negative, showing that the shell makes parallel rays diverge.

Ex. 77. In air $f = 10$. In water $\mu = 6/5, f = 30$. For second part of question, using formula (15) of § 163, we have, for the downward rays,

first, $\mu_1 = 1,\ \mu_2 = 8/5,\ p_1 = \infty,\ p_2 = x,\ r = 10;$

second, $\mu_1 = 8/5,\ \mu_2 = 4/3,\ p_1 = x,\ p_2 = f_1,\ r = -15.$

The equations are, $8/5x = 3/50$, $4/3f - 8/5x = 4/225$; giving, by addition, $4/3f_1 = 7/90, f_1 = 120/7$.

For the upward rays we have:

first, $\mu_1 = 4/3,\ \mu_2 = 8/5,\ p_1 = \infty,\ p_2 = y,\ r = 15;$

second, $\mu_1 = 8/5,\ \mu_2 = 1,\ p_1 = y,\ p_2 = f_2,\ r = -10.$

The equations are, $8/5y = 4/225$, $1/f_2 - 8/5y = 3/50$; giving, by addition, $1/f_2 = 7/90, f_2 = 90/7$.

Ex. 78. In equation (15) of § 163, $p_2 = \infty$ gives $f_1/\mu_1 = r/(\mu_1 - \mu_2)$; and $p_1 = \infty$ gives $f_2/\mu_2 = r/(\mu_2 - \mu_1) = -f_1/\mu_1$. The *minus* indicates that the two foci are on opposite sides of the surface.

Ex. 79. Using the above results, we have

$$\frac{f_1}{p_1} = \frac{\mu_1 r}{\mu_1 - \mu_2}\frac{1}{p_1}, \qquad \frac{f_2}{p_2} = \frac{\mu_2 r}{\mu_2 - \mu_1}\frac{1}{p_2}.$$

The sum is $$\frac{r}{\mu_2 - \mu_1}\left(\frac{\mu_2}{p_2} - \frac{\mu_1}{p_1}\right) = \frac{r}{\mu_2 - \mu_1}\,\frac{\mu_2 - \mu_1}{r} = 1.$$

The second result to be proved reduces to $p_1 p_2 - p_1 f_2 - p_2 f_1 = 0$, which is identical with the first result

Ex 80. The parallel rays, if reversed, would meet the axis in an external point symmetrically placed on other side. These two points are conjugate; call them P and Q Since the two principal points coincide at the centre, equation (6) of § 195 holds, with O F = O′F′, and O P = O′Q; hence O F/O P = 1/2. O F is the focal length of the equivalent lens. Since O F = F P, and the magnification at O is 1, the magnification at P is -1.

Ex 81. P, Q being the points at which $m = -1$, H is given by F H = P F, and H′ by F′H′ = Q F′

Ex 82 P, Q being positions of object and image, since F P . Q F′ = F H H′F′ $= f^2$, F P = f gives Q F′ = f, and P, Q coincide at the centre.

Again, by differentiation, increment of F P/ decrement of Q F′ = F P/Q F′ = 1

Also, $m = -$ Q F′/F′H′ $= -1$.

If H F/F′H′ is *sensibly* unity, P and Q will be at a sensibly constant distance apart, small compared with H F

Ex 83 Since first and last media are the same, F H = H′F′ = f. When P is at H, Q is at H′, and, as above, increment of F P = decrement of Q F′

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